

## Simulation

According to Shannon "Simulation is the process of designing a model of the real system by conducting experiments with this model for the purpose of designing a model of the real system." *It is imitation of reality.* Maruti car training center with various crossings and signals is a simulated model of city traffic system.

The imitation of reality which may be in the physical form or in the form of mathematical equations may be called simulation. The aircraft simulator to train pilots on new models of aircraft represents the use of physical model as a means of experimentation. The mathematical models of real-life situations in investment analysis, scheduling or inventory control can be experimented; these are known as symbolic simulation models. These models can be either deterministic or probabilistic. The deterministic models can provide answers to 'what if' type of questions in business problems. The probabilistic simulation models deal with random phenomenon and the method of simulation applied is known as Monte Carlo Simulation.

### **Advantages**

1. Behavioral Analysis - Simulation can be used to investigate the behavior of problems which are too complex to be modeled mathematically.
2. Fairly Simple - The simulation technique is easier to use than mathematical models and is considered superior to mathematical models.
3. Flexible - Simulation models are comparatively flexible and can be modified to accommodate the changing environment of the real situation.
4. Time saving - It can compress the performance of a system over several years involving large calculations into few minutes e.g. the effects of ordering, advertising or other policies over many months or years can be obtained by computer simulation in short time.
5. Imparting training - It has advantageously been used for training the operating and managerial staff in operation of complex plans. It is always advantageous to train people on simulation models before putting into their hands the real system.
6. Foresee Hindrances - Through simulation, management can foresee the difficulties and bottlenecks which may come up due to the introduction of new machines, equipment or process.

### **Limitations**

(4 Marks) Nov/02 & (3 Marks) May/04

1. Not an Optimizing technique - Simulation does not produce optimum results. It simply allows us to select the best of the alternative systems examined.
2. Costlier on occasions - It is, by no means, a cheap method of analysis. In a number of situations, such as corporate planning, simulation is comparatively costlier and time consuming.
3. Quantification - In number of situations it is not possible to quantify all the variables that affect the behavior of the system.
4. Time Consuming - On occasions simulation is very much time consuming process.
5. Useful in long term only - Reliable results are possible only if the simulation is continued for a long period.

### **Monte Carlo Simulation (a.k.a. Computer Simulation)**

The Monte Carlo method of simulation was developed by two mathematicians **John Von Neuman** and **Stainslaw Ulam**. The technique employs random numbers and is used to solve problems that involve probability and wherein physical experimentation is impracticable and formulation of mathematical formula is impossible.

The steps involved carrying out Monte Carlo simulation are:

(4 Marks) Nov/03

1. Clearly formulate the problem in order to determine the objectives and constraints.
2. Calculate respective probability.
3. Calculate cumulative probability.
4. Calculate random interval: If the cumulative probabilities are in two digits the range of random numbers to be assigned are 00 to 99; and if in three digits, the range is from 000 to 999, and so on.
5. Select a random number & check the place of that in random interval
6. Correlate the random numbers with the factors/items in the problem.
7. Summarize and examine the results in an appropriate table.

**Question 1:** What is simulation?

(1 Mark) Nov/06

**Question 2:** Write short note on Monte Carlo Simulation.

(5 Marks) Nov/01 & (2 Marks) Nov/03

**Question 3:** How would you use Monte Carlo Simulation method in inventory control?

(2 Marks) Nov/03 & (4 Marks) May/08

**Question 4:** A town has six wards and they contain 170, 510, 640, 75, 250 and 960 houses respectively. Make a random selection of 8 houses using table of random numbers. Explain the procedure adopted by you.  
(May/89)

**Question 5 (Stock simulation):** Bright Bakery keeps stocks of a popular brand of cake. Previous experience indicates the daily demand as given here.

|                     |      |      |      |      |      |      |
|---------------------|------|------|------|------|------|------|
| <b>Daily Demand</b> | 0    | 10   | 20   | 30   | 40   | 50   |
| <b>Probability</b>  | 0.01 | 0.20 | 0.15 | 0.50 | 0.12 | 0.02 |

Consider the following sequence of random numbers;

48 78 19 51 56 77 15 14 68 09

(i) Using the sequence simulate the demand for next 10 days.

(ii) Find out the stock stimulation if owner of the Bakery decides to make 30 cakes every day. Also estimate the daily Average demand for the cakes.  
(10 Marks) Nov./98

[Ans.: Average Daily Demand = 22 cakes.]

**Question 6:** A company manufactures around 200 mopeds. Depending upon the availability of raw materials and other conditions, the daily production has been varying from 196 mopeds to 204 mopeds, whose probability distribution is as given below:

|                  |      |      |      |      |      |      |      |      |      |
|------------------|------|------|------|------|------|------|------|------|------|
| Production/day : | 196  | 197  | 198  | 199  | 200  | 201  | 202  | 203  | 204  |
| Probability :    | 0.05 | 0.09 | 0.12 | 0.14 | 0.20 | 0.15 | 0.11 | 0.08 | 0.06 |

The finished mopeds are transported in a specially designed three storied lorry that can accommodate only 200 mopeds. Using the following 15 random numbers 82, 89, 78, 24, 53, 61, 18, 45, 04, 23, 50, 77, 27, 54, 10 to simulate the process to find out :

(i) What will be the average number of mopeds waiting in the factory?

(ii) What will be the average number of empty space on the lorry?

[Ans.: (i) 2.8 (ii) 0.27]

Nov./90 [Adapted] & (10 Marks) May/99

**Question 7 (Queuing Problem):** Dr. STRONG is a dentist who schedules all her patients for 30 minutes appointments. Some of the patients take more or less than 30 minutes depending on the type of dental work to be done. The following summary shows the various categories of work, their probabilities and the time needed to complete the work :

| Category   | Time required (mts) | Probability of category |
|------------|---------------------|-------------------------|
| Filling    | 45                  | 0.40                    |
| Crown      | 60                  | 0.15                    |
| Cleaning   | 15                  | 0.15                    |
| Extraction | 45                  | 0.10                    |
| Checkup    | 15                  | 0.20                    |

Simulate the dentist's clinic for four hours and determine the average waiting time for the patients as well as the idleness of the doctor. Assume that all the patients show up at the clinic at exactly their scheduled arrival time starting at 8.00 A.M. Use the following random numbers handling the above problem:

40 82 11 34 25 66 17 79 (Nov/1990)

[Ans.: Avg waiting time = 35.625 minutes; Avg idle time = nil]

**Question 8 (Service Sector):** A single counter ticket booking centre employs one booking clerk. A passenger on arrival immediately goes on the booking counter for being served if the counter is free. If, on the other hand, the counter is engaged, the passenger will have to wait. The passengers are served on first come first served basis. The time of arrival and the time of service varies from one minute to six minutes. The distribution of arrival and service time is as under:

| Arrival/Service<br>Time (Minutes) | Arrival<br>(Probability) | Service<br>(Probability) |
|-----------------------------------|--------------------------|--------------------------|
|-----------------------------------|--------------------------|--------------------------|

CA. Parag Gupta

Ph.: +91 9891 432 632

Paraggupta.ca@yahoo.co.in

Costing & O.R.

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|   |      |      |
|---|------|------|
| 1 | 0.05 | 0.10 |
| 2 | 0.20 | 0.20 |
| 3 | 0.35 | 0.40 |
| 4 | 0.25 | 0.20 |
| 5 | 0.10 | 0.10 |
| 6 | 0.05 | -    |

Required:

- (i) Simulate the arrival and service of 10 passengers starting from 9 A.M. by using the following random numbers in pairs respectively for arrival and service. Random numbers 60 09 16 12 08 18 36 65 38 25 07 11 08 79 59 61 53 77 03 10.
- (ii) Determine the total duration of
  - (1) Idle time of booking clerk and
  - (2) Waiting time of passengers.

(8 Marks) Nov/08-Old Course

[Ans.: (1) 6 Mins. (2) 6 Mins.]

**Question 9 (Service Sector):** With a view to improving the quality of customer services, a Bank is interested in making an assessment of the waiting time of its customers coming to one of its branches located in a residential area. This branch has only one teller's counter. The arrived rate of the customers and the service rate of the teller are given below:

| Time between two consecutive arrivals<br>of customers (in minutes) | Probability |
|--------------------------------------------------------------------|-------------|
| 3                                                                  | 0.17        |
| 4                                                                  | 0.25        |
| 5                                                                  | 0.25        |
| 6                                                                  | 0.20        |
| 7                                                                  | 0.13        |
| Service time by the teller<br>(in minutes)                         | Probability |
| 3                                                                  | 0.10        |
| 4                                                                  | 0.30        |
| 5                                                                  | 0.40        |
| 6                                                                  | 0.15        |
| 7                                                                  | 0.05        |

You are required to simulate 10 arrivals of customers in the system starting 11 AM and show the waiting time of the customers and idle time of the teller. Use the following random numbers taking the first two random numbers digits each for a trial and so on: 11, 56, 23, 72, 94, 83, 83, 02, 97, 99, 83, 10, 93, 34, 33, 53, 49, 94, 37 and 97.

(7 Marks) May/03

[Ans.: Total waiting time of customer is 4 minutes and total idle time of teller is 10 minutes]

**Question 10:** The occurrence of rain in a city on a day is dependent upon whether or not it rained on the previous day. If it rained on the previous day, the rain distribution is given by:

| Event      | Probability |
|------------|-------------|
| No rain    | 0.50        |
| 1 cm. Rain | 0.25        |
| 2 cm. Rain | 0.15        |
| 3 cm. Rain | 0.05        |
| 4 cm. Rain | 0.03        |
| 5 cm. Rain | 0.02        |

If it did not rain the previous day, the rain distribution is given by:

| Event      | Probability |
|------------|-------------|
| No rain    | 0.75        |
| 1 cm. Rain | 0.15        |
| 2 cm. Rain | 0.06        |
| 3 cm. Rain | 0.04        |

Simulate the city's weather for 10 days and determine by simulation the total days without rain as well as the total rainfall during the period. Use the following random numbers:

67 63 39 55 29 78 70 06 78 76

CA. Parag Gupta

Ph.: +91 9891 432 632

Paraggupta.ca@yahoo.co.in

Costing & O.R.

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for simulation. Assume that for the first day of the simulation it had not rained the day before. (Nov./93)  
**[Ans.: It didn't rain on 6 out of 10 days. Total rainfall during the period is 5cm.]**

**Question 11:** The output of a production line is checked by an inspector for one or more of three different types of defects, called defects A, B and C. If defect A occurs, the item is scrapped. If defect B or C occurs, the item must be reworked. The time required to rework a B defect is 15 minutes and the time required to rework a C defect is 30 minutes. The probabilities of an A, B and C defects are 0.15, 0.20 and 0.10 respectively. For ten items coming off the assembly line, determine the number of items without any defect, the number scrapped and the total minutes of rework time. Use the following random numbers:

|                 |    |    |    |    |    |    |    |    |    |    |
|-----------------|----|----|----|----|----|----|----|----|----|----|
| RN for defect A | 48 | 55 | 91 | 40 | 93 | 01 | 83 | 63 | 47 | 52 |
| RN for defect B | 47 | 36 | 57 | 04 | 79 | 55 | 10 | 13 | 57 | 09 |
| RN for defect C | 82 | 95 | 18 | 96 | 20 | 84 | 56 | 11 | 52 | 03 |

**[Ans.: No defect in 5 items, one item is scrapped & Rectification time required on reworking is 165 minutes]**  
 (10 Marks) May/94 & (7 marks) May/04

**Question 12 (Purchase Quantity Decision):** A company trading in motor vehicle spares wishes to determine the level of stock it should carry for the item in its range. Demand is not certain and replenishment of stock takes 3 days. For one item X, the following information is obtained:

| Demand (unit per day) | Probability |
|-----------------------|-------------|
| 1                     | .1          |
| 2                     | .2          |
| 3                     | .3          |
| 4                     | .3          |
| 5                     | .1          |

Each time an order is placed, the company incurs an ordering cost of Rs. 20 per order. The company also incurs carrying cost of Rs. 2.50 per unit per day. The inventory carrying cost is calculated on the basis of average stock.

The manager of the company wishes to compare two options for his inventory decision.

A. Order 12 units when the inventory at the beginning of the day plus order outstanding is less than 12 units.

B. Order 10 units when the inventory at the beginning of the day plus order outstanding is less than 10 units.

Currently (on first day) the company has a stock of 17 units. The sequence of random number to be used is 08, 91, 25, 18, 40, 27, 85, 75, 32, and 52 using first number for day one.

You are required to carry out a simulation run over a period of 10 days, recommend which option the manager should choose. (7 Marks) Nov./04

**[Ans.: Manager should go for Option B, as its Total Cost Rs. 228.75 is less than Option A's Rs. 276.25]**

**[Important Note:** As per a clarification (via email) from Institute, we are supposed to calculate reorder point on the basis of "inventory at end" though question has specifically asked to calculate it on basis of "inventory at the beginning". Same assumption will apply in a similar question asked in May, '00]

**Question 13:** A Publishing house has bought out a new monthly magazine, which sells at Rs. 37.5 per copy. The cost of producing it is Rs. 30 per copy. A Newsstand estimates the sales pattern of the magazine as follows:

| Demand Copies | Probability |
|---------------|-------------|
| 0 < 300       | 0.18        |
| 300 < 600     | 0.32        |
| 600 < 900     | 0.25        |
| 900 < 1200    | 0.15        |
| 1200 < 1500   | 0.06        |
| 1500 < 1800   | 0.04        |

The newsstand has contracted for 750 copies of the magazine per month from the publisher.

The unsold copies are returnable to the publisher who will take them back at cost less Rs. 4 per copy for handling charges.

The newsstand manager wants to simulate of the demand and profitability. The following random number may be used for simulation:

27, 15, 56, 17, 98, 71, 51, 32, 62, 83, 96, 69.

You are required to-

- (i) Allocate random numbers to the demand pattern forecast by the newsstand.  
(ii) Simulate twelve months sales and calculate the monthly and annual profit/loss.  
(iii) Calculate the loss on lost sales.

(8 Marks) Nov./05

[Ans.: Loss on lost sale = Rs. 15750]

**Question 14:** For a washing powder manufacturing factory, frequency distribution of contribution (=sales price-variable cost) per unit annual demand and requirement of investment were found as follows:

| Annual Demand |             | Contribution per unit |             | Required investment |             |
|---------------|-------------|-----------------------|-------------|---------------------|-------------|
| Units         | Probability | Rs.                   | Probability | Rs.                 | Probability |
| 20000         | 0.05        | 3.00                  | 0.10        | 1750000             | 0.25        |
| 25000         | 0.10        | 5.00                  | 0.20        | 2000000             | 0.50        |
| 30000         | 0.20        | 7.00                  | 0.40        | 2500000             | 0.25        |
| 35000         | 0.30        | 9.00                  | 0.20        |                     |             |
| 40000         | 0.20        | 10.00                 | 0.10        |                     |             |
| 45000         | 0.10        |                       |             |                     |             |
| 50000         | 0.05        |                       |             |                     |             |

Consider the random numbers 93,03,51,59,77,61,71,62,99,15 for using Monte-Carlo simulation for 10 runs to estimate the percentage of return on investment (ROI%) defined as

$$\text{ROI \%} = \frac{\text{Cash Inflow}}{\text{Investment}} \times 100$$

Recommend an optimum investment strategy based on modal value of ROI%.

[Ans.: Modal value of ROI % is 12.25, the optimal investment strategy is to invest Rs. 2000000]

(10 Marks) Nov.'95 & (10 Marks) May '01[Adapted]

**Question 15 (Network simulation-Very Imp. for new course-Do after PERT/CPM):** A project consists of 7 activities. The time for performance of each of the activity is as follows:-

| Activity | Immediate | Time             | Probability              |
|----------|-----------|------------------|--------------------------|
| A        | -         | 3<br>4<br>5      | 0.2<br>0.6<br>0.2        |
| B        | -         | 4                | 1.0                      |
| C        | A         | 1                | 1.0                      |
| D        | B,C       | 4<br>5           | 0.8<br>0.2               |
| E        | D         | 3<br>4<br>5<br>6 | 0.1<br>0.3<br>0.3<br>0.3 |
| F        | D         | 5<br>7           | 0.20<br>0.80             |
| G        | E,F       | 2<br>3           | 0.5<br>0.5               |

- a) Draw a network and identify critical path using expected time.  
b) Simulate the project for 5 times using random number and find the critical paths?

|    |    |    |    |    |    |    |
|----|----|----|----|----|----|----|
| 68 | 13 | 09 | 20 | 73 | 07 | 72 |
| 99 | 93 | 18 | 24 | 22 | 07 | 29 |
| 57 | 33 | 49 | 65 | 92 | 98 | 00 |
| 57 | 12 | 31 | 96 | 85 | 92 | 91 |
| 77 | 37 | 34 | 11 | 27 | 10 | 59 |