Chapter - 4

Mathematics of Finance

Simple Interest (S.I.) :-

$$S.I = \frac{PTR}{100}$$

where,

$P \rightarrow$ principal

$T \rightarrow$ Time

$R \rightarrow$ SI rate p.a

Amount $\rightarrow$ $\boxed{A = P + SI}$

Problems on S.I :-

Type 1 :-

To find S.I, when P, T, R are given

$$SI = \frac{PTR}{100}$$

Type 2 :-

To find P, when A, T and R are given

$$P = \frac{A}{1 + TR\%}$$

Type 3 :-

(a) To find R, when A, P & T are given

$$R = \left(\frac{A - P}{PT}\right) \times 100$$

(b) To find T, when A, P & R are given

$$T = \left(\frac{A - P}{PR}\right) \times 100$$

Type 4 ::

When 2 amounts (in terms of principal) and Time in the first case and a case.

shortcut :: $T_2 = \frac{A_2 - 1}{A_1 - 1} \times T_1$

Type 5 :-

When 2 amounts (in terms of Rs.) and time durations are given to find P ::

Note :: SI rate p.a $= \frac{SI \text{ for } 1yr}{P} \times 100$

SI rate for 1yr $\Rightarrow \left| \frac{A_2 - A_1}{T_2 - T_1} \right|$

formula :: $P = \left| \frac{A_2 - A_1}{T_2 - T_1} \times T_1 - A_1 \right|$

Type 6 :-

When S.I rate p.a are different for different yrs. Then to find S.I

(i) When principal is same ::

$S.I = P\left(\frac{T_1 R_1}{100} + \frac{T_2 R_2}{100} \ldots\right) \Rightarrow \frac{P}{100}(T_1 R_1 + T_2 R_2 + \ldots)$

(ii) When principal is different :

$S.I = \frac{P_1 T_1 R_1 + P_2 T_2 R_2 + \ldots}{100}$

Type 7:: To find T when an amount (in terms of P) & SI rate p.a are given.

Formula:: $T = \frac{\text{Total S.I rate}}{\text{SI rate p.a.}}$ $R = \frac{\text{Total S.I rate}}{T}$

Note::

If $A = 2P = 200$

Then the Total S.I rate is 100

Type 8:: Problems on Two SI's::

$SI_1 - SI_2 = \text{Amount}$

eg:: $SI_1 - SI_2 = 30$

$\frac{PT_1}{100}(R_1 - R_2) = 30 \Rightarrow \frac{100x\,T}{100}(6-5) = 30$

$T = \frac{30}{1} = 3\text{yrs}$

Compound Interest:: (CI)

Formula:: $CI = P[(1+i)^n - 1]$ Where,

$P \to$ principal

$i \to$ Interest rate

$n \to$ no. of compoundings

Appreciation formula $\Rightarrow A = P(1+i)^n$

$A > P$

Types of Compoundings::

(i) Annual Compounding::

i = ~~same no of yrs~~ int. rate p.a

n = no. of yrs

(ii) Half yearly Compounding::

$i = \frac{\text{int. rate p.a}}{2}$

$n = 2 \times$ no. of yrs

(iii) ~~Semi-annual~~ Quarterly Compou::

$i = \frac{\text{int. rate p.a}}{4}$

$n = 4 \times$ no. of yrs

(iv) Monthly Compounding::

$i = \frac{\text{int. rate p.a}}{12}$

$n =$ no. of yrs $\times 12$

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Problems on CI :

Type 1 :: When CI & A. (When P, i & n are given)

Note :: Default compounding is Annual Compounding

example sum :: A certain sum of money doubles itself in 5% p.a CI is.

$$A = p(1+i)^n \Rightarrow 2\cancel{p} = \cancel{p}(1.05)^n$$

$$2 = (1.05)^n$$

Type 2 :: Shortcut :: (for diff b/w time durations being 1yr)

$$i = \frac{A_2}{A_1} - 1$$

n → (diff b/w 2 time durations)

In general :: (for 3, 4, 5 yrs)

$$(1+i)^n = \frac{A_2}{A_1}$$

Type 3 :: To find A, when P, diff rate of interests for diff yrs are given.

formula :: $A = (p+i_1)\,(p+i_2)\,(p+i_3)\ldots$

Population sum → Compound Interest

Type 4 :: Important (when 2 Amt's (in terms of P) and the time period of first case is given then to find time period for 2nd case.

8 time (2^3) → $3n = 12 \Rightarrow n = 4$

128 times (2^7) → $7n = 7 \times 4 = 28$ yrs

Effective Rate of Interest :: [ERI] ::

$\% ERI = [(1+i)^n - 1] \times 100$

$ERI = [(1+i)^n - 1]$

Note :: ERI is Independent of Principal

Depreciation ::

$$A = P(1-i\%)^n$$

A → Future value or Scrap value

P → Present value

Problem on Two SI's :

eg :: 1 :: The diff b/w SI and CI for 2 yrs on a Certain sum is Rs. 25.2 at 6%. P.a.

$Diff = CI - SI$

$= P[(1+i)^n - 1] - PTR\%$

$\boxed{Diff = P[(1+i)^n - 1 - TR\%]}$

Shortcut ⇒ $P = \dfrac{\text{Diff b/w 2 Componenti}}{R^2}$ → Answer

Verification Method ::

P = 7000

i = 6% p.a

S.I = 420	SI = 420
Diff = 0	CI = 420 × 6%
	Diff = 25.2

Total CI = Total SI + Total Diff

= 840 + 25.2

= 865.2

Annuity ::

(i) Annuity Regular :: It is a fixed amount to be paid at the end of each yr / half yr / quarterly

eg:: Rent / EMI

(ii) Annuity Immediate :: It is a fixed amount to be paid at the beginning of the year.

eg:: RD / Insurance Premiums

(1) Note:: Annuity should contain every year, every month. AR is dyant

(i) Future Value of A.R $= \dfrac{a \times [(1+i)^n - 1]}{i}$

(ii) F.V of A.D $= \dfrac{a[(1+i)^n - 1]}{i} \times (1+i)$

(iii) P.V of A.R $= \dfrac{a[(1+i)^n - 1]}{i(1+i)^n}$

(iv) P.V of A.D $= \dfrac{a[(1+i)^{n-1} - 1]}{i(1+i)^{n-1}} + a$

Relationship b/w Future Value & Present Value ::

$$A = P(1+i)^n$$

$$\underset{F.V}{A} = \underset{P.V}{P}(1+i)^n$$

$$P.V = \frac{F.V}{(1+i)^n}$$

Sinking Fund::

→ It is a fixed amount to be kept apart (as an investment) in order to accumulate a
from the Income or profit for the future.
lump sum amount required

formula:: $F.V \text{ of } A.R = a\left[\dfrac{(1+i)^n - 1}{i}\right]$

Application of Annuity::

Leasing:: Leasing actually renting.

If PV of Leasing < P.V of purchasing
⇒ Leasing is Preferred

If PV of Leasing > P.V of purchasing
⇒ Purchasing is preferred

If PV of leasing = P.V of purchasing then both are goods

example::

Machine cost = 5,00,000

Purchase
⇒ 5,00,000

P.V of (A.R) $= \dfrac{a[(1+i)^n - 1]}{i(1+i)^n}$

$= 5,68,000$

Then Purchasing is Preferred.

Note:: Both leasing & Investment Decisions are Present Value problems

Investment Decision:

5,00,000 → Machine purchase (Cash outflow)

∴ PV of Cash Inflow $\geq$ P.V of Cash outflow

⇒ Then it is Good Investment

NPV = Inflow - Cash outflow

If NPV ≥ 0 it is profit

example: A machine is purchased at 80,000. If it contributes Rs. 20,000 every year. At 12% p.a for 5 yrs. Is it

P.V of Cash Outflow = 80,000

$$\text{P.V of Cash inflow} = a\left[\frac{(1+i)^n - 1}{i(1+i)^n}\right]$$

$$= 20,000\left[\frac{(1.012)^5 - 1}{0.12(1.12)^5}\right]$$

$$= 72,097.2$$

∴ It is an loss

Valuation of Bond:

* A bond is a debt Security

$$\text{P.V of Bond} = \frac{\text{F.V of 1st Yr Int}}{(1+i)} + \frac{\text{F.V of 2nd yr Int}}{(1+i)^2} + \frac{\text{F.V of 3rd yr int}}{(1+i)^3} + \frac{\text{PAR Value}}{(1+i)^3}$$

↗ FV

Note: (i) Use NRI to calculate F.V of Interest

$$\Rightarrow \text{P.V of bond} = a\left[\frac{(1+i)^n - 1}{i(1+i)^n}\right] + \frac{\text{Par Value}}{(1+i)^n}$$

a = par Value x nominal rate %.

Perpetuity:-

It is a fixed amount (receipt money) paid regularly indefinitely.

Types of perpetuity:-

(i) Normal Perpetuity:-

P.V of multiperiod perpetuity $= \frac{R}{i}$

$R \rightarrow$ Fixed payment (Rec amt)

$i \rightarrow$ Interest rate

$\therefore$ Make R is expressed in terms of annum

(ii) Growing perpetuity:-

P.V of growing perpetuity $= \frac{R}{i-g}$

where,

g - is growth rate

Compound Annual Growth Rate:-

$$A = P(1+i)^n$$

$$\frac{A}{P} = (1+i)^n$$

where $\Rightarrow$ n = End period - starting period

$$\frac{A}{P} = (1+i)^n$$

A → value of end period

P → value of starting period

n → End yr - start yr

$(1+i)$ → option