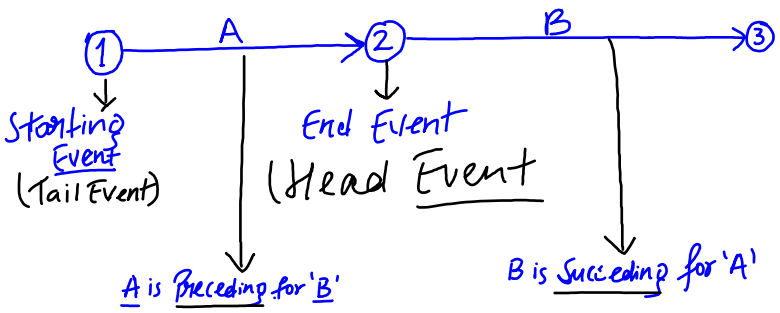
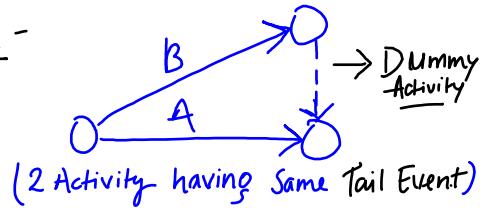
1. Construction of NETWORK

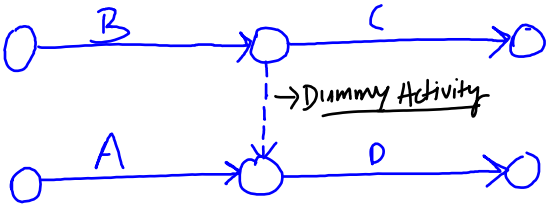


Dummy Activity - Activity which doesn't consume any Productive Time.

Type 1 -



Type 2 -



Activity	Preceding
C	B
D	A, B

(1 activity occures 2 time)

Concept of CRASHING

→ Reduction in Project Duration with change in cost (which may be ↑ or ↓)

Project Duration → Critical Path

$$\left[\text{Slop} = \frac{\text{Crash Cost} - \text{Normal Cost}}{\text{Normal Time} - \text{Crash Time}}\right]$$

Steps -

- Draw Network,
- Identify All Possible Path.
- Select Critical Path.
- Calculate Total Cost (Direct + Indirect)
- Calculate Cost Slope for each activity (Stmt of Cost Slop & Possible Reduction)
- Select which have least Cost Slop + Exist on Critical Path → Critical Path should remain Critical

PERT

(Programe Evaluation Review Technique)

- Used for 1st time project.

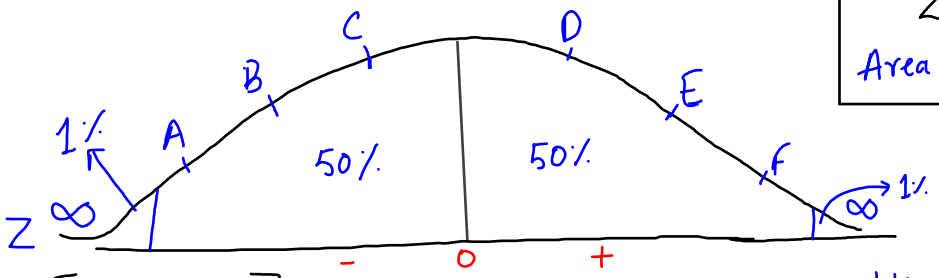
(1) Optimistic Time - (t_o) - Minimum Time to complete an activity - All event occure - Favourable Condition
(2) Pessimistic Time - (t_p) - Maximum Time to complete an Activity - All event occure - Unfavourable Condition.
(3) Most Likely - (t_m) - Normal Consumption of time to complete an activity - All Event occure in Normal Condition.

Step I →

- Average Time $= \frac{t_o + t_p + (4t_m)}{6}$
- Variance $= \left(\frac{t_o + t_p}{6}\right)^2$ [Addition of all variance = Project Variance]
- Project SD $= \sqrt{\text{Project Variance}}$

Step II - Calculation of Probability

Area Under Normal Curve :-



Z = Deviation from Average

Area to the left = 50% = Area to the Right

$$\left[Z = \frac{x - \bar{x}}{SD} \right]$$

x = Project Duration - Mgmt Want to complete.

$\bar{x}$ = Critical Path.

SD = Average of all Deviation

eg - What is the Probability to complete the project, having SD = 3, Project Duration 15 days.

Sol -

Within - 17 days

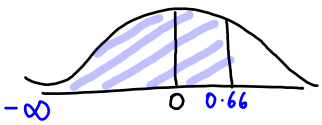
$p(z \leq 17 \text{ days})$

$p\left(z = \frac{x - \bar{x}}{SD}\right)$

$p\left(z \leq \frac{17 - 15}{3}\right)$

$p(z \leq 0.66)$

Area [-∞ to 0 + 0 to 0.66]



= 0.50 + 0.2454 (using Table)

= 0.7454 (74.54%)

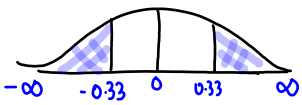
Within 14 Days

$p(z \leq 14 \text{ days})$

$p\left(z \leq \frac{14 - 15}{3}\right)$

$p(z \leq -0.33)$

$p(z \geq 0.33)$



= Area to the Right - (0 to 0.33)

= 0.50 - 0.1293 (using Table)

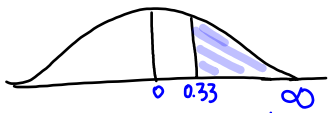
= 0.3707 (37.07%)

Probability of not meeting Due Date - 18 days

$= p(z > 18 \text{ week})$

$= p\left(z > \frac{18 - 17}{3}\right)$

$= p(z > 0.33)$

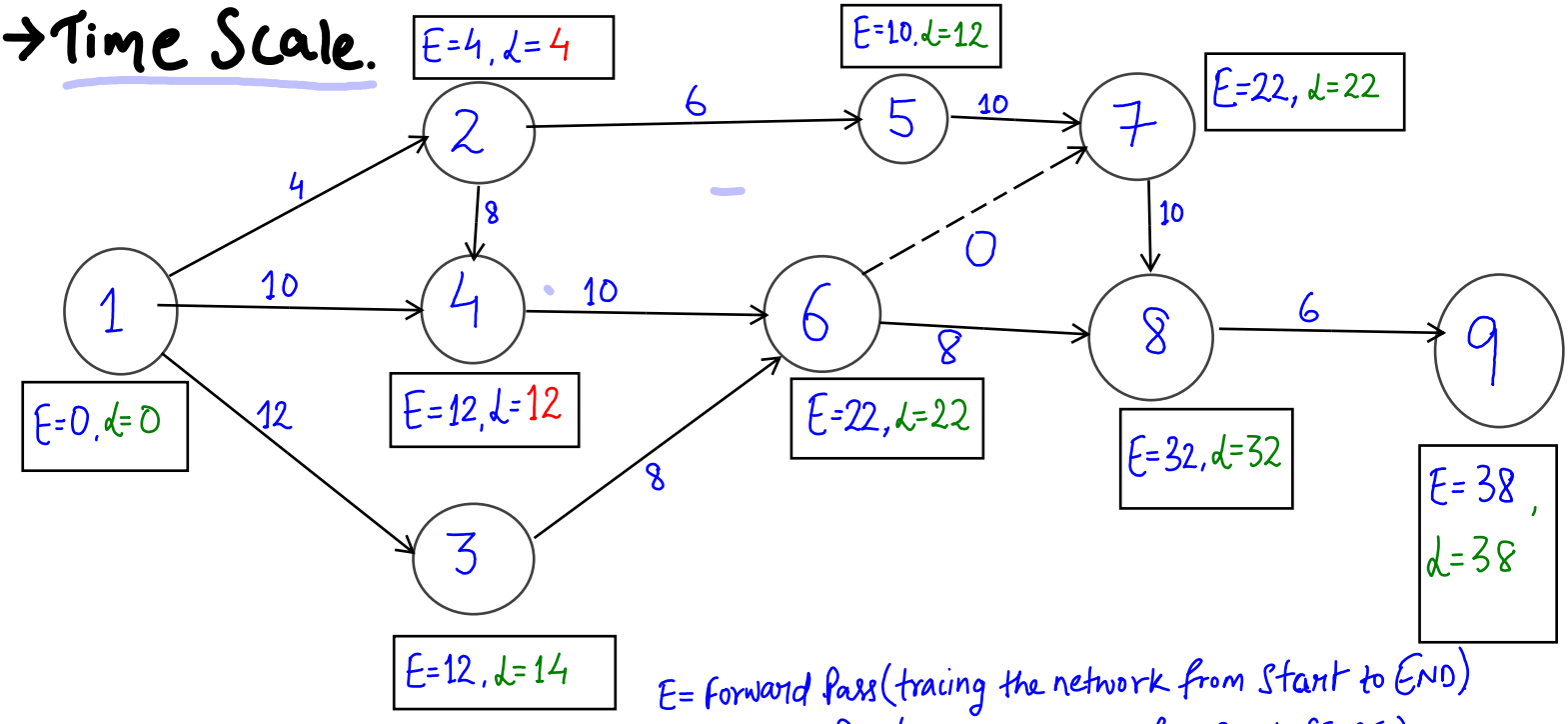


= Area to the Right - (0 to 0.33)

= 0.50 - 0.1293 (using Table)

= 0.3707 (37.07%)

→ Time Scale.



E = Forward Pass (tracing the network from Start to END)

L = Backward Pass (tracing the network from END to START.)

⇒ Condition for Critical Path -

(i) E = L (ii) E/L of Head - E/L of Tail = Activity Duration

Critical Path → 1-2-4-6-7-8-9

Project Duration = 38 days.

EST = E(Tail Event) ⇒ Early Start Time.
EFT = EST + Duration ⇒ Early Finish Time
LFT = L(Head Event) ⇒ Last Finish Time.
LST = LFT − Duration ⇒ Last Start Time.
Slack → Tail Event = L − E [eg. for activity. 3—6 ⇒ is 2]
Slack → Head Event = L − E [eg. for Activity, 1-3 ⇒ 2]
Total Free Float = LST − EST
Free Float = Total Float − Slack of Head Event.
Independent Float = Free Float − Slack of Tail Event.
Interfering Float = Total Float − Free Float.

	PERT	CPM
1.	Program Evaluation Review Technique	Critical Path Method.
2.	Suitable for R & D related Project.	Suitable for Routing Project
3.	Event Oriented.	Activity Oriented.
4.	Tool for Planning Only.	Tool used to Control both Time & Cash.
5.	Probabilistic Tool. (using Three estimate of Duration)	Deterministic. Tool (use only Single Estimate of Duration)

Job or activity	Duration of an activity D_{ij}	EST (= E_i)	EFT (E_i+D_{ij})	LST (L_j-D_{ij})	LFT (= L_j)	Slack of tail event (L_i-E_i)	Slack of head event (L_j-E_j)	Total float (LST-EST)	Free float (Total Float -Slack of head event)	Independent (Free float-slack of tail event)
1-2	4	0	4	0	4	0	0	0	0	0
1-3	12	0	12	2	14	0	2	2	0	0
1-4	10	0	10	2	12	0	0	2	2	2
2-4	8	4	12	4	12	0	0	0	0	0
2-5	6	4	10	6	12	0	2	2	0	0
3-6	8	12	20	14	22	2	0	2	2	0
4-6	10	12	22	12	22	0	0	0	0	0
5-7	10	10	20	12	22	2	0	2	2	0
6-7	0	22	22	22	22	0	0	0	0	0
6-8	8	22	30	24	32	0	0	2	2	2
7-8	10	22	32	22	32	0	0	0	0	0
8-9	6	32	38	32	38	0	0	0	0	0

Critial path is represented by 1-2-4-6-7-8-9. The project duration is 38 days.

(Note: In this table we have given detailed calculations merely for explanation.)

Illustration 5 Full Concept Question

Shown below is a PERT network and a related set of activity times:

i–j	*A*	*B*	*C*	*D*	*E*	*F*	*G*	*H*	*I*	*J*	*K*	*L*
t_o	10	12	8	4	0	12	6	9	4	0	5	9
t_m	13	15	11	7	0	18	12	12	6	0	8	12
t_p	22	18	20	16	0	36	18	27	8	0	11	33

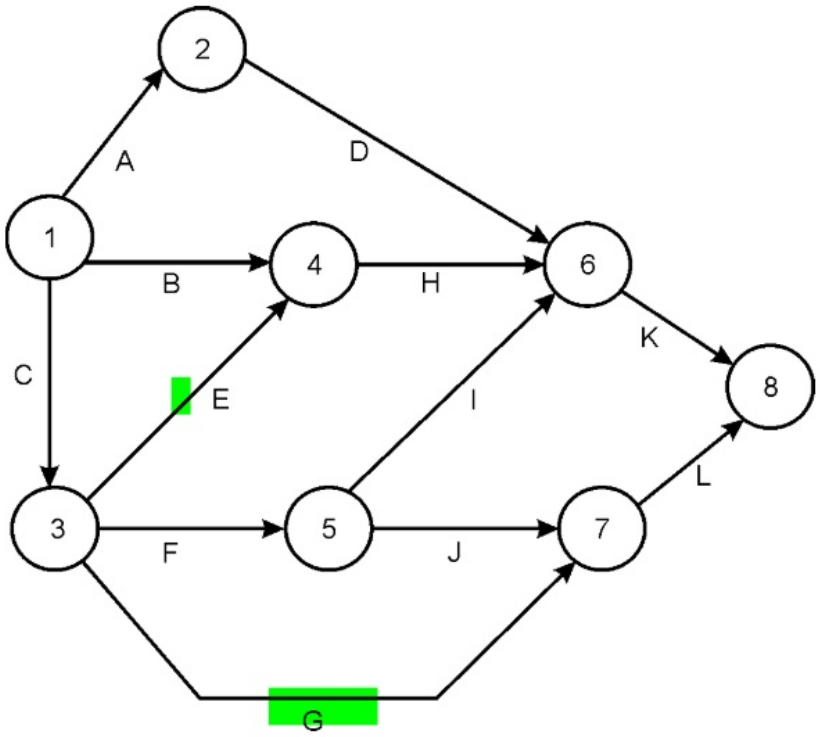


Figure 3

Required

(a) *Determine the expected completion time of each activtiy.*

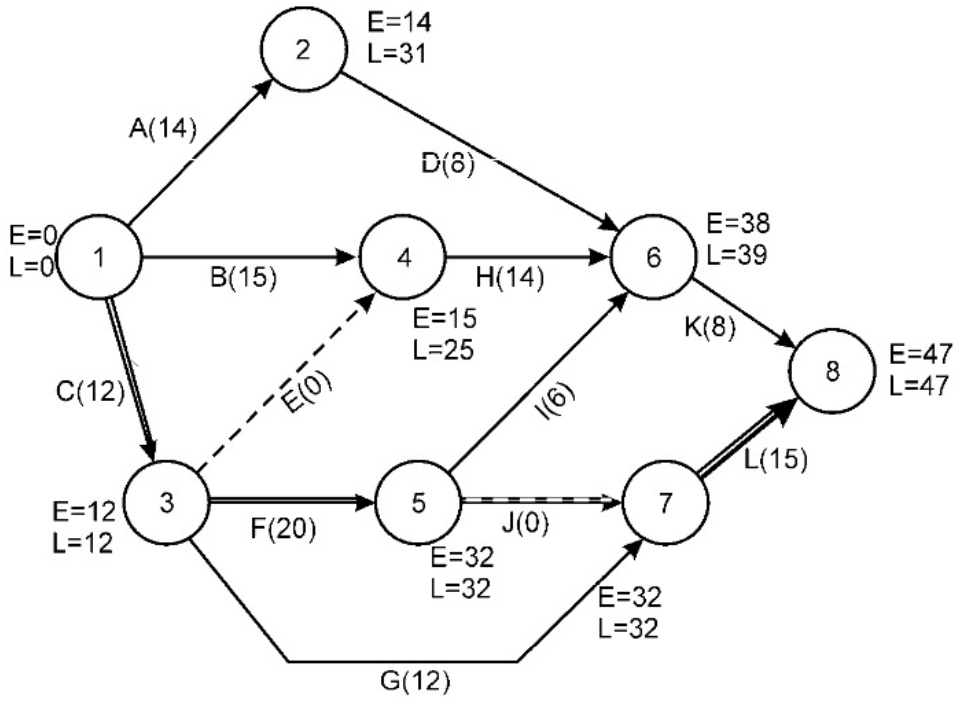
(b) *Determine the earliest expected completion time, the latest expected completion time and float of each activity.*

(c) *What is the total project completion time, and what are activities on the critical path?*

(d) *Determine S.D. of expected completion time for only those activities on the critical path.*

(e) *Determine the probability that the project will be completed within 41 weeks.*

Solution



1–3–5–7–8

Figure 4

$$T_e = \frac{t_p + 4t_m + t_0}{6} \; ; \; V_t = \left(\frac{t_p - t_0}{6}\right)^2$$

	Time estimates				Start		Finish		LST – EST		
i–j	t_o	$4t_m$	t_p	t_e	EST	LST	EFT	LFT	Float	V_t	S.D
1–2	10	52	22	14	0	17	14	31	17		
1–3	8	44	20	12	0	0	12	12	0	4	2
1–4	12	60	18	15	0	10	15	25	10		
2–6	4	28	16	8	14	31	22	39	17		
3–4	0	0	0	0	12	25	12	25	13		
3–5	12	72	36	20	12	12	32	32	0	16	4
3–7	6	48	18	12	12	20	24	32	8		
4–6	9	48	27	14	15	25	29	39	10		
5–6	4	24	8	6	32	33	38	39	1		
5–7	0	0	0	0	32	32	32	32	0	0	0
6–8	5	32	11	8	38	39	46	47	1		
7–8	9	48	33	15	32	32	47	47	0	16	4
									$\Sigma V_t = 36$		

Hence S.D. of the critical path = $\sqrt{36}$ = 6

Probability of completion critical path in 41 weeks is computed below:

$$Z = \frac{41 - 47}{6} = -1$$

∴ Probability = 0.159 (Answer).

-Pankaj Moyal.