Quantitative Techniques or Operation Research

Linear Programming Problems

Asignment Problems

- Simplest method to solve Linear Programing Problems;
- Used when the Allocation of Resources is
One To One Basis;
- This problems can also be solved by using Transporation Method or Simplex Method

Transportations Problems

- Simplified Version of Simplex Method;
- Many to Many Relationship;
- Genarally used for Cost in relation to Transportation;

Others Methods

Graphical Method

When there are Two Variables/ Constraints, it is convinient to use this method

Simplex Method

- Developed in 1946 by Mr. George Dantzig;
- it is very convinient when there are more than 2 variable / constraints;
- It provides us with a systematic way of examining the vertices of the feasible region to determine the optimal value of the objective function

Read me Please!

- **Go throught the Theory first then refer this notes for Refrence & Revisionpurpose only;**
- **It will help you to learn conceptually & what ICAI is expecting in the answer.**
- **Make sure that you follow ICAI prescribed methods to make maximum of your applied efforts.**
- **Views expressed herein are personal opinion, Efforts have been taken to eleminate error; if any please point it out!**
- **To gain confidence Practice a variety of Question & which will make sure that your marks are secured.**

This is a small complimentry contribution from my end!
I will be happy to hear your ideas!

Wishing you
a very Happy Reading
& Good Luck!

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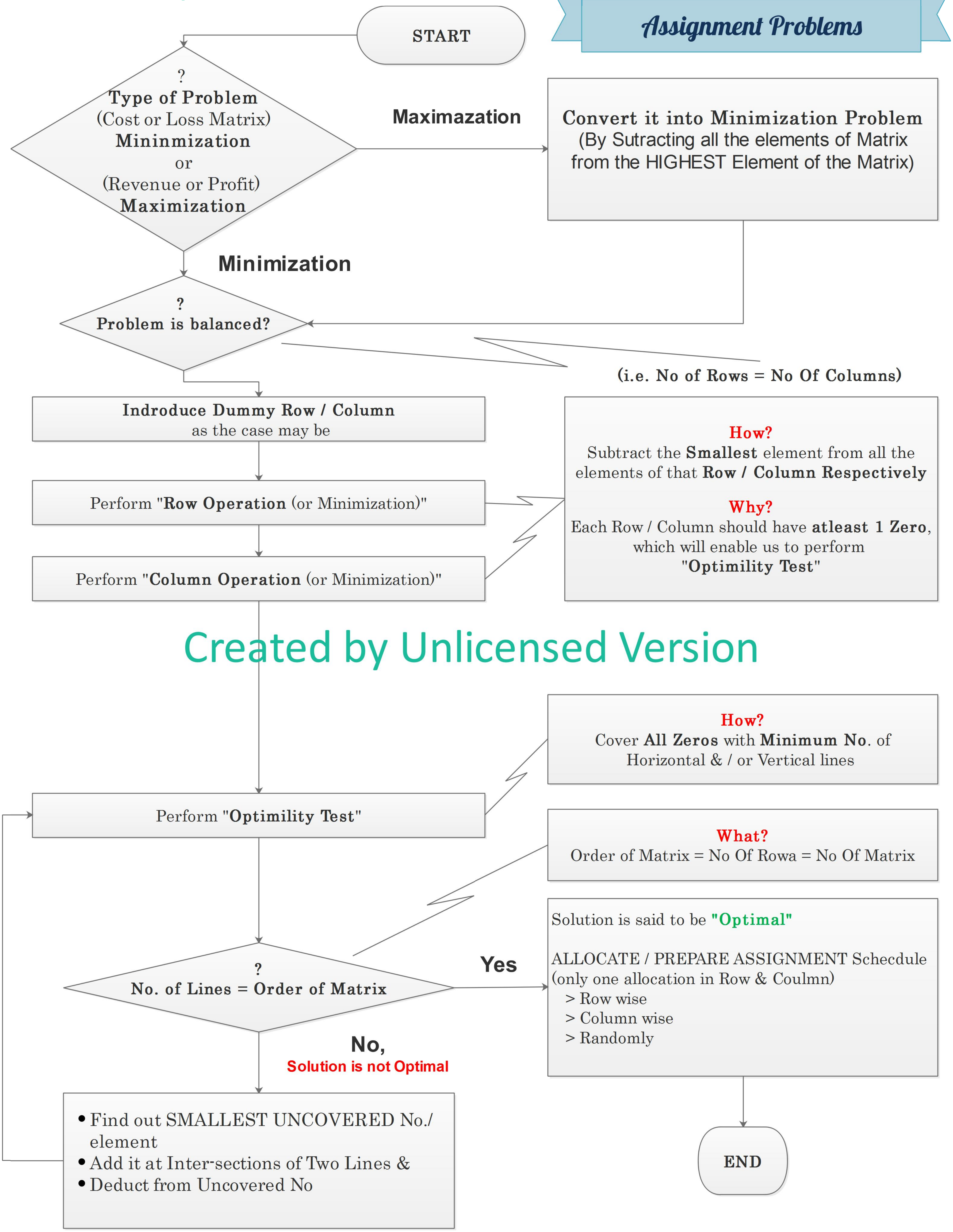
Assignment Problems



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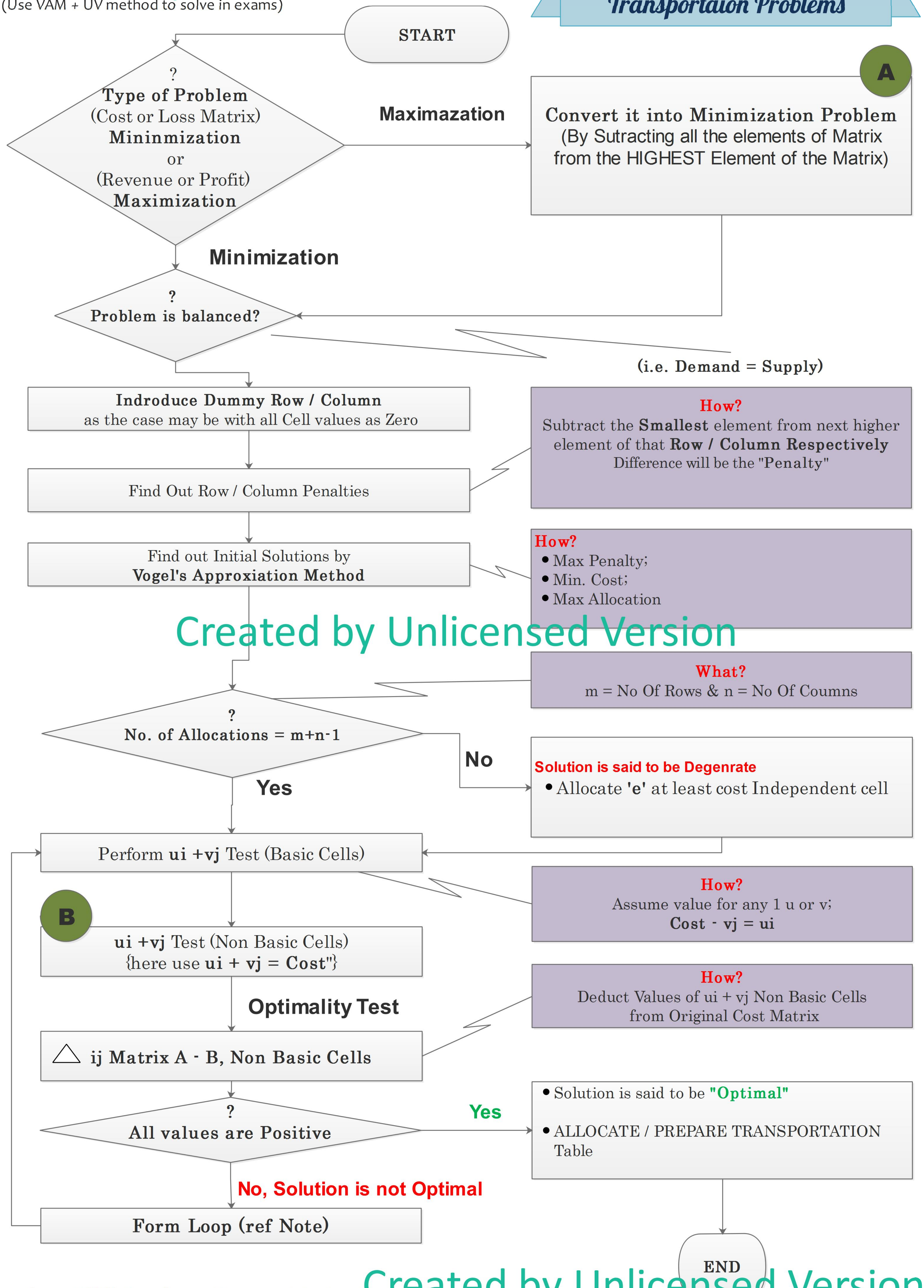
Transportaion Problems

(Use VAM + UV method to solve in exams)



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As the name suggests Simplex is one of the most simple methods, you have to keep calm & give your time atleast once. You can master yourself with Simplex, you need to practice it & Yes! your done. :)

We need to understan some new terms:

- **Slack Variable:**

>used for finding out the **Initial Feasible Solution** (i.e. if non of the operation are carried out i.e. Slack variable = RHS value, & at end they represent the balance amount of Resources left after carring out the optimal solution)
>used for **Converting inequalities into Equality** i.e to make LHS = RHS
(LHS= Required amount resources; RHS= Max amount resources available);
> genarally represnting by **S**
>**No. Of Constraints = No of Slack variables** say **Materials...S_1; Labour Hours...S_2,** & so on

- **Decion Variable:**

Usually the main variable for which decision is to be taken, represented by x,y, etc.

- **Cj - Zj = Net Evaluation row ... used to check whether the solution is optimal or not?**

(if all values are either zero or less than zero, then it is said to be Optimal Solution)

- **C_B represents the co-efficient of the basic variable in the Objective function.**

Other Terms used herein & how they are used by ICAI
Pivot Column = Key Column ... indicated by upside arrow ⇧
Pivot Row = Key Row ... indicated by " ⇐
Pivt Value = Key Value

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Simplex / Illustration:

A Company manufactures two products A & B, involving three departments - Machining, Fabrication & Assembly. The process Time, profit / unit & total capacity of each department is given as following & you are required to Set up Linear Programming Problem to Maximize Profits. What will be the product - mix so as to max. the profit ?
Solve it by Simplex Method.

Product	Machining Hrs.	Fabrication Hrs	Assesmbly Hrs	Profit Rs.
A	1	5	3	80
B	2	4	1	100
Capacity (hrs)	720	1800	900	

Solution:

OFU:
1) Arrange the above table so that it will make further operation eazy & error free... formation of basic equations
2) Steps involved are finding out Objective Function & constraints & initial feasible solution.
they are carried out as follows:

Let the no of Units of product A & B to be produced are x,y respectively.

Objective Function (Profit) : Max Z = 80x + 100y

Subject To Constraints:
1) 1x + 2y =< 720 ... Maching Hours
2) 5x + 4y =< 1800... Fabrication Hours
3) 3x+ 1y =< 900... Assembly hours
4) x,y >= 0 ... Non Negetivity condition

Converting Inequalities into Equalities by indroducing S_1, S_2, S_3 Slack Variables

Objective Function (Profit) : Max Z = 80x + 100y + 0S1 + 0S2 + 0S3
(OFU: Slack variable are nothing to do with objective function but for presentation & error free operation we write it as above)

Subject To Constraints:
1) $1x + 2y + S_1 + 0S_2 + 0S_3 = 720$ ... Maching Hours
2) $5x + 4y + 0S_1 + S_2 + 0S_3 = 1800$... Fabrication Hours
3) $3x + 1y + 0S_1 + 0S_2 + S_3 = 900$... Assembly hours
4) $x, y, S_1, S_2, S_3 > 0$ ... Non Negetivity condition

Initial feasible solution will be...
$S_1 = 720$; $S_2 = 1800$; $S_3 = 900$ by assuming x & y are equal to zero.

Now we are ready to Start with formation of Simplex Tableau as follows:
C_j row contains co-efficients of Objective functions

Simplex Tableau - I

Step 1:

(A. very initial table will look like this)

	Cj →			80	100	0	0	0	
Remarks / Rough work	CB	Basic Variables (B)	Values of Basic Variables (b)	x	y	S1	S2	S3	Minimum Ratio
Machining hr & co efficients	0	S1	720	1	2	1	0	0	
Fabrication hr & co efficients	0	S2	1800	5	4	0	1	0	
Assessmbly hr & co efficients	0	S3	900	3	1	0	0	1	
		Zj							
		Cj - Zj							

Slack Variables: S_1, S_2, S_3
Real Variable / Desion Variable: X,Y

OFU:

Cj	Co-efficients of Objective functions
Cb	Co-efficients of Slack Variables
(b)	Initial feasible solution

Step 2:

- **Calculate Zj**
  say calculated for **x** as... 0*1 + 0*5 + 0*3 = 0(hilighted by color)
  follow the same step for Y, S1, S2, S3 & obtain the Values

- **Perform Cj - Zj**
  highest positive value Cj- Zj, the variable will become entering row... 100 & y the Colum;
  The colum will called as **Pivot Column... will become entering row in next tableau**

Obtain the minmum Ratio:

- **by dividing the values of basic variable by the values Pivot column...**
  say 720 divided by 2 will be 360 & so on
- **Row containing Lowest Min. Ration is "Pivot Row" out going variable i.e. S1**
- **Pivot Value resides at the inter section of Pivot Row & Pivot Column... which will be**

	Cj →			80	100	0	0	0	
	CB	Basic Variables (B)	Values of Basic Variables (b)	x	y	S1	S2	S3	Minimum Ratio
R1	0	S1	720	1	2	1	0	0	← '720/2 = 360
R2	0	S2	1800	5	4	0	1	0	1800/4 = 450
R3	0	S3	900	3	1	0	0	1	900/1 = 900
		Zj		0	0	0	0	0	
		Cj - Zj		80	100 ↑	0	0	0	

(**Where all numbers in the Cj - Zj row are either Negative or Zero**, the solution will said to be **Optimal,** else perform next itteration)

Zj values for Slack variables will be Zero always

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Simplex Tableau - II

Step 3:

Divid Pivot Rows by the Pivot Value & obtain the following table

Operation performed	CB	Basic Variables (B)	Values of Basic Variables (b)	x	y	S1	S2	S3	Minimum Ratio
	Cj →			80	100	0	0	0	
OLD R1 / 2	100	Y	360	1/2	1	1/2	0	0	
	0	S2							
	0	S3							
		Zj							
		Cj - Zj							

Step 4:

Perform following operation & obtain the below table

Operation performed	CB	Basic Variables (B)	Values of Basic Variables (b)	x	y	S1	S2	S3	Minimum Ratio
	Cj →			80	100	0	0	0	
	100	Y	360	1/2	1	1/2	0	0	
old R2 - 4 * new R1	0	S2	360	3	0	-2	1	0	
Old R3 - 1/2 * New R2	0	S3	540	5/2	0	-1/2	0	1	
		Zj							
		Cj - Zj							

Step 5:

- **Calculate Zj**
- **Perform Cj - Zj**
- **Obtain the minmum Ratio**

	CB	Basic Variables (B)	Values of Basic Variables (b)	x	y	S1	S2	S3	Minimum Ratio
	Cj →			80	100	0	0	0	
R1	100	Y	360	1/2	1	1/2	0	0	720
R2	0	S2	360	3	0	-2	1	0	← 120
R3	0	S3	540	5/2	0	-1/2	0	1	216
		Zj		50	100	50	0	0	
		Cj - Zj		30 ↑	0	-50	0	0	

(**Where all numbers in the Cj - Zj row are either Negative or Zero**, the solution will said to be **Optimal,** else perform next itteration)

Simplex Tableau - III

Operation performed	CB	Basic Variables (B)	Values of Basic Variables (b)	x	y	S1	S2	S3	Minimum Ratio
	Cj →			80	100	0	0	0	
	100	Y							
Old R2 / 3	80	X	120	1	0	-2/3	1/3	0	
	0	S3							
		Zj							
		Cj - Zj							

Perform following operation & obtain the below table

	Cj →			80	100	0	0	0	
	CB	Basic Variables (B)	Values of Basic Variables (b)	x	y	S1	S2	S3	Minimum Ratio
old R1 - 1/2 * New R2	100	Y	300	0	1	5/6	-1/6	0	
	80	X	120	1	0	-2/3	1/3	0	
Old R3 - 5/2 * New R1	0	S3	240	0	0	7/6	-5/6	1	
		Zj		80	100	30	10	0	
		Cj - Zj		0	0	-30	-10	0	

(**Where all numbers in the Cj - Zj row are either Negative or Zero**, the solution will said to be **Optimal**)

Answer:
Y = 300 & X = 120

Your answer will look like below:

Simplex Tableau - I

Cj →			80	100	0	0	0	
CB	Basic Variables (B)	Values of Basic Variables (b)	x	y	S1	S2	S3	Minimum Ratio
0	S1	720	1	2	1	0	0	← '720/2 = 360
0	S2	1800	5	4	0	1	0	1800/4 = 450
0	S3	900	3	1	0	0	1	900/1 = 900
	Zj		0	0	0	0	0	
	Cj - Zj		80	100 ↑	0	0	0	

Authors Note:

- Same Problem can be solved by using Graphical Method.
- Operations to be performed can be written down in the extream left... need not to form part of Your final Answer.

Simplex Tableau - II

100	Y	360	1/2	1	1/2	0	0	720
0	S2	360	3	0	-2	1	0	← 120
0	S3	540	5/2	0	-1/2	0	1	216
	Zj		50	100	50	0	0	
	Cj - Zj		30 ↑	0	-50	0	0	

Simplex Tableau - II

100	Y	300	0	1	5/6	-1/6	0	
80	X	120	1	0	-2/3	1/3	0	
0	S3	240	0	0	7/6	-5/6	1	
	Zj		80	100	30	10	0	
	Cj - Zj		0	0	-30	-10	0	

(**Where all numbers in the Cj - Zj row are either Negative or Zero**, the solution will said to be **Optimal)**

X = 120, Y= 300

Profit Will be = 80 * 120 units + 100 *300 units = 39,600

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