

CRITICAL PATH ANALYSIS

Planning, Scheduling and Controlling are three important functions of management. Planning involves the formulation of objectives and goals that are subsequently translated into specific plans and projects. Scheduling is concerned about the implementation of activities necessary to achieve the laid down plans. The function of control is to institute a mechanism that can trigger a warning signal if actual performance is deviating (in terms of time, cost and some other measures of effectiveness) from the plan. The PERT and CPM models are extremely useful for the purpose of planning, scheduling and controlling the progress and completion of large and complex projects.

Network is a graphical representation of a project, depicting the flow as well as the sequence of well-defined activities and events.

Question

List the 5 steps involved in the methodology of critical path analysis.

Answer

PERT/CPM model building consists of following five steps:

- a. Analyse and break down the project in terms of specific activities and / or events.
- b. Determine the interdependence and sequence of specific activities and prepare a net- work.
- c. Assign estimates of time, cost or both to all the activities of the network.
- d. Identify the longest or critical path through the network.
- e. Monitor, evaluate and control the progress of the project by re-planning, rescheduling and reassignment of resources.

Activity is any portion of a project which consumes time or resources and has a definable beginning and ending. For example, "Installation of a D G Set" is an activity requiring the use of resource mainly effort. Activity may involve labour, paper work, contractual negotiations, machinery operations, etc. Alternative terms for "activity" are "task" and "job".

Activities are graphically represented by arrows, usually with description and time estimates written along the arrows. The tail of the arrow portraying an

activity represents the starting point of the activity and its head represents its completion. The arrow may be straight, slanting, or bent but not broken (see figure-1). The arrow is not a vector and need not be drawn to scale. Activities can be categorised into:

- Predecessor Activity;
- Successor Activity;
- Concurrent activity; &
- Dummy activity.

The beginning and ending points of an activity or a group of activities are called **Events**. Synonyms of an event are "node" and "connectors". An event is often represented graphically by a numbered circle, although any geometric figure such as square, oval, rectangle etc. also may be used. A few examples of events are as follows: (i) D G Set installed, (ii) Design completed, (iii) Bricks laid, etc.

Conventions adopted in drawing networks:

- a. Time flows from left to right.
- b. Head events always have a number higher than that of the tail events.
- c. The length and orientation of the arrows show the precedence and succession of activities. Except this there is no other significance whatsoever (chosen only for the sake of convenience of drawing).
- d. No activity may start until all previous activities in the same chain are completed.
- e. Each activity is represented by one and only one arrow in the network.
- f. Arrow direction generally indicates forward movement of time.
- g. Events are identified by numbers and numbered in a logical sequence like 10, 20, 30, . . . etc. It is not necessary and also not desirable to number them serially.
- h. Activities are identified by starting event number and ending event number.
- i. An event which represents joint completion of more than one activity is called merge event and the event from which more than one activity start is called burst event.
- j. A network should have one initial and one terminal node.
- k. All events except the first and last should have atleast one activity entering and one activity leaving them.
- l. All activities must start and end with an event.

The sequence for any activity is ascertained by answering three queries viz.:

- a. Which activity or activities must be completed before the start of a particular activity?
- b. Which activity or activities should follow this?
- c. Which activities can be accomplished simultaneously?

Errors in logical sequencing:

- Looping
- Dangling:
- Duplicate activities (or Parallel activities): If there are separate events, these sorts of situations are avoided by adding dummy activities.
- Redundancy: Unnecessary dummy activities in a network are called redundant activities.

Question

Explain Critical path in the context of a network:

Answer

Critical Path is a chain of activities that begin with the starting event and ends with ending event of a particular project. It is that path that runs through a network with the maximum length of time. Critical path indicates the minimum time that will be required to complete a project. It is determined after identifying critical events. Critical path goes through critical events. Any delay in activities on critical path lead to increase in project completion time. Further, in a network there can be more than one critical path also. The activities on critical path have zero floats.

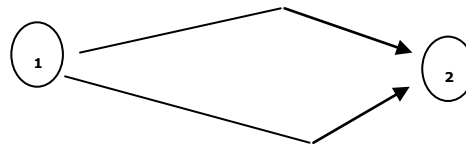
Question

What do you mean by a dummy activity? Why is it used in networking?

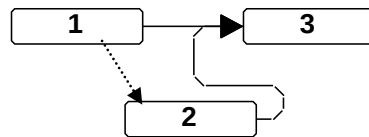
Answer

Dummy activity is a hypothetical activity which consumes no resource or time. It is represented by dotted lines and is inserted in the network to clarify an activity pattern under the following situations.

- a. To make activities with common starting and finishing events distinguishable.
- b. To identify and maintain the proper precedence relationship between activities that are not connected by events.
- c. To bring all "loose ends" to a single initial and single terminal event.



The above situation will be corrected by including a dummy (usually denoted by a dotted arrow) between 1 & 2 as below:



The basic steps involved in drawing a network are:

- Breaking up of the entire project into smaller systems known as tasks.
- For each task ascertain the activities and events to be performed.
- For each activity determine the preceding and succeeding activities.
- For each activity determine or estimate the time and other resources needed.
- Draw a network depicting the assembly of tasks into a project.

Scheduling Computations: After drawing the network, then time schedules for activities are incorporated in network. Then Forward Pass and Backward Pass calculations are made.

Forward pass calculations mean addition of activity durations from Start point to the End point in the network. Conventionally, the Start event will start at zero and further activity durations in the paths of network are added till the end. In case two or more activities end at any particular event, then the cumulative duration of the longest path is taken as the beginning point of next activity. The cumulative durations arrived at each events beginning are called Earliest Start Time of respective activities.

Backward Pass Calculations mean working from End point to the Start point and at each stage, respective activity durations are deducted from the estimated project completion time. In this case if two or more activities start at an event, then the path giving the cumulative minimum value is considered for workings to go to the preceding activity leading to the Start. The times arrived at respective nodes by deducting the cumulative activity durations from the project completion times are called latest Start Times of respective activities.

Earliest Start Time of an activity is the earliest allowable occurrence time of the tail event of the activity.

Earliest Finish Time of an activity is sum of the earliest start time of the activity and the duration of the activity.

Latest Finish Time of an activity is the latest allowable occurrence time of the head event of that activity. This is the maximum allowable time in which an activity has to be completed without affecting the project completion.

Latest Start Time of an activity is the time obtained by deducting the activity duration from its Latest Finish Time.

Question

Discuss in brief;

- a. Total float*
- b. Free float and*
- c. Independent float.*

Answer

Total Float of an activity is the difference between the earliest finish time and earliest start time. This will be same as the difference between latest finish time and latest start time. This represents the free time allowance that is available for an activity for completion without affecting project completion time.

Slack time for an event is the difference between the latest event time and earliest event time. The word slack is used for flexibility in time of events and float is used for flexibility in time of the activities. Very often these words are used synonymously in practice.

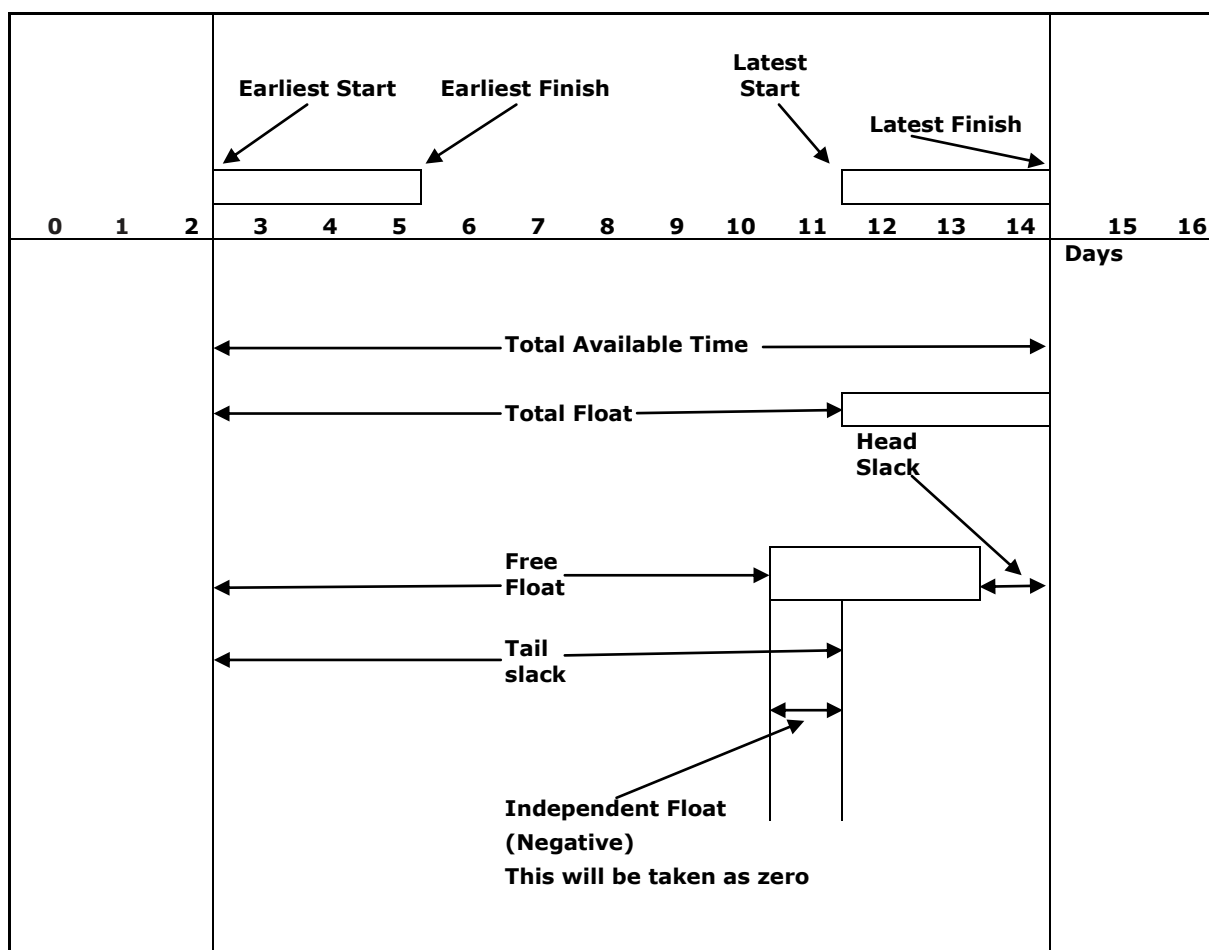
Free Float of an activity is the difference between total float and head event slack. This is the float within which an activity can be manipulated without affecting the floats of succeeding activities. This is the time with which an activity can be delayed without affecting the earliest start time of the following activities.

Independent Float of an activity is the difference between Free Float and the slack of tail event. This is that portion of the total float of an activity by which the activity can be delayed for start without affecting the floats of the

preceding activities. Value of this float will be always +ve or zero but never -ve. If the calculated value is -ve, it is taken as zero.

Interfering Float of an activity is the difference between the latest finish time of an activity and the earliest start time of the following activity. This figure represents the usage of float times of the following activities by the current activity i.e. following activities float will get reduced because of delay in execution of the current activity.

The floats and slacks are depicted in the diagram below for clear understanding.



For all activities of critical path, the earliest start time and latest start time will be same. Similarly earliest finish time and latest finish time will also be same. This implies that there will be no float for all activities on critical path.

Project crashing implies carrying activities of the project at their minimum or reduced times than the ideal times. As this involves some additional expenditure and savings in certain other expenses, depending on the objective of the management a trade off is made. Thus minimum execution crash time may not be the optimal execution time. While crashing, activities on the critical path are to be crashed as crashing elsewhere in other activities will not result in reduction of project time. Activities on critical path are prioritised for crashing depending on respective cost slopes and needs.

Activity cost slope is the figure that is obtained by dividing the difference of costs (between crash cost and normal cost) with difference of times (between crash time and normal time) i.e.

$$\text{Activity Cost Slope} = (\text{Crash Cost} - \text{Normal Cost}) / (\text{Normal time} - \text{Crash Time})$$

It will be beneficial to crash activities with lesser slope in priority to activities with higher slopes.

Updating the network is an important job while preparing CPM / PERT charts, which need to be updated on periodical basis, taking into consideration progress and developments. This is done by adding the days lapsed to the revised estimated duration of the activity and deducting from this figure the duration of the preceding activities, if any.

Time scaled Diagrams: CPM / PERT charts can be drawn on time scale sheets also. First the critical path, being the longest duration path is drawn parallel to the time scale and then other paths are drawn above or below the critical path to scale. Floats are drawn by dotted lines. Dummies are represented by dotted vertical lines. Events are entered at their earliest start times (Alternatively, Latest start times can also be used.) This will be very useful for short projects with limited activities.

Question

Define a project and briefly explain the four common implications which characterize a project.

Answer

A project can be defined as a set of activities or jobs that are performed in a certain sequence determined logically or technologically and it has to be completed within (i) a specified time, (ii) a specified cost and (iii) meeting the performance standards. Examples of a project from fairly diverse fields could be cited. Some of them are given below:

- a. Introducing a new product in the market.
- b. Construction of a new bridge over a river or construction of a 25 – storied building.
- c. Executing a large and complex order on jobbing production.
- d. Sending a spacecraft to the mars.

All these projects are characterized by the following set of common implications, although they pertain to widely different fields.

- a. The **Large-scale** characteristic: These projects are generally unusually large and complex. Thousands of suppliers, workers and other categories of persons are involved and their efforts have to be coordinated for completion of the project.
- b. The **non-recurring** characteristic: These projects are generally of a one-time nature. Neither in the past, nor in the future are they likely to be undertaken substantially in the same form.
- c. **Uncertain and critical dates:** Duration of the various activities involved in such projects are usually uncertain. Further in such type of projects, many critical dates exist by which operations must be completed in order to complete the entire project on schedule.
- d. **Completion dead line:** The fourth distinct feature of these projects is that there is dead line for the completion of the entire project. In case of any delay in the completion of the project, some penalty is levied for such delay beyond the deadline.

PROGRAM EVALUATION AND REVIEW TECHNIQUE

Question

Under what circumstance PERT is more relevant? How?

Answer

Program Evaluation and Review Technique (PERT) is used for projects which have very high uncertain activity durations. The uncertain activity durations are classified into three kinds of estimates. Viz.

- a. **Optimistic Time Estimate (t_o):** This is the shortest possible time in which an activity can be completed in ideal conditions.
- b. **Pessimistic Time Estimate (t_p):** This is the maximum possible time which an activity could take to accomplish the job. If everything went wrong and abnormal situations prevailed, this would be the time estimate.
- c. **The Most Likely Time Estimate (t_m):** This is a time estimate of an activity which lies between the optimistic and the pessimistic time estimates. It assumes that things go in a normal way with few setbacks.

Based on the above time estimates, expected time is arrived by the formula:

$$t_e = (t_o + 4t_m + t_p) / 6$$

Standard Deviation of these time estimates for an activity is calculated by dividing the range of time estimates of respective activities with 6. i.e.

$$S D = (t_p - t_o) / 6$$

Therefore, **Variance = $[(t_p - t_o) / 6]^2$**

Further, **Variance of the project** is the sum of all the activity variances on the critical path. In case there happen to be two or more critical paths (longest Paths for event variances) then the path having highest variance is treated as the variance of the project.

Square root of project variance is the **Standard Deviation of the project.**

Tail Event Variance is calculated by summing all the activity variances in the longest path to that event from the Start point (i.e. Forward pass is adopted.)

Head Event Variance is calculated by summing all the activity variances in the longest path to that event from the End point (i.e. Backward pass is adopted.)

Probability of Completion of the Project (i.e. Scheduled time, T_s):
This is found by calculating the **Standard Variate (Z)** for the project by the formula:

$$Z = (T_s - T_{cp}) / S D$$

Where T_s is scheduled time and T_{cp} is critical path duration and **S D** is the Standard Deviation of the project.

The probability for the value of standard variate (as ascertained by the above formula) is found from the **Standard Normal Distribution Table.**

Question

Write short notes on Distinction between PERT and CPM.

Answer

The PERT and CPM models are similar in terms of their basic structure, rationale and mode of analysis. Differences between PERT and CPM can be summarised as below.

Sl. #	P E R T	C P M
1	PERT is a P robabilistic model. Takes into account optimistic, pessimistic and most likely time estimates.	CPM is a deterministic model. Does not consider uncertainties.
2	PERT is E vent oriented	CPM is activity oriented
3	PERT is used for non R epetitive jobs like planning the assembly of space station, R & D Projects etc.	CPM is used for repetitive jobs where prior experience of handling such jobs exists.
4	PERT emphasises T ime only.	CPM emphasises on cost as well as time.

5	PERT incorporates Statistical analysis	CPM Does not consider statistical analysis
6	PERT serves as a Control device	It is difficult to use CPM as a control device

[Memory: P E R T & Statistical Control]

Resource smoothing: Resources refer to materials, men, machinery, money, methods etc. used in the activities of the project. Different activities require different quantum of resources. The resource requirement on any day in a project may not be uniform; e.g. 20 men required on the first day, 7 men required on the fourth day.

Resource smoothing is a network technique used for smoothing peak resource requirement during different activities of the project network.

The resources required for completing different activities of a project are smoothened (averaged) by utilising floats available on non-critical activities.

Non-critical path activities having floats are rescheduled or shifted so that a uniform demand on resources is achieved. Resource smoothing operation will be on the project duration time.

The critical path activities are not rescheduled since Total Float is Zero for these activities.

RESOURCE LEVELLING is another network technique which is used for reducing the requirement of a particular resource due to its paucity. The process of resource levelling also utilises the large floats available on non-critical activities of the project and thus cuts down the demand on the resources. In resource levelling, the maximum demand of a resource should not exceed the available limit at any point of time. Sometimes, the use of resource levelling may lead to increasing the completion time of the project. In other words, in resource levelling, constraint is on the limit of the resource availability.