RATIO

Ratio $= \frac{a}{b}$; $a = 1^{st}$ Term = Antecedent ; $b = 2^{nd}$ Term = Consequent

Illus 6 (Pg 1.3) Which Ratio is greater :- $2\frac{1}{3} : 3\frac{1}{3}$; $3.6 : 4.8$

$2\frac{1}{3} : 3\frac{1}{3}$ $\qquad$ $3.6 : 4.8$

$= \frac{2\times3+1}{3} : \frac{3\times3+1}{3}$ $\qquad$ $= 3.6 : 4.8$

$= \frac{7}{3} : \frac{10}{3}$ $\qquad$ $= \frac{36}{10} : \frac{48}{10}$

$= \frac{7}{3} \div \frac{10}{3}$ $\qquad$ $= \frac{36}{10} \div \frac{48}{10}$

$= \frac{7}{\cancel{3}} \times \frac{\cancel{3}}{10}$ $\qquad$ $= \frac{\cancel{36}^{3}}{\cancel{10}} \times \frac{\cancel{10}}{\cancel{48}_{4}}$

$= \frac{7}{10}$ $\qquad$ $= \frac{3}{4}$

$= 0.7$ $\qquad$ $= 0.75$ $\qquad$ {use Calculator}

$= 0.70$ $\qquad$ $= 0.75$

$\therefore 0.70 < 0.75$ $\qquad$ {0.70 is less than 0.75}

$\therefore$ 0.75 is greater

i.e 3.6 : 4.8 is greater

Illus 7 (Pg 1.3) $\qquad$ Original Weight $\qquad$ New weight $\qquad$ (In Ratio)

56.7 $\qquad$? $\qquad$ (Actual)

$= \frac{56.7 \times 6}{7}$

$= \boxed{48.6 \text{ kg}}$

Eg 1 (Pg 1.3) $\frac{1}{3} : \frac{1}{8} : \frac{1}{6}$

To be denominator

24	3	3	8	6
	2	1	8	2
	4	1	4	1
		1	1	1

(Make All denominators same i.e L.C.M)

$= \frac{1\times8}{3\times8} : \frac{1\times3}{8\times3} : \frac{1\times4}{6\times4}$

$= \frac{8}{24} : \frac{3}{24} : \frac{4}{24}$

$= \boxed{8:3:4}$

Eg 2 (Pg 1.3)

720

3:5

Boys 3/8 — Girls 5/8

$= \frac{3}{8} \times 720$ $= \frac{5}{8} \times 720$

Original $= \boxed{270}$ $= \boxed{450}$

(+) Admitted x (Assumed) 18 (Given)

New $270 + x$ 468

According to the condition provided :-

Ratio of the No of boys to the No of Girls after Admission = 2:3

$\therefore \frac{270+x}{468} = \frac{2}{3}$

$\therefore 3(270+x) = 2(468)$

$\therefore 3\times270 + 3\times x = 936$

$\therefore 810 + 3x = 936$

$\therefore 3x = 936 - 810$

$\therefore 3x = 126$

$\therefore \boxed{x = 42}$

Ans :- $\therefore$ 42 New boys were admitted in the school.

* Ratio = $\frac{a}{b}$ ∴ Inverse Ratio = $\frac{b}{a}$.

Product of original Ratio & Inverse Ratio is 1. i.e $\frac{a}{b} \times \frac{b}{a} = 1$

* Ratio = $\frac{a}{b}$; $\frac{c}{d}$

∴ Compound Ratio = $\frac{a \times c}{b \times d}$

* Ratio = $\frac{a}{b}$

∴ Duplicate ratio = $\frac{a^2}{b^2}$

∴ Triplicate Ratio = $\frac{a^3}{b^3}$

∴ Sub- Duplicate ratio = $\frac{\sqrt{a}}{\sqrt{b}}$

∴ Sub- Triplicate Ratio = $\frac{\sqrt[3]{a}}{\sqrt[3]{b}}$

* Commensurable Ratio = Ratio of Two Integers = $\frac{3}{2}$

∴ In-commensurable Ratio = Cannot be expressed as Integers = $\frac{\sqrt{3}}{\sqrt{2}}$

* Continued Ratio = a:b:c [3 or more]

Example 1] Pg 1.5]

Let the common multiple be 'x'.

∴ Monthly Income of Two persons are '4x' and '5x'.

Formula :- Income - Savings = Expenditure.

∴ 1st person :- Expenditure = (4x - 50)

∴ 2nd person :- Expenditure = (5x - 50)

According to the given condition provided :-

Ratio of Expenditures = 7:9

∴ $\frac{4x-50}{5x-50} = \frac{7}{9}$

∴ 9 (4x - 50) = 7 (5x - 50)

∴ 36x - 450 = 35x - 350.

$\therefore 36x - 35x = 450 - 350$

$\therefore \boxed{x = 100}$

$\therefore$ Monthly Incomes :- $4x = 4 \times 100 = \boxed{400}$

$5x = 5 \times 100 = \boxed{500}$

Eg 2] Pg 1.5] Let the common multiple be x

$\therefore$ The prices of two houses is 16x and 23x.

Price of the 1st House = Increased by 10 %.

Price of the 2nd House = Increased by Rs 477.

According to the condition provided; we get

$$\frac{16x + (10\% \text{ of } 16x)}{23x + 477} = \frac{11}{20}$$

$$\therefore \frac{16x + \left(\frac{10}{100} \times 16x\right)}{23x + 477} = \frac{11}{20}$$

$$\therefore \frac{16x + 1.6x}{23x + 477} = \frac{11}{20}$$

$$\therefore \frac{17.6x}{23x + 477} = \frac{11}{20}$$

$$\therefore 17.6x \times 20 = 11(23x + 477)$$

$$\therefore 352x = 253x + 5247$$

$$\therefore 352x - 253x = 5247$$

$$\therefore 99x = 5247$$

$$\therefore x = \frac{5247}{99}$$

$$\therefore \boxed{x = 53}$$

$\therefore$ Prices of Two Houses = $16x = 16 \times 53 = 848$

$= 23x = 23 \times 53 = 1219$

fg 3] Pg 1.5]

	Original	New
(A) No of workers	15	11
(B) Wages per worker	22	25
Total Wages (A×B)	330	275

As Seen from the above, Total Wages has decreased

∴ It has decreased in the ratio = $\frac{\text{Original Total Wages}}{\text{New Total Wages}}$

$= \frac{330}{275} = \frac{66}{55} = \frac{6}{5}$

$= \frac{6}{5} = \boxed{6:5}$

Exercise 1(A)

1) Inverse ratio of $\frac{11}{15} = \frac{15}{11}$

2) $\frac{a}{b} = \frac{3^{x5}}{4_{x5}} = \frac{15\text{ (antecedent)}}{?\text{ (consequent)}} = \frac{15}{\boxed{20}}$

3) $\frac{a}{b} = \frac{5}{7}$; ∴ Inverse Ratio :- $\frac{b}{a} = \frac{7\text{ (antecedent)}}{5\text{ (consequent)}}$ ∴ ANS = $\boxed{7}$

4) $\frac{2}{3} ; \frac{9}{4} ; \frac{5}{6} ; \frac{8}{10} = \frac{2 \times 9 \times 8 \times 8}{3 \times 4 \times 6 \times 10} = \frac{1}{1} = \boxed{1:1}$

5) $\frac{3}{4} = \left(\frac{3}{4}\right)^2 = \frac{3^2}{4^2} = \frac{9}{16} = \boxed{9:16}$

6) $\frac{25}{36} = \sqrt{\frac{25}{36}} = \frac{5}{6} = \boxed{5:6}$

7) $\frac{2}{3} = \left(\frac{2}{3}\right)^3 = \frac{2^3}{3^3} = \frac{8}{27} = \boxed{8:27}$

8) $\frac{8}{27} = \sqrt[3]{\frac{8}{27}} = \sqrt[3]{\frac{2\times2\times2}{3\times3\times3}} = \boxed{\frac{2}{3}} = \boxed{2:3}$

9] Compounded Ratio of $\left\{\frac{4}{9}\right.$ and Duplicate ratio of $\left.\frac{3}{4}\right\}$

$$= \left\{\frac{4}{9} \text{ and } \left(\frac{3}{4}\right)^2\right\}$$

$$= \frac{4^1}{9} \times \frac{9}{16} = \frac{1}{4} = \boxed{1:4}$$

10] Compounded Ratio of $\left\{\frac{4}{9}\right.$ and Duplicate of $\frac{3}{4}$ and Triplicate of $\frac{2}{3}$ and $\left.\frac{9}{7}\right\}$

$$= \left\{\frac{4}{9} \text{ and } \left(\frac{3}{4}\right)^2 \text{ and } \left(\frac{2}{3}\right)^3 \text{ and } \frac{9}{7}\right\}$$

$$= \left\{\frac{4^2}{9} \times \frac{9}{16} \times \frac{8^1}{27_3} \times \frac{9^1}{7}\right\}$$

$$= \boxed{2:21}$$

11] Compounded of $\left\{\right.$ Duplicate of $\frac{4}{5}$; Triplicate of $\frac{1}{3}$; Sub-duplicate of $\frac{81}{256}$; Ratio Sub-triplicate of $\left.\frac{125}{512}\right\}$

$$= \left\{\left(\frac{4}{5}\right)^2 \times \left(\frac{1}{3}\right)^3 \times \sqrt{\frac{81}{256}} \times \sqrt[3]{\frac{125}{512}}\right\}$$

$$= \frac{16}{25} \times \frac{1}{27} \times \frac{9^1}{256_{128}} \times \frac{5^1}{8} = \frac{1}{1920} = \boxed{1:1920}$$

12] $\frac{a}{b} = \frac{3}{4}$; $\frac{2a+3b}{3a+4b} = \frac{2(3)+3(4)}{3(3)+4(4)} = \frac{6+12}{9+16} = \frac{18}{25} = \boxed{18:25}$

13] Let the Two numbers be $2x$ and $3x$.

According to the condition provided;

$$\frac{2x-4}{3x-4} = \frac{3}{5}$$

$$\therefore 5(2x-4) = 3(3x-4)$$

$\therefore 10x - 20 = 9x - 12$

$\therefore 10x - 9x = 20 - 12$

$\therefore \boxed{x = 8}$

$\therefore$ Two Numbers are $2x = 2 \times 8 = \boxed{16}$

$3x = 3 \times 8 = \boxed{24}$

14] Let the Angles of the Triangle be $2x, 7x$ and $11x$.

$\therefore 2x + 7x + 11x = 180^\circ$ {All Angles of a triangle equals to 180°}
↳ {property of a Δ}

$\therefore 20x = 180$

$\therefore x = \frac{180}{20}$

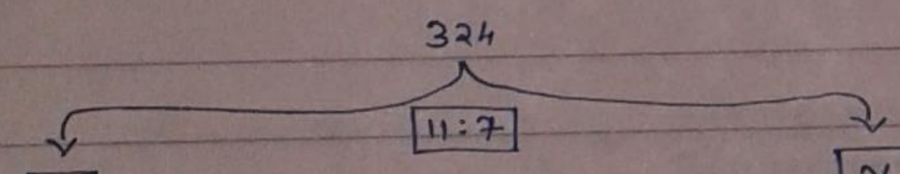
$\therefore \boxed{x = 9}$

$\therefore 2x = 2 \times 9 = \boxed{18^\circ}$

$\therefore 7x = 7 \times 9 = \boxed{63^\circ}$

$\therefore 11x = 11 \times 9 = \boxed{99^\circ}$

15]



$= \frac{11}{18}$

$= \frac{11}{\cancel{18}_{1}} \times \cancel{324}^{18}$

$= \boxed{198}$

$= \frac{7}{18}$

$= \frac{7}{\cancel{18}_{1}} \times \cancel{324}^{18}$

$= \boxed{126}$

16] Comparison can be done only if the Hours are same.

$\therefore$ Earnings in 1 hour :- Anand :- $\frac{80}{7}$

Pramode :- $\frac{90}{12}$

$\therefore$ Ratio of the Earnings in 1 hour $= \frac{\text{Anand's earnings}}{\text{Pramode's earnings}}$

$$= \frac{80}{7}$$

$$\frac{90}{12}$$

$$= \frac{80}{7} \div \frac{90}{12}$$

$$= \frac{8\cancel{0}}{7} \times \frac{\cancel{12}^{4}}{\cancel{9}\cancel{0}_{3}} = \frac{32}{21} = \boxed{32:21}$$

17] Let the two Numbers be $7x$ and $10x$

According to the condition provided;

Difference between the Two Numbers = 105

$\therefore$ Larger No − Smaller No = 105

$\therefore \quad 10x - 7x = 105$

$\therefore \quad 3x = 105$

$\therefore x = \frac{105}{3}$

$\therefore \boxed{x = 35}$

$\therefore 7x = 7 \times 35 = \boxed{245}$

$\therefore 10x = 10 \times 35 = \boxed{350}$

18] $\frac{P}{Q} = \frac{11}{12}$; $\frac{P}{R} = \frac{9}{8}$ $\quad \because$ {Given}

$\hookrightarrow \therefore \left\{\frac{Q}{P} = \frac{12}{11}\right\}$ $\qquad \therefore \left\{\frac{P}{R} = \frac{9}{8}\right\}$

Now, Make the values of 'P' Same

$\therefore \frac{Q}{P} = \frac{12 \times 9}{11 \times 9}$ $\qquad \therefore \frac{P}{R} = \frac{9 \times 11}{8 \times 11}$

$\therefore \left\{\frac{Q}{P} = \frac{108}{99}\right\}$ $\qquad \therefore \left\{\frac{P}{R} = \frac{99}{88}\right\}$

$\therefore Q : P : R = 108 : 99 : 88$

$\therefore Q:R = 108:88$

$= \frac{108}{88} \frac{54}{44} \frac{27}{22} = \frac{27}{22} = \boxed{27:22}$

19] $\frac{x}{4} = \frac{3}{4}$; $\frac{x^2y + xy^2}{x^3+y^3} = \frac{(3)^2 \times 4 + 3 \times (4)^2}{(3)^3 + (4)^3}$

$= \frac{9\times4 + 3\times16}{27+64} = \frac{36+48}{91} = \frac{\cancel{84}^{12}}{\cancel{91}_{13}} = \boxed{12:13}$

20] $\frac{p}{q}$ = Sub-duplicate ratio of $\left(\frac{p-x^2}{q-x^2}\right)$

$\therefore \frac{p}{q} = \sqrt{\frac{p-x^2}{q-x^2}}$

$\therefore \left(\frac{p}{q}\right)^2 = \frac{p-x^2}{q-x^2}$

$\therefore \frac{p^2}{q^2} = \frac{p-x^2}{q-x^2}$

$\therefore p^2(q-x^2) = q^2(p-x^2)$

$\therefore p^2q - p^2x^2 = pq^2 - q^2x^2$

$\therefore p^2q - pq^2 = p^2x^2 - q^2x^2$

$\therefore pq(p-q) = x^2(p^2-q^2)$

$\therefore \frac{pq(p-q)}{(p^2-q^2)} = x^2$

$\therefore \frac{pq\cancel{(p-q)}}{\cancel{(p+q)}\cancel{(p-q)}} = x^2$

$\therefore \boxed{x^2 = \frac{pq}{p+q}}$

21] $\frac{2s}{3t}$ = Duplicate of $\frac{2s-p}{3t-p}$

$\therefore \frac{2s}{3t} = \frac{(2s-p)^2}{(3t-p)^2}$

$\therefore \frac{2s}{3t} = \frac{(2s)^2 - 2\times2s\times p + p^2}{(3t)^2 - 2\times3t\times p + p^2}$

$\therefore \frac{2s}{3t} = \frac{4s^2 - 4sp + p^2}{9t^2 - 6tp + p^2}$

$\therefore 2s(9t^2 - 6tp + p^2) = 3t(4s^2 - 4sp + p^2)$

$\therefore 18st^2 - 12spt + 2p^2s = 12s^2t - 12spt + 3p^2t$

$\therefore 18st^2 - 12s^2t = 3p^2t - 2p^2s$

$\therefore 6st(\cancel{3t} - \cancel{2s}) = p^2(\cancel{3t} - \cancel{2s})$

$\therefore 6st = p^2$

$\therefore \boxed{p^2 = 6st}$

22] $\frac{p}{q} = \frac{2}{3}$; $\frac{x}{4} = \frac{4}{5}$

$$\frac{5px + 3qy}{10px + 4qy} = \frac{5\times2\times4 + 3\times3\times5}{10\times2\times4 + 4\times3\times5} = \frac{40+45}{80+60} = \frac{\cancel{85}^{17}}{\cancel{140}_{28}} = \frac{17}{28}$$

23] Let the No subtracted from each of the terms of the ratio be x

According to the condition provided;

$$\frac{19-x}{31-x} = \frac{1}{4}$$

$\therefore 4(19-x) = 1(31-x)$

$\therefore 76-4x = 31-x$

$\therefore 76-31 = 4x-x$

$\therefore 45 = 3x$

$\therefore \frac{45}{3} = x$

$\therefore \boxed{x = 15}$

24] Let the common multiple be 'x'

$\therefore$ Daily farnings of Two persons are $4x$ and $5x$.

(Refer fg 1, Pg 1.5 of Module - Sume Sum - Solved farlier)

25] 2nd Train

Distance = 400 kms

Time = 5 hours

$\therefore$ Speed = $\frac{\text{Distance}}{\text{Time}} = \frac{400}{5} = \boxed{80}$

$\therefore \frac{\text{Speed of 1st Train}}{\text{Speed of 2nd Train}} = \frac{7}{8}$ {Given}

$\therefore \frac{\text{Speed of 1st Train}}{80} = \frac{7}{8}$

$\therefore$ Speed of 1st Train = $\frac{7\times\cancel{80}^{10}}{\cancel{8}_{1}} = \boxed{70} = \boxed{70 \text{ km/hr}}$