

PORTFOLIO THEORY

Question 1

Consider the following information on two stocks, A and B :

Year	Return on A (%)	Return on B (%)
2006	10	12
2007	16	18

You are required to determine:

- The expected return on a portfolio containing A and B in the proportion of 40% and 60% respectively.
- The Standard Deviation of return from each of the two stocks.
- The covariance of returns from the two stocks.
- Correlation coefficient between the returns of the two stocks.
- The risk of a portfolio containing A and B in the proportion of 40% and 60%.

(5 Marks) (November 2008)

Answer

- (i) Expected return of the portfolio A and B

$$E(A) = (10 + 16) / 2 = 13\%$$

$$E(B) = (12 + 18) / 2 = 15\%$$

$$R_p = \sum_{i=1}^N X_i R_i = 0.4(13) + 0.6(15) = 14.2\%$$

- (ii) Stock A:

$$\text{Variance} = 0.5 (10 - 13)^2 + 0.5 (16 - 13)^2 = 9$$

$$\text{Standard deviation} = \sqrt{9} = 3\%$$

Stock B:

$$\text{Variance} = 0.5 (12 - 15)^2 + 0.5 (18 - 15)^2 = 9$$

Standard deviation = 3%

(iii) Covariance of stocks A and B

$$\text{Cov}_{AB} = 0.5 (10 - 13) (12 - 15) + 0.5 (16 - 13) (18 - 15) = 9$$

(iv) Correlation of coefficient

$$r_{AB} = \frac{\text{Cov}_{AB}}{S_A S_B} = \frac{9}{3 \times 3} = 1$$

(v) Portfolio Risk

$$\begin{aligned} S_P &= \sqrt{X_A^2 S_A^2 + X_B^2 S_B^2 + 2X_A X_B (S_A S_B S_{AB})} \\ &= \sqrt{(0.4)^2 (3)^2 + (0.6)^2 (3)^2 + 2(0.4)(0.6)(3)(3)(1)} \\ &= \sqrt{1.44 + 3.24 + 4.32} = 3\% \end{aligned}$$

Question 2

Mr. X owns a portfolio with the following characteristics:

	Security A	Security B	Risk Free security
Factor 1 sensitivity	0.80	1.50	0
Factor 2 sensitivity	0.60	1.20	0
Expected Return	15%	20%	10%

It is assumed that security returns are generated by a two factor model.

- If Mr. X has ₹ 1,00,000 to invest and sells short ₹ 50,000 of security B and purchases ₹ 1,50,000 of security A what is the sensitivity of Mr. X's portfolio to the two factors?
- If Mr. X borrows ₹ 1,00,000 at the risk free rate and invests the amount he borrows along with the original amount of ₹ 1,00,000 in security A and B in the same proportion as described in part (i), what is the sensitivity of the portfolio to the two factors?
- What is the expected return premium of factor 2? (8 Marks) (June 2009)

Answer

- Mr. X's position in the two securities are +1.50 in security A and -0.5 in security B. Hence the portfolio sensitivities to the two factors:-
 b prop. 1 = $1.50 \times 0.80 + (-0.50 \times 1.50) = 0.45$
 b prop. 2 = $1.50 \times 0.60 + (-0.50 \times 1.20) = 0.30$

(ii) Mr. X's current position:-

Security A ₹ 3,00,000 / ₹ 1,00,000 = 3

Security B -₹ 1,00,000 / ₹ 1,00,000 = -1

Risk free asset -₹ 100000 / ₹ 100000 = -1

b prop. 1 = $3.0 \times 0.80 + (-1 \times 1.50) + (-1 \times 0) = 0.90$

b prop. 2 = $3.0 \times 0.60 + (-1 \times 1.20) + (-1 \times 0) = 0.60$

(iii) The portfolio created in part (ii) is a pure factor 2 portfolio.

Expected return on the portfolio in part (b) is:

$R_p = 3 \times 0.15 + (-1) \times 0.20 + (-1) \times 0.10$

= 0.15 i.e. 15%

\ Expected return premium = 15% - 10% = 5%

Question 3

An investor has two portfolios known to be on minimum variance set for a population of three securities A, B and C having below mentioned weights:

	WA	WB	WC
Portfolio X	0.30	0.40	0.30
Portfolio Y	0.20	0.50	0.30

It is supposed that there are no restrictions on short sales.

- (i) What would be the weight for each stock for a portfolio constructed by investing ₹5,000 in portfolio X and ₹3,000 in portfolio Y?
- (ii) Suppose the investor invests ₹4,000 out of ₹8,000 in security A. How he will allocate the balance between security B and C to ensure that his portfolio is on minimum variance set? (6 Marks) (June 2009)

Answer

(i) Investment committed to each security would be:-

	<u>A</u> (₹)	<u>B</u> (₹)	<u>C</u> (₹)	<u>Total</u> (₹)
Portfolio X	1,500	2,000	1,500	5,000
Portfolio Y	<u>600</u>	<u>1,500</u>	<u>900</u>	<u>3,000</u>
Combined Portfolio	<u>2,100</u>	<u>3,500</u>	<u>2,400</u>	<u>8,000</u>
\ Stock weights	0.26	0.44	0.30	

(ii) The equation of critical line takes the following form:-

$$WB = a + bWA$$

Substituting the values of WA & WB from portfolio X and Y in above equation, we get

$$0.40 = a + 0.30b, \text{ and}$$

$$0.50 = a + 0.20b$$

Solving above equation we obtain the slope and intercept, $a = 0.70$ and $b = -1$ and thus, the critical line is

$$WB = 0.70 - WA$$

If half of the funds is invested in security A then,

$$WB = 0.70 - 0.50 = 0.20$$

Since $WA + WB + WC = 1$

$$WC = 1 - 0.50 - 0.20 = 0.30$$

\ Allocation of funds to security B $= 0.20 \times 8,000$

$= ₹ 1,600$, and

Security C $= 0.30 \times 8,000$

$= ₹ 2,400$

Question 4

The returns on stock A and market portfolio for a period of 6 years are as follows:

Year	Return on A (%)	Return on market portfolio (%)
1	12	8
2	15	12
3	11	11
4	2	-4
5	10	9.5
6	-12	-2

You are required to determine:

(i) Characteristic line for stock A

(ii) The systematic and unsystematic risk of stock A.

(8 Marks) (June 2009)

Answer

Characteristic line is given by

$$\alpha + \beta R_m$$

$$\beta_i = \frac{S_{xy} - n \bar{x} \bar{y}}{S_x^2 - n(\bar{x})^2}$$

$$\alpha_i = \bar{y} - \beta \bar{x}$$

Return on A (Y)	Return on market (X)	xy	x ²	(x - $\bar{x}$)	(x - $\bar{x}$) ²	(y - $\bar{y}$)	(y - $\bar{y}$) ²
12	8	96	64	2.25	5.06	5.67	32.15
15	12	180	144	6.25	39.06	8.67	75.17
11	11	121	121	5.25	27.56	4.67	21.81
2	-4	-8	16	-9.75	95.06	-4.33	18.75
10	9.5	95	90.25	3.75	14.06	3.67	13.47
<u>-12</u>	<u>-2</u>	<u>24</u>	<u>4</u>	<u>-7.75</u>	<u>60.06</u>	<u>-18.33</u>	<u>335.99</u>
38	34.5	508	439.25		240.86		497.34

$$\bar{y} = 38/6 = 6.33$$

$$\bar{x} = 34.5/6 = 5.75$$

$$\beta = \frac{S_{xy} - n \bar{x} \bar{y}}{S_x^2 - n(\bar{x})^2} = \frac{508 - 6(5.75)(6.33)}{439.25 - 6(5.75)^2} = \frac{508 - 218.385}{439.25 - 198.375}$$

$$= \frac{289.615}{240.875} = 1.202$$

$$a = \bar{y} - b \bar{x} = 6.33 - 1.202(5.75) = -0.58$$

Hence the characteristic line is $-0.58 + 1.202(R_m)$

$$\text{Total Risk of Market} = \sigma_{m^2} = \frac{S(x - \bar{x})^2}{n} = \frac{240.86}{6} = 40.14(\%)$$

$$\text{Total Risk of Stock} = \frac{497.34}{6} = 82.89(\%)$$

$$\text{Systematic Risk} = \beta_i^2 \sigma_2 = (1.202)^2 \times 40.14 = 57.99(\%)$$

$$\begin{aligned}\text{Unsystematic Risk is} &= \text{Total Risk} - \text{Systematic Risk} \\ &= 82.89 - 57.99 = 24.90(\%) \end{aligned}$$

Question 5

An investor holds two stocks A and B. An analyst prepared ex-ante probability distribution for the possible economic scenarios and the conditional returns for two stocks and the market index as shown below:

Economic scenario	Probability	Conditional Returns %		
		A	B	Market
Growth	0.40	25	20	18
Stagnation	0.30	10	15	13
Recession	0.30	-5	-8	-3

The risk free rate during the next year is expected to be around 11%. Determine whether the investor should liquidate his holdings in stocks A and B or on the contrary make fresh investments in them. CAPM assumptions are holding true. (10 Marks) (November 2009)

Answer

$$\text{Expected Return on stock A} = E(A) = \sum_{i=G,S,R} P_i A_i$$

(G,S & R, denotes Growth, Stagnation and Recession)

$$(0.40)(25) + 0.30(10) + 0.30(-5) = 11.5\%$$

Expected Return on 'B'

$$(0.40 \times 20) + (0.30 \times 15) + 0.30 \times (-8) = 10.1\%$$

Expected Return on Market index

$$(0.40 \times 18) + (0.30 \times 13) + 0.30 \times (-3) = 10.2\%$$

Variance of Market index

$$\begin{aligned} & (18 - 10.2)^2 (0.40) + (13 - 10.2)^2 (0.30) + (-3 - 10.2)^2 (0.30) \\ &= 24.34 + 2.35 + 52.27 = 78.96\% \end{aligned}$$

Covariance of stock A and Market Index M

$$\text{Cov. (AM)} = \sum_{i=G,S,R} (A_i - E(A)) [M_i - E(M)] P_i$$

$$\begin{aligned} & (25 - 11.5) (18 - 10.2)(0.40) + (10 - 11.5) (13 - 10.2) (0.30) + (-5 - 11.5) (-3 - 10.2)(0.30) \\ &= 42.12 + (-1.26) + 65.34 = 106.20 \end{aligned}$$

Covariance of stock B and Market index M

$$(20-10.1)(18-10.2)(0.40) + (15-10.1)(13-10.2)(0.30) + (-8-10.1)(-3-10.2)(0.30) = 30.89 + 4.12 + 71.67 = 106.68$$

$$\text{Beta for stock A} = \frac{\text{CoV(AM)}}{\text{VAR(M)}} = \frac{106.20}{78.96} = 1.345$$

$$\text{Beta for Stock B} = \frac{\text{CoV(BM)}}{\text{VarM}} = \frac{106.68}{78.96} = 1.351$$

Required Return for A

$$R(A) = R_f + \beta(M - R_f)$$

$$11\% + 1.345(10.2 - 11)\% = 9.924\%$$

Required Return for B

$$11\% + 1.351(10.2 - 11)\% = 9.92\%$$

Alpha for Stock A

$$E(A) - R(A) \text{ i.e. } 11.5\% - 9.924\% = 1.576\%$$

Alpha for Stock B

$$E(B) - R(B) \text{ i.e. } 10.1\% - 9.92\% = 0.18\%$$

Since stock A and B both have positive Alpha, therefore, they are UNDERPRICED. The investor should make fresh investment in them.

Question 6

A study by a Mutual fund has revealed the following data in respect of three securities:

Security	σ (%)	Correlation with Index, P_m
A	20	0.60
B	18	0.95
C	12	0.75

The standard deviation of market portfolio (BSE Sensex) is observed to be 15%.

- What is the sensitivity of returns of each stock with respect to the market?
- What are the covariances among the various stocks?
- What would be the risk of portfolio consisting of all the three stocks equally?
- What is the beta of the portfolio consisting of equal investment in each stock?
- What is the total, systematic and unsystematic risk of the portfolio in (iv) ?

(8 Marks) (November 2009)

Answer

- (i) Sensitivity of each stock with market is given by its beta.

Standard deviation of market Index = 15%

Variance of market Index = 0.0225

Beta of stocks = $\sigma_i r / \sigma_m$ A = $20 \times 0.60/15 = 0.80$ B = $18 \times 0.95/15 = 1.14$ C = $12 \times 0.75/15 = 0.60$

- (ii) Covariance between any 2 stocks =
- $\beta_1 \beta_2 \sigma_m^2$

Covariance matrix

Stock/Beta	0.80	1.14	0.60
A	400.000	205.200	108.000
B	205.200	324.000	153.900
C	108.000	153.900	144.000

- (iii) Total risk of the equally weighted portfolio (Variance) =
- $400(1/3)^2 + 324(1/3)^2 + 144(1/3)^2 + 2(205.20)(1/3)^2 + 2(108.0)(1/3)^2 + 2(153.900)(1/3)^2 = 200.244$

- (iv)
- β
- of equally weighted portfolio =
- $\beta_p = \frac{\sum \beta_i / N}{3} = \frac{0.80 + 1.14 + 0.60}{3} = 0.8467$

- (v) Systematic Risk
- $\beta_p^2 \sigma_m^2 = (0.8467)^2 (15)^2 = 161.302$

Unsystematic Risk = Total Risk – Systematic Risk
 = $200.244 - 161.302 = 38.942$

Question 7

Amal Ltd. has been maintaining a growth rate of 12% in dividends. The company has paid dividend @ ₹ 3 per share. The rate of return on market portfolio is 15% and the risk-free rate of return in the market has been observed as 10%. The beta co-efficient of the company's share is 1.2.

You are required to calculate the expected rate of return on the company's shares as per CAPM model and the equilibrium price per share by dividend growth model.

(5 Marks) (November 2010)

Answer

Capital Asset Pricing Model (CAPM) formula for calculation of expected rate of return is

$$E_R = R_f + \beta (R_m - R_f)$$

$$E_R = \text{Expected Return}$$

$$\beta = \text{Beta of Security}$$

$$R_m = \text{Market Return}$$

$$R_f = \text{Risk free Rate}$$

$$= 10 + [1.2 (15 - 10)]$$

$$= 10 + 1.2 (5)$$

$$= 10 + 6 = 16\% \text{ or } 0.16$$

Applying dividend growth mode for the calculation of per share equilibrium price:-

$$E_R = \frac{D_1}{P_0} + g$$

$$\text{or } 0.16 = \frac{3(1.12)}{P_0} + 0.12 \quad \text{or} \quad 0.16 - 0.12 = \frac{3.36}{P_0}$$

$$\text{or } 0.04 P_0 = 3.36 \quad \text{or} \quad P_0 = \frac{3.36}{0.04} = ₹ 84$$

Therefore, equilibrium price per share will be ₹ 84.

Question 8

Consider the following information on two stocks X and Y:

Year	Return on X (%)	Return on Y (%)
2008	12	10
2009	18	16

You are required to determine:

- The expected return on a portfolio containing X and Y in the proportion of 60% and 40% respectively.
- The standard deviation of return from each of the two stocks.
- The covariance of returns from the two stocks.
- Correlation co-efficient between the returns of the two stocks.
- The risk of portfolio containing X and Y in the proportion of 60% and 40%.

(8 Marks) (November 2010)

Answer

- (i) Expected return of the portfolio X and Y

$$E(X) = (12 + 18)/2 = 15\%$$

$$E(Y) = (10 + 16)/2 = 13\%$$

$$R_P = 0.6(15) + 0.4(13) = 14.2\%$$

- (ii) Stock X

$$\text{Variance} = \frac{\sum_{t=1}^n (X_t - \bar{X})^2}{N}$$

$$\text{Variance} = 0.5(12 - 15)^2 + 0.5(18 - 15)^2 = 9$$

$$\text{Standard deviation} = \sqrt{9} = 3\%$$

Stock Y

$$\text{Variance} = \frac{\sum_{t=1}^n (Y_t - \bar{Y})^2}{N}$$

$$\text{Variance} = 0.5(10 - 13)^2 + 0.5(16 - 13)^2 = 9$$

$$\text{Standard deviation} = \sqrt{9} = 3\%$$

- (iii) Covariance of Stocks X and Y

$$\text{Covariance} = \frac{\sum_{t=1}^n (X_t - \bar{X})(Y_t - \bar{Y})}{N}$$

$$\text{Cov}_{XY} = 0.5(12 - 15)(10 - 13) + 0.5(18 - 15)(16 - 13) = 9$$

- (iv) Correlation of Coefficient

$$g_{XY} = \frac{\text{COV}_{XY}}{S_X S_Y} = \frac{9}{3 \times 3} = 1$$

- (v) Portfolio Risk

$$\begin{aligned} S_P &= \sqrt{(0.6)^2(3)^2 + (0.4)^2(3)^2 + 2(0.6)(0.4)(3)(3)(1)} \\ &= \sqrt{3.24 + 1.44 + 4.32} = \sqrt{9} = 3\% \end{aligned}$$

Question 9

Mr. Tamarind intends to invest in equity shares of a company the value of which depends upon various parameters as mentioned below:

Factor	Beta	Expected value in %	Actual value in %
GNP	1.20	7.70	7.70
Inflation	1.75	5.50	7.00
Interest rate	1.30	7.75	9.00
Stock market index	1.70	10.00	12.00
Industrial production	1.00	7.00	7.50

If the risk free rate of interest be 9.25%, how much is the return of the share under Arbitrage Pricing Theory? (5 Marks)(May 2011)

Answer**Return of the stock under APT**

Factor	Actual value in %	Expected value in %	Difference	Beta	Diff. x Beta
GNP	7.70	7.70	0.00	1.20	0.00
Inflation	7.00	5.50	1.50	1.75	2.63
Interest rate	9.00	7.75	1.25	1.30	1.63
Stock index	12.00	10.00	2.00	1.70	3.40
Ind. Production	7.50	7.00	0.50	1.00	<u>0.50</u>
					8.16
Risk free rate in %					<u>9.25</u>
Return under APT					<u>17.41</u>

Question 10

Mr. Tempest has the following portfolio of four shares:

Name	Beta	Investment ₹ Lac.
Oxy Rin Ltd.	0.45	0.80
Boxed Ltd.	0.35	1.50
Square Ltd.	1.15	2.25
Ellipse Ltd.	1.85	4.50

The risk free rate of return is 7% and the market rate of return is 14%.

Required.

(i) Determine the portfolio return. (ii) Calculate the portfolio Beta. (5 Marks)(May 2011)

Answer

Market Risk Premium (A) = 14% – 7% = 7%

Share	Beta	Risk Premium (Beta x A) %	Risk Free Return %	Return %	Return ₹
Oxy Rin Ltd.	0.45	3.15	7	10.15	8,120
Boxed Ltd.	0.35	2.45	7	9.45	14,175
Square Ltd.	1.15	8.05	7	15.05	33,863
Ellipse Ltd.	1.85	12.95	7	19.95	89,775
Total Return					<u>1,45,933</u>

Total Investment ₹ 9,05,000

(i) Portfolio Return = $\frac{₹ 1,45,933}{₹ 9,05,000} \times 100 = 16.13\%$

(ii) Portfolio Beta

Portfolio Beta = Risk Free Rate + Risk Premium $\times \beta = 16.13\%$

$$7\% + 7\beta = 16.13\%$$

$$\beta = 1.30$$

Alternative Approach

First we shall compute Portfolio Beta using the weighted average method as follows:

$$\begin{aligned} \text{Beta}_P &= 0.45 \times \frac{0.80}{9.05} + 0.35 \times \frac{1.50}{9.05} + 1.15 \times \frac{2.25}{9.05} + 1.85 \times \frac{4.50}{9.05} \\ &= 0.45 \times 0.0884 + 0.35 \times 0.1657 + 1.15 \times 0.2486 + 1.85 \times 0.4972 = 0.0398 + 0.058 + 0.2859 + 0.9198 = 1.3035 \end{aligned}$$

Accordingly,

(i) Portfolio Return using CAPM formula will be as follows:

$$\begin{aligned} R_P &= R_F + \text{Beta}_P(R_M - R_F) \\ &= 7\% + 1.3035(14\% - 7\%) = 7\% + 1.3035(7\%) \\ &= 7\% + 9.1245\% = 16.1245\% \end{aligned}$$

(ii) Portfolio Beta

As calculated above 1.3035

Question 11

Discuss how the risk associated with securities is effected by Government policy.

(4 Marks)(May 2011)

Answer

The risk from Government policy to securities can be impacted by any of the following factors.

- (i) Licensing Policy
- (ii) Restrictions on commodity and stock trading in exchanges
- (iii) Changes in FDI and FII rules.
- (iv) Export and import restrictions
- (v) Restrictions on shareholding in different industry sectors
- (vi) Changes in tax laws and corporate and Securities laws.