

{ Prof.Dani's –Dani Ki Costing for C.A Final }

(Visiting Faculty of the Institute of Charter Accountant (W.I.R.C))

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Simulation Analysis

SIMULATION

Illustration 231

ABC Cooperative Bank receives and disburses different amount of cash in each month. The bank has an opening cash Balance of Rs. 15 Crore in the first month. Pattern of receipts and disbursements from past data is as follows :

Monthly Cash Receipts		Monthly Cash Disbursement	
Rs. In Crores	Probability	Rs. In Crores	Probability
30	0.20	33	0.15
42	0.40	60	0.20
36	0.25	39	0.40
99	0.15	57	0.25

Simulate the cash position over a period of 12 months.

Required :-

Calculate probability that the ABC Cooperative Bank will fall short in payments.
Calculate the average monthly shortfall.

If ABC Bank can get an overdraft facility of Rs.45 crores from other Nationalised banks.

What is the probability that they will fall short in monthly payments?

Use the following sequence (rowwise) of paired random numbers.
1778 4316 7435 3123 7244 4692 5158 6808 9358 5478 9654 0977

Illustration 232

A car rental agency has collected the following data on the demand for five seater vehicles over the past 50 days.

Daily Demand	4	5	6	7	8
No. of Days	4	10	16	14	6

The agency has only 6 cars currently.

- 1) Use the following 5 random numbers to generate 5 days of demand for the rental agency. Random No.:- 15, 48, 71, 56, 90
- 2) What is the average number of cars rented per day for the 5 days?
- 3) How many rentals will be lost over the 5 days?

Simulation Analysis

Illustration - 233

Nine villages in a certain administrative area contain 720, 130, 150, 240, 960, 100, 52, 36, 532 fields respectively. Make a random selection of six fields using the random number tables.

Illustration - 234

Frontier Bakery keeps stock of a popular brand of cake. Daily demand based on past experience is as given below:-

Experience indicates

Daily demand	0	15	25	35	45	50
Probability	0.01	0.15	0.20	0.50	0.12	0.02

Consider the following sequence of random numbers:-

R. No. 48, 78, 09, 51, 56, 77, 15, 14, 68, 09.

Using the sequence, 5 simulate the demand for the next 10 days

Find out the stock situation if the owner of the bakery decides to make 35 cakes every day.

Also estimate the daily average demand for the cakes on the basis of simulated data.

Illustration - 235

A company manufacturers around 200 mopeds. Depending upon the availability of raw materials and other conditions, the daily production has been varying from 196 mopped to 204 mopped, whose probability distribution is as given below:-

Production per day	Probability
196	0.05
197	0.09
198	0.12
199	0.14
200	0.20
201	0.15
202	0.11
203	0.08
204	0.06

The finished mopeds are transported in a specially designed three storeyed lorry that can accommodate only 200 mopped. Using the following 15 random number 82, 89, 78, 24, 53, 61, 18, 45, 04, 23, 50, 77, 27, 54, 10, simulate the process to find out:

(i) What will be the average number of mopeds waiting in the factory?

(ii) What will be the average number of empty spaces on the lorry?

Simulation Analysis

Illustration - 236

Dr. STRONG is a dentist who schedules all her patients for 30 minutes appointments. Some of the patients take more or less than 30 minutes depending on the type of dental work to be done. The following summary shows the various categories of work, their probabilities and the time needed to complete the work:

Category	Time Required	Probability of Category
Filling	45 minutes	0.40
Crown	60 minutes	0.15
Cleaning	15 minutes	0.15
Extraction	45 minutes	0.10
Checkup	15 minutes	0.20

Stimulate the dentist's clinic for four hours and determine the average waiting time for the patients as well as the idleness of the doctor. Assume that all the patients show up at the clinic at exactly their scheduled arrival time starting at 8.00 a.m. Use the following random numbers for handling the above problem.

40 82 11 34 25 66 17 79

Illustration - 237

A company manufactures 30 items per day. The sales of these items depends upon demand which has the following distribution:

Sales (Units)	Probability
27	0.10
28	0.15
29	0.20
30	0.35
31	0.15
32	0.05

The production cost and sales price of each unit are Rs. 40 and Rs. 50 respectively. Any unsold product is to be disposed off at a loss of Rs. 15 per unit. There is a penalty of Rs. 5 per unit if the demand is not met.

Using the following random numbers estimate total profit / loss for the company for the next 10 days:

10, 99, 65, 99, 95, 01, 79, 11, 16, 20

If the company decides to produce 29 items per day, what is the advantage or disadvantage to the company?

Illustration - 238

An Investment corporation wants to study the investment projects based on three factors: Market demand in units, price per unit minus cost per unit, and the investment required. These factors are felt to be independent of each other. In analyzing a new consumer product, the corporation estimates the following probability distribution:

Annual demand units	Probability	(Price - cost) per unit Rs.	Probability	Investment Rs.	Required Probability
25,000	0.05	3	0.10	27,50,000	0.25
30,000	0.10	5	0.20	30,00,000	0.50
35,000	0.20	7	0.40	35,00,000	0.25
40,000	0.30	9	0.20		
45,000	0.20	10	0.10		
50,000	0.10				
55,000	0.05				

Simulation Analysis

Using simulation process, repeat the trial 10 times, compute the return on investment for each trial, taking these three factors into account. Approximately, what is the most likely return? Use the following random number for annual demand, (Price-cost) and the investment required: 28, 57, 60, 17, 64, 20, 27, 58, 61, 30; 19, 07, 90, 02, 57, 28, 29, 83, 58, 41; 18, 67, 16, 71, 43, 68, 47, 24, 19, 97

Illustration - 239

The occurrence of rain in a city on a day is dependent upon whether or not it rained on the previous day. If it rained on the previous day, the rain distribution is given by:

Event	Probability
No rain	0.50
1 cm. rain	0.25
2 cm. rain	0.15
3 cm. rain	0.05
4 cm. rain	0.03
5 cm. rain	0.02

If it did not rain the previous day, the rain distribution is given by:

Event	Probability
No rain	0.75
1 cm. rain	0.15
2 cm. rain	0.06
3 cm. rain	0.04

Simulate the city's weather for 10 days and determine by simulation the total days without rain as well as the total rainfall during the period. Use the following random numbers.

67 63 39 55 29 78 70 06 78 76

For simulation. Assume that for the first day of the simulation it had not rained the day before.

Illustration - 240

The output of a production line is checked by an inspector for one or more of three different types of defects A, B and C. If defect A occurs, the item is scrapped. If defect B or C occurs, the item must be reworked. The time required to rework a B defect is 15 minutes and the time required to rework a C defect is 30 minutes. The probabilities of an A, B and C defects are, 0.15, 0.20, and 0.10 respectively. For ten items coming off the assembly line, determine the number of items without any defects, the number scrapped and the total minutes of rework time. Use the following random numbers,

RN for defect A
48 55 91 40 93 01 83 63 47 52

RN for defect B
47 36 57 04 79 55 10 13 57 09

RN for defect C
82 95 18 96 20 84 56 11 52 03

Illustration - 241

The management of ABC company is considering the question of marketing a new product. The fixed cost required in the project is Rs. 4,000. Three factors are uncertain viz. the selling price, variable cost and the annual sales volume. The product has a life of only one year. The management has the data on these three factors as under:

Selling price Rs.	Probability	Variable Cost Rs.	Probability	Sales volume (Units)	Probability
3	0.2	1	0.3	2,000	0.3
4	0.5	2	0.6	3,000	0.3
5	0.3	3	0.1	5,000	0.4

Simulation Analysis

Consider the following sequence of thirty random numbers:

81 32 60 04 46 31 67 25 24 10 40 02 39 68
08 59 66 90 12 64 79 31 86 68 82 89
25 11 98 16

Using the sequence (First 3 random numbers for the first trial, etc.) simulate the average profit for the above project on basis of 10 trials.

Illustration - 242

A retailer deals in a perishable commodity. The daily demand and supply are variables. The data for the past 500 days show the following demand and supply.

Supply		Demand	
Availability (kg)	No. of Days	Demand (kg)	No. of Days
10	40	10	50
20	50	20	110
30	190	30	200
40	150	40	100
50	70	50	40

The retailer buys the commodity at Rs. 20 per kg. and sells it at Rs. 30 per kg. Any commodity remains at the end of the day, has no saleable value. Moreover, the loss (unread profit) on any satisfied demand is Rs. 8 per kg. Given the following pair of random numbers, simulate 6 days sales, demand and profit.

(31, 18); (63, 84); (15, 79); (07, 32); (43, 75); (81, 27)

The first random number in the pair is for supply and the second random number is for demand viz. in the first pair (31, 18) use 31 to simulate supply and 18 to simulate demand.

The finished mopeds are transported in a specially designed three storeyed lorry that can accommodate only 200 mopeds. Using the following 15 random numbers 82, 89, 78, 24, 53, 61, 18, 45, 04, 23, 50, 77, 27, 54, 10 simulate the process to find out:

- What will be average number of mopeds, waiting in the factory?
- What will be the average number of empty spaces on the lorry?

Illustration - 243

1999 - May [4] (b) A company manufactures around 200 mopeds. Depending upon the availability of raw materials and other conditions, the daily production has been varying from 196 mopeds to 204 mopeds whose probability distribution is as given below:

Production per day	Probability
196	0.05
197	0.09
198	0.12
199	0.14
200	0.20
201	0.15
202	0.11
203	0.08
204	0.06

The finished mopeds are transported in a specially designed three storeyed lorry that can accommodate only 200 mopeds. Using the following 15 random numbers 82, 89, 78, 24, 53, 61, 18, 45, 04, 23, 50, 77, 27, 54, 10 simulate the process to find out:

- What will be the average number of mopeds, waiting in the factory?
- What will be the average number of empty spaces on the lorry?

Ans:- (i) 0.67 per day; (ii) 0.93 per day.

Simulation Analysis

Illustration - 244

1999 - Nov [6] (b) A bakery shop keeps stock of a popular brand of cake. Previous experience indicates the daily demand as given here:

Daily demand	:	0	10	20	30	40	50
Probability	:	0.01	0.20	0.15	0.50	0.12	0.02

Consider the following sequence of random numbers:

R. No. 48, 78, 19, 51, 56, 77, 15, 14, 68, 09

Using this sequence, simulate the demand for the next 10 days. Find out the stock situation if the owner of the bakery decides to make 30 cakes every day. Also, estimate the daily average demand for the cakes on the basis of simulated data. (10 marks) [S.A. 33]

Ans:- Daily average demand for cake is 22 cakes per day.

Illustration - 245

2000 - May [5] (c) A book - store wishes to carry System Analysis and Design in stock.

Demand is probabilistic and replenishment of stock takes 2 days (i.e., if an order is placed in March 1, it will be delivered at the end of the day on March 3). The probabilities of demand are given below:

Demand (daily)	:	0	1	2	3	4
Probability	:	0.05	0.10	0.30	0.45	0.10

Each time an order is placed, the store incurs an ordering cost of Rs. 10 per order. The store also incurs a carrying cost of Rs. 0.50 per book per day. The inventory carrying cost is calculated on the basis of stock at the end of each day. The manager of the book-store wishes to compare two options for his inventory decision:

- A. Order 5 books, when the inventory at the beginning of the day plus orders outstanding is less than 8 books.
- B. Order 8 books, when the inventory at the beginning of the day plus orders outstanding is less than 8 books.

Currently (beginning of the 1st day) the store has stock of 8 books plus 6 books ordered 2 days ago and expected to arrive next day. Using Monte- Carlo simulation for 10 cycles, recommend which option the manager should choose?

The two digit random number are given below:

89, 34, 78, 63, 61, 81, 39, 16, 13, 73

Ans:- The Manager should go for Option B, as its Total Cost Rs. 42.50 is less than Option A (Rs. 59.50).

Illustration - 246

2000 - Nov [6] (b) A retailer deals in a perishable commodity. The daily demand and supply are variables. The data for the past 500 days, show the following demand and supply:

Supply		Demand	
Availability (k.g.)	No. of days	Demand (kg.)	No. of days
10	40	10	50
20	50	20	110
30	190	30	200
40	150	40	100
50	70	50	40

The retailer buys the commodity at Rs. 20 per kg. and sells it at Rs. 30 per kg. Any commodity remains at the end of the day, has no saleable value. Moreover, the loss (unearned profit) on any unsatisfied demand is Rs. 8 per kg. Given the following pair of random numbers, simulate 6 days sales, demand and profit.

(31, 18): (63, 84): (15, 79): (07, 32): (43, 75): (81, 27)

The first random number in the pair is for supply and the second random number is for demand viz. in the first pair (31, 18), use 31 to simulate supply and 18 to simulate demand.

Ans: Rs. 400

Simulation Analysis

Illustration - 247

2001 – May [6] (b) An investment company wants to study the investment projects based on market demand, profit and the investment required. Which are independent of each other. Following probability distributions are estimated for each of these three factors:

Annual demand								
(Units in thousands)	:	25	30	35	40	45	50	55
Probability	:	0.05	0.10	0.20	0.30	0.20	0.10	0.05
Profit per Unit	:	3.00	5.00	7.00	9.00	10.00		
Probability	:	0.10	0.20	0.40	0.20	0.10		
Investment Required								
(in thousands of Rupees)	:	2,750	3,000	3,500				
Probability	:	0.25	0.50	0.25				

Using simulation process, repeat the trial 10 times, compute the investment on each trial taking these factors into trial. What is the most likely return?

Use the following random numbers:

(30, 12, 16);	(59, 09, 69);	(63, 94, 26);	(27, 08, 74);
(64, 60, 61);	(28, 28, 72);	(31, 23, 57);	(54, 85, 20);
(64, 68, 18);	(32, 31, 87)		

In the bracket above, the first random number is for annual demand, the second one is for profit and the last one is for the investment required. (10 marks) [S. A. 39]
 Ans:- The highest likely return is 13.33%, which corresponds to annual demand of 40,000 units resulting in a profit of Rs. 10 per unit requiring an investment of Rs. 30,00,000.

Illustration - 248

2002 – May [6] (b) A Car Manufacturing Company manufactures 40 cars per day. The sale of cars depends upon demand which has the following distribution:

Sales of Cars	Probability
37	0.10
38	0.15
39	0.20
40	0.35
41	0.15
42	0.05

The production cost and sale price of each car are Rs. 4 lakh and Rs. 5 lakh respectively. Any unsold car is to be disposed off at a loss of Rs. 2 lakh per car. There is a penalty of Re. 1 lakh per car, if the demand is not met. Using the following random number, estimate total profit / loss for the company for the next ten days:

98, 64, 98, 94, 01, 78, 10, 15, 19

If the company decides to produce 39 cars per day, what will be its impact on profitability? (10 marks) [S.A. 37]

Ans:- The profitability remains unchanged.

Simulation Analysis

Illustration - 249

2003 - May [6] (b) With a view to improving the quality of customer services, a Bank is interested in making an 'assessment of the waiting time of its' customers coming to one of its branches located in a residential area. This branch has only one teller's counter. The arrival rate of the customers and the service rate of teller are given below:

Time between two consecutive arrivals Of customers (in minutes)	Probability
3	0.17
4	0.25
5	0.25
6	0.20
7	0.13

Service time by the teller (in minutes)	Probability
3	0.10
4	0.30
5	0.40
6	0.15
7	0.05

You are required to stimulate 10 arrivals of customers in the system starting from 11 AM and show the waiting time of the customers and idle time of the teller.

Use the following random number taking the first two random numbers in two digits each for first trial and so on:

11, 56, 23, 72, 94, 83, 83, 02, 97, 99, 83, 10, 93, 34, 33, 53, 49, 94, 37 and 97.

Ans:- Total waiting time of customer is 4 minutes and total idle time of teller is 10 minutes.

Illustration - 250

2004 - May [6] (c) A company manufactures a component which requires a high degree of precision. Each unit of the component is therefore subjected to a strict quality control test to ascertain whether there is any defect in it. The defects are classified into three categories viz. A, B and C. If defect A occurs in the output, it is scrapped. If defect B or C occurs in the output, it is reworked to rectify the defect. The machine time required to rework defect B component is 30 minutes and that for defect C is 45 minutes. The probabilities are as under:

	Defect A	Defect B	Defect C
Defect occurring	0.15	0.20	0.10
Defect not occurring	0.85	0.80	0.90

Using the following random numbers, simulate a study of 10 items of output and determine the number of items with no defects, number of items scrapped due to occurrence of defect A and the total machine time required for rework due to occurrence of defect B or C.

Random number for defect A:
48 55 91 40 93 01 83 63 47 52
Random number for defect B:
47 36 57 04 79 55 10 13 57 09
Random number for defect C:
82 95 18 96 20 84 56 11 52 03

(7 marks) [S. A. 27]

Ans:- Rectification time required on networking is 165 minutes.

Simulation Analysis

Illustration - 251

2004 – Nov [6] (c) A company trading in motor vehicle spares wishes to determine the level of stock it should carry for the item in its range. Demand is not certain and replenishment of stock takes 3 days. For one item X, the following information is obtained:

Demand (Unit per day)	Probability
1	.1
2	.2
3	.3
4	.3
5	.1

Each time an order is placed, the company incurs an ordering cost of Rs. 20 per order. The company also incurs carrying cost of Rs. 2.50 per unit per day. The inventory carrying cost is calculated on the basis of average stock.

The manager of the company wishes to compare two options, for his inventory decision.

- A. Order 12 units when the inventory at the beginning of the day plus order outstanding is less than 12 units.
- B. Order 10 units when the inventory at the beginning of the day plus order outstanding is less than 10 units.

Currently (on first day) the company has a stock of 17 units. The sequence of random number to be used is 08, 91, 25, 18, 40, 27, 85, 75, 32, 52 using first number for day one.

You are required to carry out a simulation run over a period of 10 days, recommend which option the manager should choose. (7 marks) [S.A. 24]

Ans:- Option B.

Illustration - 252

2005 – Nov [6] (b) A Publishing house has bought out a new monthly magazine, which sells at Rs. 37.5 per copy. The cost of producing it is Rs. 30 per copy. A Newsstand estimates the sales pattern of the magazine as follows:

Demand Copies	Probability
0 < 300	0.18
300 < 600	0.32
600 < 900	0.25
900 < 1200	0.15
1200 < 1500	0.06
1500 < 1800	0.04

The newsstand has contracted for 750 copies of the magazine per month from the publisher. The unsold copies are returnable to the publisher who will take them back at cost less Rs. 4 per copy for handling charges.

The newsstand manager wants to simulate of the demand and profitability. The following random number may be used for simulation:

27, 13, 56, 17, 98, 71, 51, 32, 62, 83, 96, 69

You are required to –

- (i) Allocate random numbers to the demand pattern forecast by the newsstand.
- (ii) Simulate twelve months sales and calculate the monthly and annual profit / loss
- (iii) Calculate the loss on lost sales.

Ans:- Loss on lost sale = Rs. 1575

Simulation Analysis

Simulation 16.25

16.6 RANDOM NUMBERS TABLE

8481	5016	3080	4376	2579	8296	5950	1045	2650	4126
0744	3447	6173	3288	6376	6704	0900	9495	5079	1724
5556	7230	2872	4906	6134	5121	1701	6514	3421	2079
9371	1463	2164	2301	3142	3678	8707	9090	2011	5111
3033	1660	6365	9064	1155	6994	4785	9683	2004	1751
1063	7330	6532	7214	8972	6466	1217	0100	1458	9416
4389	3604	4088	9434	0136	5825	6876	7937	5478	3376
2159	8854	0634	1196	4941	2637	7497	1149	1362	3432
6749	3676	4940	1406	0614	2080	6433	1660	8975	3194
2878	3447	4804	6761	5309	0636	0622	2004	3207	4694
0591	6549	2206	6185	6198	2649	2389	9483	0924	1583
1025	3408	0546	2545	1089	1280	6701	9742	3453	5573
4244	9217	1628	4524	0163	9866	9686	2093	6458	0544
1331	9022	1386	5661	0472	1128	1902	0343	7724	0528
8853	3190	2588	8744	1221	4667	8366	4779	9937	7206
5059	4192	6331	5485	5822	0682	9350	8303	3621	2002
0821	4340	3194	0118	4773	3668	1991	7969	9190	2296
5262	1746	7108	6406	2570	4243	5029	8049	4369	5008
1210	1858	9365	6662	0239	9923	1796	6626	6591	1990
3642	6629	5775	3219	8601	4047	6681	0765	2379	3494
9508	5322	3747	0363	9225	5504	6804	7033	0957	9556
3894	3173	2853	9312	2498	6878	4366	8743	6247	0673
3600	3011	6762	0648	8316	3485	6388	8925	3790	0899
1121	2378	6313	5857	8457	1395	7240	8600	3895	6348
1900	4583	4227	4120	6893	7005	2284	6067	5827	7385
6308	9158	2830	3362	9809	4606	8699	1154	5841	7696
4460	3143	5383	0327	9268	1997	8335	0660	2156	1908
8371	5095	7273	1866	4193	4153	2035	2812	4096	7142
9397	5540	9298	9076	1229	0569	0088	1809	0631	3162
9304	1458	4013	7465	0161	6767	3781	7977	8429	4708
5606	2436	8546	3309	4802	6690	8527	2210	6700	1900
6693	9333	082	7546	2910	8553	8725	1237	4423	1570
0556	7715	8004	4245	1540	8150	3889	5273	5977	2703
6973	9299	4669	7148	1426	7086	8743	6982	5547	3004
4920	1223	5208	6661	4907	1102	0501	3625	8513	3192
0132	0038	8241	0858	7627	4174	1170	3142	2455	4391
4051	3101	9854	4488	8601	3286	3147	2500	8011	8848
0267	5912	5504	7917	7928	8034	9089	4363	2675	9497
0609	9469	3149	4086	8911	8547	3518	9048	1826	0543
2593	1666	5750	5105	4287	4390	7860	7792	1625	7659
8812	5491	2602	4100	4082	1037	9778	1778	4223	3190
3540	5095	0019	7155	1471	1851	8682	9957	3772	4708
9536	5375	1239	1624	5379	6803	7177	7911	4560	5668
3174	7677	8282	0669	5879	7874	9931	6531	9784	2607
8864	4760	1129	6205	4949	4205	0222	7479	6470	8194
5246	7341	0680	5656	6789	3071	1751	4339	5630	9498
5488	6008	4511	1440	2135	5777	9903	1048	6726	8602
3961	7928	6818	4161	4840	1382	1323	5014	7538	9654
7319	4064	4024	5401	2834	7518	3978	3742	1005	4619
5892	8731	6269	5189	2071	4064	9769	3620	9819	4548

The Assignment Problem

ASSIGNMENT

Illustration 253:-

An accounts Officer has 4 subordinates and 4 tasks. The subordinates differ in efficiency. The tasks also differ in their intrinsic difficulty. His estimates of the time each would take to perform each task is given in the matrix below. How should the tasks be allocated one to one man, so that the total man hours are minimized?

	I	II	III	IV
1	8	26	17	11
2	13	28	4	26
3	38	19	18	15
4	19	26	24	10

Let us apply the above steps take the above example.

Ans = 41 HRS.

Illustration 254:-

A manager has 5 jobs to be done. The following matrix shows the time taken by the j-th job ($j = 1, 2, \dots, 5$) on the i-th machine ($i = 1, 2, 3, \dots, 5$). Assign 5 jobs to the 5 machines so that the total time taken is minimized

	Jobs				
	1	2	3	4	5
I	9	3	4	2	10
II	12	10	8	11	9
III	11	2	9	0	8
IV	8	0	10	2	1
V	7	5	6	2	9

Ans = 19 HRS

Illustration 255:- 5 salesmen are to be assigned to 5 districts. Estimates of sales revenue in thousands of rupees for each salesmen are given below.

	A	B	C	D	E
1	32	38	40	28	40
2	40	24	28	21	36
3	41	27	33	30	37
4	22	38	41	36	36
5	29	33	40	35	39

Find the assignment pattern that maximizes the sales revenue

Ans = 191

Illustration 256:- To stimulate interest and provide an atmosphere for intellectual discussion, a finance faculty in a management school decides to hold special seminars on four contemporary topics—leasing, portfolio management, private mutual funds, swaps and options. Such seminars should be held once a week in the afternoons. However, scheduling these seminars (one for each topic, and not more than one seminar per afternoon) has to be done carefully so that the number of students unable to attend is kept to a minimum. A careful study indicates that the number of students who cannot attend a particular seminar on a specific day is as follows:

	Leasing	Portfolio Management	Private Mutual Funds	Swaps and Options
Monday	50	40	60	20
Tuesday	40	30	40	30
Wednesday	60	20	30	20
Thursday	30	30	20	30
Friday	10	20	10	30

The Assignment Problem

Find an optimal schedule of the seminars. Also find out the total number of students who will be missing at least one seminar.

Ans = 70

Illustration 257:-

A solicitor's firm employs typists on hourly piece-rate basis for their daily work. There are five typists for service and their charges and speeds are different. According to an earlier understanding only one job is given to one typist and the typist is paid for full hours even if he works for a fraction of an hour. Find the least cost allocation for the following data:

Typist	Rate per hour (Rs.)	No. of pages typed / hour
A	5	12
B	6	14
C	3	8
D	4	10
E	4	11

Job	No. of pages
P	199
Q	175
R	145
S	298
T	178

Ans = 399

Illustration 258:-

WELLDONE company has taken the third floor of a multi-storied building for rent with a view to locate one of their zonal offices. There are five main rooms in this floor to be assigned to five managers. Each room has its own advantages and disadvantages. Some have windows, some are closer to the washrooms or to the canteen or secretarial pool. The rooms are of all different sizes and shapes. Each of the five managers was asked to rank their room preferences amongst the rooms 301, 302, 303, 304 and 305. Their preferences were recorded in a table as indicated below:

MANAGER				
M ₁	M ₂	M ₃	M ₄	M ₅
302	302	303	302	301
303	304	301	305	302
304	305	304	304	304
	301	305	303	
		302		

Most of the managers did not list all the five rooms since they were not satisfied with some of these rooms and they have left off these from the list. Assuming that their preferences can be quantified by numbers, find out as to which manager should be assigned to which room so that their total preference ranking is a minimum.

Illustration 259:-

An organization producing 4 different products viz. A, B, C and D having 4 operators viz. P, Q, R and S, who are capable of producing any of the four products, works effectively 7 hours a day. The time (in minutes) required for each operator for producing each of the product are given in the cells of the following matrix along with profit (Rs. per unit).

Operator	Product			
	A	B	C	D
P	6	10	14	12
Q	7	5	3	4
R	6	7	10	10
S	20	10	15	15
Profit (Rs. / Unit)	3	2	4	1

Find out the assignment of operators to products which will maximize the profit

Ans = 918

The Assignment Problem

Illustration 260:-

A firm produces four products. There are four operators who are capable of producing any of these four products. The processing time varies from operator to operator. The firm records 8 hours a day and allow 30 minutes for lunch. The processing time in minutes and the profit for each of the products are given below:

Operator	Product			
	A	B	C	D
1	15	9	10	6
2	10	6	9	6
3	25	15	15	9
4	15	9	10	10
Profit (Rs. / Unit)	8	6	5	4

Find the optimal assignment of products to operators

Ans = 1140

Illustration 261:-

A manufacturing company, has four zones A, B, C, D and four sales engineers P, Q, R, S respectively for assignment. Since the zones are not equally rich in sales potential, therefore it is estimated that a particular engineer operating in particular zone will bring the following sales:

Zone A	:	4,20,000
Zone B	:	3,36,000
Zone C	:	2,94,000
Zone D	:	4,62,000

The engineers are having different sales ability. Working under the same conditions, their yearly sales are proportional to 14, 9, 11 and 8 respectively. The criteria of maximum expected total sales is to be met by assigning the best engineer to the richest zone, the next best to the second richest zone and so on.

Find the optimum assignment and the maximum sales.

Ans = 3,92,000

Illustration 262:-

An Electric Data Processing (ED) centre has three expert Software professionals. The centre wants three application software programs to be developed. The head of EDP centre estimates the computer time in minutes required by the experts for development of application Software Programs as follows:

Software programs	Computer time (in minutes) required by software Professionals
1	100 85 70
2	50 70 110
3	110 120 130

Assign the software professionals to the application software programs to ensure minimum usage of computer time.

Illustration 263:-

A Production supervisor is considering, how he should assign five jobs that are to be performed, to five mechanists working under him. He wants to assign the jobs to the mechanists in such a manner that the aggregate cost to perform the jobs is the least. He has following information about the wages paid to the mechanists for performing these jobs:

The Assignment Problem

	Jobs				
Mechanist	1	2	3	4	5
A	10	3	3	2	8
B	9	7	8	2	7
C	7	5	6	2	4
D	3	5	8	2	4
E	9	10	9	6	10

Assign the jobs to the mechanists so that the aggregate cost is the least.

Ans = 21

Illustration 264:-

A project consists of four (4) major jobs, for which four (4) contractors have submitted tenders. The tender amounts, in thousands of rupees, are given below.

	Jobs			
Contractors	A	B	C	D
1	120	100	80	90
2	80	90	110	70
3	110	140	120	100
4	90	90	80	90

Find the assignment, which minimizes the total cost of the project. Each contractor has to be assigned one job.

Ans = 3,50,000

Illustration 265:-

A project manager has 4 subordinates and 4 tasks. The subordinates differ in efficiency. The tasks also differ in their intrinsic difficulty. His estimates of the time each subordinate would take to perform each task is given in the matrix below. How should the task be allocated one to one man so that the total man - hours are minimized?

	I	II	III	IV
1	16	52	34	22
2	26	56	8	52
3	76	38	36	30
4	38	52	48	20

Ans = 82

Illustration 266:-

A company has four zones open and four marketing managers available for assignment. The zones are not equal in sales potentials. It is estimated that a typical marketing manager operating in each zone would bring in the following Annual Sales:

Zones	Rs.
East	2,40,000
West	1,92,000
North	1,44,000
South	1,20,000

The four marketing managers are also different in ability. It is estimated that working under the same conditions, their yearly sales would be proportionately as under:

Manager M	: 8
Manager N	: 7
Manager O	: 5
Manager P	: 4

Required:

If the criterion is maximum expected total sales, find the optimum assignment and the maximum sales

Ans = 186000

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The Assignment Problem

Illustration 267:-

A matrix giving selling costs per unit of a product by salesman A, B, C and D in regions R_1 , R_2 , R_3 and R_4 is given below:

	A	B	C	D
R_1	4	12	16	8
R_2	20	28	32	24
R_3	36	44	48	40
R_4	52	60	64	56

- Assign one salesman to one region to minimize the selling cost.
- If the selling price of the product is Rs. 200 per unit and variable cost excluding the selling cost given in the table is Rs. 100 per unit, find the assignment that would maximize the contribution.

Illustration 268:-

A Car hiring company has one car at each of five depots A, B, C, D and E. A customer in each of the five town V, W, X, Y and requires a car. The distance in kms, between depots (origin) and the towns (destination) are given in the following table:

		Depots				
		A	B	C	D	E
Towns	V	3	5	10	15	8
	W	4	7	15	18	8
	X	8	12	20	20	12
	Y	5	5	8	10	6
	Z	10	10	15	25	10

Find out as to which car should be assigned to which customer so that the total distance traveled is a minimum. How much is the total traveled distance?

Ans = 45

Illustration 269:-

XYZ airline operating 7 days a week has given the following time - table. Crews must have a minimum layover of 5 hours between flights. Obtain the pairing flights and minimize layover time away from home. For any given pairing the crew will be based at the city that results in the smaller layover: Ans = 40 HRS

Chennai Mumbai			Mumbai - Chennai		
Flight Number	Depart.	Arrive	Flight Number	Depart.	Arrive
A 1	6 AM	8 AM	B 1	8 AM	10 AM
A 2	8 AM	10 AM	B 2	9 AM	11 AM
A 3	2 PM	4 PM	B 3	2 PM	4 PM
A 4	8 PM	10 PM	B 4	7 PM	9 PM

Illustration 270:-

A factory is going to modify of a plant layout to install four new machines M1, M2, M3 and M4. There are 5 vacant places J, K, L, M and N available. Because of limited space machine M2 cannot be placed at L and M3 cannot be placed at J. The cost of locating machine to place in Rupees is shown below:

		Depots				
		J	K	L	M	N
M ₁		18	22	30	20	22
M ₂		24	18	-	20	18
M ₃		-	22	28	22	14
M ₄		28	16	24	14	16

Required:

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The Assignment Problem

Determine the optimal assignment schedule in such a manner that the total costs are kept at a minimum.

Learning Curve

LEARNING CURVE THEORY

Question 271

A company has 10 direct workers, who work for 25 days a month of 8 hours per day. The estimated down time is 25% of the total available time. The company received an order for a new product. The first unit of the new product requires 40 direct labour hours to manufacture the product. The company expects 80% (index is -0.322) learning curve for this type of work. The company uses standard absorption costing and the cost data are as under:

Direct materials	Rs. 60 per unit
Direct labour	Rs. 6 per direct labour hour
Variable overheads	Rs. 5 per direct labour hour
Fixed overheads	Rs. 7,500 per month.

- (i) Calculate the cost per unit of the first order of 30 units.
- (ii) If the company receives a repeat order for 20 units, what price will be quoted to yield a profit of 25% on selling price?

Question 272

An electronics firm which has developed a new type of fire-alarm system has been asked to quote for a prospective contract. The customer requires separate price quotations for each of the following possible orders:

	Number of fire-alarm systems
First	100
Second	60
Third	40

Learning Curve

The firm estimates the following cost per unit for the first order:

Direct materials - Rs. 500

Direct Labour -

Deptt. A (Highly automatic) 20 hours at Rs. 10 per hour.

Deptt. B (Skilled labour) 40 hours at Rs. 15 per hour.

Variable overheads 20% of direct labour

Fixed overheads absorbed:

Deptt. A Rs. 8 per hour

Deptt. B Rs. 5 per hour

Determine a price per unit for each of the three orders, assuming the firm uses a mark up of 25% on total costs and allows for an 80% learning curve.

Extract from 80% Learning curve table:

X	1.0	1.3	1.4	1.5	1.6	1.7	1.8	1.9	2.0
Y(%)	100.0	91.7	89.5	87.6	86.1	84.4	83.0	81.5	80.0

x represents the cumulative total volume produced to date expressed as a multiple of the initial order.

Y is the learning curve factor, for a given X value, expressed as a percentage of the cost of the initial order.

Learning Curve

Question 273

XYZ & Co. has given the following data:

80% Average — Time Curve

Cumulative Units (X)	Average Hours	Total-Hours	Marginal Hours
1	100	100	100
2	80	160	60
3	?	?	?
4	64	256	?

Required : Fill in the blanks.

Question 274

A customer has asked your company to prepare a bid on supplying 800 units of a new product. Production will be in batches of 100 units. You estimate that costs for the first batch of 100 units will average Rs. 100 a unit. You also expect that a 90% learning curve will apply to the cumulative labour cost on this contract.

Required :

- Prepare an estimate of the labour costs of fulfilling this contract.
- Estimate the Incremental labour cost of extending the production run to produce an additional 800 units.
- Estimate the incremental labour cost of extending the production run from 800 units to 900 units.

Learning Curve

Question 275

CYZ Co., has observed that a 90% learning curve ratio applies to all labour related costs each time a new model enters production. It is anticipated that 320 units will be manufacture during 1999. Direct labour cost for the first lot of 10 units amounts to 1,000 hours at Rs. 8 per hour. Variable overhead cost is assigned to products at the rate of Rs. 2 per direct labour hour. You are required to determine :

- i) Total labour and labour related costs to manufacture 320 units of output
- ii) Average cost of (a) the first 40 units produced (b) the first 80 units, (c) the first 100 units. -
- iii) Incremental cost of (a) units 41 – 80 and (b) units 101 – 200.

The Transportation Problem

TRANSPORTATION

- Q. 276. Solve the following transportation problem for minimum cost.

	D 1	D 2	D 3	D 4	
Q 1	5	3	6	2	19
Q 2	4	7	9	1	37
Q 3	3	4	7	5	34
	16	18	31	25	

Ans : - 355

- Q. 277.

Factory	Destination			
	1	2	3	Supply to be exhausted
A	5	1	7	10
B	6	4	5	80
C	3	2	5	15
Demand	75	20	50	

Since there is not enough supply, some of the demands at the three destinations may not be satisfied. For the unsatisfied demands, let the penalty costs be rupees 1, 2 and 3 for destinations (1), (2) and (3) respectively. Find the optimal allocation that minimizes the transportation and penalty cost.

Solution = 505

- Q. 278. The Link manufacturing company has several plants, three of which manufacture two principal products, standard card table and deluxe card table. A new deluxe card table will be introduced which much be considered in term of selling price & costs. The selling prices are : Standard Rs. 14.95, Deluxe Rs. 18.95, and new Deluxe Rs. 21.95.

Requirements		Variable costs			Available plant capacity	
Model	Quantity	Plant A	Plant B	Plant C	Plant	capacity
Standard	450	8.00	7.95	8.10	A	800
Deluxe	1,050	8.50	8.60	8.45	B	600
New Deluxe	600	9.25	9.20	9.30	C	700

Solve this problem for maximum contribution

- Q. 279. The cost per unit of transporting goods from the factories X, Y, Z to destinations. A, B and C, and the quantities demanded and supplied are tabulated below. As the company is working out the optimum logistics, the Govt. has announced a fall in oil prices. The revised unit costs are exactly half the costs given in the table. You are required to evaluate the minimum transportation cost.

Destinations Factories	A	B	C	Supply
X	15	9	6	10
Y	21	12	6	10
Z	6	18	9	10
Demand	10	10	10	30

Answer = 105

The Transportation Problem

- Q. 280. Pradeep Patil enterprises is having three plants manufacturing dry – cells, located at three different locations. Production cost differs from plant to plant. There are five sales offices of the company located in different regions of the country. The sales prices can differ from region to region. The shipping cost from each plant to each sales office and other data are given by following table:

Production cost per unit	Production data table Maximum capacity in No. of units	Plant No.
20	150	1
22	200	2
18	125	3

Shipping cost & demand & sales price table
Shipping cost

	Sales office 1	Sales office 2	Sales office 3	Sales office 4	Sales office 5
Plant 1	1	1	5	9	4
Plant 2	9	7	8	3	6
Plant 3	4	5	3	2	7

Demand & Sales prices

	Sales office 1	Sales office 2	Sales office 3	Sales office 4	Sales office 5
Demand	80	100	75	45	125
Sales price	30	32	31	34	29

Find the production and distribution schedule most profitable to the company (10 marks)

Ans : 3,170

- Q. 281. Tayal and Co. has provided the following data seeking your advice on optimum investment strategy:

Investment made at the beginning of the year	Net return data (in paise) of selected investments				Amount available (Rs. in lakhs)
	P	Q	R	S	
1	95	80	70	60	70
2	75	65	60	50	40
3	70	45	50	40	90
4	60	40	40	30	30
Maximum Investment (Lakhs)	40	50	60	60	-

Following additional information are also provided

- P, Q, R and S represents the selected investments
- The company has decided to have four years investment plan
- The policy of the company is that amount invested in any year will remain so until the end of the fourth year.
- The values (Paise) in the table represent net returns on investment of one Rupee till the end of the planning horizon (for example, a Rupee Invested in investment P at the beginning of year 1 will grow to Rs. 1.95 by the end of the fourth year, yielding a return of 95 paise). Using the above, determine the optimum investment strategy.

The Transportation Problem

Q. 282. The following table shows all the necessary information on the available supply to each warehouse, the requirement of each market and the unit transportation cost from each warehouse to each market:

	Market				
	I	II	III	IV	Supply
A	5	2	4	3	22
Warehouse B	4	8	1	6	15
C	4	6	7	5	8
Requirement	7	12	17	9	

The shipping clerk has worked out the following schedule from his experience:

12 Units from A to II

1 unit from A to III

9 Units from A to IV

15 Units from B to III

7 Units from C to I and

1 Unit from C to III

You are required to answer the following:

(i) Check and see if the clerk has the optimal schedule;

(ii) Find the optimal schedule and minimum total shipping cost; and

Answer : 104

Q. 283. A company has three warehouses W_1 , W_2 and W_3 . It is required to deliver a product from these warehouses to three customers A, B and C. These warehouses have the following units in stock.

Warehouse:	W_1	W_2	W_3
No. of units:	65	42	43

And customer requirements are:

Customer:	A	B	C
No. of units:	70	30	50

The table below shows the costs transporting one unit from warehouse to the customer:

	Warehouse		
	W_1	W_2	W_3
A	5	7	8
Customer B	4	4	6
C	6	7	7

Find the optimal transaction routes.

Answer :- Total cost = Rs. 830

The Transportation Problem

Q 284. 1999 – May [3] (b) A company has three warehouses W_1 , W_2 , and W_3 . It is required to deliver a product from these warehouses to three customers A, B and C. The warehouses have the following units in stock:

Warehouse :	W_1	W_2	W_3
No. of Units :	65	42	43

and customer requirements are:

Customer :	A	B	C
No. of Units :	70	30	50

The table below shows the costs of transporting one unit from warehouse to the customer:

		Warehouse		
		W_1	W_2	W_3
Customer	A	5	7	8
	B	4	4	6
	C	6	7	7

Find the optimal transaction route.

(10 marks) [S.A. 25]

Ans:-	Warehouse	Customer	Unit	Total Cost (Rs.)
	W_1	A	65	325
	W_2	A	5	35
	W_2	B	30	120
	W_2	C	7	49
	W_3	C	43	301
		Total		830

Q 285. 2007 – May [1] (C) (b) The initial allocation of a transportation problem, along with the unit cost of transportation from each origin to destination is given below. You are required to arrive at the minimum transportation cost by the Vogel's Approximation method and check for optimality.
(Hint: Candidates may consider $U_1 = 0$ at Row 1 for initial cell evaluation)

Requirement

11	2	8	6	2	4	18	
9	10	9	12	9	6	10	
7	6	3	8	7	7	8	
9	2	3	5	6	2	11	4

Availability

12	8	8	8	4	40
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Answer :- Optimal cost = Rs. 204

Q 286. 2008 – May [6] (a) Goods manufactured at 3 plants, A, B and C are required to be transported to sales outlets X, Y and Z. The unit of transporting the goods from the plants to the outlets are given below:

Plant	A	B	C	Total Demand
Sales outlets				
X	3	9	6	20
Y	4	4	6	40
Z	8	3	5	60
Total supply	40	50	30	120

The Transportation Problem

You are required to:

- (i) Compute the initial allocation by North – West Corner Rule.
 - (ii) Compute the initial allocation by Vogel's approximation method and check whether it is optimal.
 - (iii) State your analysis on the optionality of allocation under North – West corner Rule and Vogel's Approximation method.
- (10 marks) [S.A. 20]

Ans:- (i) Initial Total Cost 460
(ii) Initial solution 460, It is optimal
(iii) Solution under both methods is optimal

Q 287. A tonic feed for cattle is to be prepared using two types of grain X_1 and X_2 . Each unit of grain X_1 weights 4 gms, and contain 2 gms of protein. Each unit of grain X_2 weights 5 gms. and contains 1 gm. of protein. The feed is to weight at most 40 gms. and must contain at least 10 gms. of protein. If the grain X_1 cost 30 paise a unit and that X_2 costs 10 paise a unit, determine by graphical method the amount of two types of grain to be used in the preparation of the feed to minimize the cost yet meeting the requirements. Formulate LP model for this problem.

Q 288. A manufacturer makes two products P_1 and P_2 using two machinery M_1 and M_2 . Product P_1 requires 2 hours on machine and 6 hours on machine M_2 . Product P_2 requires 5 hours on machine M_1 and no time on machine M_2 . There are 16 hours of time per day available on machine M_1 and 30 hours on M_2 . Profit margin from P_1 and P_2 is Rs. 2 and Rs. 10 per unit respectively. Formulate LP model for this problem.

Q 289. A firm manufactures two products A and B on which profits earned per unit are RS. 3 and Rs. 4 respectively. Each product is processed on two machines, M_1 and M_2 . The product A requires one minute of processing time on M_1 and two minutes on M_2 while B requires one minute on M_1 and one minute on M_2 . Machine M_1 is available for use for not more than 7 hours 30 minutes, while M_2 is available for 10 hours during any working day. Formulate LP model for this problem.

Q 290. A company produces two products X and Y, each of which requires processing in three machines. The first machine can be used at most 70 hours; the second machine at most 40 hours; and the third machine at most 90 hours. The product X requires 2 hours on machine 1, 1 hour on machine 2, and 1 hour on machine 3; the product Y requires 1 hour each on machines 1 and 2, and 3 hours on the third machine. The profit is Rs. 50 per unit of X and Rs. 30 per unit of Y. Formulate LP model for this problem.

L.P.P.

Q 291. A carpenter has 90,80 and 50 running feet respectively of teak, plywood and rosewood. The product A requires 2,1 and 1 running feet and the product B requires 1,2 and 1 running feet of teak, plywood and rosewood respectively. If A would sell for Rs. 48 and B for Rs. 40 per unit. Formulate LP model for this problem.

Q 292. A machine producing either product A or B can produce A by using 2 units of chemicals and 1 unit of a compound and can produce B by using 1 unit of chemicals and 2 units of the compound. Only 800 units of chemicals and 1,000 units of the compound are available. The profits available per unit of A and B are respectively Rs. 30 and Rs. 20. Formulate LP model for this problem.

Q 293. A media specialist has decided on the allocation of advertisement in three media vehicles. Let x_i be the number of message carried on the i -th media, $i = 1,2,3$. The unit costs of a message in the 3 media are Rs. 1000, Rs. 750 and Rs. 500. The total budget available is Rs. 20,000 for the campaign period of a year. The first medium is a monthly magazine and it is desired to advertise not more than one insertion in one issue. At least six message should appear in the 2nd medium. The number of messages in the third medium should appear in between 4 and 8. The expected effective audience for unit message in the media vehicles is shown below:

Vehicle	expected effective audience
1	80,000
2	60,000
3	45,000

If is desired to build the linear programming model to maximize the total effective audience

Q 294. A fertilizer company produces two types of fertilizer called grade I and II. Each of these types is processed through two critical chemical plants. Plant A has maximum of 120 hours available in a week and plant B has a maximum of 180 hours available. Manufacturing 1,000 kg of grade I fertilizer requires approximately 6 hours in Plant A and 4 hours in Plant B; manufacturing 1,000 kg of grade II fertilizer requires 3 hours on plant A and 10 hours in plant B. The maximum that the company can sell of grade I fertilizer is 18,000 kg. If profit is Rs. 450

L.P.P.

per 1,000 kg of grade I fertilizer and Rs. 550 per 1,000 kg of grade II fertilizer, it is required to build a mathematical model for amounts of the fertilizers that maximize the profit.

Q 295. The owner of Fancy Goods shop is interested to determine, how many advertisements to re-lease in the selected three magazines A,B and C. His main purpose is to advertise in such a way that exposure to principal buyers of his goods is maximized. Percentage of readers for each magazine are known. Exposure in any particular magazine is the number of advertisements released multiplied by the number of principle buyers. The following data are available:

Particulars	Magazines		
	A	B	C
Readers	1.0 lakh	0.6 lakh	0.4 lakh
Principle Buyers	20%	15%	8%
Cost per Advertisement	8,000	6,000	5,000

The Budgeted amount at the most Rs. 1,00,000 for the advertisement. The owner has already decided that magazine a should have not more than 15 advertisement & B & C each gets at least 8 advertisement formulate the problem.

Q 296. A dealer wishes to purchase a number of fans and sewing machines. He has only Rs. 5,760 to invest and has space utmost for 20 items. A fan costs him Rs. 360 and a sewing machine Rs. 240 His expectation is that he can sell a fan at a profit of Rs. 22 and a sewing machine at a profit of Rs. 18. Assuming that he can sell all the items that he can buy, how should he invest his money in order to maximize his profit? Formulate this problem as a linear programming problem and then use graphical method to solve it.

Q 297. A company produces two types of presentation goods A and B that require gold and silver. Each unit of type A requires 3 gms. of silver and 1 gm. of gold while that of B requires 1 gm. of silver and 2 gms. of gold. The company can procure 9 gms of silver and 8 gms. of gold. If each unit of type A brings a profit of Rs. 40 and that of types B Rs. 50, determine the number of units of each type that the company should produce to maximize the profit. What is the maximum profit?

Q 298.

$$\begin{aligned} &\text{Maximize } Z = 3x_1 + 4x_2 \\ &\text{Subject to } 2x_1 + 3x_2 \leq 16 \text{ (machining time)} \\ &\quad 4x_1 + 2x_2 \leq 16 \text{ (assembly time)} \\ &\quad x_1 > 0, x_2 \geq 0 \end{aligned}$$

Q299. A farm is engaged in breeding pigs. The pigs are fed on various products grown in the farm. In view of the need to ensure certain nutrient constituents (call them X, Y and Z), it becomes necessary to buy two additional products say, A and B. One unit of product A contains 36 units of X, 3 units of Y and 20 units of Z. One unit of product B contains 6 units of X, 12 units of Y and 10 units of Z. The minimum requirement of X, Y and Z is 108 units, 36 units and 100 units respectively. Product A costs Rs. 20 per unit and product B Rs. 40 per unit.

Formulate the above as a linear programming problem to minimize the total cost and solve this problem by using graphic method.

Q 300. A local travel agent is planning a charter trip to a major sea resort. The eight day/seven – night package includes the fare for round-trip travel, surface transportation, board and lodging and selected tour options. The charter trip is restricted to 200 persons and past experience indicates that there will not be any problem for getting 200 persons. The problem for the travel agent is to determine the number of Deluxe, Standard, and Economy tour packages to offer for this charter. These three plans each differ according to seating and service for the flight, quality of accommodation, meal plans and tour options. The following table summarises the estimated prices for the three packages and the corresponding expenses for the travel agent. The travel agent has hired an aircraft for the flat fee of Rs. 2,00,000 for the entire trip.

Price and costs for tour packages per person

Tour Plan	Price (Rs.)	Hotel Costs (Rs.)	Meals & other Expenses (Rs.)
Deluxe	10,000	3,000	4,750
Standard	7,000	2,200	2,500
Economy	6,500	1,900	2,200

In planning the trip, the following consideration must be taken into account:

- (i) At least 10 percent of the packages must be of the deluxe type

- (ii) At least 35 percent but not more than 70 percent must be of the standard type.
- (iii) At least 30 percent must be of the economy type.
- (iv) The maximum number of deluxe packages available in any aircraft is restricted to 60.
- (v) The hotel desires that at least 120 of the tourists should be on the deluxe and standard packages together.
- The travel agent wishes to determine the number of packages to offer in each type so as to maximize the total profit.
- (a) Formulate the above as a linear programming problem **Do not Solve it.**

Q 301. Consider a company that must produce two products over a production period of three months of duration. The company can pay for materials and labour from two sources company funds and borrowed funds.

The firm faces three decisions:

- (1) How many units should it produce of product 1?
- (2) How many units should it produce of Product 2?
- (3) How much money should it borrow to support the production of the two products?

In making these decisions, the firm wishes to maximize the profit contribution subject to the conditions stated below:

- (i) Since the company's products are enjoying a seller's market, it can sell as many units as it can produce. The company would therefore like to produce as many units as possible subject to production capacity and financial constraints. The capacity constraints, together with cost and price data, are given in Table-1.

TABLE – 1 Capacity, Price and cost data

Product	Selling Price (Rs. per unit)	Cost of production (Rs. per unit)	Required Hours per unit in Department		
			A	B	C
1	14	10	0.5	0.3	0.2
2	11	8	0.3	0.4	0.1
Available hours per production period of three months			500	400	200

(ii) The available company funds during the production period will be Rs. 3 lakhs.

(iii) A bank will give loans upto Rs. 2 lakhs per production period at an interest rate of 20 percent per annum provided the company's acid (quick) test ratio is at least 1 to 1 while the loan is outstanding. Take a simplified acid-test ratio given by

$$\frac{\text{Surplus Cash on hand after production} + \text{Accounts receivable}}{\text{bank Borrowing} + \text{Interest accrued thereon}}$$

(iv) Also make sure that the needed funds are made available for meeting the production costs. Formulate the above as a Linear Programming Problem.

Q 302. The D.K.C Florist Company of Gokul is planning to make up 1000 arrangements for the upcoming festival Holi. The company has available the following supply of flowers at the costs shown.

Type	Number available	Cost per flower
Red roses	800	Re. 0.20
Gardenias	456	Rs. 0.25
Carnations	4,000	Re. 0.15
White roses	920	Re. 0.20
Yellow roses	422	Re. 0.22

These flowers can be used in any of the four popular arrangements whose make up and selling prices are as follows:

Arrangement	Requirements	Selling price
Economy	4 red roses 2 Gardenias 8 carnations	Rs. 6
May time	8 white roses 5 gardenias 10 carnations 4 yellow roses	Rs. 8
Spring colour	9 red roses 10 Carnations 9 white roses 6 yellow roses	Rs. 10

Deluxe rose 12 red roses Rs. 12
 12 white roses
 12 yellow roses

Formulate a linear programming problem which allows the florist company to determine how many units of each arrangement should be made up in order to maximize profits assuming all arrangements can be sold.

The formulated LP problem is not required to be solved.

Q 303. The Sonu Paper Company produces rolls of paper used in cash registers. Each roll of paper is 500 ft. in length and can be produced in widths of 1,2,3 and 5 inch. The company's production process results in 500 rolls that are 12 inches in width. Thus the company must cut its 12 inch roll to the desired width. It has six basic cutting alternatives as follows:

Cutting	No. of Rolls				Waste
Alternative	1"	2"	3"	5"	(Inches)
1	6	3	0	0	0
2	0	3	2	0	0
3	1	1	1	1	1
4	0	0	2	1	1
5	0	4	1	0	1
6	4	2	1	0	1

The maximum demand requirements for the four rolls are as follows:

Roll Width (inches)	Demand Requirements (Rolls)
1	3000
2	2000
3	1500
5	1000

The company wishes to minimize the waste generated by its production meeting its demand requirements. Formulate the LP model.

Q 304. For a company engaged in the manufacture of three products viz, X, Y and Z, the available data are give in Tables 1,2 and 3 below:

Table 1 : Minimum sales requirements

Product	Minimum Sales Requirements per month
X	10
Y	20
Z	30

Table 2: Operations, Required Processing Times and Capacity

Operations	Time (Hrs.) required per item of			Total available hours per month
	X	Y	Z	
1	1	2	2	200
2	2	1	1	220
3	3	1	2	180

Table 3: Profit (Rs.) per unit

Product	Profit (Rs.) / Unit
X	10
Y	15
Z	8

Find out the product mix to maximize profit

Q 305. A company that produces soft drinks has a contract that requires that a minimum of 80 units of the chemical A and 60 units of the chemical B into each bottle of the drink. The chemicals are available in a prepared mix from two different suppliers. Supplier X_1 has a mix of 4 units of A and 2 units of B that costs Rs. 10, and supplier X_2 has a mix of 1 unit of A and 1 unit of B that costs Rs. 4. How many mixes from company X_1 and company X_2 should the company purchase to honour contract requirement and yet minimize cost?

Q 306. A manufacturer produces two products A and B, and has his machines in operation for 24 hours a day. Production of A requires 2 hours of processing in machine M_1 and 6 hours in machine M_2 . Production of B requires 6 hours of processing in machine M_1 and 2

L.P.P.

hours in machine M_2 . The manufacturer earns a profit of Rs. 5 on each unit of A and Rs. 2 on each unit of B. How many units of each product could be produced in a day in order to achieve maximum profit?

Q 307. A small township of 15,000 people requires, on the average, 300,000 gallons of water daily. The city is supplied water from a central water – works where the water is purified by such conventional methods as filtration and chlorination. In addition, two different chemical compounds: (i) softening chemical and (ii) health chemical, are needed for softening the water and for health purposes. The waterworks plans to purchase two popular brands that contain these chemicals. One unit of Chemico Corporation's product gives 8 pounds of softening chemical and 3 pounds of health chemical. One unit of Indian chemical's product contains 4 pounds and 9 pounds per unit, respectively, for the same purpose.

To maintain the water at a minimum level of softness and to meet a minimum programme of health protection, experts have decided that 150 and 100 pounds of the two chemicals that make up each product must be added to water daily. At a cost of Rs. 8 and Rs. 10 per unit respectively for Chemico's and Indian Chemical's products, what is the optimal quantity of each product that should be used to meet the minimum level of softness and minimum health standard?

Q 303. WELL TYPE Manufacturing Company produces three types of typewriters; Manual type-writer Electronic typewriters, and Deluxe Electronic typewriters. All the three models are required to be machined first and then assembled. The time required for the various models are as follows:

Type	Machine Time (in hour)	assembly Time (in hour)
Manual Typewriter	15	4
Electronic Typewriter	12	3
Deluxe Electronic Typewriter	14	5

The total available machine time and assembly time are 3,000 hours and 1,200 hours respectively. The data regarding the selling price and variable costs for the three types are:

	Manual	Electronic	Deluxe Electronic
Selling Price (Rs.)	4,100	7,500	14,600
Labour Material and other			
Variable costs (Rs.)	2,500	4,500	9,000

L.P.P.

The company sells all the three types on credit basis, but will collect the amounts on the first of next month. The labour, material and other variable expenses will have to be paid in cash. This company has taken a loan of Rs. 40,000 from a co-operative bank and this company will have to repay it to be bank on 1st April, 1999. The TNC bank from whom this company has borrowed Rs. 60,000 has expressed its approval to renew the loan.

The Balance Sheet of this company as on 31.3.99 is as follows:

Liabilities	Rs.	Assets	Rs.
Equity share capital	1,50,000	Land	90,000
Capital Reserve	15,000	Buildings	70,000
General Reserve	1,10,000	Plant & Machinery	1,00,000
Profit & Loss A/c	25,000	Furniture & Fixtures	15,000
Loan term loan	1,00,000	Vehicles	30,000
Loan from TNC Bank	60,000	Inventory	5,000
Loan from Co-op bank	40,000	Receivables	50,000
		Cash	1,40,000
Total	5,00,000	Total	5,00,000

The company will have to pay a sum of Rs. 10,000 towards the salary from top management executives and other fixed overheads for the month. Interest on long terms loans is to be paid every month at 24% per annum. Interest on loan from TNC and Co-operative Banks may be taken to be Rs. 1,200 for the month. Also this company has promised to deliver 2 Manual typewriters and 8 Deluxe Electronic typewriters to one of its valued customers next month.

Also make sure that the level of operations in this company is subject to the availability of cash next month. This company will also be able to sell all their types of typewriter in the market. The senior Manager of this company desires to know as to how many units of each typewriter must be manufactured in the factory next month so as to maximize the profits of the company. Formulate this as a linear programming problem. The formulated problem need not be solved.

Q.309. The manager of Sohan Oil Co., wishes to find the optimal mix of two possible blending processes. For process 1, an input of 1 unit of crude oil A and three units crude oil B produces an output of 5 units of gasoline X and two units of gasoline Y. For process 2, an input

L.P.P.

Q 309. The manager of Sohan Oil Co., wishes to find the optimal mix of two possible blending processes. For process 1, an input of 1 unit of crude oil A and three units crude oil B produces an out put of 5 units of gasoline X and two units of gasoline Y. For process 2, an input of 4 units of crude oil A and 2 unit of crude oil B produces an output of 3 units of gasoline X and 8 units of gasoline Y. Let x_1 and x_2 be the number of units the company decides to use of process 1 and process, 2 respectively. The maximum amount of crude oil A available is 100 units and that of crude oil B is 150 units. Sales commitments require that at least 200 units of gasoline X and 75 units of gasoline Y are produced. The unit profits of process 1 and process 2 are p_1 and p_2 respectively. Formulate the blending problem as a linear programming model.

Q 310. A computer Company produces three types of models, which are first required to be machined and then assembled. The time (in hours) for these operations for each model is give below:

Model	Machine Time	Assembly Time
P III	20	5
P II	15	4
Celeron	12	3

The total available machine time and assembly time are 1,000 hours and 1,500 hours respectively. The selling price and other variable costs for three models are:

	P III	P II	Celeron
Selling Price (Rs.)	3,000	5,000	15,000
Labour, Material and other Variable Cots (Rs.)	2,000	4,000	8,000

The company has taken a loan of Rs. 50,000 from a Nationalized Bank, which is required to be repaid on 1.4.2001. In addition, the company has borrowed Rs. 1,00,000 from XYZ Cooperative Bank. However, this bank has given its consent to renew the loan:

L.P.P.

The balance sheet of the company as on 31.3.2001 is as follows:

Liabilities	Rs.	Assets	Rs.
Equity share capital	1,00,000	Land	80,000
Capital reserve	20,000	Buildings	50,000
Profit & loss Account	30,000	Plant & Machinery	1,00,000
Long-term loan	2,00,000	Furniture etc.	20,000
Loan from XYZ Cooperative Bank	1,00,000	Cash	2,10,000
Loan from Nationalized Bank	50,000		
	5,00,000		5,00,000

The company is required to pay a sum of Rs. 15,000 towards the salary. Interest on long-term loan is to be paid every month @ 18% per annum. Interest on loan from XYZ Cooperative and Nationalised Banks may be taken as Rs. 1,500 per month. The company has already promised to deliver three P III, Two P II and five Celeron type of computers to M/s. ABC Ltd. next month. The level of operation of the company is subject to the availability of cash next month.

The Company Manager is willing to know that how many units of each model must be manufactured next month, so as to maximize the profit.

Linear Programming

Q 311. 2009 - May [6] (a) Formulate the dual for the following linear program:

Minimise: $100x_1 + 90x_2 + 40x_3 + 60x_4$

Subject to

$$6x_1 + 4x_2 + 8x_3 + 4x_4 \leq 140$$

$$10x_1 + 10x_2 + 2x_3 + 6x_4 \leq 120$$

$$10x_1 + 12x_2 + 6x_3 + 2x_4 \leq 50$$

$$x_1, x_2, x_3, x_4 \geq 0$$

(Only formulation is required. Please do not solve).

(6 marks) [S. A. 20]

Ans:- Minimise $140y_1 + 120y_2 + 50y_3$, Subject to; $6y_1 + 10y_2 + 10y_3 \geq 100$; $4y_1 + 10y_2 + 12y_3 \geq 90$; $8y_1 + 2y_2 + 6y_3 \geq 40$; $4y_1 + 6y_2 + 2y_3 \geq 60$; $y_1, y_2, y_3, y_4 > 0$.

----- Space to write important points for revision -----

Q 312. 2000 - Nov [1] (C) (b) A manufacture produces three products three products Y_1, Y_2, Y_3 from three raw materials X_1, X_2, X_3 . The cost of raw materials X_1, X_2 and X_3 is Rs. 30, Rs. 50 and Rs. 120 per kg respectively and they are available in a limited quantity viz. 20 kg of X_1 , 15 kg of X_2 and X_3 . The selling price of Y_1, Y_2 and Y_3 is Rs. 90, Rs. 100 and Rs. 120 per kg respectively. In order to produce 1 kg of $Y_1, \frac{1}{2}$ kg of $X_1, \frac{1}{4}$ kg of X_2 and $\frac{1}{4}$ kg of X_3 are required. Similarly to produce 1 kg of $Y_2, \frac{3}{7}$ kg of $X_1, \frac{2}{7}$ kg of X_2 and $\frac{2}{7}$ kg of X_3 and to produce 1 kg $Y_3, \frac{2}{3}$ kg of X_2 and $\frac{1}{3}$ kg of X_3 will be required.

Formulate the linear programming problem to maximize the profit. (10 marks) [S.A. 22]

Ans:- Maximise $Z = 32.50y_1 + 38.57y_2 + 46.67y_3$

Subject to Constraints

$$7y_1 + 6y_2 \leq 280$$

$$21y_1 + 24y_2 + 56y_3 \leq 1260$$

$$21y_1 + 24y_2 + 28y_3 \leq 840$$

Q 313. 2001 - May [1] (C) (b) A Computer Company produces three types of models, which are first required to be machined and then assembled. The time (in hours) for these operations for each model is given below:

Model	Machine Time	Assembly Time
P III	20	5
PII	15	4
Celeron	12	3

The total available machine time and assembly time are 1,000 hours and 1,500 hours respectively. The selling price and other variable costs for three models are:

	PIII	PII	Celeron
Selling Price (Rs.)	3,000	5,000	15,000
Labour, Material and			
Other variable Costs (Rs.)	2,000	4,000	8,000

The company has taken a long of Rs. 50,000 from a Nationalized Bank, which is required to be repaid on 1.4.2001. In addition, the company has borrowed RS. 1,00,000 from XYZ cooperative Bank. However, this bank has given its consent to renew the loan.

The Balance Sheet of the company as on 31.3.2001 is as follows:

Liabilities	Rs.	Assets	Rs.
Equity Share capital	1,00,000	Land	80,000
Capital reserve	20,000	Buildings	50,000
Profit & Loss Account	30,000	Plant & Machinery	1,00,000
Long term Loan	2,00,000	Furniture etc.	20,000

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Linear Programming

Loan from XYZ Cooperative Bank	1,00,000	Vehicles	40,000
Loan from National Bank	<u>50,000</u>	Cash	2,10,000
Total	<u>5,00,000</u>	total	<u>5,00,000</u>

The company is required to pay a sum of Rs. 15,000 towards the salary. Interest on long-term loan is to be paid every month @ 18% per annum. Interest on loan from XYZ Cooperative and Nationalised Banks may be taken as Rs. 1,500 per month. The company has already promised to deliver three P III, Two P II and five Celeron type of Computers to M/s ABC Ltd. next month. The level of operation in the company is subject to the availability of cash next month.

The Company Manager is willing to know that how many units of each model must be manufactured next month, so as to maximize the profit.

Formulate a linear programming problem for the above.

(10 marks) [S.A, 24]

Ans:- Maximise $Z = 1000 X_1 + 1000 X_2 + 7000 X_3$ - (Rs. 15000 + Rs. 3000 + Rs. 1500)

Subject to Constraints

$$20X_1 + 15X_2 + 12X_3 < 1000$$

$$5X_1 + 4X_2 + 3X_3 < 1500$$

$$2000 X_1 + 4000 X_2 + 8000 X_3 < \text{Rs. } 1,40,500$$

Where $X_1 > 3$, $X_2 > 2$ and $X_3 > 5$

----- Spaceto Write important points for revision -----

Q 314. 2001 - Nov [1] {C} (b) A Sports Club is engaged in the development of their players by feeding them certain minimum amount of Vitamins (say A, B and C), in addition to their normal diet. In view of this, two types of products X and Y are purchased from the market. The contents of Vitamin constituents per unit, are shown in the following table:

Vitamin Constituents	Vitamin contents in products		Minimum requirement for each player
	X	Y	
A	36	06	108
B	03	12	36
C	20	10	100

The cost product X is RS. 20 and that of Y is Rs. 40.

Formulate the linear programming problem for the above and minimize the total cost, and solve problem by using graphic method.

(10 marks) [S.A. 21]

Ans:- The optimal solution is to purchase 4 units of product X and 2 units of product Y.

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Q 315. 2002 - May [1] {C} (b) A farm is engaged in breeding pigs. The pigs are fed on various products grown in the farm. In view of the need to ensure certain nutrient constituents (call them X, Y and Z), it becomes necessary to buy two additional products say, A and B. One unit of product A contains 36 units of X, 3 units of Y and 20 units of Z. One unit of product B contains 6 units of X, 12 units of Y and 10 units of Z. The minimum requirement of X, Y and Z is 108 units, 36 units and 100 units respectively. Product A costs Rs. 20 per unit and product B Rs. 40 per unit.

Formulate the above as a linear programming problem to minimize the total cost and solve this problem by using graphic method.

(10 marks) [S.A. 22]

Ans:- The optimal solution is to purchase 4 units of product A and 2 units of product B in order to have minimum cost of Rs. 160.

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Linear Programming

Q 316. 2002 – Nov [6] (c) A company manufactures two products namely A and B. The contribution per kg of output is Rs. 240 and Rs. 140 respectively for products A and B. The total fixed costs amount to Rs. 1,200 per week

The production of two products is restricted by limited supplies of three items of raw materials namely P, Q and R. The quantities of P, Q and R which are necessary to produce single units of the products A and B together with the total stock of materials available each week are given below:

	Product A	Product B	Total Quantity available per week kg
Raw material P	16	20	160
Q	10	25	150
R	4	0	32

Using the graphical approach of Linear Programming (LP), calculate the maximum Profit per week. (8 marks) [S. A. 19]

Ans:- Maximum Profit is Rs. 944

----- Space to write important points for revision -----

Q 317. 2003 – Nov [5] (b) A manufacture produces two products D₁ and D₂ using two machines R₁ and R₂. Products D₁ requires 2 hours on machine R₁ and 6 hours on machine R₂. Product D₂ utilises 5 hours of machine R₁ only. Total hours available per day on machine R₁ is 16 and R₂ is 30. Profit margin from D₁ and D₂ is Rs. 2 and Rs. 10 per unit respectively. Using simplex method find out the daily production mix to optimis profit. (8 marks) [S.A. 17]

Ans:- 32

----- Space to write important points for revision -----

Q 318. 2004 – May [5] (c) The budgeted data relating to two products manufactured by a company for a month are as under:

	Product A	Product B
Selling price	300	200
Variable manufacturing costs	160	60
Sales commission	60	40

Each unit of product incurs costs in the company's two departments P and Q. The total capacity available for the month under review is budgeted to be 1,400 hours in department P and 2,000 hours in department Q. The capacity costs amount to Rs. 14,000 and Rs. 20,000 respectively per month for P and Q irrespective of the level of usage made of it. The number of hours required in each of these departments to complete one unit of output is as under:

		A	B
Department	P	2	4
Department	Q	5	4

The maximum output which the company can sell in the month is restricted to 400 units of either of the products.

You are required to formulate the Linear Programming (LP) model and solve it graphically to determine the optimal product mix and profit. (8 marks) [S.A. 22]

Ans:- (i) Optimal Profit Rs. 7,000

----- Space to write important points for revision -----

Linear Programming

Q 319. 2004 - Nov [3] (b) A manufacturing company produces two types of product the SUPER and REGULAR. Resource requirements for production are given below in the table. There are 1,600 hours of assembly worker hours available per week. 790 hours of paint time and 300 hours of inspection time. Regular customers bill demand at least 150 units of the REGULAR type and 90 units of the SUPER type.

Product	Profit / Contribution Rs.	Table Assembly time Hrs.	Paint time Hrs.	Inspection time Hrs.
REGULAR	50	1.2	0.8	0.2
SUPER	75	1.6	0.9	0.2

Formulate the Linear programming problem to determine product mix on a weekly basis.

----- Space to write important points for revision -----

Q 320. 2005 - May [6] (b) A company manufactures two products A and B, involving three departments - Machining, Fabrication and Assembly. The process time, profit / unit and total capacity of each department is given in the following table.

	Machining (Hours)	Fabrication (Hours)	Assembly (Hours)	Profit Rs.
A	1	5	3	80
B	2	4	1	100
Capacity	720	1,800	900	

Set up Linear Programming Problem to maximize profit. What will be the product Mix at Maximum profit level? (9 marks) [S.A. 23]

Ans:- Maximize $z = 80x + 10y$ subject to

$$X + 2y < 720$$

$$5x + 4y < 1800$$

$$3x + y < 900$$

$$X > 0 \ y > 0$$

where

X = No. of units of A

Y = No. of units of B.

$X = 120$ units, $y = 300$ units and the maximum profit Rs. 39,600

----- Space to write important points for revision -----

Q 321. 2006 - Nov [5] (c) A Ltd. makes 2 products, Tables (T) and Chairs (C), which must be processed through Assembly (A) and Finishing (F) departments. Assembly has 60 hours available per week; finishing can handle upto 48 hours a week.

Manufacture of one table requires 4 hours of assembly and 2 hours in finishing. Each chair requires 2 hours in assembly and 4 hours in finish.

Profit is Rs. 80 per table and Rs. 60 per chair. Choose the best combination of chairs and tables to be produced to maximize profit.

(i) Formulate the Linear Programming model equations.

(2 marks)

(ii) Present graphically.

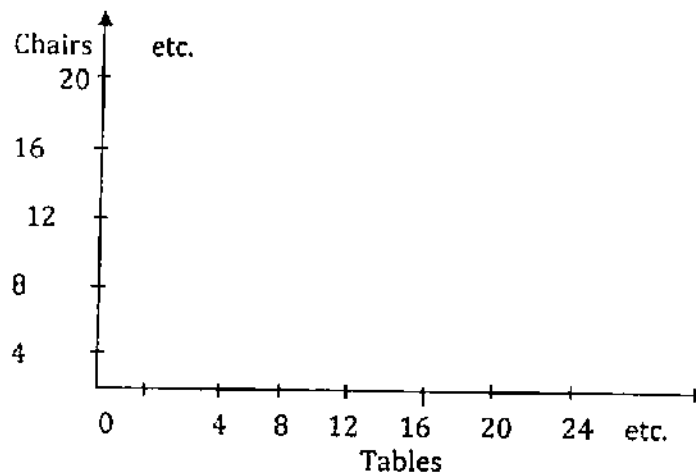
(5 marks)

(iii) What is the maximum profit?

(1 marks) [S.A. 17]

Linear Programming

Use the graph provided as follows:



Ans:-

- (i) If $x \rightarrow$ No of Tables and $y \rightarrow$ no. of chairs produced.
 Maximise $Z = 80x + 60y$ Subject to constraints
 $4x + 2y < 60$
 $2x + 4y < 48$
 $x \geq 0, y \geq 0$

- (ii) Maximum profit $80 \times 12 + 60 \times 6 = \text{Rs. } 1,320$

----- Space to write important points for revision -----

Q 322. 2007 - May [4] (a) A gear manufacturing company makes two types of gears - A and B. Both gears are processed on 3 machines, Hobbing M/c, Shaping M/c and Grinding M/c. The time required by each gear and total time available per week on each M/c is as follows:

Machine	Gear (A) (Hours)		Gear (B) (Hours)		Available Hours
Hobbing M/s	3		3		36
Shaping M/s	5	2	60		
Grinding M/c	2	6	60		

Other data:

Selling price (Rs.)	820	960
Variable cost (Rs.)	780	900

Determine the optimum production plan and the maximum contribution for the next by simplex method. The initial table is given below:

		C ₁	40	60	0	0	0
		Qty.					
C ₁	Variable		x ₁	x ₂	x ₃	x ₄	x ₅
0	x ₃	36	3	3	1	0	0
0	x ₄	60	5	2	0	1	0
0	x ₅	60	2	6	0	0	1

Ans:- Optimum $Z = 660$ with $x_1 = 40$ and $x_2 = 60$

----- Space to write important points for revision -----

Linear Programming

Q. 3. 2008 – May [5] (a) Transport Ltd. Provides tourist vehicles of 3 types – 20 – seater vans, 8-seater big cars and 5-seater small cars. These seating capacities are excluding the drivers. The company has 4 vehicles of the 20-seater van type, 10 vehicles of the eight-seater big car types and 20 vehicles of the 5-seater small car types. These vehicles have to be used to transport employees of their client company from their residences to their offices and back. All the residences are in the same housing colony. The offices are at two different places, one in is the Head Office and the other is the Branch. Each vehicle plies only one round trip per day, if residence to office in the morning and office to residence in the evening. Each day, 180 officials need to be transported in Route 1 (from residence to Head office and back) and 40 officials need to be transported in Route II (from Residence to Branch office and back). The cost per round trip for each type of vehicle along each route is given below.

You are required to formulate the information as a linear programming problem, with the objective of minimizing the total cost of hiring vehicles for the client company, subject to the constraints mentioned above. (only formulation is required. Solution is not needed)

	20 – Seater Vans		Figs. – Rs. / round trip 8 – seater big cars 5 – seater small cars	
Route I – Residence – Head Office and Back	600	400	300	
Route II – Residence – Branch Office and Bank	500		300	200

(8 marks) [S.A. 18]

Ans:- (Minimise Cost $Z = 600S_{11} + 400S_{12} + 300S_{13} + 500S_{21} + 300S_{22} + 200S_{23}$)

Subject to

$$20S_{11} + 8S_{12} + 5S_{13} = 180$$

$$20S_{21} + 8S_{22} + 5S_{23} = 40$$

$$S_{11} + S_{21} \leq 4$$

$$S_{12} + S_{22} \leq 10$$

$$S_{13} + S_{23} \leq 20$$

Chapter 6 & 7 CPM PERT

Q 324.

Activity	Duration
1 - 2	3 days
1 - 3	7 days
2 - 3	5 days
2 - 4	8 days
3 - 4	4 days

With the help of the activities given above draw a network. Determine its critical path, earliest start time, earliest finish time, latest start time, latest finish time, total float, free float and independent float.

Q 325.

Activity	Duration
1 - 2	9 days
1 - 3	8 days
1 - 4	15 days
2 - 4	5 days
3 - 4	10 days
4 - 5	2 days

With the help of the activities given above draw a network. Determine its critical path, earliest start time, earliest finish time, latest start time, latest finish time, total float, free float and independent float.

Q 326.

Activity	Duration
1 - 2	4 days
1 - 3	12 days
1 - 4	10 days
2 - 4	8 days
2 - 5	6 days
3 - 6	8 days
4 - 6	10 days
5 - 7	10 days
6 - 7	0 days
6 - 8	8 days
7 - 8	10 days
8 - 9	6 days

With the help of the activities given above draw a network. Determine its critical path, earliest start time, earliest finish time, latest start time, latest finish time, total float, free float and independent float.

CPM PERT

Q 327.

A small maintenance project consists of the following twelve jobs whose precedence relations are defined with their node numbers:

Job (i - j)	(1, 2)	(1, 3)	(1, 4)	(2, 3)	(2, 5)	(2, 6)
Duration (in days)	10	4	6	5	12	9
Job (i - j)	(3, 7)	(4, 5)	(5, 6)	(6, 7)	(6, 8)	(7, 8)
Duration (in days)	12	15	6	5	4	7

- (i) Draw an arrow diagram representing the project.

Q 328.

The following information is given below:

Activity	(1-2)	(2-3)	(2-4)	(3-5)	(4-6)	(5-6)	(5-7)	(6-7)
Optimistic time (in weeks)	3	9	6	8	8	0	5	8
Most likely time (in weeks)	3	6	4	6	6	0	4	5
Pessimistic time (in weeks)	3	3	2	4	4	0	3	2

Draw the network diagram for the above. Calculate:

- Variance to each activity
- Critical path and expected project length
- The probability that the project will be completed in 23 weeks

Q 329.

A small project consists of seven activities for which the relevant data are given below:

Activity	Preceding Activities	Activity Duration (Days)
A	-	4
B	-	7
C	-	6
D	A, B	5
E	A, B	7
F	C, D, E	6
G	C, D, E	5

- Draw the network and find the project completion time
- Calculate total float for each of the activities

Q 330.

The following table gives the activities in a construction project and the time duration of each

Activity	Preceding Activities	Activity Duration (Days)
A	-	16
B	-	20
C	A	8
D	A	10
E	B, C	6
F	D, E	12

Required:

- Draw the activity network of the project
- Find critical path
- Find the total float and free-float for each activity

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Q 331.

A project consist of eight activities with the following relevant information:

Activity	Immediate Predecessor	Estimated Duration (days)		
		Optimistic	Most Likely	Pessimistic
A	-	1	1	7
B	-	1	4	7
C	-	2	2	8
D	A	1	1	1
E	B	2	5	14
F	C	2	5	8
G	D, E	3	6	15
H	F, G	1	2	3

- (i) Draw the PERT network and find out the expected project completion time.
- (ii) What duration will have 95% confidence for project completion?

Q 332.

A project consists of seven activities and the time estimates of the activities are furnished as under:

Activity	Optimistic Days	Most likely days	Pessimistic days
1 - 2	4	10	16
1 - 3	3	6	9
1 - 4	4	7	16
2 - 5	5	5	5
3 - 5	8	11	32
4 - 6	4	10	16
5 - 6	2	5	8

Required:

- (i) Draw the network diagram
- (ii) Identify the critical path and its duration
- (iii) What is the probability that the project will be completed in 5 days earlier than the critical path duration?
- (iv) What project duration will provide 95% confidence level of completion ($Z_{0.95} = 1.65$)?

Z	1.00	1.09	1.18	1.25	1.33
Probability	0.1587	0.1379	0.1190	0.1056	0.0918

Q 333.

A small project is composed of seven activities, whose time estimates are listed below: Activities are identified by their beginning (i) and ending (j) node numbers.

Activity (i - j)	Estimated durations (in days)		
	Optimistic	Most likely	Pessimistic
1 - 2	2	2	14
1 - 3	2	8	14
1 - 4	4	4	16
2 - 5	2	2	2
3 - 5	4	10	28
4 - 6	4	10	16
5 - 6	6	12	30

- (a) Draw the project network
- (b) Find the expected duration and variance for each activity. What is the expected project length?

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- (c) If the project due date is 38 days, what is the probability of not meeting the due date?

Z	0.50	0.67	1.00	1.33	2.00
P	0.3085	0.2514	0.1587	0.0918	0.0228

Q 334.

An Engineering Project has the following activities, whose time estimates are listed below:

Activity (I - j)	Estimated duration (in months)		
	Optimistic	Most likely	Pessimistic
1 - 2	2	2	14
1 - 3	2	8	14
1 - 4	4	4	16
2 - 5	2	2	2
3 - 5	4	10	28
4 - 6	4	10	16
5 - 6	6	12	30

- Draw the project network and find the critical path
- Find the expected duration and variance for each activity. What is the expected project length?
- Calculate the variance and standard deviation of the project length
- What is the probability that the project will be completed at least eight months earlier than expected time?

- (v) If the project due date is 38 months, what is probability of not meeting the due date?

Given:

Z:	0.50	0.67	1.00	1.33	2.00
Prob:	0.3085	0.2514	0.1587	0.0918	0.0228

Q 335.

The following data pertains to the network. It is desired to compress the project to the least possible duration day by day and estimate the extra cost.

I - j	T_n	T_c	cost slope
1 - 2	3	2	700
1 - 3	7	4	200
2 - 3	5	3	100
2 - 4	8	6	200
3 - 4	4	2	400

Q 336.

A small maintenance project consists of jobs in the table below. With each job is listed its normal time and a minimum, or crash time, in days. The cost (in Rs. per day) of each job is also given.

Job I - j	Normal day duration	Minimum (Crash) day duration	Cost /day
1 - 2	9	6	20
1 - 3	8	5	25
1 - 4	15	10	30
2 - 4	5	3	10
3 - 4	10	6	15
4 - 5	2	1	40

- What is the normal project length and the minimum project length?
- Determine the minimum crashing cost of schedules ranging from normal length down to, and including, the minimum length schedule. That is, if L = length of the schedule, find the costs of schedules which are L , $L - 1$, $L - 2$ and so on days long.
- Overhead costs total Rs. 60 per day. What is the optimum length schedule in terms of both crashing and overhead cost? List the scheduled duration of each job for your solution:

Q 337.

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5] (b) Given the following information:

	0-1	1-2	1-3	2-4	2-5	3-4	3-6	4-7	5-7	6-7
Duration	2	8	10	6	3	3	7	5	2	8

(in days)

- Draw the arrow diagram.
 - Identify critical path and find the total project duration.
 - Determine total, free and independent floats.
- (10 marks)

Q 338.

Nov 14] (b) A project has the following time schedule:

Activity	Time in Weeks	Activity	Time in Weeks
1-2	2	4-6	3
1-3	2	5-8	1
1-4	1	6-9	5
2-5	4	7-8	4
3-6	8	8-9	3
3-7	5		

Construct PERT network and compute:

- total float for each activity; and
 - critical path and its duration.
- (10 marks)

Q 339.

2000 - May [3] (b) A project has the following time schedule:

Activity	Time in weeks	Activity	Time in weeks
1-2	4	5-7	8
1-3	1	6-8	1
2-4	1	7-8	2
3-4	1	8-9	1
3-5	6	8-10	8
4-9	15	9-10	7
5-6	2		

Construct a PERT network and compute:

- T_E and T_L for each event
 - Float for each activity; and
 - Critical path and its duration.
- (10 marks)

CPM PERT

Q 340.

The time schedule for different activities of a project is given below:

Activity (i-j)	Time in days
1-2	8
1-3	10
1-4	8
2-3	10
2-6	16
3-5	17
4-5	18
4-8	14
5-6	9

Construct the PERT network and compute

- Critical path and its duration.
- Total and free float for each activity.

(10 marks)

2005-06 [5] (c) A project consists of the following activities with the time estimates noted against each.

Activity	Time Estimate (Weeks)	Activity	Time Estimate (Weeks)
1-2	2	3-7	5
1-3	2	4-6	3
1-4	1	5-8	1
2-5	4	6-9	5
3-5	8	7-8	4
		8-9	3

Required :

- Draw a network diagram
- Determine the critical path and its duration
- Determine the total float for each activity.

(7 marks)

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2004 - May [6] (b) A project is composed of seven activities as per details given below:

Activity	Normal Time (days)	Crash Time (days)	Normal Cost ₹	Crash Cost ₹
1-2	4	3	1,500	2,000
1-3	2	2	1,000	1,000
1-4	5	4	1,875	2,250
2-3	7	5	1,000	1,500
2-5	7	6	2,000	2,500
3-5	2	1	1,250	1,625
4-5	5	4	1,500	2,125

Indirect cost per day of the project is ₹ 500.

Required:

- Draw the project network.
- Determine the critical path and its duration.
- Find the optimum duration and the resultant cost of the project. (8 marks)

Q 343.

2010 - May [5] (b) A small project is composed of seven activities, whose time estimates are listed below. Activities are identified by their beginning (i) and ending (j) node numbers:

Activity (i-j)	Estimated durations (in days)		
	Optimistic	Most likely	Pessimistic
1-2	2	2	14
1-3	2	8	14
1-4	4	4	16
2-5	2	2	2
3-5	4	10	28
4-6	4	10	16
5-6	6	12	30

- Draw the project network.
- Find the expected duration and variance for each activity. What is the expected project length?

Given: Z	0.50	0.67	1.00	1.33	2.00
P	0.3085	0.2514	0.1587	0.0918	0.0228

(4 marks)

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2001 - May [5] (b) Consider the schedule of activities and related information as given below, for the construction of a Plant:

Activity	Expected Time (Months)	Variance	Expected Cost (Millions of ₹)
1-2	4	1	5
2-3	2	1	3
3-6	3	1	4
2-4	6	2	9
1-5	2	1	2
5-6	5	1	12
4-6	9	5	20
5-7	7	8	7
7-8	10	16	14
6-8	1	1	4

Assuming that the cost and time required for one activity is independent of the time and cost of any other activity and variations are expected to follow normal distribution.

Draw a network based on the above data and calculate:

- Critical path
- Expected cost of construction of the plant
- Expected time required to build the plant
- The standard deviation of the expected time.

(10 marks)

Q 345.

2001 - Nov [5] (b) An Engineering Project has the following activities, whose time estimates are listed below:

Activity	Estimated duration (In months)		
	Optimistic	Most likely	Pessimistic
(i - j)			
1-2	2	2	14
1-3	2	8	14
1-4	4	4	16
2-5	2	2	2
3-5	4	10	28
4-6	4	10	16
5-6	6	12	30

- Draw the project network and find the critical path.
- Find the expected duration and variance for each activity. What is the expected project length?
- Calculate the variance and standard deviation of the project length.
- What is the probability that the project will be completed at least eight months earlier than expected time?
- If the project due date is 38 months, what is the probability of not meeting the due date? Given:

Z	0.50	0.67	1.00	1.33	2.00
Prob.	0.785	0.851	0.954	0.991	0.999

(10 marks)

CPM PERT

Q 346.

2002 - May [5] (b) A small project consists of jobs as given in the table below. Each job is listed with its normal time and a minimum of crash time (in days). The cost (in ₹ per day) for each job is also given:

Job (i—j)	Normal duration (in days)	Minimum (crash) Duration (in days)	Cost of Crashing (₹ per day)
1—2	9	6	20
1—3	8	5	25
1—4	15	10	30
2—4	5	3	10
3—4	10	6	15
4—5	2	1	40

- What is the normal project length and the minimum project length?
- Determine the minimum crashing cost of schedules ranging from normal length down to, and including, the minimum length schedule. That is, if L = Length of the schedule, find the costs of schedules which are L , $L-1$, $L-2$ and so on.
- Overhead costs total ₹ 60 per day. What is the optimum length schedule in terms of both crashing and overhead cost? List the schedule duration of each job for your solution.

(10 marks)

Q 347.

2003 - May [6] (c) A project consists of the following activities, whose time estimates are given against each as under:

Activity	Estimated duration (weeks)		
	Optimistic	Most likely	Pessimistic
1-2	3	6	15
1-3	2	5	14
1-4	6	12	30
2-5	2	5	8
2-6	5	11	17
3-6	3	6	15
4-7	3	9	27
5-7	1	4	7
6-7	4	19	28

Required:

- Draw the project network.
- Find the expected duration and variance of each activity.
- Determine the critical path and the expected project duration.
- What is the probability that the project will be completed in 38 weeks?
- What project duration will have 95% chance of completion. ($Z_{0.95} = 1.65$).

Given:

Z	0.21	0.41	0.82
ϕ	0.0832	0.1591	0.2939

(8 marks)

LOGARITHMS

	1	2	3	4	5	6	7	8	9	Mean Difference
0	0.0000	0.0004	0.0008	0.0012	0.0016	0.0020	0.0024	0.0028	0.0032	0.0004
1	0.0036	0.0040	0.0044	0.0048	0.0052	0.0056	0.0060	0.0064	0.0068	0.0004
2	0.0072	0.0076	0.0080	0.0084	0.0088	0.0092	0.0096	0.0100	0.0104	0.0004
3	0.0108	0.0112	0.0116	0.0120	0.0124	0.0128	0.0132	0.0136	0.0140	0.0004
4	0.0144	0.0148	0.0152	0.0156	0.0160	0.0164	0.0168	0.0172	0.0176	0.0004
5	0.0180	0.0184	0.0188	0.0192	0.0196	0.0200	0.0204	0.0208	0.0212	0.0004
6	0.0216	0.0220	0.0224	0.0228	0.0232	0.0236	0.0240	0.0244	0.0248	0.0004
7	0.0252	0.0256	0.0260	0.0264	0.0268	0.0272	0.0276	0.0280	0.0284	0.0004
8	0.0288	0.0292	0.0296	0.0300	0.0304	0.0308	0.0312	0.0316	0.0320	0.0004
9	0.0324	0.0328	0.0332	0.0336	0.0340	0.0344	0.0348	0.0352	0.0356	0.0004
10	0.0360	0.0364	0.0368	0.0372	0.0376	0.0380	0.0384	0.0388	0.0392	0.0004
11	0.0396	0.0400	0.0404	0.0408	0.0412	0.0416	0.0420	0.0424	0.0428	0.0004
12	0.0432	0.0436	0.0440	0.0444	0.0448	0.0452	0.0456	0.0460	0.0464	0.0004
13	0.0468	0.0472	0.0476	0.0480	0.0484	0.0488	0.0492	0.0496	0.0500	0.0004
14	0.0504	0.0508	0.0512	0.0516	0.0520	0.0524	0.0528	0.0532	0.0536	0.0004
15	0.0540	0.0544	0.0548	0.0552	0.0556	0.0560	0.0564	0.0568	0.0572	0.0004
16	0.0576	0.0580	0.0584	0.0588	0.0592	0.0596	0.0600	0.0604	0.0608	0.0004
17	0.0612	0.0616	0.0620	0.0624	0.0628	0.0632	0.0636	0.0640	0.0644	0.0004
18	0.0648	0.0652	0.0656	0.0660	0.0664	0.0668	0.0672	0.0676	0.0680	0.0004
19	0.0684	0.0688	0.0692	0.0696	0.0700	0.0704	0.0708	0.0712	0.0716	0.0004
20	0.0720	0.0724	0.0728	0.0732	0.0736	0.0740	0.0744	0.0748	0.0752	0.0004
21	0.0756	0.0760	0.0764	0.0768	0.0772	0.0776	0.0780	0.0784	0.0788	0.0004
22	0.0792	0.0796	0.0800	0.0804	0.0808	0.0812	0.0816	0.0820	0.0824	0.0004
23	0.0828	0.0832	0.0836	0.0840	0.0844	0.0848	0.0852	0.0856	0.0860	0.0004
24	0.0864	0.0868	0.0872	0.0876	0.0880	0.0884	0.0888	0.0892	0.0896	0.0004
25	0.0900	0.0904	0.0908	0.0912	0.0916	0.0920	0.0924	0.0928	0.0932	0.0004
26	0.0936	0.0940	0.0944	0.0948	0.0952	0.0956	0.0960	0.0964	0.0968	0.0004
27	0.0972	0.0976	0.0980	0.0984	0.0988	0.0992	0.0996	0.1000		0.0004

LOGARITHMS

	1	2	3	4	5	6	7	8	9	Mean Difference
30	0.4771	0.4779	0.4787	0.4796	0.4804	0.4812	0.4820	0.4828	0.4836	0.0008
31	0.4844	0.4852	0.4860	0.4868	0.4876	0.4884	0.4892	0.4900	0.4908	0.0008
32	0.4916	0.4924	0.4932	0.4940	0.4948	0.4956	0.4964	0.4972	0.4980	0.0008
33	0.4988	0.4996	0.5004	0.5012	0.5020	0.5028	0.5036	0.5044	0.5052	0.0008
34	0.5060	0.5068	0.5076	0.5084	0.5092	0.5100	0.5108	0.5116	0.5124	0.0008
35	0.5132	0.5140	0.5148	0.5156	0.5164	0.5172	0.5180	0.5188	0.5196	0.0008
36	0.5204	0.5212	0.5220	0.5228	0.5236	0.5244	0.5252	0.5260	0.5268	0.0008
37	0.5276	0.5284	0.5292	0.5300	0.5308	0.5316	0.5324	0.5332	0.5340	0.0008
38	0.5348	0.5356	0.5364	0.5372	0.5380	0.5388	0.5396	0.5404	0.5412	0.0008
39	0.5420	0.5428	0.5436	0.5444	0.5452	0.5460	0.5468	0.5476	0.5484	0.0008
40	0.5492	0.5500	0.5508	0.5516	0.5524	0.5532	0.5540	0.5548	0.5556	0.0008
41	0.5564	0.5572	0.5580	0.5588	0.5596	0.5604	0.5612	0.5620	0.5628	0.0008
42	0.5636	0.5644	0.5652	0.5660	0.5668	0.5676	0.5684	0.5692	0.5700	0.0008
43	0.5708	0.5716	0.5724	0.5732	0.5740	0.5748	0.5756	0.5764	0.5772	0.0008
44	0.5780	0.5788	0.5796	0.5804	0.5812	0.5820	0.5828	0.5836	0.5844	0.0008
45	0.5852	0.5860	0.5868	0.5876	0.5884	0.5892	0.5900	0.5908	0.5916	0.0008
46	0.5924	0.5932	0.5940	0.5948	0.5956	0.5964	0.5972	0.5980	0.5988	0.0008
47	0.5996	0.6004	0.6012	0.6020	0.6028	0.6036	0.6044	0.6052	0.6060	0.0008
48	0.6068	0.6076	0.6084	0.6092	0.6100	0.6108	0.6116	0.6124	0.6132	0.0008
49	0.6140	0.6148	0.6156	0.6164	0.6172	0.6180	0.6188	0.6196	0.6204	0.0008
50	0.6212	0.6220	0.6228	0.6236	0.6244	0.6252	0.6260	0.6268	0.6276	0.0008
51	0.6284	0.6292	0.6300	0.6308	0.6316	0.6324	0.6332	0.6340	0.6348	0.0008
52	0.6356	0.6364	0.6372	0.6380	0.6388	0.6396	0.6404	0.6412	0.6420	0.0008
53	0.6428	0.6436	0.6444	0.6452	0.6460	0.6468	0.6476	0.6484	0.6492	0.0008
54	0.6500	0.6508	0.6516	0.6524	0.6532	0.6540	0.6548	0.6556	0.6564	0.0008
55	0.6572	0.6580	0.6588	0.6596	0.6604	0.6612	0.6620	0.6628	0.6636	0.0008
56	0.6644	0.6652	0.6660	0.6668	0.6676	0.6684	0.6692	0.6700	0.6708	0.0008
57	0.6716	0.6724	0.6732	0.6740	0.6748	0.6756	0.6764	0.6772	0.6780	0.0008
58	0.6788	0.6796	0.6804	0.6812	0.6820	0.6828	0.6836	0.6844	0.6852	0.0008
59	0.6860	0.6868	0.6876	0.6884	0.6892	0.6900	0.6908	0.6916	0.6924	0.0008
60	0.6932	0.6940	0.6948	0.6956	0.6964	0.6972	0.6980	0.6988	0.6996	0.0008
61	0.7004	0.7012	0.7020	0.7028	0.7036	0.7044	0.7052	0.7060	0.7068	0.0008
62	0.7076	0.7084	0.7092	0.7100	0.7108	0.7116	0.7124	0.7132	0.7140	0.0008
63	0.7148	0.7156	0.7164	0.7172	0.7180	0.7188	0.7196	0.7204	0.7212	0.0008
64	0.7220	0.7228	0.7236	0.7244	0.7252	0.7260	0.7268	0.7276	0.7284	0.0008
65	0.7292	0.7300	0.7308	0.7316	0.7324	0.7332	0.7340	0.7348	0.7356	0.0008
66	0.7364	0.7372	0.7380	0.7388	0.7396	0.7404	0.7412	0.7420	0.7428	0.0008
67	0.7436	0.7444	0.7452	0.7460	0.7468	0.7476	0.7484	0.7492	0.7500	0.0008
68	0.7508	0.7516	0.7524	0.7532	0.7540	0.7548	0.7556	0.7564	0.7572	0.0008
69	0.7580	0.7588	0.7596	0.7604	0.7612	0.7620	0.7628	0.7636	0.7644	0.0008
70	0.7652	0.7660	0.7668	0.7676	0.7684	0.7692	0.7700	0.7708	0.7716	0.0008
71	0.7724	0.7732	0.7740	0.7748	0.7756	0.7764	0.7772	0.7780	0.7788	0.0008
72	0.7796	0.7804	0.7812	0.7820	0.7828	0.7836	0.7844	0.7852	0.7860	0.0008
73	0.7868	0.7876	0.7884	0.7892	0.7900	0.7908	0.7916	0.7924	0.7932	0.0008
74	0.7940	0.7948	0.7956	0.7964	0.7972	0.7980	0.7988	0.7996	0.8004	0.0008
75	0.8012	0.8020	0.8028	0.8036	0.8044	0.8052	0.8060	0.8068	0.8076	0.0008
76	0.8084	0.8092	0.8100	0.8108	0.8116	0.8124	0.8132	0.8140	0.8148	0.0008
77	0.8156	0.8164	0.8172	0.8180	0.8188	0.8196	0.8204	0.8212	0.8220	0.0008
78	0.8228	0.8236	0.8244	0.8252	0.8260	0.8268	0.8276	0.8284	0.8292	0.0008
79	0.8300	0.8308	0.8316	0.8324	0.8332	0.8340	0.8348	0.8356	0.8364	0.0008
80	0.8372	0.8380	0.8388	0.8396	0.8404	0.8412	0.8420	0.8428	0.8436	0.0008
81	0.8444	0.8452	0.8460	0.8468	0.8476	0.8484	0.8492	0.8500		0.0008

1. 2. 3. 4. 5. 6. 7. 8. 9. 10. 11. 12. 13. 14. 15. 16. 17. 18. 19. 20. 21. 22. 23. 24. 25. 26. 27. 28. 29. 30. 31. 32. 33. 34. 35. 36. 37. 38. 39. 40. 41. 42. 43. 44. 45. 46. 47. 48. 49. 50. 51. 52. 53. 54. 55. 56. 57. 58. 59. 60. 61. 62. 63. 64. 65. 66. 67. 68. 69. 70. 71. 72. 73. 74. 75. 76. 77. 78. 79. 80. 81. 82. 83. 84. 85. 86. 87. 88. 89. 90. 91. 92. 93. 94. 95. 96. 97. 98. 99. 100.

1. 2. 3. 4. 5. 6. 7. 8. 9. 10. 11. 12. 13. 14. 15. 16. 17. 18. 19. 20. 21. 22. 23. 24. 25. 26. 27. 28. 29. 30. 31. 32. 33. 34. 35. 36. 37. 38. 39. 40. 41. 42. 43. 44. 45. 46. 47. 48. 49. 50. 51. 52. 53. 54. 55. 56. 57. 58. 59. 60. 61. 62. 63. 64. 65. 66. 67. 68. 69. 70. 71. 72. 73. 74. 75. 76. 77. 78. 79. 80. 81. 82. 83. 84. 85. 86. 87. 88. 89. 90. 91. 92. 93. 94. 95. 96. 97. 98. 99. 100.