# Portfolio Management – (Introduction to Capital Market)

## Return & Risk of 'Individual Security'

**Return :** $R_x = \dfrac{D_1 + (P_1 - P_0)}{P_0} \times 100$

**Risk :**

$\text{Standard Deviation}(\sigma) = \sqrt{\sum (x_i - \bar{x})^2 P_i}$

$\underset{(COV)}{\text{Co-efficient of variance}} = \dfrac{\sigma}{\bar{X}}$ ← standard deviation ($\sigma$); ← Mean ($\bar{X}$)

## Return & Risk of Portfolio

**Return ($R_p$) :** $R_x w_x + R_y w_y + \ldots + R_n w_n$ ($w$ = weight)

**Risk :**

Standard Deviation – $(\sigma_p) = \sqrt{\sigma_x^2 w_x^2 + \sigma_y^2 w_y^2 + 2\sigma_x \sigma_y w_x w_y CORR_{xy}}$

$CoV_{xy} = \sum (x_i - \bar{x})(y_i - \bar{y}) P_i$

or

$CoV_{xy} = \sigma_x \sigma_y CORR_{xy}$

variance $= \sigma^2$

If one security is risky & other security is risk free then Standard Deviation ($\sigma$)

$\boxed{\sigma_p = \sigma_{Risky} w_{Risky}}$

$\because \sigma_{Rf} = 0$

so $\sigma_p = \sqrt{\sigma_R^2 w_R^2 + \underset{\downarrow\,0\,(zero)}{\sigma_{Rf}^2} w_{Rf}^2 + 2\sigma_R w_R \sigma_{Rf} \underset{\downarrow\,0\,(zero)}{\sigma_{Rf}} CORR_{RfR}}$

If $CORR_{xy} = +1$ ⇓

$\boxed{\sigma_p = \pm(\sigma_x w_x + \sigma_y w_y)}$

If $CORR_{xy} = -1$ ⇓

$\boxed{\sigma_p = \pm\, \sigma_x w_x - \sigma_y w_y}$

## Determination of proportion of amount to be invested in two Securities (i.e. weights of securities) to minimise the Standard Deviation of portfolio. (Risk Averter / Risk Avoider)

$$\boxed{w_x = \frac{\sigma_y^2 - COV_{xy}}{\sigma_x^2 + \sigma_y^2 - 2COV_{xy}}} \qquad \boxed{w_y = 1 - w_x}$$

If $CORR_{xy}$ is $+1$

$$\boxed{w_x = \frac{\sigma_y}{\sigma_x - \sigma_y}}$$

If $CORR_{xy} = -1$

$$\boxed{w_x = \frac{\sigma_y}{\sigma_x + \sigma_y}}$$

★ If $CORR_{xy}$ is $+1$ or $-1$ (i.e. perfectly positive or perfectly negative respectively) & weights are properly assigned then Standard Deviation of portfolio will always be Zero (0)

## Standard Deviation of portfolio in three Security case

$$\boxed{\sigma_{xyz} = \sqrt{\underset{①}{\sigma_x^2 w_x^2} + \underset{②}{\sigma_y^2 w_y^2} + \underset{③}{\sigma_z^2 w_z^2} + \underset{④}{2\sigma_x\sigma_y w_x w_y CORR_{xy}} + \underset{⑤}{2\sigma_y\sigma_z w_y w_z CORR_{yz}} + \underset{⑥}{2\sigma_x\sigma_z w_x w_z CORR_{xz}}}}$$

If 'n' no. of Securities are there in portfolio then there will be $\boxed{\frac{n(n-1)}{2}}$ formulations

($2\sigma_x\sigma_y w_x w_y CORR_{xy}$ वाला Part)

& Total formulations $= \boxed{n + \frac{n(n-1)}{2}}$

For example

there are (in above case) 3 securities

So No. of formulations will be $\frac{n(n-1)}{2} = \frac{3(3-1)}{2} = \frac{6}{2} = \underline{3}$ (i.e. 4, 5, 6)

& total formulations $= 3 + 3 = \underline{6}$ (1, 2, 3, 4, 5, 6)

## Standard Deviation of portfolio of 'n' Securities
(where 'n' is comparatively large number)

$$\boxed{\sigma_p = \sqrt{(\sigma_1^2 w_1^2 \times n) + 2\sigma_1\sigma_2 w_1 w_2 \, CORR_{12} \times \frac{n(n-1)}{2}}}$$

It has been assumed that σ (standard deviation), weights, & CORR between securities are 'same'

## Standard Deviation of portfolio of 'n' Securities
(where 'n' is a very large number)

In this case we will calculate variance (i.e. $\sigma^2$)

$$\boxed{\sigma_p^2 = \frac{1}{n}(\text{Average variance}) + \left(1 - \frac{1}{n}\right)(\text{Average co-variance})}$$

∵ n is a very large no.
so $\frac{1}{n}$ will be a very small no. (almost equals to zero)

So $\boxed{\sigma_p^2 = \text{Average co-variance}}$

Remember :-

1) If Return of two securities are different but 'σ' is same then Security with Higher Return is preferred.

2) If return of two securities are same but 'σ' (SD) are different then Security with Lower 'σ' (SD) is preferred

3) If return & Risk (i.e. σ) both are different then COV will be calculated & Lower CoV should be preferred.

# Capital Asset Pricing model (CAPM)

$$\boxed{R_p = R_f + \beta(R_m - R_f)}$$

- $R_f$ → Risk free rate of return
- $\beta$ → Sensitivity of a security
- $R_m$ → Market Rate of Return

$$\boxed{\frac{R_p - R_f}{\beta} = R_m - R_f}$$

- $R_m - R_f$ → Market Risk Premium
- $\frac{R_p - R_f}{\beta}$ → Reward to Risk Ratio

If Reward to Risk Ratio of a security = Market Risk Premium
↳ Security is '<u>correctly priced</u>'

If Reward to Risk Ratio of a security > Market Risk Premium
↳ Security is '<u>under-priced</u>' & should be '<u>Purchased</u>'.

If Reward to Risk Ratio of a security < Market Risk Premium
↳ <u>Security</u> is Over-priced & should not be Purchased

While calculating Reward to Risk Ratio i.e. $\frac{R_p - R_f}{\beta}$
<u>$R_p$ will not be calculated as per CAPM (i.e. $R_p = R_f + \beta(R_m - R_f)$)</u>
Because if we calculate $R_p$ as per CAPM then security will always be correctly priced, So instead of CAPM return we will use likely return [i.e. $R_p = D_1 + (P_1 - P_0)$]

## Market Model
(Critism of CAPM)

as per this model $\boxed{R_p = \alpha^* + R_f + \beta(R_m - R_f)}$

Jensen's alpha

$$\boxed{\alpha^* = \alpha - R_f(1-\beta)}$$

$$\boxed{\alpha = \overline{R_p} - \beta(\overline{R_m})}$$

Based on Average past performance

- $\alpha^*$ can be negative also

- Characteristic line or line of Regration

assuming that weights are equal (of past years)

then $\alpha = \overline{R_p} - \beta(\overline{R_m})$

$\Rightarrow \alpha = R_p - \beta(R_m)$

& $\boxed{R_p = \alpha + \beta(R_m)}$

This is called as characteristic line or line of regration.

- Portfolio Risk Premium (as per market model)

$$\boxed{R_p - R_f = \alpha^* + \beta(R_m - R_f)}$$

- Risk premium will be different in CAPM & market model (except in case $\alpha^* = 0$)

But relationship between change in Risk premium will always same in CAPM & market model.

**Example :** Let $\alpha = 4$, $R_f = 6\%$, $\beta = 2$, $R_m = 10$

So as per CAPM

$$\text{Risk premium} = \beta(R_m - R_f)$$
$$= 2(10-6) = 8\%$$

Now if $R_m$ increases to 12% than

$$\text{Risk premium} = \beta(R_m - R_f)$$
$$= 2(12-6) = 12\%$$

Change in Risk premium = $12 - 8 = 4\%$

as per market model

$$\text{Risk premium} = \alpha^* + \beta(R_m - R_f)$$
$$\alpha^* = \alpha - R_f(1-\beta)$$
$$= 4 - 6(1-2)$$
$$= 4 + 6 = 10$$
$$\Rightarrow 10 + 2(10-6)$$
$$\Rightarrow 18\%$$

If $R_m$ increases to 12% than

$$\text{Risk premium} = 10 + 2(12-6)$$
$$= 10 + 12 = 22\%$$

Change in Risk premium = $22 - 18 = 4\%$

Same

## $\beta$ (Beta)

$$\boxed{\beta_A = \beta_e w_e + \beta_d w_d}$$

$\beta_e$ → β of equity
$\beta_d$ → β of debt
(w = weights)

$\beta_A$ → β of Total company (or β of asset)

Change in capital structure will only change equity and debt 'β' But not overall 'β' means overall 'β' will be same It is not affected by change in capital structure.

- Higher the 'β' more is the Risk.
- β can be negative (But rarely, when the security is negatively or inversely co-related with market) example: Gold
- Market β is always assumed to be '1'

## Ungeared v/s Geared Beta

Ungeared co. = Unleveraged co. = means co. financed by equity only.

$\beta_{uge}$ = $\beta_e$ of an Ungeared co.

Geared co. & leverage co. = means co. financed by debt also

$\beta_{ge}$ = β of geared co.

$$\boxed{\beta_{uge} = \beta_{ge} \times \frac{E}{E + D(1-t)}}$$



## Some Important Relationships

a)
$$\boxed{\beta_s = CORR_{sm} \times \frac{\sigma_s}{\sigma_m}}$$

Or $\beta_s = CORR_{sm} \times \frac{\sigma_s}{\sigma_m} \times \frac{\sigma_m}{\sigma_m}$

or
$$\boxed{\beta_s = \frac{COV_{sm}}{\sigma_m^2}}$$

b)
$$\boxed{COV_{AB} = CORR_{AB} \times \sigma_A \times \sigma_B}$$

or

$$\boxed{CORR_{AB} = \frac{COV_{AB}}{\sigma_A \times \sigma_B}}$$

$$\boxed{COV_{AB} = \frac{COV_{Am} \times COV_{Bm}}{\sigma_m^2}}$$

$$\left(\because CORR_{AB}\,\cancel{\sigma_A}\,\cancel{\sigma_B} = \frac{CORR_{Am}\,\cancel{\sigma_A}\,\cancel{\sigma_m} \times CORR_{Bm}\,\cancel{\sigma_B}\,\cancel{\sigma_m}}{\cancel{\sigma_m^2}}\right)$$

so $\boxed{CORR_{AB} = CORR_{Am} \times CORR_{Bm}}$

$$\boxed{COV_{AB} = \beta_A \beta_B \sigma_m^2}$$

$$\left(\because COV_{AB} = \underbrace{\frac{COV_{Am}}{\sigma_m^2}}_{\beta_A} \times \underbrace{\frac{COV_{Bm}}{\sigma_m^2}}_{\beta_B} \times \sigma_m^2\right)$$

## Systematic Risk & Unsystematic Risk

Total Risk $= \boxed{\sigma_s^2}$ or $\boxed{\sigma_s}$

Systematic Risk $= \boxed{\beta_s^2 \sigma_m^2}$ or $\boxed{\beta_s \sigma_m}$

Unsystematic Risk $= \boxed{\sigma_s^2 - \beta_s^2 \sigma_m^2}$ or $\boxed{\sigma_s - \beta_s \sigma_m}$

## Calculation of Overall cost of capital of company

Method I

$$\boxed{K_o = K_e W_e + K_d W_d}$$

(Ke & Kd can be calculated Ke by CAPM & Kd normally)

Method II

$$\boxed{K_o = R_f + \beta_A (R_m - R_f)}$$

## Arbitrage Pricing Theory (APT)

$$\text{Expected Return (as per APT)} = R_f + \left[\beta_1 \times \text{Change in Risk factor}_1 + \beta_2 \times \text{Change in Risk factor}_2 + \ldots + \beta_n \times \text{Change in Risk factor}_n\right]$$

## Arbitrage pricing line

$$\boxed{R_p = R_f + (\text{Change in Risk factor} \times \beta)}$$

## Random Walk Theory

This theory states that behaviour of stock market-prices is unpredictable & there is no relationship between the present prices of shares and their future prices.

In the layman's language "prices of stock market behave exactly the way a drunk would behave while walking in a blind lane.

As per this theory no connection can be established between two successive peaks (high price of stock) & troughs (low price of stock).

## Markowitz model of Risk - Return optimization

As per Harry Markowitz (father of modern portfolio theory)

- Investors are mainly concerned with two properties of an asset : Risk & Return
- By diversification of portfolio it is possible to trade off between them.
- Risk of an individual asset hardly matters to an investor. What really matters is the contribution it makes to investors' Risk.

## Sharpe's optimal portfolio.

- Sharpe provided a model for selection of appropriate securities in a portfolio.
- The selection of any stock is directly related to its reward to Risk Ratio i.e. $\boxed{\dfrac{R_p - R_f}{\beta}}$

- cut off point ⇒ $\boxed{C = \dfrac{\sigma_m^2 \times \sum \dfrac{(R_i - R_f)\beta}{\sigma_{ei}^2}}{1 + \left[\sigma_m^2 \times \sum \dfrac{\beta_i^2}{\sigma_{ei}^2}\right]}}$

$C^*$ (i.e. Highest value of C)

($\sigma_{ei}^2$ = Unsystematic Risk of stock)

C will be calculated for each stock & highest value of C will be cut off point i.e. $C^*$

The stock ranked above $C^*$ will be selected.

If No. of stocks is larger, there is no need to calculate $C_i$ value of all stocks after ranking has been done.

Once $C^*$ found the calculatn can be terminated.

Now the proportions of each of the selected securities (ranked above $C^*$) will be calculated

$$\Rightarrow \boxed{W_i = \frac{Z_i}{\Sigma Z_i}}$$

$$\Rightarrow \boxed{Z_i = \frac{\beta_i^2}{\sigma_{ei}^2}\left[\left(\frac{R_i - R_f}{\beta_i}\right) - C^*\right]}$$



Diversificatn means "Don't put all your eggs in one Basket".
means investing in more than one security
It reduces risk.
Greater the diversificatn lower will be the risk.