Chapter 1
Linear Programming

Simplex Method:-

Steps:-

1. Determine the objective function Z. Objective may be maximization or minimization.

2. For maximation problem the constraints would be ≤ sign.
   For minimization problem the constraints would be ≥ sign.

3. Introduce slack variable
   For ≤ sign – add the slack variable ie. Add S1
   For ≥ sign – subtract the slack variable and add artificial variable
   ie. Subtract S1, add A1.
4. Change the Objective function
   For S1 – Add '0S1'
   For A1 – Add 'MA1'
5. Simplex table format:-

		Cj						
Quantity	Variable	Const.	X	Y	Z	S1	S2	RR
	S1							
	S2							
		Zj						
		Cj – Zj						

6. Zj is arrived by summation of constant column with X,Y,Z columns

7. Criteria for selecting the key column :-
   For Maxima ion Problem – Highest value of Cj – Zj
   For Minimization Problem – Lowest Value of Cj – Zj

8. Divide the Quantity Column with Key column to arrive at RR

9. Criteria for Selecting the Key row :-
   For Maximation & Minimization Problem – Lowest Positive RR is selected

10. The Meeting Point is key Element

11. Criteria for deciding the optimal solution
    For Maximation Problem – All elements in Cj – Zj row is negative or zero.
    For Minimization Problem – All elements in Cj – Zj row is positive or zero

Note – For finding whether all the elements in Cj – Zj row is positive or zero

for minimization problem substitute all the 'M' with highest value.

12. If solution is not reached next table is formed.

13. Input for next table is
First key row in the next table is filled by dividing all the numbers in the key row of the previous table with the key element.
Remaining all the rows is arrived as follows: -

Corresponding previous Table row element	_ (Value relating to that row in the key column	* Corresponding element in key row in the 2nd table as filled in previous step)

14. Check the optimal solution, if not reached form the third table.

15. If solution is reached then answer is amount in quantity column corresponding to the variable.

Other Points : -

- We can convert the Minimization Problem into Maximation Problem. This is known as duality.
- We can change the $\geq$ sign to $\leq$ sign to match the problem
  E.g. $X + Y \leq 100$
  is converted into $-X - Y \geq -100$

Question 1
What are the practical applications of Linear programming?
Answer
Linear programming can be used to find optional solutions under constraints.

I. **In production:**
   a. pdt. mix under capacity constraints to minimise costs/maximise profits along with marginal costing.
   b. Invent-ory management to minimise holding cost, warehousing / transporting from factories to warehouses etc.
II. **Sensitivity Analysis:**
   a. By providing a range of feasible solutions to decide on discounts on selling price, decisions to make or buy.
III. **Blending:** Optional blending of raw materials under supply constraints.
IV. **Finance:** Portfolio management, interest/receivables management.
V. **Advertisement mix:** In advertising campaign – analogous to pdn. management and pdt. mix. Assignment of personnel to jobs and resource allocation problems. However, the validity will depend on the manager's ability to establish a proper linear relationship among variables considered.

Question 2
Explain the concept and aim of theory of constraints. What are the key measures of theory of constraints?
Answer
The theory of constraints focuses its attention on constraints and bottlenecks within organisation which hinder speedy production. The main concept is to maximize the rate of manufacturing output is the

throughput of the organisation. This requires to examine the bottlenecks and constraints. A bottleneck is an activity within the organization where the demand for that resource is more than its capacity to supply.

A constraint is a situational factor which makes the achievement of objectives / throughput more difficult than it would otherwise, for example of constraint may be lack of skilled labour, lack of customer orders, or the need to achieve high quality in product output.

For example let meeting the customers' delivery schedule be a major constraint in an organisation. The bottleneck may be a certain machine in the factory. Thus bottlenecks and constraints are closely examined to increase throughput.

Key measures of theory of constraints:

i. **Throughput contribution:** It is the rate at which the system generates profits through sales. It is defined as, sales less completely variable cost, sales – direct are excluded. Labour costs tend to be partially fixed and conferred are excluded normally.
ii. **Investments:** This is the sum of material costs of direct materials, inventory, WIP, finished goods inventory, R & D costs and costs of equipment and buildings.
iii. **Other operating costs:** This equals all operating costs (other than direct materials) incurred to earn throughput contribution. Other operating costs include salaries and wages, rent, utilities and depreciation.

Question 3

Write short notes on the characteristics of the dual problem.

Answer

Characteristics of the dual problem:

i. For any linear programming model called primal model, there exists a companion model called the dual model.
ii. The number of constraints in the primal model equals the number of variables in the dual model.
iii. The number of variables in the primal problem equals the number of constraints in the dual model.
iv. If the primal model is a maximization problem then the dual model will be of the form less than or equal to, ">=" while the restrictions in the dual problem will be of the form-greater than or equal to, ">="
v. The solution of the prima; model yields the solution of the dual model. Also, an optimal simplex table for the dual model yields the optimal solution to the primal model. Further, the objective functions of the two optimal tables will have identical values.
vi. Dual of the prima's dual problem is the primal problem itself. Feasible solutions to a primal and dual problem are both optimal if the complementary slackness conditions hold, that is, (value of a primal variable) x (value of the corresponding dual surplus variable) = 0 or (value of a primal slack variable) x (value of the corresponding dual variable) = 0.
If this relationship does not hold, than either the primal solution or the dual solution or both are no optimal.
vii. If the primal problem has no optimal solution because of infeasibility, then the dual problem will have no optimal solution because of unboundedness.
viii. If the primal has no optimal solution because of unboundedness, then the dual will have no optimal solution because of infeasibility.

Chapter 2
The Transportation Problem

- The procedure followed is Minimization Procedure
- Problem is generally solved in Vogel's Approximation Method(VAM)
- **Steps for the problem is : -**
  1. Convert profit matrix into loss matrix.
  2. Balance the problem.
  3. Arrive at Row penalty and column penalty

  Row penalty and column penalty is calculated at (2^{nd} least – 1^{st} least) in the corresponding row or column.
  4. Select from the entire Row penalty and column penalty maximum number.
  5. From the entire Row or Column minimum is selected.
  6. Strike the row or column which gets eliminated.
  7. Continue until the entire item in the table is strike.
  8. Write separately Initial solution table.
  9. Check for Degeneracy. Degeneracy occurs when all the elements in the initial solution is equal to (Row + column – 1)
  10. If degeneracy occurs introduce efcilon – 'e'. 'e' is introduced in least independent cell.
  11. Form UV Matrix. It is formed by the element in the original solution corresponding to the element in the Initial solution.
  12. Find unalloted elements in the UV Matrix
  13. Find △Ij i.e.(Original Matrix element – Unalloted element found above)
  14. Check for optimal solution ie. All items must be zero or positive.
  15. If not reached select the maximum negative in △Ij matrix.
  16. Form a loop and reallocate the solution.
  17. Repeat from step 9.

Notes: -

1. If there is zero in △Ij matrix while arriving at optimal solution then there is another solution for the problem.
2. Dummy column can be introduced in profit or loss matrix.
3. If there is penalty/redundancy payment for unsatisfying demand etc. is given then fills the dummy row or column with that amount or fill it with zeros.
4. If there is constraint in the problem first satisfy the constraint and then solve.
5. various other methods for solving the problem is
   - Least cost method
   - North west corner rule
6. Generally VAM method is used

Question 1

State the methods in which initial feasible solution can be arrived at in a transportation problem

Answer

The methods by which initial feasible solution can be arrived at in a transportation model are as under:
(i) North West Corner Method.
(ii) Least Cost Method
(iii) Vogel's pproximation Method (V M)

Question 2
Explain the term degeneracy in a transportation problem.

Answer
If a basic feasible solution of transportation problem with m origins and n destinations has fewer than m + n – 1 positive xij (occupied cells) the problem is said to be a degenerate transportation problem. Such a situation may be handled by introducing an infinitesimally small allocation e in the least cost and independent cell. While in the simple computation degeneracy does not cause any serious difficulty, it can
cause computational problem in transportation problem. If we apply modified distribution method, then the dual variable ui and vj are obtained from the Cij value to locate one or more Cij value which should be equated to corresponding Cij + Vij.

Question 3
Will the initial solution for a minimization problem obtained by Vogel's pproximation Method and the Least Cost Method be the same ? Why?
Answer
The initial solution need not be the same under both methods. Vogel's pproximation Method uses the diff-erences between the minimum and the next minimum costs for each row and column.This is the penalty or opportunity cost of not utilising the next best alterative. The highest penalty is given the 1st preference . This need not be the lowest cost.
For example if a row has minimum cost as 3, and the next minimum as 2, penalty is 1; whereas if another row has minimum 4 and next minimum 6, penalty is 2, and this row is given preference. But least cost given preference to the lowest cost cell, irrespective of the next cost.
Vogel's pproximation Method will to result in a more optimal solution than least cost They will be the same only when the maximum penalty and the minimum cost coincide.

Chapter 3
The Assignment Problem

1) Basis of Technique used is minimization Technique
2) It can also be done in maximation Technique
3) Various steps in Assignment Problem are

Step 1: Check whether the problem is balanced or unbalanced by checking whether row is equal to column, if unbalanced add dummy column or row to balance the problem
Step 2: Identify Least Number in each row and subtract with all number in that Row.
Step 3: Identify least number of each column and subtract with all number in that column.
Step 4: Check whether solution is reached with zero selection in one row and column, ie. Cover all the zero with minimum number of lines, solution is reached only when selected zeros is equal to number of rows or columns or number of lines is equal to order of matrix.
Step 5: If solution is not reached so maximum sticking
Step 6: Select the least element from the uncovered area Element
Step 7: The element selected above is

i) Subtracted with all the uncovered area item element
ii) Added to all the intersection point element (Intersection of two lines)

Step 8: Check the solution

Step 9: If solution is not reached continue with the process from step 5.

Question 1

In an assignment problem to assign jobs to men to minimize the time taken, suppose that one man does not know how to do a particular job, how will you eliminate this allocation from the solution?

Answer

In an assignment minimization problem, if one task cannot be assigned to one person, introduce a prohibit-tively large cost for that allocation, say M, where M has a high the value. Then, while doing the row minimum and column minimum operations, automatically this allocation will get eliminated.

Chapter 4
Critical Path Analysis

Network Analysis (CPM/PERT)

CPM

- Total float = LS – ES (or) LF – EF
- Free float = Total float – Head event slack
- Independent float = Free float – Tail event slack
- In the diagram Es = Lf in the critical path
- Critical path is the longest duration
- To find the minimum time associated cost (i.e. Additional cost incurred per unit of time saved) following formula is used :-
  Crash cost per day (or) Activity cost supply
  $$= \frac{\text{Crash cost – Normal cost}}{\text{Normal time – Crash time}}$$
- Interfacing float = It is the part of the total float which causes reduction in the float of the succession activities. In other words it is the portion of activity float which cannot be continued without affecting adversely the float of the subsequent activity or activities.
- Steps in proceeding the problem : -
  1. First find and fill the ES and LF column from the diagram.
  2. Then find LS and EF as follows :-
     Ls = Lf – Duration
     Ef = Es + Duration
  3. Find total float
  4. Find free float. Wherever total float column has zero free float column is also taken has zero and remaining elements is filled as said above
  5. Find Independent float. Wherever free float column has zero Independent float column is also taken has zero and remaining elements is filled as said above

Notes: -

1. ES = Earliest Start. Indicates earliest time that the given activity can be scheduled
2. EF = Earliest Finish. Time by which the activity can be completed at the earliest.
3. LF = Latest Finish. Latest allowable occurrence time of the head event of the activity.
4. LS = Latest Start.
5. Total duration of the critical path is the maximum time/amount consumed for the activity. This should be crashed with respect to crashing days and crashing cost. This crashing should not change the critical path.

PERT : -

- Expected (or) Average time is found by assigning weights as follows : -

  1 for optimistic
  4 for Most likely
  1 for pessimistic

  $$\text{Average time} = \frac{1 \text{ optimistic} + 4 \text{ most likely} + 1 \text{ pessimistic}}{6}$$

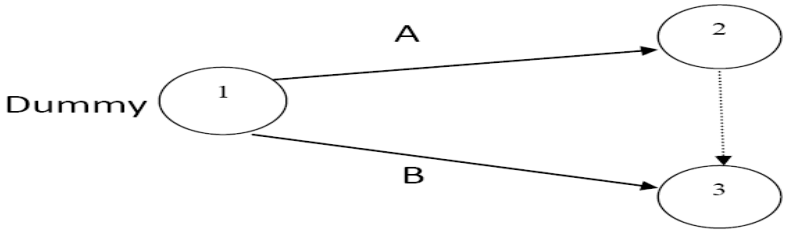
- Standard Deviation = $\frac{\text{Pessimistic time} - \text{Optimistic time}}{6}$
- Variance = $(\text{Standard Deviation})^2$
- Probability of completing the project in N days

  $$= \frac{\text{Required time(N) (-) Expected time (critical path duration)}}{\text{Standard Deviation}}$$

  [Nothing but Z = (X - Mean) / Standard deviation]

  = Y (say)
  = Find Z(y)
  = Probability %
  - If required time > Expected time then = 0.5 + Z(Y)
  - If required time < Expected time then = 0.5 – Z(Y)

Question 1
Explain the following in the context of a network:
(i) Critical path
(ii) Dummy activity.
Answer
(i) Critical Path: Critical Path is a chain of activities that begin with the starting event and ends with ending
event of a particular project. It is that path that runs through a network with the maximum length of time or it indicates the maximum possible time required for completion of a project. Critical path indicates the minimum time that will be required to complete a project. It is determined after identifying critical events. Critical path goes through critical events.
(ii) Dummy Activities: Dummy Activity is that activity which does not consume time or resources. It is used when two or more activities have same initial and terminal events. As a result of using dummy activities, other activities can be identified by unique end events. These are usually shown by arrows with dashed lines.



Question 2

List the 5 steps involved in the methodology of critical path analysis.

Answer

Working Methodology of PERT:

The working methodology of PERT which includes both CPM and PERT, consists of following five steps:

- Analyze and break down the project in terms of specific activities and/or events.
- Determine the interdependence and sequence of specific activities and prepare a net-work.
- Assign estimates of time, cost or both to all the activities of the network.
- Identify the longest or critical path through the network.
- Monitor, evaluate and control the progress of the project by re-planning, rescheduling and reassignment of resources.

Chapter 5
Program Evaluation and Review Technique

Question 1

Write short notes on Distinction between PERT and CPM.

Answer

Distinction between PERT and CPM:

The PERT and CPM models are similar in terms of their basic structure, rationale and mode of analysis. However, there are certain distinctions between PERT and CPM networks which are enumerated below:

i. CPM is activity oriented i.e. CPM network is built on the basis of activities. Also results of various calculations are considered in terms of activities of the project. On the other hand, PERT is even oriented.

ii. CPM is a deterministic model i.e. it does not take into account the uncertainties involved in the estimation of time for execution of a job or an activity. It completely ignores the probabilistic element of the problem. PERT, however, is a probabilistic model. It uses three estimates of the activity time; optimistic, pessimistic and most likely, with a view to take into account time uncertainty. Thus, the expected duration for each activity is probabilistic and expected duration indicates that there is fifty per probability of getting the job done within that time.

iii. CPM laces dual emphasis on time and cost and evaluates the trade-off between project cost and project item. By deploying additional resources, it allows the critical path project manager to manipulate project duration within certain limits so that project duration can be shortened at an optimal cost. On the other hand, PERT is primarily concerned with time. It helps the manger to schedule and coordinate various activities so that the project can be completed on scheduled time.

iv. CPM is commonly used for those projects which are repetitive in nature and where one has prior experience of handling similar projects. PERT is generally used for those projects where time required to complete various activities are not known as prior. Thus, PERT is widely used for planning and scheduling research and development project.

Question 2

Define a project and briefly explain the four common implications which characterize a project, and state the five steps of the working methodology of critical path analysis.

Answer

A project can be defined as a set of activities or jobs that are performed in a certain sequence determined logically or technologically and it has to be completed within

(i) a specified time,
(ii) a specified cost and
(iii) meeting the performance standards.

Examples of a project from fairly diverse fields could be cited. Some of them are given below:

1. Introducing a new product in the market.
2. Construction of a new bridge over a river or construction of a 25 – storied building.
3. Executing a large and complex order on jobbing production.
4. Sending a spacecraft to the mars.

All these projects are characterized by the following set of common implications, although they pertain to widely different fields.

i. The Large-scale characteristic: These projects are generally unusually large and complex. Thousands of suppliers, workers and other categories of persons are involved and their efforts have to be coordinated for completion of the project.
ii. The non-recurring characteristic: These projects are generally of a one -time nature. Neither in the past, nor in the future they are likely to undertaken substantially in the same form.
iii. Uncertain and critical dates: During of the various activities involved in such projects are usually uncertain. Further in such type of projects, many critical dates exits by which operations must be completed in order to complete the entire project on schedule.
iv. Completion dead line: The fourth distinct feature of these projects is that there is dead line for the completion of the entire project. In case of any delay in the completion of the project, some penalty is levied for such delay beyond the dead line.

The working methodology of Critical Path Analysis (CPA) which includes both CPM and PERT, consists of following five steps:

- Analyse and breakdown the project in terms of specific activities and / or events.
- Determine the interdependence and sequence of specific activities and prepare a network, Assign estimates of time, cost or both to all the activities of the network.
- Identify the longest or critical path through the network.
- Monitor, evaluate and control the progress of the project by re -planning, rescheduling and reassignment of resources.

Chapter 6
SIMULATION

Question 1
(i) What is simulation?
(ii) What are the steps in simulation?
Answer
Simulation is a quantitative procedure which describes a process by developing a model of that process and then conducting a series of organized trial and error experiments to product the behaviour of the process over time.
Steps in the simulation process:

- Define the problem and system you intend to simulate.
- Formulate the model you intend to use.
- Test the model, compare with behaviour of the actual problem environment.
- Identify and collect data to test the model.
- Run the simulation.
- Analyse the results of the simulation and, if desired, change the solution you are evaluating.
- Rerun the simulation to tests the new solution.
- Validate the simulation i.e., increase the chances of valid inferences.

Question 2
How would you use the Monte Carlo Simulation method in inventory control?
Answer
The Monte Carlo Simulation:
It is the earliest mathematical Model of real situations in inventory control:
Steps involved in carrying out Monte Carlo simulation are:

i. Define the problem and select the measure of effectiveness of the problem that might be inventory shortages per period.
ii. Identify the variables which influence the measure of effectiveness significantly for example, number of units in inventory.
iii. Determine the proper cumulative probability distribution of each variable selected with the probability on vertical axis and the values of variables on horizontal axis.
iv. Get a set of random numbers.
v. Consider each random number as a decimal value of the cumulative probability distribution with the decimal enter the cumulative distribution plot from the vertical axis. Project this point horizontally, until it intersects cumulative probability distribution curve. Then project the point of intersection down into the vertical axis.
vi. Then record the value generated into the formula derived from the chosen measure of effectiveness. Solve and record the value. This value is the measure of effectiveness for that simulated value. Repeat above steps until sample is large enough for the satisfaction of the decision maker.

Question 3
State major reasons for using simulation technique to solve a problem and also describe basic steps in a general simulation process.
Answer
Reasons:

i. It is not possible to develop a mathematical model and solutions with out some basic assumptions.
ii. It may be too costly to actually observe a system.
iii. Sufficient time may not be available to allow the system to operate for a very long time.
iv. Actual operation and observation of a real system may be too disruptive.

Steps:

i. Define the problem or system which we want to simulate.
ii. Formulate an appropriate model of the given problem.
iii. Ensure that model represents the real situation/ test the model, compare its behavior with the behavior of actual problem environment.
iv. Identify and collect the data needed to list the model.
v. Run the simulation
vi. Analysis the results of the simulation and if desired, change the solution.
vii. Return and validate the simulation.

Question 4

Write a short note on the advantages of simulation.

Answer

Advantages of simulation are enumerated below:

i. Simulation techniques allow experimentation with a model of the system rather than the actual operating system. Sometimes experimenting with the actual system itself could prove to be too costly and, in many cases too disruptive. For example, if you are comparing two ways of providing food service in a hospital, the confusion that would result from operating two different systems long enough to get valid observations might be too great. Similarly, the operation of a large computer centralunder a number of different operating alternatives might be too expensive to be feasible.
ii. The non-technical manage can comprehend simulation more easily than a complex mathematical model. Simulation does not require simplifications and assumptions to the extent required in analytical solutions. A simulation model is easier to explain to management personnel since it is a description of the behaviour of some system or process.
iii. Sometimes there is not sufficient time to allow the actual system to operate extensively. For example, if we were studying long-term trends in world population, we simply could not wait the required number of years to see results. Simulation allows the manger to incorporate time into an analysis. In a computer simulation of business operation the manager can compress the result of several years or periods into a few minutes of running time.
iv. Simulation allows a user to analyze these large complex problems for which analytical results are not available. For example, in an inventory problem if the distribution for demand and lead time for an item follow a standard distribution, such as the poison distribution, then a mathematical or analytical solution can be found. However, when mathematically convenient distributions are not applicable to the problem, an analytical analysis of the problem may be impossible. A simulation model is a useful solution procedure for such problemes

Chapter 7
Learning Curve Theory

Learning is the process of acquiring skill, Knowledge, and ability by an individual. According to learning curve theory the productivity of the worker increases with increase in experience due to learning effect. The learning theory suggests that the best way to master a task is to "learn by doing". In other words, as people gain experience with a particular job or project they can produce each unit more efficiently than the preceding one.

The speeding up of a job with repeated performance is known as the learning effect or learning curve effect.

The cumulative average time per unit produced is assumed to fall by a constant percentage every time the total output is doubled. So generally learning effect is found in the multiples of 2. If learning curve effect is asked between two even numbers then Learning curve equation is formed ie. Learning curve effect is expressed mathematically as follows:

Learning curve equation =

$Y = a(x)^{-b}$ Where Y = Average time per unit
a = Total time for first unit
x = Cumulative number of units manufactured
b = the learning curve index

$$\text{Learning curve index (b)} = \frac{\log (1 - \%\ \text{decrease})}{\log 2}$$

Question 1

Discuss the application of the learning curve.

Answer

Application of Learning curve: Learning curve helps to analyse cost-volume profit relationships during familiarisation phase of product or process to arrive at cost estimates. It helps in budgeting and profit planning.

It helps in pricing and consequent decision making – e.g. acceptance of an order, negotiations in establishing contract prices etc. with the advantage of the knowledge of decreasing unit cost.

It helps in setting standards in the learning phase.

Question 2

What are the distinctive features of learning curve theory in manufacturing environment? Explain the learning curve ratio.

Answer

As the production quantity of a given item is doubled, the cost of the item decreases at a fixed rate. This phenomenon is the basic premise on which the theory of learning curve has been formulated. As the quantity produced doubles, the absolute amount of cost increase will be successively smaller but the rate of decrease will remain fixed. It occurs due to the following distinctive features of manufacturing environment:

1. Better tooling methods are developed and used.
2. More productive equipments are designed and used to make the product.
3. Design bugs are detected and corrected.
4. Engineering changes decrease over time.
5. Earlier teething problems are overcome.

6. Rejections and rework tend to diminish over time.

In the initial stage of a new product or a new process, the learning effect pattern is so regular that the rate of decline established at the outset can be used to predict labour cost well in advance. The effect of experience on cost is summarized in the learning curve ratio or improvement ratio.

$$\text{Learning Curve ratio} = \frac{Average\ Labour\ cost\ of\ first\ 2N\ units}{Average\ labour\ cost\ of\ first\ N\ units}$$

Question 3

The following information is provided by a firm. The factory manager wants to use appropriate average learning rate on activities, so that he may forecast costs and prices for certain levels of activity.

(i) A set of very experienced people feed data into the computer for processing inventory records in the factory. The manager wishes to apply 80% learning rate on data entry and calculation of inventory.

(ii) A new type of machinery is to be installed in the factory. This is patented process and the output may take a year for full fledged production. The factory manager wants to use a learning rate on the workers at the new machine.

(iii)An operation uses contract labour. The contractor shifts people among various jobs once in two days. The labour force performs one task in 3 days. The manager wants to apply an average learning rate for these workers.

You are required to advise to the manager with reasons on the applicability of the learning curve theory on the above information.

Answer

The learning curve does not apply to very experienced people for the same job, since time taken can never tend to become zero or reduce very considerably after a certain range of output. This is the limitation of the learning curve.

- Data entry is a manual job so learning rate theory may be applied. Calculation of inventory is a computerized job. Learning rate applies only to manual labour.
- Learning rate should not be applied to a new process which the firm has never tried before.
- The workers are shifted even before completion of one unit of work. Hence learning rate will not apply.

Question 4

Explain distinctive features of learning curve theory in manufacturing environment.

Answer

The production quantity of a given item doubled the cost of that item decrease at a fixed rate. This phenomenon is the basic premise on which the theory of learning curve has been formulated.

The distinctive features of a learning curve are:

1. Better tooling methods are developed and used
2. More productive equipments are designed and used to make the product.
3. Design bugs are detected and corrected.
4. Better design engineering reduces material and labour costs.
5. Early teething problems are overcome. As production progresses management is prompted to achieve better planning and better management.
6. Rejections and rework tend to diminish over time.
7. As quantity produced increases the Cost per unit decreases, since each unit entails:
   - (i) Lesser labour
   - (ii) Greater productivity of material and labour
   - (iii) Fewer delays and lesser time losses.