

- **Return on a Security** consists of the dividend yield and capital gain.
- **Expected Rate of Return on a Security** is the sum of the products of possible rates of return and their probabilities. Thus

$$E(R) = R_1P_1 + R_2P_2 + \dots + R_nP_n = \sum_{i=1}^n R_iP_i$$

The expected rate of return is an average rate of return. This average rate may deviate from the possible outcomes (rates of return).

- **Variance and Standard Deviation of Returns of a Security** can be calculated as follows:

$$\sigma^2 = [R_1 - E(R)]^2 P_1 + [R_2 - E(R)]^2 P_2 + \dots + [R_n - E(R)]^2 P_n = \sum_{i=1}^n [R_i - E(R)]^2 P_i$$

$$\sigma = \sqrt{\sigma^2}$$

Variance or standard deviation is a measure of the risk of returns on a security.

- **Portfolios** Generally, investors in practice hold multiple securities. Combinations of multiple securities are called portfolios.

- **Expected Return on a Portfolio** is the sum of the returns on individual securities multiplied by their respective weights (proportionate investment). That is, it is a weighted average rate of return. In the case of a two-security portfolio, the portfolio return is given by the following equation:

$$E(R_p) = wR_x + (1 - w)R_y$$

In the case of n -security portfolio, the portfolio return will be as follows:

$$E(R_p) = w_1R_1 + w_2R_2 + \dots + w_nR_n = \sum_{i=1}^n w_iR_i$$

- **Covariance** is the product of the standard deviations of individual securities times their correlation coefficient.

$$\text{Cov}_{xy} = \text{cor}_{xy} s_x s_y$$

$$\text{Cor}_{xy} = \frac{\text{cov}_{xy}}{\sigma_x \sigma_y}$$

- **Portfolio Risk** is not a weighted average risk. Securities included in a portfolio are associated with each other. Therefore, the portfolio risk also accounts for the covariance between the returns of securities. The portfolio risk in the case of a two-security portfolio can be computed as follows:

$$\begin{aligned}
 s_p^2 &= w^2 s_x^2 + (1-w)^2 s_y^2 + 2(w)(1-w)\text{Cov}_{xy} \\
 &= w^2 s_x^2 + (1-w)^2 s_y^2 + 2(w)(1-w)s_x s_y \text{Cor}_{xy} \\
 s_p &= \sqrt{s_p^2}
 \end{aligned}$$

We may observe that the portfolio risk consists of the risk of individual securities plus the covariance between securities. Covariance depends on the standard deviation of individual securities and their correlation.

- **Magnitude of Portfolio Risk** will depend on the correlation between the securities. The portfolio risk will be equal to the weighted risk of individual securities if the correlation coefficient is +1.0. For correlation coefficient of less than 1, the portfolio risk will be less than the weighted average risk. When the two securities are perfectly negatively correlated, i.e., the correlation coefficient is -1.0, the portfolio risk becomes zero.
- **Minimum Variance Portfolio** is called the optimum portfolio. The following formula can be used to determine the optimum weights of securities in a two-security portfolio:

$$w_x^* = \frac{\sigma_y^2 - \text{Cov}_{xy}}{\sigma_x^2 + \sigma_y^2 - 2 \text{Cov}_{xy}}$$

w_x^* is the optimum weight of security x and $1 - w_x$ of security y .

- **Portfolio Risk in Case of n -security Portfolio** the portfolio risk can be calculated as follows:

$$\sigma_p^2 = \frac{1}{n} \times \text{average risk} + \left[1 - \frac{1}{n}\right] \times \text{average covariance}$$

As the number of securities in the portfolio increases, the portfolio variance approaches the average covariance. Thus diversification helps in reducing the risk.

- **Investment or Portfolio Opportunity Set** represents all possible combinations of risk and return resulting from portfolios formed by varying proportions of individual securities. It presents the investor with the risk-return trade-off.
- **Mean-Variance Criterion** for a given risk, an investor would prefer a portfolio with higher expected rate of return. Similarly, when the expected returns are

same, she would prefer a portfolio with lower risk. The choice between high risk–high return or low risk–low return portfolios will depend on the investor's risk preference. This is referred to as the mean-variance criterion.

- **Efficient Portfolio** is one that has the highest expected returns for a given level of risk. The efficient frontier is the frontier formed by the set of efficient portfolios.
- **Optimum Risky Portfolio and Separation Theorem** is the market portfolio of all risky assets where each asset is held in proportion of its market value. It is the best portfolio since it dominates all other portfolios. An investor can thus mix her borrowing and lending with the best portfolio according to her risk preferences. She can invest in two separate investments—a risk free asset and a portfolio of risky securities. This is known as the *separation theorem*.
- **Unsystematic Risk** can be eliminated through diversification. It is a risk unique to a specific security. When individual securities are combined, their unique risks cancel out.
- **Systematic Risk** cannot be eliminated through diversification. It is a market-related risk. It arises because individual securities move with the changes in the

market. Investors are risk averse. They will take risk only if they are compensated for the risk, which they bear. Since systematic risk can be eliminated through diversification, they will be compensated for assuming systematic risk.

- **Capital Market Line** the market prices securities in a manner that they yield higher expected returns than the risk-free securities. The risk-averse investors can be induced to hold risky securities when they are offered risk premium. The capital market line (CML) defines this relationship. The equation for CML is:

$$E(R_p) = R_f + \left[\frac{E(R_m) - R_f}{\sigma_m} \right] \sigma_p$$

where $E(R_p)$ is the portfolio return, R_f the risk-free return, $E(R_m)$ the return on market portfolio, σ_m the standard deviation of market portfolio and, σ_p the standard deviation of the portfolio.

Capital Market Line (CML) is an efficient set of risk-free and risky securities, and it shows the risk-return trade-off in the market equilibrium.

- **Capital Asset Pricing Model (CAPM)** the model explaining the risk-return relationship is called the capital asset pricing model (CAPM). It provides that in

a well-functioning capital market, the risk premium varies in direct proportion to risk.

- **Security's Beta** CAPM provides a measure of risk and a method of estimating the market's risk-return line. The market (systematic) risk of a security is measured in terms of its sensitivity to the market movements. This sensitivity is referred to the security's beta.

Beta reflects the systematic risk, which cannot be reduced. Investors can eliminate unsystematic risk when they invest their wealth in a well-diversified market portfolio. A beta of 1.0 indicates average level of risk while more than 1.0 means that the security's return fluctuates more than that of the market portfolio. A zero beta means no risk.

In terms of the security market line, beta is a ratio of the covariance of returns of a security, j , and the market portfolio, m , to the variance of return of the market portfolio:

$$b_j = \frac{\text{Cov}_{jm}}{\text{Var}_m} = \frac{s_j s_m \text{Cor}_{jm}}{s_m^2} = \frac{s_j \text{Cor}_{jm}}{s_m}$$

where β_j is beta of the security, σ_j the standard deviation of return of security, σ_m

the standard deviation of returns of the market portfolio, σ_m^2 the variance of returns of the market portfolio m and Cor_{jm} the correlation coefficient between the returns of the security j and the market portfolio m .

- **Characteristics Line** a line that is called the characteristics line can represent the relationship between the security returns and the market returns. The slope of the characteristics line is the sensitivity coefficient, which, as stated earlier, is referred to as *beta*.
- **Expected Return on a Security (*Security Market Line (SML)*)** is given by the following equation:

$$E(R_j) = R_f + (R_m - R_f)b_j$$

where R_f is the risk-free rate, R_m the market return and the measure of the security's systematic risk. This equation gives a line called the *security market line (SML)*.

- **Arbitrage Pricing Theory (APT)** the differences of securities' returns may not be fully explained by their betas. The arbitrage pricing theory (APT), resulting

from the limitations of CAPM, assumes that many macro-economic factors may affect the system risk of a security (or an asset). Thus, APM is a multi-factor model to explain the return and return of a security. The factors influencing security return may include industrial production, growth in gross domestic product, the interest rate, inflation, default premium, and the real rate of return. Price-to-book-value ratio and size have also been found to explain to the differences in the security returns.