FOUNDATION

PAPER 4

Business Mathematics
&
Statistics Fundamentals

COMPILATION

PAPER 4

BUSINESS MATHEMATICS & STATISTICS FUNDAMENTALS

COMPILATION

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Business Mathematics and Statistics Fundamentals

SECTION – I

ARITHMETIC

June 2010 Examination

Question:

1. Answer *any two* of the following:

   Choose the correct option showing the proper reasons/calculations.

   (a) If x is the mean proportional between x – 2 and x + 6 then the value of x is

   (i) 4 (ii) 3 (iii) 2 (iv) none of these

   (b) Of the five numbers the average of first four numbers is 8 and the average of the last four numbers is 6. Then the difference of the first and the fifth number is

   (i) 6 (ii) 8 (iii) 10 (iv) none of these

   (c) The true discount on a bill due in 6 months at 8% p.a. is Rs. 40. Then the amount of the bill is

   (i) Rs. 1,000 (ii) Rs. 1,200 (iii) 1,040 (iv) none of these

Answer to Question No. 1:

(a) $\frac{x-2}{x} = \frac{x}{x+6} \Rightarrow x^2 + 4x - 12 = x^2 \Rightarrow x = 3$ Ans (ii)

(b) Difference between the first and the fifth number is (8 x 4) – (6 x 4) = 8 Ans (ii)

(c) Here $TD = \frac{Ani}{1+ni}, TD = 40, n = \frac{6}{12} = \frac{1}{2}, i = \frac{8}{100}$

$$\therefore 40 = \frac{A.\frac{1}{2}.\frac{8}{100}}{1+\frac{1}{2}.\frac{8}{100}} = \frac{A}{26} \Rightarrow A = 26 \times 40 = 1040$$

Ans (iii)

Question:

2. Answer *any one* of the following:

   (a) Divide Rs. 6,200 in 3 parts such that the interest for the three parts for 2, 3 and 5 years respectively at 5% simple interest p.a. are same.

   (b) A dealer mixes two varieties of teas costing Rs. 100 per kg. and Rs. 160 per kg. in the proportion 5:1. He sold the 6 kg. mixture at the rate of Rs. 120 per kg. Find his profit.

Answer to Question No. 2(a):

1^{st} part is x, 2^{nd} part is y, 3^{rd} part is z

$\therefore\ x + y + z = 6200$

Interest on 1^{st} part = $x \times \frac{5}{100} \times 2 = \frac{x}{10}$

Interest on 2^{nd} part = $y \times \frac{5}{100} \times 3 = \frac{3y}{20}$

Interest on 3^{rd} part = $z \times \frac{5}{100} \times 5 = \frac{z}{4}$

$$\therefore \frac{x}{10} = \frac{3y}{20} = \frac{z}{4} = K$$

$$\therefore\ x = 10K, y = \frac{20K}{3}, z = 4K$$

$$\therefore 10K + \frac{20K}{3} + 4K = 6200 \Rightarrow K = 300$$

$$1^{st}\ \text{part} = 10 \times 300 = 3000, 2^{nd}\ \text{part} = 20 \times \frac{300}{3} = 2000, 3^{rd}\ \text{part} = 4300 = 1200$$

Answer to Question No. 2 (b):

Of 6 kg. mixture tea there are 5 kg of 1^{st} type costing Rs. 500 and 1kg. of 2^{nd} type costing Rs. 160. Thus cost price of 6 kg. of given mixture = 500 + 160 = 660 Rs. Their selling price = 6×120 = 720 Rs. His profit = 720 – 660 = 60 Rs.

December 2009 Examination

Question : 1.

(a) Answer *any two* of the following:

Choose the correct option showing the proper reasons / calculations.

(a) The number to be added to each term of the ratio 3:7 to make it 1:2 is

(i) 2, (ii) 1, (iii) 3, (iv) none of these.

(b) The average of 7 numbers is 27. If one number is included, the average becomes 25. The included number is

(i) 11, (ii) 10, (iii) 12, (iv) none of these.

(c) The time in which a sum of money becomes double at 10% p.a., simple interest is

(i) 8 years, (ii) 10 years, (iii) 12 years, (iv) none of these.

Answer to Question no. 1.

(a) $\frac{3+x}{7+x}=\frac{1}{2}\Rightarrow x=1$ Ans (ii)

(b) $\frac{\sum x}{7}=27\Rightarrow\sum x=189$

$25=\frac{189+x}{8}\Rightarrow x=200-189=11$ Ans (i)

(c) $A=2P, P=\text{Principal},\ i=\frac{10.0}{100}=0.1$

We know $A=P(1+in)\Rightarrow 2P=P(1+0.1n)\Rightarrow n=\frac{1}{0.1}=10(\text{in yrs}).$ Ans(ii)

Question : 2.

Answer *any one* of the following

(a) A bill for Rs. 2060 is due in 6 months. Calculate the difference between True Discount (TD) and Banker's Discount (BD), the rate of interest being 6%.

(b) In a liquid mixture 20% is water and in another mixture water is 25%. These two mixtures are mixed in the ratio 5:3. Find the percentage of water in the final mixture.

Answer to Question no. 2(a).

$$\text{T.D.}=\frac{\text{B.V}\times ni}{1+ni}=\frac{2060\times 1x6}{2\times 100\left(1+\frac{0.06}{2}\right)}=\frac{1030\times 6}{103}=\text{Rs.}60$$

$$\text{B.D.}=\text{B.V.}\times ni=\frac{2060\times 1\times 0.06}{2}=\frac{1030\times 6}{100}=\text{Rs.}61.80$$

B.D. – T.D. = 61.80 – 60 = Rs. 1.80

Answer to Question no. 2(b).

In the 1st pot, liquid : water = 80 : 20 = 4:1 as water = 20% $\therefore$ liquid = 80%

In the 2nd pot, liquid : water = 3:1

Let 5 litres of mixture from 1st pot be mixed with 3 litres of second pot

In $5l$ mixture, water $=\frac{1}{1+4}\times 5=1$

In $3l$ mixture, water $=\frac{1}{1+3}\times 3=\frac{3}{4}$

In 8l mixture, total water = $1+\frac{3}{4}=\frac{7}{4}$

In 100% mixture, total water = $\frac{7}{4}\times\frac{100}{8}=21\frac{7}{8}$

$\therefore$ Required percentage = $21\frac{7}{8}$

June 2009 Examination

Question: 1.

Answer *any two* of the following:

Choose the correct option showing the proper reasons / calculations.

(a) Let marks obtained by Ram, Rahim and Jodu be A, B and C respectively. Given A:B = 1:2, B:C = 3:4. The combined ratio A:B:C is

(i) 1:2:4, (ii) 3:6:8, (iii) 1:6:8, (iv) none of them.

(b) if $\frac{\sqrt{a}+\sqrt{b}}{\sqrt{a}-\sqrt{b}}=\frac{2}{1}$ then $\frac{a+b}{a-b}$ is equal to

(i) 5/4, (ii) 4/5, (iii) 3, (iv) none of them.

(c) The time, in which the true discount on amount Rs. 550 due is Rs. 50 at 4% per annum, is

(i) 2 yrs., (ii) 3 yrs., (iii) 2.5 yrs., (iv) none of them.

Answer to Question 1:

(a) B:C = 3:4 = $3\times\frac{2}{3}:4\times\frac{2}{3}=2:\frac{8}{3}$

A:B:C = 1:2: $\frac{8}{3}$ = 3:6:8 **Ans. (ii)**

(b) $\frac{\sqrt{a}+\sqrt{b}}{\sqrt{a}-\sqrt{b}}=\frac{2}{1}$. Then by componendo and dividendo

$$\frac{(\sqrt{a}+\sqrt{b})+(\sqrt{a}-\sqrt{b})}{(\sqrt{a}+\sqrt{b})-(\sqrt{a}-\sqrt{b})}=\frac{2+1}{2-1}=\frac{3}{1}\Rightarrow\frac{\sqrt{a}}{\sqrt{b}}=\frac{3}{1}\Rightarrow\frac{a}{b}=\frac{9}{1}$$

Again by componendo and dividendo $\frac{a+b}{a-b}=\frac{9+1}{9-1}=\frac{10}{8}=\frac{5}{4}$ **Ans. (i)**

(c) Let A = amount due at n years at the rate of r% per annum

True discount $= \frac{Ani}{1+ni}$ i.e., $50 = \frac{550 \times n \times 0.04}{1+n \times 0.04}$ A = 550

i = r%= 4% So 50(1+ 0.04n) = 22n ⇒50 + 2n = 22n

$$\Rightarrow n = \frac{50}{20} = 2.5 \text{ yrs.}$$ **Ans. (iii)**

Question: 2.

Answer *any one* of the following:

(a) if $\frac{x}{b+c} = \frac{y}{c+a} = \frac{z}{a+b}$ then show that

(b - c) (x - a) + (c - a) (y - b) + (a - b) (z - c)= 0

(b) A person borrowed Rs. 10,000 at some simple interest rate for 2 years. After expiry of one year he borrowed another Rs. 20,000 at 1% lower intrest rate for 1 year. At the end he paid fully Rs. 33,000. Find the rate of interest at which he borrowed first.

Answer to Question 2(a):

Let $\frac{x}{b+c} = \frac{y}{c+a} = \frac{z}{a+b} = k$ (constant). say

Then x = k(b + c), y= k(c + a), z = k(a + b)

So, (b - c) (x - a) + (c - a) (y - b) + (a - b) (z - c)

= [x(b - c) + y (c - a) + z(a - b)] - [a(b - c) + b(c - a) + c(a - b)]

= [k(b + c) (b - c) + k(c + a) (c - a) + k(a + b) (a - b)] - [ab - ac + bc - ab + ac - bc]

$$= \left[k(b^2 - c^2) + k(c^2 - a^2) + k(a^2 - b^2)\right] - 0$$

$$= \left[k(b^2 - c^2 + c^2 - a^2 + a^2 - b^2)\right] - 0$$

= k × 0 - 0 = 0 - 0 = 0 Proved

Answer to Question 2(b):

Let he borrowed Rs. 10,000/- @ r% per annum for 2 years and at the end of one year he borrowed Rs. 20,000/- @ (r - 1)% per annum for 1 year.

Then at the end of 2 years he paid Rs. 10,000/- with interest (simple) for 2 years @ r% and Rs. 20,000/- with interest (simple) for 1 year @ (r - 1)%.

Then the amount he paid fully at the end

$$= 10000\left(1+\frac{2r}{100}\right)+20000\left(1+\frac{r-1}{100}\right) = 30000+200r+200(r-1)$$

= 30000 + 400 r - 200 Rs.

Thus 30000 + 400 r - 200 = 33000

$\Rightarrow 400r = 3200 \Rightarrow r = 8$

So he borrowed first Rs. 10,000/- @ 8% per annum.

December 2008 Examination

Question: 1.

1. Answer *any three* of the following:

Choose the correct option showing the proper reasons/calculations.

(a) Two numbers are in the ratio 3:4. If 10 is subtracted from both of them the ratio will be 1:2. So the numbers are

(A) 15 and 20, (B) 12 and 16, (C) 30 and 40, (D) none of them.

(b) The mean of age of 5 men is 40 years. Three of them are of same age and they are excluded. The mean of the remaining two is 25. Age of one of the excluded person in years is

(A) 20, (B) 25, (C) 40, (D) none of them.

(c) A man bought three qualities of tea in the ratio 5:4:3 with prices per kg. Rs. 390, Rs. 375 and Rs. 450 respectively and mixed them together. The cost price of the mixture per kg. in Rs. is

(A) 395, (B) 420, (C) 400, (D) none of them.

(d) Ram lends Hari Rs. 1000 and Hari repays Rs. 13000 to Ram at the end of 3 years in simple interestfully. The rate of interest Ram charged to Hari per annum for repayment of loan is

(A 13%, (B) 12%, (C) 10%, (D) none of them.

(e) A bill of Rs. 1020 is due in 6 months. True discount in rupees at interest rate 4% per annum is

(A) 25, (B) 20, (C) 20.4, (D) none of them.

Answer to Question No. 1

(a) $\rightarrow$ **(A):** *Let x, y* are numbers, So $\frac{x}{y}=\frac{3}{4}$ and $\frac{x-10}{y-10}=\frac{1}{2}$

i.e., 4x = 3y and 2x20 = y-10 or, 2x= y +10
or, 4x = 2y + 20 or, 3y = 2y + 20 or, y = 20

and $x = \frac{3}{4}y \therefore x = 15$

So, the numbers are 15 and 20.

(b) $\rightarrow$ **(D):** Total age = $40 \times 5 = 200$ years

Let, 3 equal ages are x, x, x years.

Then, $\frac{200-3x}{2}$ = 25 or, 3x = 200 - 50 or, 3x = 150 or, x = 50

(c) $\rightarrow$ **(C):** Cost price of mixture = $\frac{5\times 390\times 4\times 375+3\times 450}{5+4+3}$

$= \frac{1950+1500+1350}{12} = \frac{4800}{12}$ = Rs. 400 per kg.

(d) $\rightarrow$ **(C):** Rs. 1300 = Rs. 1000 (1 + 3i)

where i = simple interest rate per annum.

So, 1300 = 1000 (1 + 3i) or, 300 = 3000i

or, $i = \frac{300}{3000} = \frac{1}{10} = \frac{1}{10}\times 100\% = 10\%$

(e) $\rightarrow$ **(B):** True discount TD = $\frac{Ani}{1+ni}$ where A = Amount of Bill,

n = time period, i = rate of interest p.a.

Thus TD = $\frac{1020\times\frac{1}{2}\times\frac{4}{100}}{1+\frac{1}{2}\times\frac{4}{100}} = \frac{\frac{2040}{100}}{\frac{102}{100}}$ = Rs. 20.

Question : 2.

Answer any one of the following:

(a) The proportion of liquid I and liquid II in four samples are 2:1, 3:2, 5:3 and 7:5. A mixture is prepared by taking equal quantities of the samples. Find the ratio of liquid I to liquid II in the final mixture.

(b) If the difference between true discount and banker's discount on a sum due in 3 months 4% per annum is Rs. 20, find the amount of bill.

Answer to Question No. 2(a):

$$\frac{\text{Liquid I}}{\text{Liquid II}} = \left[\frac{2}{3}x+\frac{3}{5}x+\frac{5}{8}x\ \frac{7}{12}x\right] \Big/ \left[\frac{1}{3}x+\frac{2}{5}x+\frac{3}{8}x\ \frac{5}{12}x\right]$$

$$= \frac{80+72+75+70}{40+48+45+50} = \frac{297}{183} = \frac{99}{61}$$

$\therefore$ Ratio of Liquid I and Liquid II in final mixture is 99:61.

Answer to Question No. 2(b):

A = Amount due at the end of n years = P (1 + ni) where

P = Present value, i = rate of interest, n = $\frac{3}{12} = \frac{1}{4}$ year

BD = Ani = P (1 + ni)ni, TD = Pni

$$BD - TD = P(ni)^2 = 20 \Rightarrow P = \frac{20}{\left(\frac{1}{4} \times \frac{4}{100}\right)^2} = 20 \times 100^2 = 200000 \text{ (in Rs.)}$$

$$A = P(1+ni) = 200000\left(1+\frac{1}{4}\times\frac{4}{100}\right) = 200000 \times 1.01 = 202000$$

Amount of bill: Rs. 202000.

June 2008 Examination

Question:

1. Answer *any two* of the following:

   Choose the correct option showing the proper reasons/calculations.

   (i) If x be added to the numbers 10 and 30, then they will be in the ratio 3:4 So x is

   (A) 20 (B) 30 (C) 50 (D) none of these

   (ii) Sub triplicate ratio of 8:27 is

   (A) 2:3 (B) 512:19683 (C) $\sqrt{2}:3\sqrt{3}$ (D) none of these

   (iii) Rs. 20 is decreased at the rate of 5% to

   (A) Rs.21 (B) Rs.19 (C) Rs.15 (D) none of these

   (iv) The mean of 4 numbers is 10. If one number is excluded the mean will be 8. The excluded number is.

   (A) 10 (B) 8 (C) 9 (D) none of these

2. Answer *any two* of the following:

   (i) If $\frac{a}{2} = \frac{b}{3} = \frac{c}{4}$ then find the value of $\frac{a+b+c}{b}$ where none of a, b, c is zero.

   (ii) A bill for Rs. 816 is due in 6 months. Find the true discount if the rate of interest is 4% per annum.

   (iii) A man mixed 3 litres of kerosene oil, purchased ar Rs. 2 per litre and 2 litres of kerosene oil, purchased ar Rs. 4.50 per litre. FInd the cost price of the mixture per litre.

Answer to Question No. 1(a):

(i) $\frac{10+x}{30+x}=\frac{3}{4}$ or, 40 + 4x = 90 + 3x or, x = 50. **Correct Answer is (C).**

(ii) Subtriplicate ratio of 8:27 is $8^{1/3}:27^{1/3}=\left(2^3\right)^{\frac{1}{3}}:\left(3^3\right)^{\frac{1}{3}}=2:3$. **Correct Answer is (A)**

(iii) $20-20\times\frac{5}{100}=20-1=19$ **Correct Answer is (B).**

(iv) Total of 4 numbers = 40. Total of 3 numbers after exclusion = 24
So excluded number = 40 – 24 = 16. **Correct Answer is (D)**

Answer to Question No. 1(b):

(i) Let $\frac{a}{2}=\frac{b}{3}=\frac{c}{4}=k\neq 0.$ Then a = 2k, b = 3k, c = 4k.

$$\frac{a+b+c}{b}=\frac{2k+3k+4k}{3k}=\frac{9k}{3k}=3$$

(ii) A = amount of bill, n = period, i = rate of interest

(T.D.) = True discount = $\frac{Ani}{1+ni}$, A = Rs.816, n = 6month = 0.50 yrs.

$$i=\frac{4}{100}=0.04, \text{So T.D.}=\frac{816\times0.50\times0.04}{1+0.50\times0.04}=\frac{816\times0.02}{1+0.02}=\frac{16.32}{1.02}=\text{Rs.}16$$

(iii) Cost price of mixture per litre = $\frac{3\times2+2\times4.5}{3+2}=\frac{15}{5}$ = Rs.3.

Question: 2

Answer *any two* of the following:

(a) A man spent 20% of his money and Rs. 50 after it. Then he spent 20% of the remainder. If he had Rs. 1980 left, what was his original money?

(b) A class has 3 divsions. Average marks of the students of the class, first division, second division and third division are 47, 44, 50 and 45 respectively in Mathematics. If first two division have 30 and 40 students, find the number of students in third division when all the students of the class have Mathematics as a subject.

(c) Two vessels contain mixtures of milk and water in the proportion 1:2 and 3:2 respectively. In what proportion should the two mixtures be mixed together so as to form a new mixture containing equal quantity of milk and water?

(d) If the banker's gain on a bill, due for six months at the rate of 4% per annum be Rs. 10, find the bill value and the present value.

Answer to Question No. 2(a):

Let Rs.x be the orginal money of a man. He spent 0.2x first and then Rs.50.

Remainder = x – (0.2x + 50) = 0.8x – Rs. 50.

Then he spent $\frac{20}{100} \times (0.8x - 50)$ Rs.

Then he had

(0.8x – 50) – 0.2(0.8x – 50) = 0.8(0.8x – 50) Rs. left

So, 0.8(0.8x – 50) = 1980

or, 0.64 – 40 = 1980

or, 0.64x = 2020

$$\text{or, } x = \frac{2020}{0.64} = 3156.25$$

Hence his original money was Rs. 3156.25

Answer to Question No. 2(b):

Let the no. of students be x in third division.

Students have total marks in Mathematics in 1st division $= 30 \times 44 = 1320$

Students have total marks in Mathematics in 2nd division $= 40 \times 50 = 2000$

Students have total marks in Mathematics in 3rd division $= x \times 45 = 45x$

Total marks in Mathematics in the whole class = 3320 + 45x

Total number of students in the class = 30 + 40 + x = 70 + x

Then average marks of the students in the class =

$$\text{Thus } \frac{3320 + 45x}{70 + x}, = 47$$

So, 3320 + 45x = 3290 + 47x

or, 30 = 2x

or, x = 15

Hance the number of students in third division is 15.

Answer to Question No. 2(c):

Let x litre of mixture of 1st vessel be mixed with y litres of mixture of 2nd vessel

This x litres of mixture contains $\frac{x}{3}$ litres of milk and $\frac{2x}{3}$ litres of water.

Also y litres of mixtures contains $\frac{3y}{5}$ litres of milk and $\frac{2y}{5}$ litres of water.

Final mixture contains $\left(\frac{x}{3}+\frac{3y}{5}\right)$ litres of milk and

$\left(\frac{2x}{3}+\frac{2y}{5}\right)$ litres of water

For equal quantity of milk and water

$$\frac{x}{3}+\frac{3y}{5}=\frac{2x}{3}+\frac{2y}{5}$$

or, $\frac{3y}{5}-\frac{2y}{5}=\frac{2x}{3}-\frac{x}{3}$

or, $\frac{y}{5}=\frac{x}{3}$

or, $\frac{x}{y}=\frac{3}{5}$

So, the two mixtures are mixed in the proportion 3:5

Answer to Question No. 2(d):

Banker's Gain = Banker's discount – True Discount

$= \text{Ani} - \text{Pni} = (A - P)\,ni$

Where, A = sum due at the end of n years for i = rate of interest on present value P.

Thus $A = P(1 + ni)$. So, $A - P = Pni$,

Here $n = \frac{1}{2} = 0.50$ yr ; $i = \frac{4}{100} = 0.04$

Thus B.G. = $P(ni)^2$

or, $10 = P(0.50 \times 0.04)^2 = 0.0004P$

or, $P = \frac{10}{0.0004} = \frac{100000}{4} = \text{Rs.}25000 =$ present value

$A = P(1 + ni) = 25000(1 + 0.50 \times 0.04) = 25000(1 + 0.02)$

$= 25000 \times 1.02 = 25500$ Rs.

Thus, Bill Value = Rs. 25500 and

Present Value = 25000.

December 2007 Examination

Question:

(a) 1. Answer a*ny three* of the following:

Choose the correct option showing the proper reason.

(i) If $\frac{2x-3y}{2x+3y}=\frac{2}{5}$, then $x:y$ is

(A) 2:7, (B) 7:2 (C) 2:5, (D) none of these.

(ii) The duplicate ratio of 2:3 is

(A) $\sqrt{2}:\sqrt{3}$, (B) 4:6 (C) 4:9, (D) none of these.

(iii) Compounded ratio of 3:7, 21:25 is

(A) 25:9 (B) 7:21, (C) 3:25, (D) none of these.

(iv) A person takes loan Rs. 3,000/- at 11% per annum from a bank. He repays the loan afer 2 years. Then he pays

(A) Rs. 3,300/- (B) Rs. 3,660/- (C) Rs. 4,000/- (D) none of these

(b) Answer *any two* of the following:

(i) The ages of 5 boys are 5, 8, 10, 13 and 14 years. What is their average age?

(ii) The true discount on a bill due in 9 months at 4% per annum is Rs.60. Find the amount of the bill.

(iii) If $\frac{\sqrt{x}-\sqrt{y}}{\sqrt{x}+\sqrt{y}}=\frac{1}{2}$ find $\frac{x}{y}$

Answer to Question No. 1(a):

(i) $\rightarrow$ (B): $\frac{2x-3y}{2x+3y}=\frac{2}{5}\Rightarrow\frac{4x}{-6y}=\frac{7}{-3}\Rightarrow\frac{x}{y}=\frac{7}{2}\Rightarrow x:y=7:2$

(ii) $\rightarrow$ (C): Duplicate ratio of 2:3 is 4 : 9

(iii) $\rightarrow$ (D): $\frac{3}{7}\times\frac{21}{25}=\frac{9}{25}$ [None of these]

(iv) $\rightarrow$ (B): $A=3000+\frac{3000\times 2\times 11}{100}$ = Rs. 3,660/-

Answer to Question No. 1 (b):

(i) Average age = $\dfrac{5+8+10+13+14}{5} = \dfrac{50}{5} = 10$ years

(ii) True discount, TD = $\dfrac{Ani}{1+ni}$

Where A = amount of bill due, n = time period, i = rate of interest

Now TD = Rs. 60, n = 9 months = $\dfrac{9}{12}$ years, i = $\dfrac{4}{100}$

$$\text{Thus } 60 = \frac{A.\frac{9}{12}.\frac{4}{100}}{1+\frac{9}{12}.\frac{4}{100}} = \frac{3A/100}{\frac{103}{100}} = \frac{3A}{103}$$

$$\Rightarrow A = \frac{60 \times 103}{3} \quad \text{Rs. } 2060$$

Hence the amount of Bill = Rs. 2,060

(iii) $\dfrac{\sqrt{x}-\sqrt{y}}{\sqrt{x}+\sqrt{y}} = \dfrac{1}{2}$, By componendo and dividendo, we have,

$$\frac{\sqrt{x}-\sqrt{y}+\sqrt{x}+\sqrt{y}}{\sqrt{x}-\sqrt{y}-\sqrt{x}-\sqrt{y}} = \frac{1+2}{1-2}$$

$\dfrac{2\sqrt{x}}{-2\sqrt{y}} = \dfrac{3}{-1}$, squaring both sides, we get,

$$\frac{4x}{4y} = \frac{9}{1} \therefore \frac{x}{y} = \frac{9}{1}$$

Question 2 :

2. Answer any *two* of the following

(a) If $\dfrac{x}{y+z} = \dfrac{y}{z+x} = \dfrac{z}{x+y} = K$ prove that $x+y+z=0$ if $K \neq \dfrac{1}{2}$

(b) In an examination 20% of the candidates fail in subject A, 25% in subject B and 10% in both the subjects. Find the percentage of those who fail exactly in one subject. Also find the percentage of candiates passed in both the subjects.

(c) A car travels a distance of 40 km at a speed of 20 km per hour, a second distance of 50 km at a speed of 25 km per hour and a third distance of 210 km at a speed of 35 km per hour. Find the average speed of the car driving the whole distance.

(d) A shop owner sells his goods at 17.5 % discount. What price should he mark on an article that costs him Rs. 1,200 to make a profit of 37.5% on his cost?

Answer to Question No. 2 (a).

$x = K(y + z)$

$y = K(z + x)$

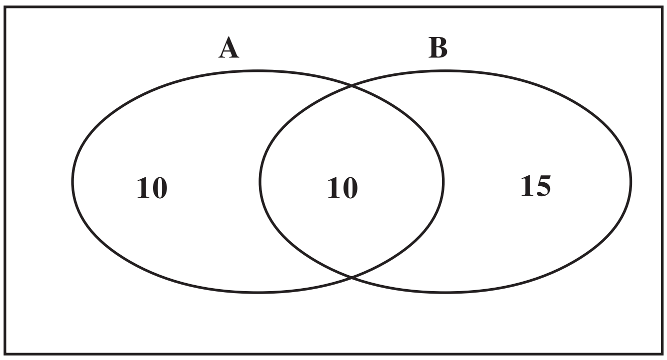
$z = K(x + y)$

Adding them $(x + y + z) = 2K(x + y + z)$

$\Rightarrow (2K - 1)(x + y + z) = 0$

$\left(K - \frac{1}{2}\right)(x + y + z) = 0$ (Dividing by 2)

Answer to Question No. 2(b):



% of students failed in exactly one subject = 10 + 15 = 25%

% of students passed in both the subjects is 100 - 35 = 65%

Answer to Question No. 2(c):

1st distance 40 km is travelled in $\frac{40}{20}$ hr = 2 hr

2nd distance 50 km is travelled in $\frac{50}{25}$ hr = 2 hr

3rd distance 210 km is travelled in $\frac{210}{35}$ hr = 6 hr

The total distance 40 + 50 + 210 = 300 km is travelled in 2 + 2 + 6 = 10 hr

Thus average speed during the car's travel is $\frac{300}{10}$ = 30 km p.h.

Answer to Question No. 2(d):

Given CP = Rs. 1200   Profit = 37.5 % of CP = Rs. 450

N.S.P. = CP + Profit = 1200 + 450 = Rs. 1,650

also NSP = MP – Discount

$\therefore$ 1650 = MP – $\frac{17.5}{100}$ × MP = MP (1 – 0.175) = MP(0.825)

MP = $\frac{1650}{0.825}$ = Rs. 2,000

$\therefore$ The shop owner should mark his article for Rs. 2000

June 2007 Examination

Question:1.

(a) if the mean proportional between x and 2 is 4, find x.

(b) The mean of 4 numbers is 9. If one number is excluded the mean becomes 8. Find the excluded number.

(c) Compute the simple interest on Rs. 5,700 for 2 years at 2.5% p.a.

(d) A number is increased by $33\frac{1}{3}\%$. If the number thus obtained is 600, find the number.

Answer to Question 1:

(a) By question, mean proportional of x and 2 is 4 i.e. $\sqrt{2x} = 4 \Rightarrow x = 8$

(b) The excluded number = 4 × 9 – 3 × 8 = 36 – 24 = 12

(c) $SI = \frac{PRT}{100} = \frac{5700 \times 2.5 \times 2}{100} = Rs.285.00$ [SI = Simple Interest]

(d) Let x be the number,

By question, $x + \frac{100x}{3 \times 100} = 600 \Rightarrow \frac{4}{3}x = 600 \Rightarrow x = 450$

the number is 450.

Question: 2.

(a) If, $\frac{x}{a}=\frac{y}{b}=\frac{z}{c}$ prove that $\frac{x^2-yz}{a^2-bc}=\frac{y^2-zx}{b^2-ca}=\frac{z^2-xy}{c^2-ab}$

(b) The prime cost of an article was three times the value of the materials used. The cost of raw materials was increased in the ratio 3:4 and the productive wage was increased in the ration 4:5. Find the present prime cost of an article, which could formerly be made for Rs. 180.

Answer to Question 2:

(a) $\frac{x}{a}=\frac{y}{b}=\frac{z}{c}=k$ where k is a constant $\neq 0$

$x = ak,\ y= bk,\ z = ck$

Now, $\frac{x^2-yz}{a^2-bc}=\frac{a^2k^2-bk.ck}{a^2-bc}=\frac{k^2(a^2-bc)}{a^2-bc}=k^2$

Similarly, $\frac{y^2-zx}{b^2-ca}=\frac{b^2k^2-bk.ck}{a^2-bc}=\frac{k^2(b^2-ca)}{b^2-ca}=k^2$

$\frac{z^2-xy}{c^2-ab}=\frac{c^2k^2-ak.bk}{c^2-ab}=\frac{k^2(c^2-ab)}{c^2-ab}=k^2$

Hence, $\frac{x^2-yz}{a^2-bc}=\frac{y^2-zx}{b^2-ca}=\frac{z^2-xy}{c^2-ab}$

(b) Let C_1= Raw material cost

C_2 = productive wage

C = prime cost of the article

Given: $C = C_1 + C_2$ .... eqn. no. (i)

$C = 3C_1$ .... eqn. no. (ii)

So, from (ii), $180 = 3C_1 \Rightarrow C_1 = 60$

from (i) , $180 = 60 + C_2 \Rightarrow C_2 = 120$

Present Prime Cost of the article $=\frac{4}{3}C_1+\frac{5}{4}C_2$

$=\frac{4}{3}\times 60+\frac{5}{4}\times 120$

$= 80 + 150$

= Rs. 230

Question:3.

(a) A dealer of radio offers radio for Rs. 2,720 cash down or for Rs. 720 cash down and 24 monthly installments of Rs. 100 each. Find the rate of simple interest charged per annum. 5

(b) If the Banker's gain on a sum due in 6 months at 4% per annum is Rs. 100, find the amount of bill. 5

Answer to Question 3:

(a) The simple interest charged per annum = i%

Then, the simple interest charged per month = $\frac{i}{2}\%$

Price of the radio = Rs. 2720. Offer is Rs. 720 cash down and the remaining Rs. 2000 is to be cleared up in 24 equal monthly installments of Rs. 100 each with respective earning of interest.

Then,

$$2000\left(1+\frac{24i}{1200}\right) = 100 + 100\left(1+\frac{i}{1200}\right) + ... + 100\left(1+\frac{23i}{1200}\right)$$

$$= 2400 + \frac{100i}{1200}(1+2+.......+23)$$

$$\text{or, } 40i = 2400 - 2000 + \frac{i}{12} \times \frac{23 \times 24}{2}$$

$$\text{or}, (40-23)i = 400 \quad \text{or}, i = \frac{400}{17} = 23\frac{9}{17}$$

Thus, the required simple interest rate = $23\frac{9}{17}\%$ per annum

(b) B.G. = Banker's gain

B. D.= Banker's discount

T. D. = True discount

A = Amount of Bill

P = Principal

n = No. of years

i = rate of interest per annum

$$\text{Here, } n = \frac{1}{2},\ i = \frac{4}{100} = 0.04$$

$$\text{Thus } 100 = (A-P).\frac{1}{2}.\frac{4}{100} = \frac{2}{100}(A-P)$$

$$\text{So,}(A-P)=\frac{100\times 100}{2}=5000=\text{T.D.}$$

Again , T.D. = Pni or, $5000 = P.\frac{1}{2}.\frac{4}{100}$

or, P=5000 50 = 250000

Then A = P + Pni = 250000 + 5000 = Rs. 255000

So, the amount of bill = Rs. 255000

December 2006 Examination

Question: 1.

(a) If the two numbers 20 and x + 2 are in the ratio 2:3, find x

(b) If 15% of a particular number is 45, find the number.

(c) If $\frac{\sqrt{a}-\sqrt{b}}{\sqrt{a}+\sqrt{b}}=\frac{1}{2}$, find, the value of $\frac{a}{b}$.

(d) What principal will be increased to Rs. 4,600 after 3 years at the rate of 5% per anum simple interest?

Answer to Question No. 1

(a) $\frac{20}{x+2}=\frac{2}{3}\Rightarrow 60=2x+4\Rightarrow 2x=56\Rightarrow x=28$

(b) Let the number be x. Then $\frac{15}{100}x=45\Rightarrow x=\frac{4500}{15}=300$

(c) $\frac{\sqrt{a}-\sqrt{b}}{\sqrt{a}+\sqrt{b}}=\frac{1}{2}$

Then $2\sqrt{a}-2\sqrt{b}=\sqrt{a}+\sqrt{b}\Rightarrow\sqrt{a}=3\sqrt{b}\Rightarrow a=9b$

i.e. $\frac{a}{b}=9$ Hence, the value of $\frac{a}{b}$ is 9

(d) P = principal, A = amount = Rs. 4600

Then $4600-P=P\times 3\times\frac{5}{100}\Rightarrow 4600=P\left(1+\frac{15}{100}\right)$

i.e., $\frac{115}{100}P=4600\Rightarrow P=\frac{4600\times 100}{115}=4000$

$\therefore$ Principal (P) is Rs. 4000.

Question: 2.

(a) If $\frac{p}{b-c}=\frac{q}{c-a}=\frac{r}{a-b}$, prove that p + q + r = pa + qb + rc.

(b) The average score of boys is 60, that of girls is 70 and that of all the candidates is 64 appearing in Mathematics of annual. examination. Find the ratio of number of boys and number of girls there. If the total number of candidates appearing in Mathematics is 150, find the number of boys there.

Answer to Question No. 2:

(a) $\frac{p}{b-c}=\frac{q}{c-a}=\frac{r}{a-b}=k$ where k is constant $\neq 0$

$\therefore p = k(b-c),\ q = k(c-a),\ r = k(a-b)$

$\therefore p + q + r = k(b-c) + k(c-a) + k(a-b)$

$= k(b - c + c - a + a - b) = k.\ 0 = 0$

$pa + qb + rc = ak(b-c) + bk(c-a) + ck(a-b)$

$= k[ab - ac + bc - ab + ca - bc] = k\ .\ 0 = 0$

(b) Let and n be no.. of boys and girls respectively appearing in Mathematics of Annual Examination.

Then total score of all the candidates = score of boys + score of girls

= 60m + 70n

Again total score = 64 (m + n)

Thus 64 (m + n) = 60m + 70n $\Rightarrow$ 4m = 6n $\Rightarrow \frac{m}{n}=\frac{3}{2}$

Now m + n = 150, Then $1+\frac{n}{m}=1+\frac{2}{3}\Rightarrow\frac{m+n}{m}=\frac{5}{3}$

$\Rightarrow\frac{150}{m}=\frac{5}{3}\Rightarrow m=150\times\frac{3}{5}=90$

Thus, the number of boys (m) = 90

Question: 3.

(a) A dealer mixed two varieties of teas having costs Rs. 1,200 and Rs. 2,500 each per kg in such a way that he can gain 20% by selling the resultant mixture at Rs. 1,800 per kg. Find the proportion in which the two types of tea are mixed.

(b) If the Banker's gain on a bill, due in 4 months at the rate 6% per annum, be Rs. 200, find the bill value, Banker's discount and true discount of the bill.

Answer to Question No. 3:

(a) Let cost price = Rs. 100. On 20% profit, sale price = 1.20 × (100) = Rs. 120

Then, for sale price Rs. 1,800, cost price = $\frac{1800\times100}{120}$ = Rs. 1,500

Let m kg of 1^{st} variety is mixed with n kg of 2^{nd} variety

Then 1200m + 2500n = (m + n) 1500

$\Rightarrow$ 1000n = 300m

$$\frac{m}{n} = \frac{1000}{300} = \frac{10}{3}$$

So, the two varieties are mixed in the ratio 10:3

(b) BG = BD – TD = 200, n = 4 months = $\frac{1}{3}$ year

$$i = \frac{6}{100} = 0.06, \quad A = P(1 + ni)$$

where BG = Banker's gain,
BD = Banker's discount,
TD = true discount,
n = time period,
i = rate of interest
A = amount at the eand of time period = BV = Bill value
P = Present value

$$P = \frac{A}{l+ni}, \qquad TD = Pni = \frac{Ani}{l+ni}, \; BD = Ani$$

$$BG = BD - TD = Ani - Pni = \left(A - \frac{A}{l+ni}\right)ni = \frac{A(ni)^2}{l+ni}$$

$$\text{Thus, } 200 = \frac{A\left(\frac{1}{3}\right)^2\left(\frac{6}{100}\right)^2}{1+\frac{1}{3}\times\frac{6}{100}} = \frac{36A}{306\times300}$$

$$\Rightarrow \text{Bill value} = A = = \frac{200\times306\times300}{36} = 200\times51\times50 = \text{Rs. } 510000$$

$$\text{Banker's discount} = BD = Ani = 510000\times\frac{1}{3}\times\frac{6}{100} = \text{Rs. } 10200$$

$$TD = \frac{Ani}{l+ni} = \frac{10200}{1+\frac{1}{3}\times\frac{6}{100}} = \frac{10200}{1.02} = \text{Rs. } 10000.$$

June 2006 Examination

Q1. (a) If 3, *x* and 27 are in continued proportion. Find *x*.

(b) What number is to be added to each term of the ratio 2:5 to make it 3:4?

(c) If P exceeds q by 20%, find the percentage by which q is less then P?

(d) An employer pays wages Rs. 60 per male worker and Rs. 45 per female worker each per day. If he engaze 8 male and 4 female workers on some day then find average wage per worker on that day.

Answer to Question 1:

(a) 3, x and 27 are in continued proportion.

So, $\frac{3}{x}=\frac{x}{27}$ or $x^2=81 \Rightarrow x=9$

(b) Let x be the number.

$\frac{2+x}{5+x}=\frac{3}{4} \Rightarrow x=7$

(c) By question $p=q+\frac{q}{5} \Rightarrow 5p=6q=\frac{5}{6}p=p-\frac{p}{6}$

So q is less than $p\frac{1}{6}\times 100=16\frac{2}{3}\%$

(d) Average wage paid $=\frac{60\times 8+45\times 4}{12}=\frac{480+180}{12}=\frac{660}{12}=55(\text{in Rs.})$

Q2. (a) At the same rate of simple interest a principal to Rs. 2,056 in 4 years and amounts to Rs. 2248 in 7 years. Find the rate of interest and the principal.

(b) Two men will have equal income if the income of one be increased by 7% and that of the other be reduced by $7\frac{1}{2}\%$. If their total income is Rs. 8,379. Find their incomes.

Answer to Question 2:

(a) Simple interest for 3 years = Rs. (2,248-2,056) = Rs. 192

,, ,, 4 years = $192\times\frac{4}{3}$ = Rs. 256

Prinipal amount = Rs. (2056-256) = Rs. 1,800

Rate of interest = $\frac{100\times 1}{P\times T}=\frac{100\times 256}{1800\times 4}=\frac{32}{9}=3\frac{5}{9}\%$

(b) Let the salary of first man be Rs x and that of second man by y.

By question x + y = 8,379 ..... (i)

Also $x+\frac{7}{100}x=y-\frac{15}{2\times 100}y$

$\Rightarrow 107x = 92.5y$....... (ii)v

Solving (i) and (ii), we get y = 4,494 (in Rs) and x = 3,885 (in Rs.)

The required salary of 1st man = Rs. 3,885

and ,, ,, of 2nd man = Rs. 4,494

Q3. (a) If the differene between true discount and Banker's discount is Rs. 20, find the amount of the bill for a sum due in 6 months at 4% per annum.

(b) A person drove his car for first 20 km and then 30 km at an average speed of 20 km and 30 km per hour respectively. At what speed must he drive next 50 km if the average speed of the whole distance of his driving is 40 km per hour?

Answer to Question 3:

(a) Let A = Amount of bill, P = Present value, i = Rate of intest and n = no. of years.

Then A = P+Pni, T.D = Tni, BD = Ani

Thus B.D – TD = (A – P) ni

$$\therefore (A-P)\times\frac{1}{2}\times.04=20\Rightarrow A-P={}^{20}\!/_{.02}=1000$$

$$\text{Again } A-P=Pni\Rightarrow 1000=P\times\frac{1}{2}\times.04\Rightarrow P=\frac{1000}{0.02}=50{,}000$$

Hence, Amount of bill = P+Pni = $50{,}000+50{,}000\ \frac{1}{2}\times 0.04=51{,}000$ (in Rs.)

(b) Let av.speed of the last 50 km be x kmph.

Total distance he drove = 20 + 30 + 50 = 100km

$$\text{Total time taken}=\frac{20}{20}+\frac{30}{30}+\frac{50}{x}\text{ hours}=2+\frac{50}{x}\text{ hours.}$$

$$\text{Average speed}=\frac{100}{2+\frac{50}{x}}=\frac{100x}{2x+50}=40$$

or, 100x - 80x = 2000

$\Rightarrow$ x = 100 km per hour. So average speed is 100 km per hour.

December 2005 Examination

Question:1

(a) If $\frac{a+b}{a-b}=2$, find the value of $\frac{a^2-ab+b^2}{a^2+ab+b^2}$.

(b) At what rate per annum will a sum of money double itself in 10 years with simple interest?

(c) What will be the cost price per KG of the mixture of two types of teas, mixed in the ratio 3 : 2 if the first type is purchased in Rs. 200 per kg and the second in Rs. 300 per kg?

Answer to Q. No. 1:

(a) a + b = 2a – 2b or 3b = a

Then $\frac{a^2-ab+b^2}{a^2+ab+b^2}=\frac{9b^2-3b\cdot b+b^2}{9b^2+3b\cdot b+b^2}=\frac{7b^2}{13b^2}=\frac{7}{13}$

(b) $A=P\left(1+\frac{nr}{100}\right)$, A = amount, P = Principal, r = Rate of interest

per annum and n = no. of years.

$\therefore 2P=P\left(1+\frac{10r}{100}\right)$ or $2=1+\frac{r}{10}$ $P>O$ Then $1=\frac{r}{10}$ or $r=10$

$\therefore$ Rate of interest = 10%

(c) 3 Kg. of 1st tea costs Rs. 600, 2 Kg. of second tea costs Rs. 600 and 5 Kg. of mixture costs Rs. 1200. Thus 1 Kg. of mixture costs $\frac{1200}{5}=$ Rs. 240.

Question 2:

(a) If $\frac{x}{b+c}=\frac{y}{c+a}=\frac{z}{a+b}$, prove that $\frac{x(y-z)}{b^2-c^2}=\frac{y(z-x)}{c^2-a^2}=\frac{z(x-y)}{a^2-b^2}$

(b) A sum of money is to be divided among three sons in such a way that the first son is to get 30% of the whole, the second son is to get 40% of the remainder and the third son the rest. If the third son gets Rs. 21,000, what would be the total sum divided and the amounts in the respective shares of the first and second son?

Answer to Q. No. 2:

(a) $\frac{x}{b+c}=\frac{y}{c+a}=\frac{z}{a+b}=k\,(\text{const.})$ say,

i.e. $x=k(b+c), y=k(c+a), z=k(a+b)$

or, $x-y=k(b+c)-k(c+a)=k(b-a)=-k(a-b)$ or $\frac{x-y}{a-b}=-k$

$$y - z = k(c + a) - k(a + b) = k(c - b) = -k(b - c) \text{ or } \frac{y - z}{b - c} = -k$$

$$z - x = k(a + b) - k(b + c) = k(a - c) = -k(c - a) \text{ or } \frac{z - x}{c - a} = -k$$

Thus

$$\frac{x(y-z)}{b^2 - c^2} = \frac{x}{b+c} \cdot \frac{y-z}{b-c} = k \cdot (-k) = -k^2,$$

$$\frac{y(z-x)}{c^2 - a^2} = \frac{y}{c+a} \cdot \frac{z-x}{c-a} = k \cdot (-k) = -k^2,$$

$$\frac{z(x-y)}{a^2 - b^2} = \frac{z}{a+b} \cdot \frac{(x-y)}{a-b} = k \cdot (-k) = -k^2,$$

$$\text{So, } \frac{x(y-z)}{b^2 - c^2} = \frac{y(z-x)}{c^2 - a^2} = \frac{z(x-y)}{a^2 - b^2} \rightarrow \text{Proved}$$

(b) Second son gets 40% of $70\% = \text{or } \frac{40}{100} \times \frac{70}{100} = \frac{28}{100}$ or 28% of total sum

Third son gets (100–30–28) = 42% of total sum of money.

Now 42% of total sum of money = Rs. 21,000

So total sum of money $= \text{Rs. } 21{,}000 \times \frac{100}{42} = \text{Rs. } 50{,}000$

First son gets $50000 \times \frac{30}{100} = \text{Rs.}15{,}000$

Second son gets $50000 \times \frac{28}{100} = \text{Rs.}14{,}000$

Question 3:

(a) Two vessels contain mixture of milk and water in the proportions 2 : 3 and 4 : 3 respectively. In what proportion should the two mixtures be mixed so as to form new mixture containing equal quantities of milk and water?

(b) The difference between interest and true discount on a sum due in 5 years at 5% per annum is Rs. 50. Find the sum.

Answer to Q. No. : 3

(a) Let the two mixtures are in the ratio of x:y.

Let x litre of first mixture is mixed with y litre of second mixture.

Then, x litre of first mixture contains $\frac{2}{5}x$ litre of milk and $\frac{2}{3}x$ litre of water and y litre of second mixture contains $\frac{4}{7}y$ litre of milk and $\frac{3}{7}y$ litre of water.

Then last mixture contains $\frac{2}{5}x+\frac{4}{7}y$ litre of milk

and $\frac{3}{5}x+\frac{3}{7}y$ litre of water

Also it is given that $\frac{2}{5}x+\frac{4}{7}y=\frac{3}{5}x+\frac{3}{7}y$

So, $\frac{1}{7}y=\frac{1}{5}x$ or, $7x=5y$ or, $\frac{x}{y}=\frac{5}{7}$

Hence, the two mixtures should be mixed in the ratio of 5 : 7

(b) Let the sum of money be Rs. 100

Interest on Rs. 100 for 5 years @ 5% is $100\times5\times\frac{5}{100}=\text{Rs.}25$

True discount (TD) $=\frac{100\times5\times\frac{5}{100}}{1+5\times\frac{5}{100}}=\frac{25}{1+\frac{1}{4}}=\frac{100}{5}=\text{Rs.}20$

Difference between interest and true discount is $25-20=\text{Rs. }5$

The difference between interest and TD is Rs. 5 when sum of money is Rs. 100

The difference between interest and TD is Rs. 50 when sum of money is $\frac{100\times50}{5}=\text{Rs.}1000$

So, the required sum is Rs. 1000.

June 2005 Examination

Question : 1.

(a) An article was marked to sale for Rs. 1240. But it was sold at a discount 22.5%. Compute its selling price.

(b) If $\frac{4x-3z}{4c}-\frac{4z-3y}{3b}-\frac{4y-3x}{2a}$ show that each ratio is equal to $\frac{x+y+z}{2a+3b+4c}$

(c) The simple intrest on Rs. 300 at the rate of 4% per annum with that on Rs. 500 at the rate of 3% per annum, both for the same period, is Rs. 162. Find the time period.

Answer to Q. No. 1:

(a) Discount = 22.5% of 1240 = 22.5 / 100 $\times$ 1240 = Rs. 279

Hence, S.P. = 1240 – 279 = Rs. 961

(b) $$\frac{4x-3z}{4c}=\frac{4z-3y}{3b}-\frac{4y-3x}{2a}=\frac{4x-3z+4z-3y+4y-3x}{4c+3b+2a}=\frac{x+y+z}{2a+3b+4c}$$

(c) Let time period = n years. Using $I=p\times\frac{r}{100}\times n$ formula, we get,

$300\times\frac{4}{100}\times n+500\times\frac{3}{100}\times n=162$ or, 12n + 15n = 162 or, 27n = 162

or, n = 6 years.

Hence, required time period = 6 years.

Question: 2.

(a) The average score of girls in HSC examination is 75 and that of boys is 70. The average score of all the candidates in the examination is 72. Find the ratio of number of girls and boys that appeared in the examination.

(b) A person drove his car for 20 km. at an average speed of 25 km. per hour. At what average speed must he drive for the next 20 km., if his average speed for the whole distance is to be 30 km. per hour?

Answer to Q. N. 2:

(a) No. of girls = m, and no. of boys = n (say)

Total score of all girls = 75m; that of boys = 70n

Now total score of all girls and boys together = 75 m + 70 n

Total no. of student = m + n

Hence, average score = $\frac{75m+70n}{m+n}=72\,(\text{given})$

Or, 75m + 70n = 72m + 72n or, 3m = 2n

or, m/n = 2/3 Hence, reqd. ratio = 2.3

(b) $\text{Time}=\frac{\text{dis tan ce}}{\text{speed}}$

For the first 20 km. drive, time = 20/25 hrs.

For the second 20 km. drive time = 20/x hrs *where*, x = speed of drive in second case.

Total time taken to cover 40 km. Distance $=\left(\frac{20}{25}+\frac{20}{x}\right)$ hrs

Since average speed for whole journey = 30 km/hr.

Hence, time taken = 40/30

$$\text{Now,} \frac{20}{25}+\frac{20}{x}=\frac{40}{30} \quad \text{or,} \frac{1}{25}+\frac{1}{x}=\frac{2}{30}=\frac{1}{15}$$

$$\text{or,} \frac{1}{x}=\frac{1}{15}-\frac{1}{25}=\frac{5-3}{75}=\frac{2}{75}$$

$$\text{or, } x=\frac{75}{2}=37.5 \text{km / hr.}$$

Hence, reqd. speed = 37.5 km/hr

Question: 3.

(a) At what simple interest rate percent per annum a sum of money will be doubled of it self in 25 years?

(b) A dealer mixes 90 litres of wine contaning 10% of water with 60 litres of another wine contaning 20% of water. What is the percentage of water in the mixture? If 50 litres pure water is added to the mixture, find the percentage of water in the final mixture.

Answer to Q. No. 3:

(a) We know, $A = P\left(1+\frac{n \times r}{100}\right)$, here A = 2P, n = 25, r = ?

$$\text{i.e. } 2P = P\left(1+25 \times \frac{r}{100}\right)$$

$$\text{or, } 2 = 1+25 \times \frac{r}{100} \quad \text{or, } 1=\frac{r}{4} \quad \text{or, } r = 4.$$

Hence, required rate of interrest = 4%.

(b) Quantity of water in 90 litres of mixture = 10/100 × 90 = 9 litres.

Quantity of water in 60 litres of mixture = 20/100 × 60 = 12 litres.

Total quantity of water in 150 litres of mixture = 21 litres.

Hence, percetage of water = 21/150 × 100 = 14%

After adding 50 litres of pure water in the above mixtures, quantity of water = 21 + 50 = 71 litres and total quantity of mixture = 150 + 50 = 200 litres.

Hence, percentage of water in 200 litres of final mixture = 71/200 × 100 = 35.5%

December 2004 Examination

Question :

1. (a) The ratio of present age of mother to her daughter is 5:3. Ten years hence the ratio would be 3:2. Find their present ages.

   (b) Find a mean proportional between 27 and 243.

   (c) In an examination 20% of the candidates failed in English, 30% in Mathematics and 10% in both. Find the percentage of those who passed in both subjects.

Answer to Question No. 1(a):

Let present age of mother be 5x and that of her daughter be 3x years.

10 years hence age of mother will be (5x + 10) years and that of daughter be (3x + 10) years.

By question $\frac{5x+10}{3x+10} = \frac{3}{2}$ or, $2(5x+10) = 3(3x+10)$ or, $10x + 20 = 9x + 30$

or, $x = 10$

$\therefore$ Reqd. ages are 5 x 10 = 50 years and 3 x 10 = 30 years.

Answer to Question No. 1(b):

Let mean proportional be x

Now $27 : x = x : 243$ or, $x^2 = 27 \times 243 = 3^3 \times 3 \times 81 = 81 \times 81 = 81^2$

$\therefore x = 81$

Answer to Question No. 1(c):

Let the number of candidates be 100.
Now number of candidates failed only in English = 20 – 10 = 10
Again number of candidates failed only in Mathematics = 30 – 10 = 20
Number of candidates failed in English and Mathematics = 10 + 20 + 10 = 40
So number of candidates passed = 100 – 40 = 60
$\therefore$ Reqd. percentage = 60%.

Question :

2. (a) Monthly incomes of two persons are in the ratio 2 : 3 and their monthly expenditures are in the ratio 4 : 7. If each saves Rs. 50 a month, find their monthly incomes and expenditures. 5

   (b) The price of cooking gas is increased by 20%. Find out how much percent a man must reduce his consumption so that the expenditure on cooking gas may increase only by 8%. 5

Answer to Question No. 2 (a):

Monthly incomes of two persons be 2x and 3x (in Rs.)

Each saves Rs. 50

So respective expenditures will be (2x – 50) and (3x – 50)

By question, $\frac{2x-50}{3x-50} = \frac{4}{7}$ or, 7(2x – 50) = 4(3x – 50) or, 14x – 350 = 12x – 200

or, x = 75

$\therefore$ Income of two persons = 2 x 75 = 150 (Rs.) and 3 x 75 = 225 (Rs.)

and expenditures are 150 – 50 = Rs. 100 and 225 – 50 = Rs. 175.

Answer to Question No. 2 (b):

Let his expenditure on cooking gas = Rs 100 and cost of gas per c.c. Re. 1. So he consumes 100 c.c. of gas. Increased price of 100 c.c. of gas = 100 + 20 = Rs. 120

His expenditure increases by 8%. So his present expenditure = 100 + 8 = Rs. 108.

amount (Rs.)	gas (c.c.)
120	100
108	?

$$? = \frac{100}{120} \times 108 = 90$$

He can purchase now 90 c.c. of gas by Rs. 108.

$\therefore$ Reduction of consumption = 100 – 90 = 10 c.c. i.e. 10%

Question :

3. (a) If the difference between true discount and banker's discount on a sum due in 6 months at 4% per annum is Rs. 20, find the amount of the bill.

   (b) A sum of money becomes double in 20 years at simple interest. Find the number of years by which the sum will be triple.

Answer to Question No. 3 (a):

B.G. = B.D. – T.D. = 20, n = 6/12 = 1/2; i = 4/100 = 0.04

B.D. = Ani, T.D. = pni

$$\text{B.G.} = \text{Ani} - \text{P ni} = (\text{A} - \text{P}) \times \frac{1}{2} \times = 20 (\text{given}).$$

or, A – P = 40/0.04 = 1000........... (i) i.e. T.D. = 1000 ....(ii)

$$\text{Again T.D.} = \text{Pni} = \text{P} \times \frac{1}{2} \times 0.04 = 1000 \text{ or, P} = 50,000$$

So A – 50,000 = 1000 by (i) or, A = 51,000

$\therefore$ Reqd. amount of the bill = 51,000.

<u>Alternatively</u> : Let sum = Rs. 100

Int. on Rs. 100 for 6 months $@\,4\% = 100.\frac{6}{12}.\frac{4}{100} = \text{Rs. } 2$

Int	Sum	
2	100	$? = \frac{100}{2} \times 20 = 1000$
20	?	

$\therefore$ T.D. = Rs. 1000, as B.G. is int. on T.D.

Again, Int.	Sum	
2	100	$? = \frac{100}{2} \times 1000 = 50,000$
1000	?	$\therefore$ P.V. = Rs. 50,000 as T.D. is int. on P.V.

$\therefore$ B.V. = P.V. + T.D. = 50,000 + 1000 = Rs. 51,000.

Answer to Question No. 3 (b):

From A = P(1 + ni), we get, 2P = P(1 + ni) or, 2 = 1 + 20 i or, 20 i = 1 or, i = 1/20

Again, 3P = P(1 + ni) or, $3 = 1 + n.\frac{1}{20}$, putting A = 3P, i = 1/20, n = ?

or, n/20 = 2 or, n = 40

SECTION - II

ALGEBRA

NUMBER SYSTEM
INDICES
SURDS
VARIATION
EQUATION
PERMUTATION & COMBINATION
LOGARITHM
COMPOUND INTEREST
DEPRECIATION
SET THEORY
INEQUALITIES

PAPER P-4

Business Mathematics and Statistics Fundamentals

SECTION – II

ALGEBRA

June 2010 Examination

Question:

3. Answer *any three* of the following:

   Choose the correct option showing necessary reasons/calculations.

   (a) In a class of 80 students, 52 read Mathematics, 36 read Statistics and 20 read both Mathematics and Statistics. Then number of students who read neither Mathematics nor Statistics is

   (i) 60 (ii) 8 (iii) 12 (iv) none of these

   (b) If logarithm of a number to the base $\sqrt{2}$ is 4, then the logarithm of the same number to the base $2\sqrt{2}$ is

   (i) 4/3 (ii) 4 (iii) 8 (iv) none of these

   (c) The number of ways in which letters of the word Monday be arranged beginning with the letter O and ending the the letter Y is

   (i) 120 (ii) 24 (iii) 96 (iv) none of these

   (d) If p and q be two logical statements then (pVq) V : p is

   (i) (FFFF) (ii) (TFFT) (iii) (FTTT) (iv) none of these

   (e) The area of a circle varies directly with square of its diameter. Area of the circle is 38.5 sq.cm. when diameter is 7 cm. If diameter of the circle is 1 cm then area of the circle in sq.cm. is

   (i) -5.5/7 (ii) 11/7 (iii) 22/7 (iv) none of these

Answer to Question No. 3(a):

n(M), n(S), $n(M \cap S)$, $n(M \cup S)$, $n(M^c \cup S^c)$ be the numbers of students reading Mathematics, Statistics, Maths and Stat both, either Maths or Stat, none of Maths and Stat.

Then n(M) = 52, n(S) = 36, $n(M \cap S) = 20$.

$n(M \cup S) = n(M) + n(S) - n(M \cap S) = 52 + 36 - 20 = 68$

$n(M^c \cup S^c) = 80 - n(M \cup S) = 80 - 68 = 12$ Ans (iii)

Answer to Question No. 3(b):

$$x = \text{number} \log_{\sqrt{2}} x = 4 \Rightarrow x = \left(\sqrt{2}\right)^4 = 4$$

$$\log_{2\sqrt{2}} x = \log_{2\sqrt{2}} 4 = \frac{\log_2 4}{\log_2 2\sqrt{2}} = \frac{\log_2 2^2}{\log_2 2^{3/2}} = \frac{2}{3/2} = \frac{4}{3} \text{ Ans (i)}$$

Answer to Question No. 3(c):

As beginning and ending letters of the arrangements are fixed, the remaining 4 letters can be arranged among themselves in 4! = 24 ways Ans (ii)

Answer to Question No. 3(d):

P	q	: p	pvq	(pvq) v : p
T	T	F	T	T
T	F	F	T	T
F	T	T	T	T
F	F	T	F	T

None of these. Ans (iv)

Answer to Question No. 3(e):

Area of a circle = A, diameter of the circle = d, K = constant

Then $A \propto d^2$ or $A = Kd^2$. A = 38.5 sq. cm. when d = 7 cm.

$$\text{Thus } K = \frac{38.5}{49} = \frac{5.5}{7}$$

$$\text{When } d = 1\text{cm}, A = \frac{5.5}{7} d^2 = \frac{5.5}{7} \text{sq.cm.} \quad \text{Ans (i)}$$

Question:

4. Answer *any two* of the following: 3 x 2

   (a) If w be an imaginary cube root of unity then find the value of $(1 - \omega + \omega^2)(1 + \omega - \omega^2)$

   (b) Simple interest and compound interest in 2 years for same principal are Rs. 200 and Rs. 210 at the same rate of interest per annum. Find the principal amount.

   (c) The volume of a gas varies directly as the absolute temperature and inversely as pressure. When the pressure is 15 units and the temperature is 260 units the volume is 200 units. What will be the volume when the pressure is 18 units and the temperature is 195 units?

Answer to Question No. 4(a):

$$1+\omega+\omega^2=0$$
$$1-\omega+\omega^2=1+\omega^2-\omega=-\omega-\omega=-2\omega$$
$$1+\omega-\omega^2=-\omega^2-\omega^2=-2\omega^2$$
$$\therefore\left(1-\omega+\omega^2\right)(1+\omega-\omega^2)=(-2\omega)(-2\omega^2)=4\omega^3=4\times1=4$$

Answer to Question No. 4(b):

Let x = Principal amount and r% = rate of interest per annum

The simple interest = $\text{Rs.}200=x\times\frac{r}{100}\times2=\frac{rx}{50}\Rightarrow rx=10000$

The compound interest = $\text{Rs.}210=x\left(1+\frac{r}{100}\right)^2-x$

$$=x\left(\frac{2r}{100}+\frac{r^2}{10000}\right)=\frac{rx}{50}+\frac{r^2x}{10000}$$

$$\Rightarrow 210=200+\frac{r^2x}{10000}\Rightarrow 10=\frac{10000r}{10000}\Rightarrow r=10$$

$$\text{So, } x=\frac{200\times50}{r}=\frac{10000}{10}=1{,}000\,\text{Rs.}$$

Answer to Question No. 4(c):

Volume = V, Pressure = P, Absolute Temp = T

$$\therefore V\propto T\ \&\ V\propto\frac{1}{P}\Rightarrow V\propto\frac{T}{P}\Rightarrow V=K\frac{T}{P}\quad K=\text{constant}$$

When P = 15, T = 260 then V = 200

$$200=K\frac{260}{15}\Rightarrow K=\frac{150}{13}$$

When $P=15, T=260$ then $V=\frac{150}{13}\times\frac{195}{18}=125$ units

December 2009 Examination

Question : 3

Answer *any three* of the following :

Choose the correct option showig necessary reasons/calculations.

(a) If $x = 2+\sqrt{3}$ then the value of $x^4 + \frac{1}{x^4}$ is

(i) 98, (ii) 196, (iii) 194, (iv) none of these

(b) ${}^nC_r + {}^nC_{r-1}$ is equal to

(i) ${}^{n-1}C_r$, (ii) ${}^{n+1}C_r$, (iii) ${}^nC_{r+1}$, (iv) none of these

(c) Given a varies as $bx + c$. Value of a is 3 when $b = 1$, $c = 2$ and is 5 when $b = 2$, $c = 3$. The value of x would be

(i) –1, (ii) 2, (iii) 3, (iv) none of these

(d) If one root of the equation $x^2 – bx + K = 0$ is twice the other root then the value of K is

(i) – 8, (ii) 8, (iii) 4, (iv) none of these

(e) If $\frac{\log x}{y-z} = \frac{\log y}{z-x} = \frac{\log z}{x-y}$ then the value of xyz is

(i) 1, (ii) 0, (iii) –1, (iv) none of these.

Answer to Question no. 3.

(a) $\frac{1}{x} = \frac{1}{2+\sqrt{3}} = \frac{2-\sqrt{3}}{(2+\sqrt{3})(2-\sqrt{3})} = \frac{2-\sqrt{3}}{4-3} = 2-\sqrt{3}$

$$\therefore x + \frac{1}{x} = (2+\sqrt{3}) + (2-\sqrt{3}) = 4$$

$$x^2 + \frac{1}{x^2} = \left(x + \frac{1}{x}\right)^2 - 2 = 4^2 - 2 = 14$$

$$x^4 + \frac{1}{x^4} = \left(x^2 + \frac{1}{x^2}\right)^2 - 2 = 14^2 - 2 = 196 - 2 = 194 \quad \text{Ans(iii)}$$

b) ${}^nC_r + {}^nC_{r-1} = \frac{\underline{|n}}{\underline{|r}\,\underline{|n-r}} + \frac{\underline{|n}}{\underline{|r-1}\,\underline{|n-r+1}}$

$$=\frac{\underline{|n}}{\underline{|r}\,\underline{|n-r}}\left[\frac{n-r+1+r}{r(n-r+1)}\right]$$

$$=\frac{(n+1))\underline{|n}}{\underline{|r}\,\underline{|n-r+1}}=\frac{\underline{|n+1}}{\underline{|r}\,\underline{|n-r+1}}={}^{n+1}C_r \text{ Ans(ii)}$$

(c) a = K (bx + c) where *K* is constant of variation

So 3 = $K(x + 2)$ and 5 = $K(2x + 3)$

$$\therefore \frac{3}{5}=\frac{x+2}{2x+3}\Rightarrow x=1 \text{ Ans(iv)}$$

(d) Let α and 2 α be the roots of the equation. Then

$\alpha + 2\alpha = 6 \Rightarrow \alpha = 2,\ \alpha(2\alpha) = K \Rightarrow K = 2.4 = 8$ Ans (ii)

(e) $\frac{\log x}{y-z}=\frac{\log y}{z-x}=\frac{\log z}{x-y}=K(\text{say})$

Then $\log x = K(y - z)$

$\log y = K(z - x)$

$\log z = K(x - y)$

Adding $\log x + \log y + \log z = K(y - z + z - x + x - y) = K \times 0 = 0$

or, $\log xyz = \log 1$

or, $xyz = 1$ Ans (v)

Question : 4

Answer *any two* of the following:

(a) If universal set is { 1, 2, 3, 4, 5, 6}, A = {2, 4, 5}, B = {1, 3, 5}, C = {5, 6} then find

(i) $B - A$, (ii) $A \cup B - C$, (iii) $(A \cup B \cup C)'$, where A' represent complement of A.

(b) Find the square root of $16 - 30i$.

(c) The number of handshakes in a party was counted as 66. Determine the number of guests attending the party, asuming all guests shake hands with each other.

Answer to Question no. 4(a).

S = {1, 2, 3, 4, 5, 6},

A = {2, 4, 5}, B = {1, 3, 5}, C= {5, 6}

Then B – A = {1, 3}

$A \cup B$ = {1, 2, 3, 4, 5}

$A \cup B - C = \{1, 2, 3, 4, 5\} - \{5, 6\} = \{1, 2, 3, 4\}$

$(A \cup B \cup C) = \{1, 2, 3, 4, 5, 6\} = S$

$(A \cup B \cup C)' = S - S = f$, null set

Answer to Question no. 4(b).

$16 - 30i$

$$= \sqrt{25 - 9 - 30i} = \sqrt{25 + 9i^2 - 30i}$$

$$= \sqrt{\left((5 - 3i)^2\right)} = \pm(5 - 3i)$$

Answer to Question no. 4(c).

${}^nC_2 = 66$, n = number of guests

$$\Rightarrow \frac{n(n-1)}{2} = 66$$

$\Rightarrow n^2 - n = 132$
$\Rightarrow n^2 - n - 132 = 0$
$\Rightarrow n^2 - 12n + 11n - 132 = 0$
$\Rightarrow (n - 12)(n - 11) = 0$
$\Rightarrow n = 12$ or -11
$n =$ is inadmisible
So $n = 12$

June 2009 Examination

Question: 3

Answer *any three* of the following:

Choose the correct option showing necessary reasons/calculations.

(a) After rationalization $\frac{\sqrt{3} + \sqrt{2i}}{\sqrt{3} - \sqrt{2i}}$ will be

(i) $1 + 2\sqrt{6i}$, (ii) $\frac{5 + 2\sqrt{6i}}{5}$, (iii) $1 - 2\sqrt{6i}$, (iv) $\frac{1 + 2\sqrt{6i}}{5}$.

(b) $\frac{\left(2^{n+1}\right) + \left(2^{n+2}\right)}{\left(2^{n+2}\right) - 2\left(\frac{1}{2}\right)^{1-n}}$ simplifies to

(i) 4, (ii) 2, (iii) 8, (iv) 20.

(c) The value of $\log_2 \log_2 \log_3 81$ is

(i) 1, (ii) 4, (iii) 3, (iv) 2.

(d) The value of $\times$ satisfies the equation $\sqrt{\frac{x}{1-x}}+\sqrt{\frac{1-x}{x}}=\frac{13}{6}$ is

(i) $\left(\frac{2}{3},\frac{3}{2}\right)$, (ii) $\left(\frac{4}{9},\frac{9}{4}\right)$, (iii) (4,9), (iv) none of these.

(e) If ${}^{m}c_6 : {}^{m-3}c_3 = 91:4$, then the value of m is

(i) 13, (ii) 15, (iii) 14, (iv) none of these.

Answer to Question 3:

(a) $$\frac{\sqrt{3}+\sqrt{2i}}{\sqrt{3}-\sqrt{2i}}=\frac{\left(\sqrt{3}+\sqrt{2i}\right)\left(\sqrt{3}+\sqrt{2i}\right)}{\left(\sqrt{3}-\sqrt{2i}\right)\left(\sqrt{3}+\sqrt{2i}\right)}=\frac{\left(\sqrt{3}\right)^2+\left(\sqrt{2i}\right)^2+2\sqrt{3}\sqrt{2i}}{\left(\sqrt{3}\right)^2-\left(\sqrt{2i}\right)^2}$$

$$=\frac{3-2+2\sqrt{6i}}{3-(-2)}=\frac{1+2\sqrt{6i}}{5}$$ **Ans. (iv)**

(b) $$\frac{\left(2^{n+1}\right)+\left(2^{n+2}\right)}{\left(2^{n+2}\right)-2\left(\frac{1}{2}\right)^{1-n}}=\frac{2.2^n+4.2^n}{4.2^n-2.2^{n-1}}=\frac{6.2^n}{4.2^n-2^n}=\frac{6.2^n}{3.2^n}=2$$ **Ans. (ii)**

(c) $\log_2 \log_2 \log_3 81 = \log_2 \log_2 \log_3 3^4 = \log_2 \log_2 2^2 = \log_2 2 = 1$ **Ans. (i)**

(d) $$\sqrt{\frac{x}{1-x}}+\sqrt{\frac{1-x}{x}}=\frac{13}{6}$$

$$\Rightarrow t+\frac{1}{t}=\frac{13}{6}\Rightarrow 6t^2-13t+6=0\Rightarrow t=\frac{2}{3},\frac{3}{2}$$

$$\therefore \frac{x}{1-x}=\frac{4}{9}\Rightarrow 13\times=4\Rightarrow\times=\frac{4}{13}$$

and

$$\frac{x}{1-x}=\frac{9}{4}\Rightarrow 13x=9\Rightarrow x=\frac{9}{13}$$ **Ans. (iv) None of these**

(e) $$\frac{{}^{m}c_6}{{}^{m-3}c_3} = \frac{91}{4} \Rightarrow \frac{\dfrac{\lfloor m}{\lfloor 6 \lfloor m-6}}{\dfrac{\lfloor m-6}{\lfloor 3 \lfloor m-6}} = \frac{91}{4} \Rightarrow \frac{6\lfloor m}{720\lfloor m-3}\,\frac{91}{4}$$

$$\Rightarrow \frac{m(m-1)(m-2)}{120} = \frac{91}{4}$$

$\Rightarrow m(m - 1)(m - 2) = 91 \times 30 = 15 \times 14 \times 13$

$\Rightarrow m = 15$ **Ans. (ii)**

Questions: 4.

Answer *any two* of the following:

(a) Find the square root of $x + \sqrt{x^2 - y^2}$.

(b) The total expenses of a boarding house varies partly with the number of boarders and partly fixed. The total expenses are Rs. 10,000 for 25 boarders and Rs. 11,500 for 30 boarders. Find the fixed expenses.

(c) Of 50 students appearing in the examination 20 failed in Mathematics, 25 failed in English and 10 failed in both. Find the number of students of those 50 students who passed in both Mathematics and English. Write the formula you use completely.

Answer to Question 4(a):

$$x + \sqrt{x^2 - y^2} = \frac{1}{2}\left(2x + 2\sqrt{x+y}\sqrt{x-y}\right)$$

$$= \frac{1}{2}\left(x + y + x - y + 2\sqrt{x+y}\sqrt{x-y}\right)$$

$$= \frac{1}{2}\left(\sqrt{x+y} + \sqrt{x-y}\right)^2$$

$$\therefore \sqrt{x + \sqrt{x^2 - y^2}} = \pm\frac{1}{\sqrt{2}}\left(\sqrt{x+y} + \sqrt{x-y}\right)$$

Answer to Question 4(b):

Let total expenses = Rs. E, no. of boarders = n, charge per boarder = Rs. x

Fixed expenses = Rs. F. Then $(E - F) \propto n$ i.e., $E-F = Kn$, $K=$ constant

Thus $E = F + Kn$. Thus $10000 = F + 25K$

and $11500 = F + 30K$

Subtracting 1st eqn. from the second 1500 = 5K $\Rightarrow$ K = 300

Then F = 10000 - 25300 =10000 - 7500 = 2500 Rs.

Answer to Question 4(c):

Let M = set of candidates failed in Mathematics,

n(M) = no. of candidates failed in Mathematics = 20

E = set of candidates failed in English

($E \cap M$ = set of candidates failed in both Mathematics and English

S = set of candidates appeared in the exam. Similarly n(E) = 25,

$n(E \cap M) = 10$, n(S) = 50

$n(E^C \cap M^C) = n(S) - n(E \cup M) = 50 - [n(E) + n(M) - n(E \cap M)]$

$= 50 - [25 + 20 - 10] = 50 - 35 = 15$

so, no. of candidates passed in both the subjects = 15

December 2008 Examination

Question: 3.

(a) Answer *any three* of the following:

Choose the correct option showing necessary reasons/calculations.

(i) After arranging 5, $3\sqrt{3}$, $2\sqrt{6}$ in descending order they are

(A) $3\sqrt{3}, 5, 2\sqrt{6}$, (B) $2\sqrt{6}, 3\sqrt{3}, 5$, (C) $3\sqrt{3}, 2\sqrt{6}, 5$, (D) none of them.

(ii) If $y \propto \frac{1}{x^3}$ and x = 2 when y = 3, then for x = 3 the value of y is

(A) $\frac{4}{3}$, (B) $\frac{8}{9}$, (C) $\frac{4}{9}$, (D) none of them.

(iii) The number of ways in which the letters of word the COLLEGE can be arranged is

(A) 240, (B) 2520, (C) 5040, (D) none of them.

(iv) The number of digits in is (given 0.30103)

(A) 12, (B) 11, (C) 13, (D) none of them.

(v) Correct statement among $1 \subset \{1, 3, 4\}$, $\{1, 3\} \in \{1, 3, 4\}$ and $\{1, 4\} \subset \{1,3,4\}$ is

(A) $1 \subset \{1,3,4\}$, (B) $\{1, 4\} \subset \{1, 3, 4\}$ (C) $\{1, 3\} \in \{1, 3, 4\}$, (D) none of them.

(b) Answer *any three* of the following:

(i) Find the value of $\left(\frac{1}{35}\right)^{-2/5}$

(ii) Evaluate modulus of 3 -2*i*.

(iii) Determine the quadratic equation whose roots are 3 and -2.

(iv) Draw the graph of x ≤ -3 in XOY plane.

(v) If log p – log q = log (p – q), show that p = $\frac{q^2}{q-1}$.

Answer to Question No. 2(a):

(i) $\rightarrow$ (A) :

$5=\sqrt{25},\ 3\sqrt{3}=\sqrt{3\times9}=\sqrt{27},2\sqrt{6}=\sqrt{4\times6}=\sqrt{24}.$

As $24 < 25 < 27,\ \therefore \sqrt{24}<\sqrt{25}<\sqrt{27}$

Thus order is, $3\sqrt{3}$, 5, $2\sqrt{6}$

(ii) $\rightarrow$ (B) : $y=\frac{k}{x^3}, k$ = constant. $3=\frac{k}{8}$ or , k = 24

For x = 3, $y=\frac{24}{x^3}=\frac{24}{27}=\frac{8}{9}$

(iii) $\rightarrow$ (D) : (non of them) : Requied number of ways

$=\frac{\underline{|7}}{\underline{|2}\underline{|2}}=\frac{7\times6\times5\times4\times3\times2\times1}{2\times2}=1260,$

Since there are 7 letters and 2 L's, 2 E's and other 1 each.

(iv) $\rightarrow$ (C) : $\log_{10} 2^{40} = 40 \log_{10} 2 = 40 \times 0.30103 = 12.0412$.

As characteristic is 12, no. of digits in 2^{40} is 12 + 1 = 13.

(v) $\rightarrow$ (B) : 1 is a member of {1,3,4}. So, first statement is worng, {1,3} is not a member but is a subset of {1,3,4}. So, second statement is worng. {1,4} is a subset of {1,3,4}. So, is correct.

Answer to Question No. 3(b):

(i) $\left(\frac{1}{32}\right)^{-\frac{2}{5}}=(32)^{\frac{2}{5}}=\left(2^5\right)^{2/5}=2^{5\times\frac{2}{5}}=2^2=4$.

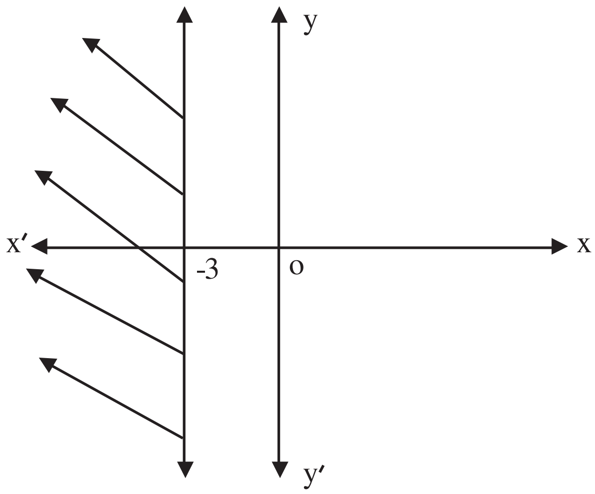
(ii) mod $(3-2i)=\sqrt{3^2+(-2)^2}=\sqrt{9+4}=\sqrt{13}$.

(iii) The required quadratic equation is $(x - 3)(x - (-2)) = 0$

or, $(x - 3)(x + 2) = 0$ or $x^2 + (-3 + 2)x - 6 = 0$

or, $x^2 - x - 6 = 0$.

(iv)



Here XO is x-axis and YO is y-axis.

(v) $\frac{p}{q} = p - q \Rightarrow p = \frac{q^2}{q - 1}$

Questions: 4.

Answer *any two* of the following:

(a) Solve: $2^{5x}.4^{4x-2} = \frac{8^{3x-8}}{16^{-3x}}$.

(b) If $x = 7 + 4\sqrt{3}$ find the value of $\sqrt{x} + \frac{1}{\sqrt{x}}$

(c) The volume of a gas varies directly as the absolute temperature and inversely as pressure. When the pressure is 10 units and the temperature is 200 units, the volume is 160 units. What will be the volume when presure is 12 units and temperature is 480 units?

(d) From 7 gentlemen and 4 ladies a committee of 5 is to be formed. In how many ways can this be done to include atleast one leady?

Answer to Question No.4 (a):

$$2^{5x}(2^2)^{4x-2} = \frac{(2^3)^{3x-8}}{(2^4)^{-3x}}$$

$$\text{or, } 2^{5x}.2^{2(4x-2)} = \frac{2^{3(3x-8)}}{2^{-4x3x}}$$

$$\text{or, } 2^{5x+8x-4} = 2^{9x-24+12x} \text{ or, } 2^{13x-4} = 2^{21x-24}$$

or, $13x - 4 = 21x - 24$ [as bases are same]

$$\text{or, } 24-4 = 21x - 13x \text{ or, } 8x = 20 \text{ or, } x = \frac{20}{8} = \frac{5}{2} = 2.5$$

Answer to Question No. 4(b):

$$\sqrt{x} = \sqrt{4+3+2(2\sqrt{3})} = \sqrt{(2+\sqrt{3})^2} = 2+\sqrt{3}$$

$$\frac{1}{\sqrt{x}} = \frac{1}{2+\sqrt{3}} = \frac{2-\sqrt{3}}{(2+\sqrt{3})(2-\sqrt{3})} = \frac{2-\sqrt{3}}{2^2-\left(\sqrt{3}\right)^2} = \frac{2-\sqrt{3}}{4-3} = 2-\sqrt{3}$$

$$\text{Thus, } \sqrt{x} + \frac{1}{\sqrt{x}} = 2+\sqrt{3}+2-\sqrt{3} = 4$$

Answer to Question No. 4(c):

Let, V = volume, T = temperature (absolute), P = pressure

Then, $V \propto \frac{T}{P}$ i.e., $V = \frac{KT}{P}$, K = constant.

For P = 10 unit, T= 200 unit, V = 160 unit, $160 = \frac{200K}{10}$

or, $K = \frac{1600}{200}$ or, K = 8 i.e., $V = \frac{8T}{P}$

Hence, required volume (V) = $\frac{8\times480}{12} = 8\times40 = 320$ cubic units.

Answer to Question No. 4(d):

1st case ${}^4C_1 \times {}^7C_4 = 140$

2nd case ${}^4C_2 \times {}^7C_3 = 210$

3rd case ${}^4C_3 \times {}^7C_2 = 84$

4th case ${}^4C_4 \times {}^7C_1 = 7$

$\therefore$ Total no. of selection = 140 + 210 + 84 + 7 = 441

June 2008 Examination

Question: 3.

(a) Answer *any five* of the following:

Choose the correct option showing necessary reasons/calculations.

(i) The value of is $2-\sqrt{-16}\div\sqrt{-4}$

(A) –1 (B) 0 (C) –(2 + *i*) (D) none of these

(ii) If ω is an imaginary cube root of unity the value of $\omega(1+\omega-\omega^2)$ is

(A) 1 (B) 0 (C) 2 (D) none of these

(iii) if both *a* and *b* are rational numbers the value of (*a, b*) in $\frac{\sqrt{3}+1}{\sqrt{3}-1}=a+b\sqrt{3}$

(A) (2, –1) (B) (–2, 1) (C) (2, 1) (D) none of these

(iv) Quardratic equation having one root $2+\sqrt{3}$ is

(A) $x^2 + 1 = 4x$ (B) $x^2 + 4x + 1 = 0$ (C) $x^2 + 4x = 1$ (D) none of these

(v) The value of x in ${}^{10}C_x = 10x\,{}^{9}C_{x-1}$ is

(A) 0 (B) 1 (C) 2 (D) none of these

(vi) The logarithm of 25 to the base $\sqrt{5}$ is

(A) 2 (B) 3 (C) 4 (D) none of these

(vii) The number of digits in 2^{20}, where $\log^{10} 2 = 0.30103$, is

(A) 7 (B) 6 (C) 5 (D) none of these

(viii) If $M = \{1, 2, 3, 5\}$, $N = \{2, 3, 4, 5\}$, $P = \{3, 5, 6\}$ then $M \cap (N - P)$ is

(A) {5} (B) {3} (C) {2, 4} (D) none of these

(b) Answer *any three* of the following:

(i) Simplify: $3\sqrt{48}-2\sqrt{75}$

(ii) Write the set $\{x: x$ is an integer, $-2 < x \le 2\}$ in Roster form.

(iii) If 5 is one root of the quadratic equation $x^2 - ax = 15$ find the value of *a*.

(iv) The following statements are given:

p : A number is an odd number

q : A number is greater than 5.

State the truth value of the conjuction

Answer to Question No. 3(a):

(i) $2-\sqrt{16}\div\sqrt{-4}=2-\sqrt{(4i)^2}\div\sqrt{(2i)^2}$

$=2-4i\div 2i=2-2=0$

Correct answer is (B)

(ii) w stisfies $w^2 + w + 1 = 0$ and $w^3 = 1$

Thus $w(1 + w - w^2) = w + w^2 - w^3 = -1 - 1 = -2$ ($\because$ $1 + w + w^2 = 0$)

Correect answer is (D)

(iii) $\dfrac{\sqrt{3}+1}{\sqrt{3}-1}=\dfrac{\left(\sqrt{3}+1\right)^2}{\left(\sqrt{3}-1\right)\left(\sqrt{3}+1\right)}=\dfrac{3+1+2\sqrt{3}}{\left(\sqrt{3}\right)^2-1}=\dfrac{2(2+\sqrt{3})}{3-1}=2+\sqrt{3}$

So, $2+\sqrt{3}=a+b\sqrt{3}$

Correct answer is $(a, b) = (2, 1)$ i.e., (C)

(iv) Other root is $2-\sqrt{3}$ So root has sum = 4, product = 1

Then the equation is $x^2 - 4x + 1 = 0$ or, $x^2 + 1 = 4x$

Correct answer is (A)

(v) $\dfrac{\underline{|10}}{\underline{|x}\,\underline{|10-x}}=\dfrac{10\times\underline{|9}}{\underline{|x-1}\,\underline{|10-x}}=\dfrac{\underline{|10}}{\underline{|x-1}\,\underline{|10-x}}$. so, $\underline{|x}=\underline{|x-1}$ is possible only

when $\underline{|1}=\underline{|1-1}$ or, $x = 1$

Correct answer is (B)

(vi) $\log_{\sqrt{5}} 25 = x$ say. Then $\left(\sqrt{5}\right)^x = 25$ or, $\left(5^{\frac{1}{2}}\right)^x = 5^2$

or, $5^{\frac{x}{2}} = 5^2$ or, $\dfrac{x}{2} = 2$ or, $x = 4$ as bases are same

So, Correct answer is (C)

(vii) $\text{Log}_{10}2^{20} = 20\log_{10}2 = 20\times 0.30103 = 6.02060$

i.e., characteristic is 6. Thus there are 7 digits.

So, Correct answer is (A)

(viii) N–P = (2,3,4,5) – (3,5,6)

= Set of elements belonging to N but not to P

= (2,4)

$M \cap (N - P) = (1,2,3,5) \cap (2,4) = (2)$

So, Correct answer is none of these i.e., (D)

Answer to Question No. 3(b):

(i) $3\sqrt{48} - 2\sqrt{75}$

$= 3\sqrt{16 \times 3} - 2\sqrt{25 \times 3}$

$= 3\sqrt{16}.\sqrt{3} - 2.\sqrt{25}\sqrt{3}$

$= 3 \times 4\sqrt{3} - 2 \times 5\sqrt{3} = (12 - 10) \times \sqrt{3} = 2\sqrt{3}$

(ii) $\{-1, 0, 1, 2\}$

(iii) 5 satisfies $x^2 - ax = 15$

or, $5^2 - 5a = 15$

or, $a = 2$

Thus the value of a is 2.

(iv) A number greater than 5 is an odd number

Question: 4.

Answer *any three* of the following:

(a) Find the number of years at the end of which Rs. 100 will become Rs. 1000 allowing compound interest at 4% per annum. [Given log 1.04 = 2.0170333]

(b) Solve : $xy = 6$, $yz = 2$, $zx = 3$

(c) The monthly expenses of a boarding house are partly fixed and partly varied with the number of boarders. The monthly charges are Rs. 100 per head when there are 25 boarders and Rs. 80 per head when there are 15 boarders. If the monthly charge per head is Rs. 70 find the number of boarders.

(d) Form a quadratic equation whose roots are cube of the roots of $2x^2 + 4 = 6x$.

(e) Out of 5 diffrent consonants and 4 different vowels how many different words can be formed each containing 3 consonants and 2 vowels?

(f) Prove that $x^{\log y - \log z}\, y^{\log z - \log x}\, z^{\log x - \log y} = 1$

Answer to Question No. 4 (a):

$$A = P\left(1 + \frac{R}{100}\right)^n$$

$$\Rightarrow 1000 = 100\left(1 + \frac{4}{100}\right)^n \Rightarrow 10 = (1.04)^n$$

Taking logarithm on both sides we get,

$$n \log 1.04 = \log 10 \text{ or, } n = \frac{\log 10}{\log 1.04} = \frac{1}{0.017033339} = 58.71$$

or, $n = 58.71$ years (approx.)

$\therefore$ The required number of years = 58.71 years (approx.)

Answer to Question No. 4(b):

Multiplying both sides of the equations

$z^2y^2z^2 = zy.yz.zx = 6.2.3 = 6^2$

or $(xyz)^2 = 6^2$ or, $xyz = \pm 6$

$$x = \frac{xyz}{yz} = \frac{\pm 6}{2} = \pm 3$$

$$y = \frac{xyz}{zx} = \frac{\pm 6}{3} = \pm 2$$

$$z = \frac{xyz}{xy} = \frac{\pm 6}{6} = \pm 1$$

(x, y, z) = (3, 2, 1) or (-3, -2, -1)

Hence, x = (3, -3), y = (2, -2), z = (1, -1)

Answer to Question No. 4(c):

In a month per head x = fixed expenses

y = variable expenses,

$y \infty n$ (n = no. of boarders), y = kn, (k=constant)

In a month total expenses = E

Thus E = x + kn

Given E = 100 Rs. when n = 25 and E = 80 Rs. when n = 15

So 100 = x + 25k and 80 x + 15k

Substracting 20 = 10 k or k =2. Then x = 100 - 25 x 2 = 50

So E = 50 +2n

For, E = 70, 70 = 50 + 2n or, 20 = 2n or, n = 10

Thus required number of boarders = 10

Answer to Question No. 4(d):

Let α and β be the roots of $x^2 - 3x + 2 = 0$ or, $2x^2 - 6x + 4 = 0$

So, $\alpha + \beta = 3$, $\alpha\beta = 2$

Then $\alpha^3 + \beta^3 = (\alpha + \beta)^3 - 3\alpha\beta(\alpha + \beta) = 3^3 - 3 \times 2 \times 3 = 27 - 18 = 9$

$\alpha^3 \beta^3 = (\alpha \beta)^3 = 2^3 = 8$

Thus require quadratic equation is

$x^2 - (\alpha^3 + \beta^3)\,x + \alpha^3 \ \alpha^3 \beta^3 = 0$ or, $x^2 - 9x + 8 = 0$

Answer to Question No. 4(e):

Out of 5 different consonants, 3 consonants can be chosen in ${}^5C_3 = \frac{5 \times 4}{2} = 10$ Ways

Out of 4 different vowels, 2 vowels can be chosen in ${}^4C_2 = \frac{4 \times 3}{2} = 6$ Ways

So, 3 consonants and 2 vowels can be chosen for a words in 10 x 6 = 60 ways

Then, number of possible words = No. of arrangements of them

Numer of required words which can be formed = 60 x 5! = 60 x 120 = 7200

Answer to Question No. 4(f):

$$\log\left[x^{\log y - \log z}\, y^{\log y - \log x}\, z^{\log x - \log y}\right]$$
$$= (\log y - \log z)\log x + (\log z - \log x)\log y + (\log x - \log y)\log z$$
$$= \log x \log y - \log z \log x + \log y \log z - \log x \log y + \log z \log x - \log y \log z$$
$$= o = \log 1$$

Taking antiog (considering commo log arith)

$$x^{\log y - \log z}\, y^{\log z - \log x}\, Z^{\log x - \log y} = 1 \text{(proved)}$$

December 2007 Examination

Question: 3:

(a) Answer any *five* of the following

Choose the correct option showing necessary reasons/calculations.

(i) The conjugate complex number of $2 + 3i$ is

(A) 2 – 3i, (B) 3 – 2i, (C) 3 + 2i, (D) none of these.

(ii) if α and β be the roots of a quadratic equation $x^2 - 3x + 5 = 0$ then the value of $\alpha^2 + \beta^2$ is

(A) 1, (B) – 1, (C) 3, (D) none of these

(iii) If ${}^n p_x = 336$ and ${}^n C_x = 56$, then (n, x) will be in order.

(A) (8, 3), (B) (3, 8), (C) (8, 8), (D) none of these.

(iv) The logarithm of 400 to the base $2\sqrt{5}$ is

(A) 2,6020, (B) 2, (C) 4, (D) none of these.

(v) The difference between simple interest (SI) and compound interest (CI) on Rs. 1,000 for 2 years at 4% p.a. payable quarterly is

(A) Rs. 15 (B) Rs. 81 (C) Rs. 2 (D) none of these.

(vi) If A = {1,2,3,4,5} and B = {0,1,2,3,4,5,6,7,8,9}, then $A \Delta B(=(A : B) \cup (B : A)$ is

(A) {4,2,3,5) (B) {6,7,8,9) (C) {0,6,7,8,9} (D) none of these

(vii) If *p* and *q* be the proportions, then De Morgan's Laws are stated as

(A) $\begin{array}{l} p \vee q = p \wedge q \\ : p \vee q = : p \vee q \end{array}$ (B) $\begin{array}{l} :(p \vee q) = : p \wedge : q \\ :(p \vee q) = : p \vee : q \end{array}$

(C) $\begin{array}{l} p \vee : q = p \wedge : q \\ : p \wedge q = : (p \vee q) \end{array}$ (D) None of these.

(b) Answer any *three* of the following:

(i) Express $\sqrt[3]{108}$ as mixed surd

(ii) Find the value of $\left(\frac{1}{81}\right)^{\frac{3}{4}} \times \left(\frac{1}{8}\right)^{\frac{1}{3}}$

(iii) If a varies as *b* then show that a + *b* varies as a - b

(iv) Represent the following by Venn diagram $(A \cap B) \cup (A \cap B)$

Answer to Question No. 3 (a):

(i) $\rightarrow$A : Conjugate complex number of 2 + 3i is 2 - 3i

(ii) $\rightarrow$B : $\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta = 3^2 - 2 \times 5 = 9 - 10 = -1$

(iii) $\rightarrow$A : ${}^nC_x = 56 \Rightarrow \frac{{}^nP_x}{\underline{|x}} = 56 \Rightarrow \underline{|x} = \frac{336}{56} = 6 = \underline{|x} \Rightarrow x = 3$

Now, ${}^nC_x = 336 \Rightarrow {}^np_3 = 336 \Rightarrow$ n (n -1)(n - 2) = 8 x 7 x 6 $\Rightarrow$ n = 8.

Hence, (n, x) = (8, 3)

(iv) $\rightarrow C : \log_{2\sqrt{5}} 400 = \frac{\log 400}{\log 2\sqrt{5}} = \frac{2\log 20}{\frac{1}{2}\log 20} = 4$

$$\rightarrow \text{C :SI} = \frac{\text{PRT}}{100} = \frac{1000 \times 4 \times 2}{100} = \text{Rs.80}$$

(where SI = Simple Interest)

$$\text{CI} = \text{A} - \text{P} = \text{P}\left(1 + \frac{\text{R}}{100}\right)^{\text{n}} - P = 1000[(1.01)^{\text{n}} - 1]$$

$= 1000[1.082 - 1] = \text{Rs. } 82$

(where CI = Compound Interest)

$\text{CI} - \text{SI} = \text{Rs. } 82 - \text{Rs. } 80 = \text{Rs. } 2$

(vi) $\rightarrow$C : $\text{A} : \text{B} = \phi$ and $\text{B} : \text{A} = \{0,6,7,8,9\}$

$\text{A} \Delta \text{B} = \{0,6,7,8,9\}$

(vii) $\rightarrow \text{B}:(\text{A} \cup B)' = \text{A}' \cap \text{B}'$ BY DeMorgan's Law

$: (\text{p} \vee \text{q}) = : \text{p} \wedge : \text{q}$

$: (\text{p} \wedge \text{q}) = : \text{p} \wedge : \text{q}$

Answer to Qustion No. 3(b):

(i) $\sqrt[3]{108} = \sqrt[3]{3^3.4} = \left(3^3\right)^{\frac{1}{3}} . 4^{\frac{1}{3}} = 3.4^{\frac{1}{3}} = 3\sqrt[3]{4}$

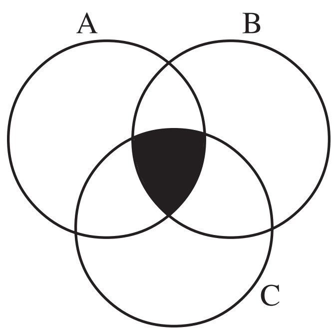
(ii) $\left(\frac{1}{81}\right)^{-\frac{3}{4}} \times \left(\frac{1}{8}\right)^{-\frac{1}{3}} = (81)^{\frac{3}{4}} \times 8^{\frac{1}{3}} = \left(3^4\right)^{\frac{3}{4}} \times (2)^{3 \times \frac{1}{3}} = 3^3 \times 2 = 27 \times 2 = 54$

(iii) $\text{a} \infty \text{b} \Rightarrow \frac{1}{\text{b}} = k$ = constant of variation, i.e. , a = kb

Then $\frac{\text{a} + \text{b}}{\text{a} - \text{b}} = \frac{\text{kb} + \text{b}}{\text{kb} - \text{b}} = \frac{\text{k} + 1}{\text{k} - 1} = \text{cons tan t}$

So $(\text{a} + \text{b}) \; \alpha \; (\text{a} - \text{b})$

(iv)



Question: 4:

Answer *any three* of the following

(a) Find the squrare root of $17 + 12\sqrt{2}$

(b) If w be an imaginary cube root of 1, show that

$(1 + w - w^2)(1 - w + w^2) = 4$

(c) In how many ways the letters of the word BALLOON be arranged so that two L's do not come together?

(d) If α, β be the roots of the equation $x^2+px+7 = 0$ and $\alpha^2 + \beta^2 = 22$, find the value of p.

(e) In a certain population the annual birth and death rates per 1000 are 39.4 and 19.4 respectively. Find the number of years in which the population will be double assuming that there is no immigration or emigration.

Answer to Question No. 4(a):

Let $\sqrt{17 + 2\sqrt{2}} = \sqrt{x} + \sqrt{y}$

Squaring $17 + 12\sqrt{2} = x + y + 2\sqrt{xy}$

Equaling rational and irrational parts

$X + y = 17$ and $\sqrt{xy} = 6\sqrt{2}$

i.e., $X + y = 17$ and $xy = 72$

to solve them we proceed as follows:

$x(17 - x) = 72$

or, $x^2 - 17X + 72 = 0$

or, $(x - 8)(x - 9) = 0$

i.e. $x = 8,9$

Then $y = 9, 8$

so, the required square roots = $\pm\left(\sqrt{9} + \sqrt{8}\right)$ or, $\pm\left(3 + 2\sqrt{2}\right)$

Answer to Question No. 4 (b):

$(1 + w - w^2)(1 - w + w^2) = (-w^2 - w^2)(-w - w) = 4w^3 = 4$

Answer to Question No. 4(c):

There are 7 letters of which 2 are L's, 2 are O's and others are 1 each.

So, letters of the said words BALLOON can be arranged in $\frac{7!}{2!2!}$ ways

If L's come together then consider 2 L's as one letter.

Then there will be 6 letters. No. of arrangements 2 L's come together is $\frac{6!}{2!}$, since 2 L's are arranged among themselves in 1 way.

So required no. of arrangements $= \frac{7!}{2!2!} - \frac{6!}{2!} = 900$

Answer to Question No. 4 (d):

$\alpha + \beta = -p, \alpha\beta = 7$

$\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta$

$\Rightarrow 22 = p^2 - 14 \Rightarrow p^2 = 36 \therefore p = \pm 6$

Answer to Question No. 4(c):

$$\text{Rate per annum} = \frac{(39.4 - 19.4) \times 100}{1000} = \frac{20 \times 100}{1000} = 2\% \text{ per annum}$$

$$A = p\left(1 + \frac{R}{100}\right)^n \Rightarrow 2p = p\left(1 + \frac{2}{100}\right)^n \Rightarrow 2 = (1.02)^n$$

$$\Rightarrow n = \frac{\log 2}{\log 1.02} = \frac{0.3010}{0.0086} = 36 \text{ years}$$

June 2007 Examination

Question: 4.

Answer any five of the following:

(a) Find the value of $\left(\frac{1}{243}\right)^{-3/5}$

(b) Simplify : $3\sqrt{48}-4\sqrt{75}+\sqrt{192}$

(c) Find the conjugate of 3+ 2 *i*.

(d) If $a + b$ varies as a - b, prove that $a \propto b$.

(e) Find the roots of the equation $2x^2$-5x +3=0

(f) In XOY plane draw the graph of x = 2.

(g) Find the value of $\log_2 \log_3 81$.

Answer to Question 4:

(a) $\left(\frac{1}{243}\right)^{-3/5}=\left(\frac{1}{3^5}\right)^{-3/5}=\left(3^{-5}\right)^{-3/5}=3^3=27$

(b) $$3\sqrt{48}-4\sqrt{75}+\sqrt{192}=3\sqrt{16\times3}-4\sqrt{25\times3}+\sqrt{64\times3}$$
$$=3\sqrt{4^2\times3}-4\sqrt{5^2\times3}+\sqrt{8^2\times3}$$
$$=12\sqrt{3}-20\sqrt{3}+8\sqrt{3}=0$$

(c) The conjugate of $3+2i=3\text{-}2i$

(d) Given : $a+b \propto a-b \therefore \frac{a+b}{a-b}=k$ (constant)

$$\Rightarrow (a+b)=k(a-b)$$
$$\Rightarrow a(1-k)=-(k+1)b$$
$$\Rightarrow \frac{a}{b}=\frac{k+1}{k-1}=\text{constant}$$
$$\Rightarrow a \propto b$$

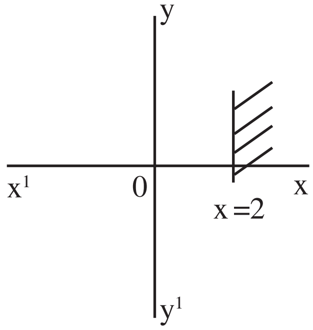
(e) The quadratic equation is $2x^2-5x+3=0$

$$\Rightarrow 2x^2-2x-3x+3=0$$
$$\Rightarrow 2x(x-1)-3(x-1)=0$$
$$\Rightarrow (x-1)(2x-3)=0$$

$$\Rightarrow 2\left(x-\frac{3}{2}\right)(x-1)=0$$

So, the roots are 1, $\frac{3}{2}$

(f)



Shaded portion is the graph of $x \geq 2$

(g) $\log_2 \log_3 81 = \log_2 \log_3 3^4 = \log_2 (4 \log_3 3) = \log_2 4 = \log_2 2^2 = 2 \log_2 2 = 2$

$[\Theta \log_a a = 1]$

Question: 5.

(a) Solve : $\sqrt{\frac{x}{1-x}} + \sqrt{\frac{1-x}{x}} = \frac{5}{2}$

(b) A question paper of an examination is divided into 3 groups A, B, and C containing 4, 5 and 3 questions respectively. In how many ways an examinee can answer 6 questions taking at least 2 from group A, at least 2 from group B and at least 1 from group C?

Answer to Question 5:

(a) Let $\sqrt{\frac{x}{x-1}} = y$

So, $y + \frac{1}{y} = \frac{5}{2}$ ....(1)

Then $\left(y - \frac{1}{y}\right)^2 = \left(y + \frac{1}{y}\right)^2 - 4 = \frac{25}{4} - 4 = \frac{9}{4}$

$y - \frac{1}{y} = \pm \frac{3}{2}$ ....(2)

Adding (1) and (2)

$2y = \frac{8}{2}$ or 1 i.e., $y = 2$ or $\frac{1}{2}$

So, $\sqrt{\frac{x}{1-x}} = 2$ or $\frac{1}{2}$

$\frac{x}{1-2} = 4$ or $\frac{1}{4}$

When, $\frac{x}{1-x} = 4,\ x = 4-4x,\ 5x = 4$ or $x = \frac{4}{5}$

When, $\frac{x}{1-x} = \frac{1}{4}, 4x = 1-x,\ 5x = 1,$ or $x = \frac{1}{5}$

(b) n examinee can answer 6 questions (Q) in following ways:

(i) 3Q from Gr. A, 2Q from Gr. B, 1Q from Gr. C i.e. in ${}^4C_3 \times {}^5C_2 \times {}^3C_1$ ways

(ii) 2Q from Gr. A, 3Q from Gr. B, 1Q from Gr. C i.e. in ${}^4C_2 \times {}^5C_3 \times {}^3C_1$ ways

(iii) 2Q from Gr. A, 2Q from Gr. B, 2Q from Gr. C i.e. in ${}^4C_2 \times {}^5C_2 \times {}^3C_2$ ways

So, he can answer 6 questions in

${}^4C_3 \times {}^5C_2 \times {}^3C_1 + {}^4C_2 \times {}^5C_3 \times {}^3C_1 \times {}^4C_2 \times {}^5C_2 \times {}^3C_2$

$= 4 \times \frac{5.4}{2} \times 3 + 6 \times \frac{5.4}{2} \times 3 + 6 \frac{5.4}{2} \times 3$

$= 120 + 180 + 180 = 480$ ways

Question: 6.

(a) Prove that the proposition pv~ (p∧q) is tautology.

(b) One of the roots of $x^2 + ax + 8 = 0$ is 2 and the roots of $x^2+ax+b = 0$ are equal. Find b.

Answer to Question 6:

(a)

p	q	p ∧ q	~ (p ∧ q)	pv ~ (p ∧ q)
T	T	T	F	T
T	F	F	T	T
F	T	F	T	T
F	F	F	T	T

(b) Let ∝ and 2 be the roots of the equation $x^2 + ax + 8=0$

Then, sum of the roots = ∝+ 2 = - a and product of the roots 2 ∝ = 8

$\Rightarrow \alpha = \frac{8}{2} = 4$ and

hence, $\alpha + 2 = -a = 4 + 2 = 6$ i.e. $a = -6$

Let, β and â be the roots of equation $x^2 + ax + b = 0$ i.e., $x^2 - 6x + b = 0$

As before, $\beta + \beta = 6$ and $\beta^2 = b$. Now, $2\beta = 6$ or, $\beta = 3$. Hence $b = 3^2 = 9$

Aliter, when $x = 2$, $2^2 + a\,2 + 8 = 0 \Rightarrow a = -6$

Again since roots of ., $x^2 - 6x + b = 0$ are equal,

Discriminant = 0, So, $(-6)^2 - 4.1.\,b = 0 \Rightarrow 36 - 4b = 0 \Rightarrow b = 9$

Question : 7.

(a) Determine the time period during which a sum of Rs. 1,234 amounts to Rs. 5,678 at 8% p.a. compound interest, payable quarterly. [Given: log 1234 = 3.0913, log 5678 = 3.7542 and log 1.02 = 0.0086]

(b) If $\frac{a(b+c-a)}{\log a} = \frac{b(c+a-b)}{\log b} = \frac{c(a+b-c)}{\log c}$, show that $b^c c^b = c^a a^c = a^b b^a$

Answer to Question 7:

(a) $A = P\left(1 + \frac{i}{4}\right)^{4n}$

Here $A = 5678$, $P = 1234$, $i = 8\%$, n = Time Period = ?

$$\text{So}, 5678 = 1234\left(1 + \frac{0.08}{4}\right)^{4n} = 1234(1.02)^{4n}$$

$$\Rightarrow \log 5678 = \log 1234 + 4n \log 1.02$$

$$\Rightarrow 3.7542 = 3.0913 + 4n \times 0.0086$$

$$\Rightarrow n = \frac{3.7542 - 3.0913}{4 \times 0.0086} = \frac{0.6629}{0.0344} = 19.27 \text{ years;}$$

(b) $\frac{a(b+c-a)}{\log a} = \frac{b(c+a-b)}{\log b} = \frac{c(a+b-c)}{\log c} = k\,(\text{say})$

$$\text{Then } \frac{\log a}{a} = \frac{b+c-a}{k}, \frac{\log b}{b} = \frac{c+a-c}{k}, \frac{\log c}{c} = \frac{a+b-c}{k}$$

$$\text{Now } \log(a^b b^a) = b\log a + a\log b = ab\left(\frac{\log a}{a} + \frac{\log b}{b}\right)$$

$$= ab\left(\frac{b+c-a}{k} + \frac{c+a-b}{k}\right) = ab\left[\frac{2c}{k}\right] = \frac{2abc}{k}$$

$$\text{Similarly, } \log(b^c c^b) = \frac{2abc}{k} \text{ and } \log(c^a a^c) = \frac{2abc}{k}$$

$$\text{So, } \log\left(a^b b^a\right) = \log\left(b^c c^b\right) = \log\left(c^a a^c\right)$$

$$\text{Taking antilogarithm, } \left(a^b b^a\right) = \left(b^c c^b\right) = \left(c^a a^c\right)$$

December 2006 Examination

Question: 4.

Answer any five of the following

(a) Express $2\sqrt{2}$ as a surd of sixth order.

(b) find the modules of the complex number $3 + 4i$.

(c) If x varies as y then show that $x^2 + y^2$ varies as $x^2 - y^2$.

(d) Find the logarithm of 125 to the base $5\sqrt{5}$.

(e) Find the quadratic equation whose one root is $3 + \sqrt{2}$.

(f) Prove that ${}^{10}P_{10} \times {}^{22}C_{12} = {}^{22}P_{10}$.

(g) If S be the set of all prime numbers and M = {0, 1, 2, 3}, find S^{ε} M.

Answer to Question No. 4:

(a) $2\sqrt{2} = \sqrt{8} = 8^{\frac{1}{2}} = 8^{\frac{3}{6}} = \left(8^3\right)^{\frac{1}{6}} = (512)^{\frac{1}{6}} = \sqrt[6]{512}$

(b) $|3+4i| = \sqrt{3^2 + 4^2} = \sqrt{9+16} = \sqrt{25} = 5$

(c) $\frac{x}{y} = k$ (= constant). Then $\frac{x^2+y^2}{x^2-y^2} = \frac{k^2y^2+y^2}{k^2y^2-y^2} = \frac{k^2+1}{k^2-1} = \text{constant}$

$\Rightarrow x^2 + y^2$ varies as $x^2 - y^2$

(d) $\log_{5\sqrt{5}} 125 = x, \therefore \left(5\sqrt{5}\right)^x = 125 = \left(5\sqrt{5}\right)^2 \therefore x = 2$

$\therefore$ Logarithm is 2.

(e) One root is $3+\sqrt{2}$, other root is $3-\sqrt{2}$

The required equation : $x^2-\left(3+\sqrt{2}+3-\sqrt{2}\right)^{x}+\left(3+\sqrt{2}\right)\left(3-\sqrt{2}\right)=0$

(f) $^{10}P_{10}\times{}^{22}C_{22}=\dfrac{|\underline{10}}{|\underline{10-10}}\times\dfrac{|\underline{22}}{|\underline{12}\,|\underline{10}}=\dfrac{|\underline{10}}{|\underline{0}}\times\dfrac{|\underline{22}}{|\underline{12}\,|\underline{10}}=\dfrac{|\underline{22}}{|\underline{12}}$ $\quad|\underline{0}=1$

$=\dfrac{|\underline{22}}{|\underline{22-10}}={}^{22}P_{10}$

(g) Let S = set of prime no's = {2, 3, 5, 7, ...}
S ∩ M = {2, 3, 5, 7, ...} ∩ {0, 1, 2, 3}
= {2, 3}

Question: 5.

(a) For what value of m, $4x^2-10mx+4=0$ and $4x^2-9x+2=0$ have a common root?

(b) Out of 5 gentlmen and 3 ladies, a committee of 6 persons is to be selected. Find the number of committees to be formed (i) when there are 4 gents; (ii) when there is a majority of gents.

Answer to Question No. 5:

(a) $4x^2-9x+2=0$ i.e., $(4x-1)(x-2)=0$

Then x = $\dfrac{1}{4}$ or 2

Roots satisfy the equations and the given equations have a common root. So, if the common root is $\dfrac{1}{4}$ then $x=\dfrac{1}{4}$ satisfies.

$4x^2-10mx+4=0$, So, $\dfrac{4}{16}-\dfrac{10}{4}m+4=0$ i.e. $\dfrac{10}{4}m=\dfrac{17}{4}$

$\Rightarrow m=\dfrac{17}{10}$

Again if the common root is 2, then $x=2$ satisfies $4x^2-10mx+4=0$
So, $16-20m+4=0$ $\quad\Rightarrow m=1$

i.e., $m=\dfrac{17}{10}$ or 1

Hence, the value of $m=\dfrac{17}{10}$ or 1

(b) (i) The number of committees is $^5C_4\times{}^3C_2$
(when there are 4 gents) $=5\times3=15$

(ii) No. of committees $= {}^5C_4 \times {}^3C_2 + {}^5C_5 \times {}^3C_1$
(when there is a majority of gents) $= 5 \times 3 + 1 \times 3 = 15 + 3 = 18$

Question: 6.

(a) Publisher of a book pays a lump sum plus an amount for every copy he sold, to the author. If 1000 copies were sold the author would receive Rs. 2,500 and if 2700 copies were sold the author would receive Rs. 5,900. How much the author would receive if 5000 copies were sold?

(b) Construct truth table for $(p \wedge q) \wedge \sim (p \wedge q)$

Answer to Question No. 6:

(a) Let, x = lump sum amount, the author received
y = variable amount, the author received
n = no. of copies of books sold
Then, $y \propto n$ or, y = kn, k = constant
A = The amount author received = x + kn
Thus 2500 = x + 1000k
5900 = x + 2700k
$\therefore$ 3400 = 1700 k k = 2 Then x = 500
i.e., A = 500 + 2n.
Hence required amount the author received = Rs. 10500

(b) TRUTHTABLE

p	q	$(p \wedge q)$	$\sim (p \wedge q)$	$(p \wedge q) \wedge \sim (p \wedge q)$
T	T	T	F	F
T	F	F	T	F
F	T	F	T	F
F	F	F	T	F

Hence, $(p \wedge q) \wedge \sim (p \wedge q)$ is a contradiction.

Question: 7.

(a) Determine the time period by which a sum of money would be three times of itself at 8% p.a. C.I. [given $\log_{10} 3 = 0.4771$, $\log^{10} 1.08 = 0.0334$]

(b) Solve: $x(x + y + z) = 6$; $y (x + y + z) = 12$; $z (x + y + z) = 18$

Answer to Question No. 7:

(a) $A = P\left(1+\dfrac{R}{100}\right)^n \Rightarrow 3P = P(1+0.08)^n$

$\Rightarrow 3 = (1.08)^n$

$$\Rightarrow n = \frac{\log 3}{\log 1.08} = \frac{0.4771}{0.0334} = 14.28 \text{ years}$$

(b) $x\,(x + y + z) = 6 \qquad \therefore x + y + z = \dfrac{6}{x}$

$y\,(x + y + z) = 12 \qquad \therefore x + y + z = \dfrac{12}{y}$

$z\,(x + y + z) = 18 \qquad \therefore x + y + z = \dfrac{18}{z}$

$\dfrac{6}{y} = \dfrac{12}{y} = \dfrac{18}{z} \quad \Rightarrow \dfrac{1}{x} = \dfrac{2}{y} = \dfrac{3}{z} = \dfrac{1}{k}$

$\therefore x = k,\ y = 2k,\ z = 3k$

$\therefore \mathrm{x\,(x + y + z) = 6} \quad \Rightarrow k\,(k + 2k + 3k) = 6$

$\Rightarrow 6k^2 = 6 \Rightarrow k^2 = 1 \therefore k \pm 1$

The required solution is

$x = 1, y = 2, z = 3$, or, $x = -1, y = -2, z = -3$

Aliter,

Adding the given three equations we get

$(x + y + z)^2 = 36$

$\therefore x + y + z = \pm 6$

Putting $x + y + z = 6$ in the three equations, we get the solution as

$x = 1, y = 2, z = 3$

Putting $x + y + z = -6$ in the three equations, we get the solution as

$x = -1, y = -2, z = -3$

June 2006 Examination

Answer Question No. 4 (Compulsory - 5 marks) and any two (10x 2=20 marks) from the rest.

Q4. Answer *any five* of the following:

(a) Simplify : $3\sqrt{8} \div 2\sqrt{-18}$

(b) Express $\sqrt{3}$ as a surd of twelfth order.

(c) Evaluate $\log_{0.01} (0.001)$.

(d) Form a quadratic equation whose roots are 2 and 3.

(e) If (a+b) varies as (a–b), Prove that a^2+b^2 varies as b^2

(f) Simplify $4P_2 \div 4C_2$

(g) If A = {1,2,3,4}, B= {2,4,5,8}, C= {3,4,5,6,7}, find $A \cup (B \cup C)$

Answer to Question 4:

(a) $$\frac{3\sqrt{-8}}{2\sqrt{-18}}=\frac{3\sqrt{(2\sqrt{2i})^2}}{2\sqrt{(3\sqrt{2i})^2}}=\frac{3\times 2\sqrt{2i}}{2\times 3\sqrt{-18}}=1$$

(b) $$\sqrt{3}=3\frac{6}{12}=12\sqrt{3^6}=12\sqrt{729}$$

(c) $$\log_{0.01}(0.001)=\frac{\log 10^{-3}}{\log 10^{-2}}=\frac{-3\log 10}{-2\log 10}=\frac{3}{2}$$

(d) The required equation is (x-2) (x-3) = 0 $\Rightarrow x^2 - 5x + 6 = 0$

(e) a+b = K(a-b), k = constant of variation

$$\Rightarrow (k-1)a=(k+1)b \Rightarrow a=\frac{K+1}{K-1}b$$

$$\therefore a^2+b^2=\left(\frac{K+1}{K-1}\right)^2 b^2+b^2=\frac{(K+1)^2+(K-1)^2}{(K-1)^2}b^2=\frac{2(K+1)^2}{(K-1)_2}b^2=k^1b^2$$

Where K^1 is a constant

So, a^2+b^2 varies as b^2

(f) $$\frac{4p_2}{4c_2}=\frac{4!/2!}{4!/2!2!}=2!=2$$

(g) $B \cup C = \{2,3,4,5,6,7,8\} : A \cup (B \cup C) = \{1,2,3,4,5,6,7,8\}$

Q5. (a) The equation $x^2 + 2(p+2)x + 9p = 0$ has equal roots, Find the values of p.

(b) Prove that $2^nP_n = 2^n\{1,3,5....(2n-1)\}$

Answer to Question 5:

(a) For equal roots $b^2 - 4ac = 0$

$\Rightarrow 4(P+2)^2 - 4 \times 9p = 0$

$\Rightarrow P^2 - 5p + 4 = 0 \Rightarrow (p-1)(p-4)=0 \Rightarrow p=1, 4$

$\therefore$ value of P is 1 and 4

Alter : $2\alpha = -2(P+2) \Rightarrow \alpha = -(p+2)$; $\alpha^2 = 9p \Rightarrow (p+2)^2 = 9p \Rightarrow p^2 - 5p + 4 = 0$

$\Rightarrow (p-1)(p-4) = 0 \ \therefore p = 1, 4$

(b) $2n\,P_n = \frac{2n!}{n!} = \frac{\{1,3,5....(2n-1)\}\{2,4,6....(2n-2),2n\}}{n!}$

$= \frac{\{(1,3,5....(2n-1)\}2^n\{1,2,3(n-1).n\}}{n!} = \frac{2^n\{1,3,5.....(2n-1)\}n!}{n!}$

$= 2^n\{1,3,5....(2n-1)\}$

Hence, the equality follows.

Q6. (a) The expenses of a boarding house are partly fixed and partly varies with the number of boarders. The charge is Rs. 70 per head when there are 20 boarders and Rs. 60 per head when there are 40 boarders. Find the charge per head when there are 50 boarders.

(b) Show by the truth table. $p \wedge q = \sim(\sim p \vee \sim q)$

Answer to Question 6 :

(a) Let x = fixed monthly expense of the boarding house

n = no. of boarders

y = variable monthly expense for boarder

e = monthly expense

Then e = x + y = x + kn since $y \;\alpha\; n \Rightarrow y = kn$, k being a constant.

Given : 70 x 20 = x + 20k .... (i)

60 x 40 = x + 40k..............(ii)

Solving (i) and (ii) k = 50

x = 1400 – 20k = 400

Thuse = 400 + 50n

For n = 50, e = 400 + 50 x 50 = 2900 (in Rs.)

$\therefore$ Charge per head = $\frac{2900}{50}$ = 58 (in Rs.)

(b) Truth Table

Col. no.	P (1)	q (2)	$p \wedge q$ (3)	~p (4)	~q (5)	$\sim p \vee \sim q$ (6)	$\sim(\sim p \vee \sim q)$ (7)
	T	T	T	F	F	F	T
	T	F	F	F	T	T	F
	F	T	F	T	F	T	F
	F	F	F	T	T	T	F

Entries of Col (3) and Col (7) are alike.

So. $p \wedge q = \sim(\sim p \vee \sim q)$

Q7. (a) A machine is depriciated at the rate of 10% on reducing balance. The original cost of which Rs. 1,00,000 and the ultimate scrap value wa Rs. 37,500. Estimate the effective life of the machine. [Given log 2 = 0.3010 and log 3 = 0.4771]

Answer to Question 7:

(a) $\text{We know}, S = c\left(1-\frac{D}{100}\right)^n$

$\therefore 37500 = 100000\left(1-\frac{10}{100}\right)^n$

Where S = Scrap Value

C = Cost of machine

D = Depreciation

n = life of machine

$\Rightarrow 375 = 1000\ (0.9)^n$

$\Rightarrow 3000 = 8000\ (0.9)^n$

Taking log, log 3 = 3 log 2 + n log.9

$\Rightarrow n\frac{\log 3 - 3\log 2}{\log(0.9)} = \frac{0.4771 - 3x.3010}{(2x0.47711-1)}$

$= \frac{0.4259}{0.0458} = 9.30 \text{ years}$

$$\left[\begin{array}{l} \log 0.9 = \log\frac{9}{10} \\ = \log 3^2 - \log 10 \\ = 2\log 3 - 1 \end{array}\right]$$

December 2005 Examination

Question 4:

Answer any five of the following:

(a) Express $\sqrt[3]{135}$ as mixed surd.

(b) If a+2b varies as a –2b, prove that a varies as b.

(c) From a quadratic equation whose one root is $2+\sqrt{3}$

(d) Evaluate $(243)^{-\frac{1}{5}}$

(e) If ${}^{n}p_{2}=56,\text{find n}.$

(f) Prove that log (1+2+3) = log 1+ log 2 + log 3.

(g) If A={1, 2, 3} and B = {2, 3, 4}, find (A–B) ∪ (B–A).

Answer to Q. No 4 :

(a) $\sqrt[3]{135}=\sqrt[3]{27\times 5}=3\sqrt[3]{5}$

(b) (a+2b) = k(a – 2b), k is a constant i.e. $a=\dfrac{2(k+1)}{k-1}b=\text{const.}\times b$

Thus a varies as b

(c) Roots are : $2+\sqrt{3}$ and $2-\sqrt{3}$ (conjugate root)

Required Equation : $\{x-(2+\sqrt{3}\}\{x+(2-\sqrt{3})=0$

$$\Rightarrow x^2-\{2+\sqrt{3}+2-\sqrt{3}\}x+(2+\sqrt{3})(2-\sqrt{3})=0$$

$$\Rightarrow x^2-4x+1=0$$

(d) $(243)^{-\frac{1}{5}}=(3^5)^{-\frac{1}{5}}=3^{-1}=\dfrac{1}{3}$

(e) ${}^{n}p_{2}=56\Rightarrow\dfrac{n!}{(n-2)!}=56\Rightarrow n(n-1)=8\cdot(8-1)\Rightarrow n=8$

(f) LHS = log (1+2+3) = log 6 = log (1×2×3) = log 1+ log 2 +log 3 proved

(g) Now, A – B = {1} · {B–A} ={4}

∴ (A – B)∪ (B – A) = {1} ∪{4} = {1, 4}

Question 5:

(a) Construct a quadratic equation whose roots are α^3 and β^3 when α and β are the roots of $x^2+3x+2=0$ 5

(b) If $2^x = 3^y = 6^z$, prove that $\frac{1}{x}+\frac{1}{y}=\frac{1}{z}\cdot$ 5

Answer to Q. No. 5:

(a) Given α and β are the roots of $x^2 + 3x + 2 = 0$

So $\alpha + \beta = -3$ and $\alpha\beta = 2$

Now $\alpha^3 + \beta^3 = (\alpha + \beta)^3 - 3\alpha (\alpha + \beta) = (-3)^3 - 3.2.(-3)$

$= -27 + 18 = -9$

$\alpha^3 \cdot \beta^3 = 2^3 = 8$

So, the required equation : $x^2 - (\alpha^3+\beta^3) \times + \alpha^3 \ \beta^3 = 0$

$$\Rightarrow x^2 + 9x + 8=0.$$

(b) $2^x = 6^z$ or $2 = 6^{\frac{z}{x}}$

$\text{Also}, 3^y = 6^z \cdot \text{or}, 3 = 6^{\frac{z}{y}}$

$\text{Then}, 2 \times 3 = 6^{\frac{z}{x}} \times 6^{\frac{z}{y}}$

$\text{or } 6 = 6^{\frac{z}{x}+\frac{z}{y}}$

$\text{Thus}, \frac{z}{x}+\frac{z}{y} = 1 \text{(as base are same)}$

$\text{or}, \frac{1}{x}+\frac{1}{y}=\frac{1}{z}.\text{Proved.}$

Question 6:

(a) Solve $\text{Log}_x 2 = \frac{\log_{4x} 2}{\log_x 2}\cdot$ 5

(b) As the number of units manufactured in a factory is increased from 200 to 300, the total cost of production increases from Rs. 16,000 to Rs. 20,000. If the total cost of production is partly fixed and other part varies as number of units produced, find the total cost for producing 500 units.

Answer to Q. No. 6 :

(a) $$\frac{1}{\log_2 x} = \frac{\frac{1}{\log_2 4x}}{\frac{1}{\log_2 x}} = \frac{\log_2 x}{\log_2 4x}$$

i.e., $(\log_2 x)^2 = \log_2 4x = \log_2 4 + \log_2 x$

$= 2 + \log_2 x$ ($\log_2 \cdot 4 = 2\log_2 2 = 2$)

Let, $\log_2 x = a$. Then, $a^2 = 2 + a$ or $a^2 - a - 2 = 0$

or, $(a-2)(a+1) = 0$ i.e. $a = 2$ or -1

Thus $\log_2 x = 2$ or -1 i.e., $x = 2^2$ or 2^{-1}, i.e., $x = 4$ or $\frac{1}{2}$.

(b) Let, C= total cost, a = fixed cost, n = no. of units produced C = a + kn. where k is constant of variation.

Then $a + 200k = 16000$... (i) and $a + 300k = 20000$ .....(ii)

substracting first equation from 2nd, we get, $100k = 4000$ or, $k = 40$ and $a = 16000 - 200 \times 40 = 8000$.

Thus, $C = 8000 + 40n$

For $n = 500$, $C = 8000 + 40 \times 500 = 8000 + 20000 =$ Rs. 28,000

The required Total Cost = Rs. 28,000.

Question 7 :

(a) If a group of 13 workers contains 5 women, in how many ways can a sub-group of 10 workers be selected so as to include atleast 6 men? 5

(b) In a class test of 70 students, 23 and 30 students passed in Mathematics and in Statistics respectively and 15 passed in Mathematics but not passed in Statistics. Using set theory result, find the number of students who passed in both the subjects and who did not pass in both the subjects. 5

Answer to Q. No. 7

(a) New group of 10 workers with atleast 6 men can be formed in following ways (Here among 13 workers 5 are women and 8 are men).

Group Contents	No. of ways of forming groups
6 men and 4 women	${}^8C_6 \times {}^5C_4 = \frac{8!}{6!2!} \times \frac{5!}{4!1!} = 28 \times 5 = 140$
7 men and 3 women	${}^8C_7 \times {}^5C_3 = \frac{8!}{7!1!} \times \frac{5!}{3!2!} = 8 \times 10 = 80$
8 men and 2 women	${}^8C_8 \times {}^5C_2 = \frac{8!}{8!0!} \times \frac{5!}{2!3!} = 1 \times 10 = 10$

Thus, required number of ways of forming new group of 10 workers $= 140 + 80 + 10 = 230$

(b) Let A, B, U be the set of students passing Mathematics, set of students passing Statistics and set of students appeared in class test,

n (A), n (B), n (U) be the sizes of A, B and U,

Here, n (A) = 23, n (B) = 30, n(U) = 70

$n(A \cap B^c)$ = no. of students who passed in Mathematics but not in Statistic = 15

$n(A \cap B^c) = n(A) - n(A \cap B)$

$15 = 23 - n(A \cap B)$

or $n(A \cap B) = 23 - 15 = 8$

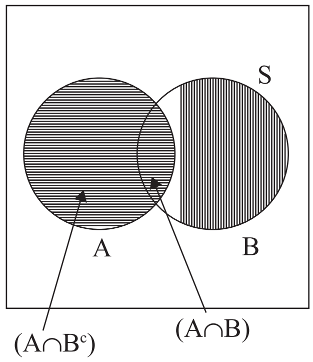
No. of students passed in both the subjects = 8

Again, $n(A^c \cap B^c) = n[(A \cup B)^c] = n(\cup) - n(A \cup B)$

$= 70 - [n(A) + n(B) - n(A \cap B)]$

$= 70 - 23 - 30 + 8 = 78 - 53 = 25$

Number of students who failed in both the subjects = 25



June 2005 Examination

Question: 4.

Answer any five of the following:

(a) find p if $(P)^{p\sqrt{p}} = \left(p\sqrt{p}\right)^p$

(b) Solve: $x^2 + 9 = 0.$

(c) If n (S) = 12, n (T) = 20 and $S \subset T$, find $n\left(S \cup T\right)$

(d) If a varies as b prove that $a + b$ varies as $a - b$

(e) If the roots of a quadratic equation be 3 and 5, find the quadratic equation.

(f) Find the logarithm of 125 to the base $5\sqrt{5}$

(g) If ${}^{r}C_{12} = {}^{r}C_{8}$, find ${}^{22}C_{r}$.

Answer to Q. No. 4:

(a) $(p)^{p\sqrt{p}} = \left(p\sqrt{p}\right)^p$ or, $p^{p^{3/2}} = \left(p^{3/2}\right)^p = p^{3/2p}$

$$\text{or, } p^{3/2} = \frac{3p}{2} \text{ or, } p^{\frac{3}{2}-1} = \frac{3}{2} \text{ or, } p^{\frac{1}{2}} = \frac{3}{2} \text{ or, } p = \left(\frac{3}{2}\right)^2 = \frac{9}{4}$$

(b) $x^2 + 9 = 0$ Or, $x^2 = -9$ or, $x^2 = 9i^2 = (\pm 3i)^2$ or, $x = \pm 3i$

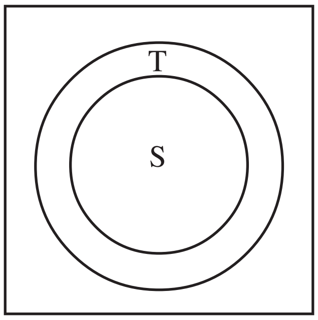
(c) $S \subset T$, so $n(S \cap T) = 12$ as $n(S) = 12$

Now, $n(S \cup T) = n(S) + n(T) - n(S \cap T) = 12 + 20 - 12 = 20$

Alternative way:

Using Venn diagram

Refer fig. $n(S \cup T) = n(T) = 20$



(d) $a \propto b$, or, $a = kb$, where k = constant

$$\text{Now, } \frac{a+b}{a-b} = \frac{kb+b}{kb-b} = \frac{(k+1)b}{(k-1)b} = \frac{k+1}{k-1} = \text{a constant}$$

$\therefore (a + b) \propto (a - b)$

(e) Sum of roots = 3 + 5 = 8, product of roots = $3 \times 5 = 15$

Hence, required equation is $x^2 - 8x + 15 = 0$

(f) Let $\log_{5\sqrt{5}} 125 = x$, then $\left(5\sqrt{5}^{x}\right) = 125 = \left(5\sqrt{5}\right)^2$ or, $x = 2 \therefore \log_{5\sqrt{5}} 125 = 2$

(g) ${}^{r}C_{12} = {}^{r}C_{8} = {}^{r}C_{r-8}$ Or, $12 = r - 8$ or, $r = 20$

$$\text{Now, } {}^{22}C_r = {}^{22}C_{20} = {}^{22}C_2 = \frac{22.21}{2} = 11 \times 21 = 231$$

Question: 5.

(a) Prove that $\dfrac{\log\sqrt{27} + \log 8 + \log\sqrt{100}}{\log 14400} = \dfrac{3}{4}$

Answer to Q. No. 5:

(a) $$\frac{\log\sqrt{27} + \log 8 + \log\sqrt{100}}{\log 14400} = \frac{\log 3^{3/2} + \log 2^3 + \log 10^{3/2}}{\log(120)^2} = \frac{\frac{3}{2}\log 3 + 3\log 2 + \frac{3}{2}\log 10}{2\log 120}$$

$$\frac{\frac{3}{2}(\log 3 + 2\log 2 + \log 10)}{2\log(3 \times 4 \times 10)} = \frac{3}{4}\frac{(\log 3 + \log 4 + \log 10)}{(\log 3 + \log 4 + \log 10)} = \frac{3}{4} = \text{R.H.S.}$$

Question : 6.

(a) Prove that "CALCUTTA" is twice of "AMERICA" in respect of number of arrangements of letters.

(b) If S is the set of all prime numbers, $M = \{x | 0 \leq x \leq 9\}$, exhibit

(i) $M-(S \cap M)$ (ii) $M \cup N$ where $N = \{0, 1, 2, ..., 20\}$

(c) Show by the truth table: $\therefore p \Rightarrow q =: q \Rightarrow: p$

Answer to Q. No. 6:

(a) In CALCUTTA, each of the letters C, A, T occurs twice and total number of letter is 8.

So, number of arrangements is $\dfrac{8!}{2!2!2!} = \dfrac{8 \times 7!}{8} = 7!$

And in AMERICA, A occurs twice and total number of letters is 7, so no. of arrangements is $\dfrac{7!}{2}$. Hence in respect of arrangments of letters, CALCUTTA is twice of AMERICA.

(b) $M = \{0, 1, 2, ..., 8, 9\}$; $S = \{2, 3, 5, 7, 11, 13, 17, 23, ...\}$

$S \cap M = \{2,3,5,7\}$; $M–S \cap M = \{0, 1, 4, 6, 8, 9\}$

Again, $M \subset N$, so $M \cap U = \{0, 1, 2, ...19, 20\} = N$

(c)

p	q	: p	: q	p⇒q	: p⇒ : q
1	2	3	4	5	6
T	T	F	F	T	T
T	F	F	T	F	F
F	T	T	F	T	T
F	F	T	T	T	T

Since, Col. (5) and Col. (6) are identical, so $p \Rightarrow q =: q \Rightarrow : p$

Question:7.

(a) Out of 6 ladies and 3 gentlemen, a committee of six is to be selected. In how many ways can this committee containing at least 4 ladies be formed?

(b) If the roots of the equation $ax^2 + bx + c = 0$ be in the ratio 2:3 then show that $6b^2 = 25ca$.

Answer to Q. No. 7:

(a)

	Possible cases		Selections
	Ladies (6)	Gentlemen (3)	
(i)	4	2	${}^6C_4 \times {}^3C_2 = {}^6C_2 \times {}^3C_1 = \frac{6\times5}{2}\times3 = 45$
(ii)	5	1	${}^6C_5 \times {}^3C_1 = {}^6C_1 \times {}^3C_1 = 6\times3 = 18$
(iii)	6	0	${}^6C_6 \times {}^3C_0 = 1\times1 = 1$

For case (i), each way of selecting ladies can be associated with each way of selecteing gentlemen, so total number of ways = ${}^6C_4 \times {}^3C_2$ simillarly for case (ii) & (iii)

$\therefore$ Required no. of ways = 45 + 18 + 1 = 64

(b) Let α, β be the roots of the equation $ax^2 + bx + c = 0$ so that

$$\alpha+\beta = \frac{-b}{a}.....(i); \qquad \alpha\beta = \frac{c}{a}.....(ii)$$

Again, $\frac{\alpha}{\beta} = \frac{2}{3}$;or $\qquad \alpha = \frac{2}{3}\beta.....(iii)$

From (i), $\frac{2}{3}\beta + \beta = \frac{-b}{a}$;or $\qquad \frac{5\beta}{3} = \frac{-b}{a}$ or, $\qquad \beta = \frac{3}{5}\times\frac{-b}{a} = \frac{-3b}{5a}$

From (iii), $\alpha = \frac{2}{3}\times\frac{-3b}{5a} = \frac{-2b}{5a}$

From (ii), $\frac{-2b}{5a}\times\frac{-3b}{5a} = \frac{c}{a}$; or, $\frac{6b^2}{25a^2} = \frac{c}{a}$; or, $6b^2 = 25ac\,[\text{as}, a \neq 0]$

(c) Alternative way

let the roots be 2α and 3α

$$\text{So } 2\alpha + 3\alpha = \frac{-b}{a} \Rightarrow 5\alpha = \frac{-b}{a} \quad \text{(i)} \quad 2\alpha\times3\alpha = \frac{c}{a} \Rightarrow 6\alpha^2 = \frac{c}{a}\text{(ii)}$$

From (i) and (ii) above

$$\frac{b^2}{25a^2} = \frac{c}{6a}; \quad \text{or,} \quad 6b^2 = 25ac$$

December 2004 Examination

Question :

4. Answer *any five* parts of the following:

(a) Find the modulus of $\frac{24+7i}{4+3i}$.

(b) Simplify $\frac{x^{a+b}x^{a-b}x^{m-2a}}{x^{m-a}}$

(c) Show that log (1 + 2 + 3) = log 1 + log 2 + log 3.

(d) Find the value of $3\sqrt{-4} \div 2\sqrt{-9}$.

(e) Solve for p if $2^{p+3} + 2^{p+1} = 320$.

(f) If $7P_r = 2520$, find the value of r.

Answer to Question No. 4 (a):

$$\text{Let}, z = \frac{24+7i}{4+3i}, \text{ now} |z| = \left|\frac{24+7i}{4+3i}\right| = \left|\frac{24+7i}{4+3i}\right| = \frac{\sqrt{24^2+7^2}}{\sqrt{4^2+3^2}}$$

$$= \frac{\sqrt{576+49}}{\sqrt{16+9}} = \frac{\sqrt{625}}{\sqrt{25}} = \frac{25}{5} = 5$$

Answer to Question No. 4 (b):

$$\text{Expr.} = \frac{x^{a+b+a-b+m-2a}}{x^{m-a}} = \frac{x^m}{x^{m-a}} = x^{m-(m-a)} = x^a$$

Answer to Question No. 4 (c):

L.H.S. = log 6 = log (1.2.3) = log 1 + log 2 + log 3.

Answer to Question No. 4 (d):

$$\text{Expr.} = \frac{3\sqrt{4}\sqrt{-1}}{2\sqrt{9}\sqrt{-1}} = \frac{3.2.i}{2.3.i} 6/6 = 1. = 6/6 = 1.$$

Answer to Question No. 4 (e):

$2^{p+3} + 2^{P+1} = 320$ or, $2^p.2^3 + 2^p.2 = 320$ or, $2^p (2^3 + 2) = 320$ or, $2^p. 10 = 320$ or, $2^p = 32 = 2^5$ or, p = 5 (as base is same)

Answer to Question No. 4 (f):

$$7p_r = 2520 \text{ or, } \frac{\lfloor 7}{\lfloor 7-r} = \frac{\lfloor 7}{\lfloor 2} \text{ or, } \lfloor 7-r = \lfloor 2 \text{ or, } 7-r = 2 \text{ or, } r = 5$$

Question :

5. (a) Given $p^2 = 7p - 3$ and $q^2 = 7q - 3$ where $p \neq q$. Form a quadratic equation whose roots are p/q and q/p.

(b) Solve for x: $(\sqrt{5})^{4(x-1)} = 5^{2x-3} + 20$.

Answer to Question No. 5 (a):

$p^2 = 7p - 3$ or, $p^2 - 7p + 3 = 0$

Again, $q^2 = 7q - 3$ or, $q^2 - 7q + 3 = 0$ which means that p and q are the roots of the equation $x^2 - 7x + 3 = 0$. Now $p + q = 7$ and $p.q = 3$

Given roots are p/q and q/p

$$\frac{p}{q} + \frac{q}{p} = \frac{p^2+q^2}{pq} = \frac{(p+q)^2 - 2pq}{pq} = \frac{7^2 - 2.3}{3} = \frac{49-6}{3} = \frac{43}{3} \text{ and } \frac{p}{q} \times \frac{q}{p} = 1$$

So the reqd. equation is $x^2 - \frac{43}{3}x + 1 = 0$ i.e. $3x^2 - 43x + 3 = 0$

Answer to Question No. 5 (b):

$$5^{\frac{1}{2}.4(x-1)} = 5^{2x-3} + 20 \text{ or, } 5^{2x-2} = 5^{2x-3} + 20 \text{ or, } 5^{2x-2} - 5^{2x-3} = 20$$

or, $5^{2x}(5^{-2} - 5^{-3}) = 20$ or, $5^{2x}\left(\frac{1}{25} - \frac{1}{125}\right) = 20$ or, $5^{2x}(4/125) = 20$

or, $5^{2x} = 5.125 = 5.5^3 = 5^4$ or, $2x = 4$ or, $x = 2$.

Question :

6. (a) If $\frac{\log x}{y^2 + z^2 + yz} = \frac{\log y}{z^2 + x^2 + zx} = \frac{\log z}{x^2 + y^2 + xy}$

show that $x^{y-z} y^{z-x} z^{x-y} = 1$.

(b) An engine without any wagons can run 24 km/hr and its speed is diminished by a quantity varying as the square root of the number of wagons attached to it. With 4 wagons its speed becomes 20 km/hr. Find the maximum number of wagons with which the engine can move.

Answer to Question No. 6 (a):

$$\frac{\log x}{y^2+z^2+yz}=\frac{\log y}{z^2+x^2+zx}=\frac{\log z}{x^2+y^2+xy}=k \text{ (say)}$$

or, $\log x = k(y^2 + z^2 + yz)$, $\log y = k(z^2 + x^2 + zx)$, $\log z = k(x^2 + y^2 + xy)$ ....(i)

To show $x^{y-z}y^{z-x}z^{x-y} = 1$, taking logarithm both sides

$\log (x^{y-z}.\ y^{z-x}.\ z^{x-y}) = \log 1 = 0$ i.e. to show

$(y - z) \log x + (z - x) \log y + (x - y) \log z = 0$

L.H.S. $= (y - z).\ k.\ (y^2 + z^2 + + yz) + (z - x).k.(z^2 + x^2 + zx) + (x - y).k.(x^2 + y^2 + xy)$

$= k(y^3 - z^3 - x^3 + x^3 - y^3) = k.0 = 0$, hence proved.

Answer to Question No. 6 (b):

Let x = no. of wagons attached, u = quantity of diminished speed, so that $u \propto \sqrt{x}$ or $u = m\sqrt{x}$, m = constant of variation.

Now distance (in km.) travelled by the train per hour (its speed) $= 24 - m\sqrt{x}$

When x = 4, speed = 20 km per hour. We find $20 = 24 - m\sqrt{x}$ or, $20 = 24 - m\sqrt{4}$

or, $20 = 24 - 2m$ or, $m = 2$

Hence the speed of the engine with x wagons $= 24 - 2\sqrt{x}$. Now as x increases, speed diminishes. Let the speed become zero for $x = x_i$. So, $0 = 24 - 2\sqrt{x_1}$ or, $\sqrt{x_1} = 12$

or, $x_1 = 144$. Thus for 144 wagons attached, the engine fails to move the train. So the engine can move with 144 – 1 = 143 wagons.

Question :

7. (a) A question paper is divided into three groups A, B and C, each of which contains 3 questions, each of 25 marks. One examinee is required to answer 4 questions taking at least one from each group. In how many ways he can choose the questions to answer 100 marks?

   (b) In a survey of 100 students it was found that 60 read Economics, 70 read Mathematics, 50 read Statistics, 27 read Mathematics and Statistics, 25 read Statistics and Economics and 35 read Mathematics and Economics and 4 read none. How many students read all three subjects? 5

Answer to Question No. 7 (a):

Possible cases	Grp. A	Grp. B	Grp. C	Selections
1	1	1	2	${}^3c_1 \times {}^3c_1 \times {}^3c_2 = 3 \times 3 \times 3 = 27$
2	1	2	1	${}^3c_1 \times {}^3c_2 \times {}^3c_1 = 3 \times 3 \times 3 = 27$
3	2	1	1	${}^3c_2 \times {}^3c_1 \times {}^3c_1 = 3 \times 3 \times 3 = 27$
Total				81

For case 1, 1 question can be selected out of 3 questions of group A in ${}^3c_1$, 1 question from group B in ${}^3c_1$, and 2 questions of group C in ${}^3c_2$. Each way of selecting question of group A can be associated with each way of selecting questions from group B and group C. So we get ${}^3c_1 \times {}^3c_1 \times {}^3c_2$. Similarly for cases 2 and 3.

$\therefore$ Reqd. no. of ways = 27 + 27 + 27 = 81.

Answer to Question No. 7 (b):

Let E stands for Economics, M stands for Mathematics and S stands for Statistics.

Now $n(E) = 60$, $n(M) = 70$, $n(S) = 50$, $n(M \cap S) = 27$, $n(S \cap E) = 25$, $n(M \cap E)=35$, $n(E \cap M \cap S) = ?$

Using $n(E \cap M \cap S) = n(E) + n(M) + n(S) - n(E \cap M) - n(S \cap E) + n(E \cap M \cap S)$

We get $100 - 4 = 60 + 70 + 50 - 35 - 27 - 25 + n(E \cap M \cap S)$ (as 4 students read nothing)

or, $96 = 180 - 87 + n(E \cap M \cap S)$ or, $n(E \cap M \cap S) = 96 - 93 = 3$

$\therefore$ Reqd. no. of students = 3

SECTION - III

MENSURATION

AREA & PARAMETER

VOLUME, SURFACE AREA OF SOLID FIGURES

PAPER P-4

Business Mathematics and Statistics Fundamentals

SECTION – III

MENSURATION

June 2010 Examination

Question:

5. Answer *any three* of the following:

   Choose the correct option showing necessary reasons/calculations:

   (a) The perimeter of an equilateral triangle is 36 cm. Then the area of the triangle is

   (i) 30”3 sq.cm (ii) 36”3 sq.cm (iii) 36”2 sq.cm (iv) none of these

   (b) The sum of the interior angles and each interior angle of a pentagon is

   (i) (540^0, 180^0) (ii) (450^0, 90^0) (iii) (720^0, 144^0) (iv) none of these

   (c) The sides of a cubiod are 40 cm, 20 cm and 10 cm. It is melted to form a new cube. The surface area of the new cube in sq.com is

   (i) 1400 (ii) 2800 (iii) 4000 (iv) none of these

   (d) The volume of a hollow right circular cylinder of height 14 cm with iternal and external radii of base 8 cm and 10 cm respectively, has the volume is cu.cm as (Take π = 22/7)

   (i) 4400 (ii) 1584 (iii) 88 (iv) none of these

   (e) A right prism has triangle base whose sides are 13 cm, 20 cm and 21 cm. If the altitude of the prism is 9 cm then the volume of the prism is

   (i) 1134 c.c. (ii) 1200 c.c. (iii) 1000 c.c. (iv) none of these

Answer to Question No. 5(a):

Let the side of the equilateral triangle be a cm.

$\therefore 3a = 36 \Rightarrow a = 12\text{cm}$

$\text{Area} = \frac{\sqrt{3}}{4}a^2 = 36\sqrt{3}\,\text{sq.cm}$ Ans (ii)

Answer to Question No. 5(b):

Sum of the interior angles of a polygon = (2n – 4) 90^0

For pentagon, sum of the interior angles = (25 – 4) 90^0 = 690^0 = 540^0

Each interior angle = 540^0/5 = 108^0 Ans (i)

Answer to Question No. 5(c):

Side length of a new cube = a cm

Surface area of the cube = $6a^2 = 6400 = 2400$ sq. cm

(Since $a^3 = 40 \times 30 \times 10 \Rightarrow a = 20$ cm) Ans (iv)

Answer to Question No. 5(d):

R, r = external, internal radii of base, h = height of cone

Required volume = $\pi\left(R^2 - r^2\right)h = \frac{22}{7}\left(10^2 - 8^2\right) \times 14 = 1584$ cucm Ans(ii)

Answer to Question No. 5(e):

$$\text{Here } \frac{\frac{4}{3}\pi r_1^3}{\frac{4}{3}\pi r_2^3} = \frac{64}{27} \Rightarrow \frac{r_1}{r_2} = \frac{4}{3} \Rightarrow 3r_1 = 4r_2$$

Volume = area of base $\times$ height = $126 \times 9 = 1134$ c.c Ans (i)

Question:

6. Answer *any two* of the following:

   (a) A right pyramid stands on a base 16 cm square and its height is 15 cm. Find the slant surface and volume of the pyramid.

   (b) A road of one meter wide is developed around a circular garden with diameter 20m @ Rs. 100 per sq. m. Find the cost of development of the road. (Take $\pi = 22/7$)

   (c) The volume of two spheres are in the ratio 64:27. Find their radii if the sum of their radii is 21 cm. (Take $\pi = 22/7$)

Answer to Question No. 6(a):

Here $1^2 = h^2 + r^2 = 15^2 + 8^2 = 289 \therefore 1 = \sqrt{289} = 17$ cm

Slant surface area = $\frac{1}{2} \times 64 \times 17 = 544$ sq.cm

Volume of the pyramid = $\frac{1}{3} \times 16^2 \times 15 = 1280$ c.c.

Answer to Question No. 6(b):

Radius of circular garden = $\frac{20}{2} = 10$ cm

Radius of circular garden with road = $\frac{20+2}{2}$ = 11 cm

Area of the road = $\pi\left(11^2 - 10^2\right) = \frac{22}{7}(121-100) = 66$sq.cm

So cost of development of road = 66 $\times$ 100 = Rs. 6600

Answer to Question No. 6 (c):

Here $\frac{\frac{4}{3}\pi r_1^3}{\frac{4}{3}\pi r_2^3} = \frac{64}{27} \Rightarrow \frac{r_1}{r_2} = \frac{4}{3} \Rightarrow 3r_1 = 4r_2$

Also $r_1 + r_2 = 21$ ....(2)

Solving (1) and (2) we get r_1 = 12 cm, r_2 = 9 cm

December 2009 Examination

Question : 5.

Answer *any three* of the following

Choose the correct option showing necessary reasons/calculations.

(a) If two adjacent sides of righ angle of a right-angled triangle are such that the length of one side is twice the other and the hypotenuse is 5 cm then area of the triangle in sq.cim is

(i) 10, (ii) 5, (iii) 2.5, (iv) none of these.

(b) If the parameter of a semicircle is 36 cm then area of that semicircle in sq. cm is (Given) $\pi = \frac{22}{7}$

(i) 144, (ii) 22, (iii) 77, (iv) none of these.

(c) Surfaces of a cube of volume 125 cu. ft. are painted with black colour at cost of Rs. 10 per sq. ft. The amount required to paint the outer surfaces of the cube in Rs. is

(i) 1500, (ii) 1250, (iii) 1000, (iv) none of these.

(d) If 3 solid spheres of radii 3ft., 4ft. and 5ft. of iron are melted to form a new sphere, the surface area of the new sphere in square feet is (Given) $\pi = \frac{22}{7}$

(i) $\frac{264}{7}$, (ii) $\frac{528}{7}$, (iii) $\frac{792}{7}$, (iv) none of these.

(e) A right pyramid stands on a base of 12 cm Square and its height is 8 cm. Then its total Surface area in sq. cm. is

(i) 240 (ii) 384 (iii) 624 (iv) None of these.

Answer to Question no. 5.

(a) Let the adjacent sides of the right angle of the right angled triangle be *a* and 2*a* cm. The hypotenuse has length $= \sqrt{a^2+(2a)^2} = \sqrt{5a^2} = \sqrt{5}a$

So $\sqrt{5}a = 5 \Rightarrow a = \sqrt{5}$

Then area of the triangle $= \frac{1}{2}a \times 2a = a^2 = (\sqrt{5})^2 = 5$ sqcm Ans(ii)

(b) Let r = radius of the semicircle in cm. Then perimeter

$$2r + \frac{1}{2} \times 2\pi r = 2r + \pi r = \left(2 + \frac{22}{7}\right) r = \frac{36}{7} r \text{ cm.}$$

Thus $\frac{36}{7} r = 36$ or, r = 7 cm

Then area of that semicircle $= \frac{1}{2}\pi r^2 = \frac{1}{2} \times \frac{22}{7} \times 49 = 77$ sq.cm. Ans(iii)

(c) Let cube has a side = x cm. Then volumn = x---^{3} cu ft

So $x^3 = 125$ or x = 5. Its surface area = $6x^2 = 6 \times 25 = 150$ sq. ft.

Required cost of painting = 150×10 = Rs. 1500/- Ans(i)

(d) Volume of the new sphere $= \frac{4}{3}\pi\left(3^3+4^3+5^3\right) = \frac{4}{3}\pi(27+64+125)$

$$= \frac{4}{3}\pi \times 216 = \frac{4}{3}\pi(6)^3$$

Also radius of the new sphere = r ft

Then $\frac{4}{3}\pi r^3 = \frac{4}{3}\pi(6)^3 \Rightarrow r = 6$

So surface area of new sphere $= 4\pi \times 6^2 = \frac{4 \times 22 \times 36}{7} = \frac{3168}{7}$ = none of these Ans(iv)

(e) (Slant height)^2 = (height)^2 + $\left(\frac{1}{2} \times \text{base}\right)^2 = 8^2 + 6^2 = 100 = 10^2$

So slant height = 10 cm

Total surface area = $\frac{1}{2}$ × perimeter of baseslant height + area of base

$$\frac{1}{2} \times 48 \times 10 + 12 \times 12 = 240 + 144 = 384 = \text{sq. cm.} \quad \text{Ans (ii)}$$

Question : 6.

Answer *any two* of the following

(a) The areas of three adjacent sides of a cuboid are 15 sq. cm, 10 sq. cm. and 6 sq. cm. Find the voulume of the cuboid.

(b) The height and slant height of a right circular cone is 24 cm and 25 cm respectively. Find the area of the curved surface and volume. $\left(\text{Given } \pi = \frac{22}{7}\right)$.

(c) The sum of length, breadth and height of a rectangular parallelepiped is 24 cm and its diagonal is 15 cm. Find the area of the whole surface of the parallelepiped.

Answer to Question no. 6(a).

Let length, breadth and height of the cuboid be a, b, c cm.

Then area of 3 adjacent sides are ab, ac, bc cm

So ab = 15, ac = 10, bc = 6

Multiply them $ab \times ac \times bc = 15 \times 10 \times 6$

or, $(a\,b\,c)^2 = 900 = (30)^2$

or, abc = 30

So volume of the cuboid = abc = 30 cu. cm.

Answer to Question no. 6(b).

Height h = 24 cm, slant height l = 25 cm

Radius $r = \sqrt{l^2 - h^2} = \sqrt{25^2 - 24^2} = 7\,\text{cm}$

Area of the curved surface $\pi rl = \frac{22}{7} \times 7 \times 25$

$= 550$ sq. cm.

Volume of the cone = $\frac{1}{3}\pi r^2 h$

$= \frac{1}{3} \times \frac{22}{7} \times 7^2 \times 24$

$= 1232$ c.c

Answer to Question no. 6(c).

Let length = a cm, breadth = b cm, height = c cm of the rectangular parallelepiped.

$\therefore a + b + c = 24, \sqrt{a^2 + b^2 + c^2} = 15$

$\therefore\ 2(ab+bc+ca) = (a+b+c)^2 - (a^2+b^2+c^2)$

$= (24)^2 - (15)^2 = 351$

$\therefore$ Area of the whole surface = 351 sq. cm.

June 2009 Examination

Question: 5.

Answer *any three* of the following:

Choose the correct option showing necessary reasons/calculations.

(a) The area of the equilateral triangle with a side of length 2 cm is

(i) $\frac{\sqrt{3}}{2}$ sq. cm, (ii) $\sqrt{3}$ sq. cm, (iii) $\frac{\sqrt{3}}{4}$ sq. cm (iv) none of these.

(b) A circular garden having diameter 60 ft. has a path of width 10 ft. surrounding outside the garden. Area of the path is

i) 2200 sq. ft., (ii) $\frac{28600}{7}$ sq. ft., (iii) $\frac{1600}{7}$ sq. ft., (iv) none of these.

(c) A rectangular parallelepiped has length 20 cm, breadth 10 cm, and height 5 cm, the total surface area of it is

(i) 350 sq. cm (ii) 1000 sq. cm, (iii) 700 sq.cm (iv) none of these.

(d) For a solid right circular cylinder of height 9 cm and radius of base 7 cm the total surface area is

(i) 352 sq. cm, (ii) 550 sq.cm, (iii) 700 cu.ft., (iv) none of these.

(e) The diameter of the base of a conical tent is 14 ft. ant height of the tent is 15 ft. The volume of the space covered by the tent is

(i) 2130 cu.ft., (ii) 8520 cu.ft., (iii) 700 cu.ft., (iv) none of these.

Answer to Question 5:

(a) Let a be length of a side

Area of the equilateral tirangle = $\frac{\sqrt{3}}{4}a^2 = \frac{\sqrt{3}}{4}2^2 = \sqrt{3}$ sq. cm. **Ans. (ii)**

(b) Radius of circular garden = $\frac{1}{2} \times 60 = 30$ ft.

Radius of circular garden along with path = 30 + 10 = 40 ft.

Then area of the path

= Area of the garden with path - Area of the garden

$= \pi\, 40^2 - \pi\, 30^2 = \frac{22}{7}(1600 - 900) = \frac{22}{7} \times 700 = 2200$ sq. ft. **Ans (i)**

(c) Surface area of the rectangular parallelepiped

= 2 (length × breadth + lenght × height + heightbreadth)

= 2 (2010 + 205 + 105) = 700 sq. cm. **Ans (iii)**

(d) Total surface area = $2\pi r^2 + 2\pi rh$ where r = radius of base = 7 cm

and height of the cylinder h = 9 cm

So total surface area = $2\pi r\,(r + h)$

$= 2 \times \frac{22}{7} \times 7(7 + 9) = 44 \times 16 = 704$ sq. cm. **Ans (iii)**

(e) Let r = radius of base of conical tent and h = its height

So r = 7 ft and h = 15 ft

Space covered by the tent = $\frac{1}{3}\pi r^2 h = \frac{1}{3} \times \frac{22}{7} \times 49 \times 15 = 770$ cu ft Ans. (iv) None of these

Question: 6.

Answer *any two* of the following:

(a) The area of a rectangle is 96 sq. cm and its perimeter is 40 cm, what are its length and breadth?

(b) Find the quantity of water in litre flowing out of a pipe of cross-section area 5 cm^2 in 1 minute if the speed of the water in the pipe is 30 cm/sec.

(c) The volumes of two spheres are in the ratio 8:27 and the difference of their radii is 3 cm. Find the radii of both the spheres.

Answer to Question 6(a):

(a) Let ℓ = length and b = breadth of rectangle in cm.

Then perimeter = $2(\ell + b) = 40$ cm i.e. $\ell + b = 20$.............(1)

And area = $\ell b = 96$ sq cm i.e. $\ell b = 96$...............(2)

So $\ell(20 - \ell) = 96 \Rightarrow 20\ell - \ell^2 = 96 \Rightarrow \ell^2 - 20\ell + 96 = 0$

i.e. $\ell = 12$ or 8

Then b = 20 - ℓ = 8 or 12

But b < ℓ so ℓ = 12 cm and b = 8 cm

Answer to question 6(b):

Volume of water flowing in 1sec = 5 × 30 = 150 c.c.

Volume of water flowing in 1 min = 150 × 60 = 9000 c.c. = 9 litre

Answer to Questions 6(c):

$$\frac{\frac{4}{3}\pi r_1^3}{\frac{4}{3}\pi r_2^3} = \frac{8}{27} \Rightarrow \frac{r_1}{r_2} = \frac{2}{3} \Rightarrow r_1 = 2k, r_2 = 3k$$

Now 3k - 2k = 3 $\Rightarrow$ k = 3

So, the radii (r_1) of 1st sphere = 6 cm and

the radii (r_2) of 2nd sphere = 9cm

December 2008 Examination

Question : 5.

(a) Answer *any three* of the following:

Choose the correct option showing necessary reasons/calculations.

(i) The area of the triangle with sides of length 3 cm, 4 cm and 5 cm, (in sq. com) is

(A) 12 (B) 6 (C) 24 (D) none of them

(ii) The perimeter in cm of a semicircle of diameter 14 cm is

(A) 25 (B) 44 (C) 36 (D) none of them

(iii) The volume in cu. ft of a right pyramid having altitude 6 f t and square base with length of a side 4 ft is.

(A) 32 (B) 96 (C) 48 (D) none of them

(iv) A path of 5 ft wide is to be laid just outside round the square garden with length of a side 50 ft. the area of the path in sq. ft would be

(A) 1000 (B) 2500 (C) 1200 (D) none of them

(v) The volume of a solid sphere is $\frac{32}{3}\pi$ cu. cm. Surface area of the sphere in sq. cm. is

(A) 12π (B) 8π (C) 16π (D) none of them

(a) Answer *any three* of the following:

(i) Find the total surface area of a cube whose volume is 64 cu. cm.

(ii) Find the hypotensuse of a right angled isosceles triangle having area $\frac{9}{2}$ sq. ft.

(iii) A bicycle wheel of radius 35 cm makes 5000 revolutions to cover x km. Find the value of x.

(iv) Two solid sphere of radii 3 cm and 2 cm are melted any by them another solid shpere in formed.

Find the volume of the new sphere.

(v) A solid right circular cone having 7 cm. height and 3 cm radius of base. Find its volume.

(vi) Find the length of a side of rhombus having diagonals 6 cm and 8 cm.

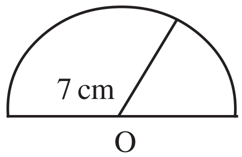
Answer to Question No. 5 (a):

(i) $\rightarrow$ (B) : As $3^2 + 4^2 = 5^2$, the triangle is right angled triangle.

Two adjuscent sides of the right angle are 3 cm and 4 cm.

So, required area = $\frac{1}{2} \times 3 \times 4 = 6$ sq. cm.

(ii) $\rightarrow$ (C) :



Radius = $\frac{14}{2} = 7$ cm.

So, the perimeter $= \frac{1}{2} \times 2\pi \times 7 + 14$ (i.e., $\frac{1}{2}$ circumference + diameter)

$= \frac{22}{7} \times 7 + 14 = 22 + 14 = 36$ cm.

(iii) $\rightarrow$ (A) : Area of base = 4^2 = 16 sq. ft., height = 6ft.

Volume = $\frac{1}{3} \times$ Area of base $\times$ height = $\frac{1}{3} \times 16 \times 6 = 32$ cu.ft.

(iv) $\rightarrow$ (D): The garden is square with length of a side 50 ft.

The garden with the path is also square

with length of a side = $50 + 2 \times 5 = 60$ ft.

So, area of the path = Area of the garden with the path – Area of the garden without the path
= $60 \times 60 - 50 \times 50 = 3600 - 2500 = 1100$ sq. ft.

(v) $\rightarrow$ (C): Let r = radius, volume = $\frac{4}{3}\pi r^3$ surface area = $4\pi r^2$

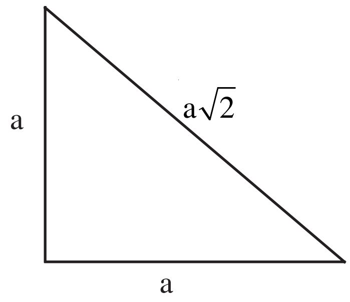
Thus, $\frac{4}{3}\pi r^3 = \frac{32}{3}\pi$ then $r^3 = \frac{32}{3} \times \frac{3}{4} = 8$ or, r = 2

So, surface area = $4\pi r^2 = 4 \times 4 \times \pi = 16\pi$ sq.cm.

Answer to Question No. 5(b):

(i) Volume = a^3 = 64 where a = side of the cube = 4 cm.

Total surface areas = $6a^2 = 4^2$ = 96 sq. cm.

(ii) 

$a^2 + a^2 = 2a^2$,

a = length of equal sides Area of the isosceles triangle (right angled) = sq. ft.

So, $a\frac{a^2}{2} = \frac{9}{2} = 3\text{ft}$.

Then, hypotenuse is of length =

$a\sqrt{2} = 3\times\sqrt{2}\,\text{ft} = 3\sqrt{2}\,\text{ft}$.

(iii) Let r be the radius. by question

$$2\pi r\times 5000 = x\times 1000\times 100 \Rightarrow 2\times\frac{22}{7}\times 35\times 5000 = x\times 100000$$

$\Rightarrow x = 11\text{km}$.

(iv) Volume of new sphere $=\frac{4}{3}\pi\times 3^3 + \frac{4}{3}\pi\times 2^3$

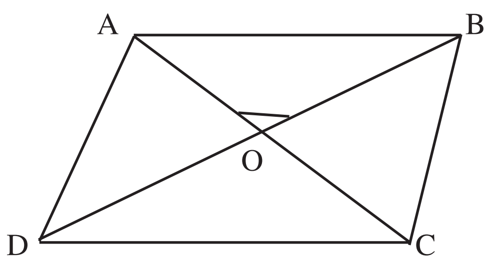
$$=\frac{4}{3}\pi(27+8) = \frac{4\times 35}{3}\pi$$

$$=\frac{140}{3}\pi\text{ cu.cm.} = 342.22\text{cu.cm}$$

(v) Volume of the right circular cone $=\frac{1}{3}\pi r^2 h = \frac{1}{3}\times\frac{22}{7}\times 9\times 7$

= 66 cu. cm.

Since, r = radius of base = 3 cm. h = height = 7 cm.

(vi) 

Rhombus ABCD has triangle AOB,

$AO = \frac{6}{2} = 3$ cm

$BO = \frac{8}{2} = 4$ cm & $\angle AOB = 90^{O}$

So, $AB^2 = AO^2 + OB^2 = 3^2 + 4^2 = 25 = 5^2$

So, length of one side of rhombus = 5cm.

Question : 6.

Answer *any two* of the following:

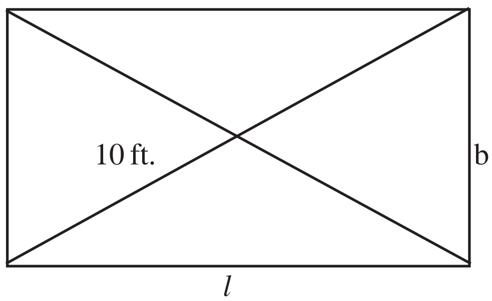
(a) One diagonal of a rectangle is 10 ft. If the perimeter is 28 ft, find its length and breadth.

(b) A circle of radius 7 cm is inscribed within a square touching the sides. Find the area of one fillet thus formed.

(c) Find the volume and surface area of a hollow cylinder wih height 7 inches internal an external radii of base 5 inches and 3 inches respectively.

(d) A conical tent is required to accommodate 11 people. Each person must have 14 sq. ft. of space on the ground and 140 cu. ft. of air to breadthe. Find the height, slant height and width of the tent.

Answer to Question No. 6(a):



Let length = l, breadth = b of the rectangle

Then $l^2 + b^2 = 10^2$

Again $2(l + b)$ = perimeter = 28

or, $l + b = 14$ .... (1)

$l^2 + b^2 + 2\ lb = 196$ (by quaring)

So, $2lb = 196 - 100 = 96$

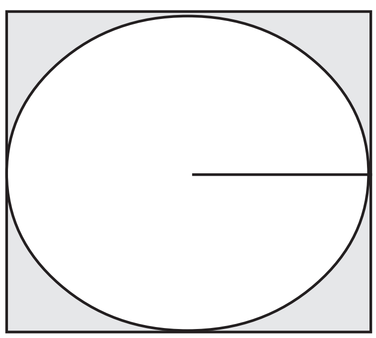
i.e. $(l - b)^2 = (l + b)^2 - 4lb = 14^2 - 192 = 196 - 192 = 4$

or, $l - b = 2$ ....(2)

Then adding (1) & (2), $2l = 16$ or, $l = 8$ and Subtracting (2) from (1), $2b = 12$ or, $b = 6$.

Rquired length is 8 ft and breadth is 6 ft.

Answer to Question No. 6(b):



Radius of circle = 7 cm

So, length of a side of the square = 7 cm x 2 = 14 cm.

The area of the square = 196 sq. cm.

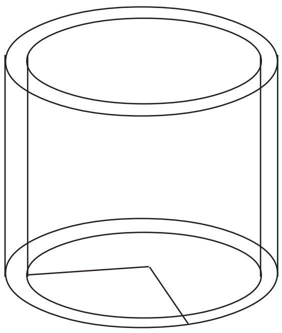
The area of the circle = $\pi \times 7^2 = \frac{22}{7} \times 49 = 154$ sq.cm.

Fillets are of equal size

The area of 4 fillets = 196 - 154 = 42 sq. cm.

Area of one fillet = $\frac{42}{4}$ = 10.5 sq. cm.

Answer to Question No. 6(c):



Volume of hollow sphere = $\pi(R^2 - r^2)h$

Surface area of hollow sphere = $2\pi(R + r)h$

where h = height, R = external radius and r = internal radius of the base. h= 7 inches, R = 5 inches, r = 3 inches

$$\text{Volume } = \frac{22}{7}\left(5^2 - 3^2\right) \times 7 = 22 \times 16 = 352 \text{ cu.inches}$$

$$\text{Surface area } = = 2 \times \frac{22}{7}(5+3) \times 7 = 16 \times 22 = 352 \text{ inches}$$

Answer to Question No. 6(d):

$$\frac{1}{3}\pi r^2 h / \pi r^2 = \frac{140 \times 11}{14 \times 11} \Rightarrow h = 30 \text{ ft}$$

$$r^2 = 14 \times 11 \times \frac{7}{22} = 7^2 \Rightarrow r = 7 \text{ ft}$$

$$\text{Slant height} = \sqrt{h^2 + r^2} = \sqrt{30^2 + 7^2} = \sqrt{949} = 30.8 \text{ ft}$$

Width of the tent = 2r = 2×7 = 14 ft

SECTION - IV

CO-ORDINATE GEOMETRY

COORDINATES
STRAIGHT LINE
CIRCLE
PARABOLA
ELLIPSE
HYPERBOLA

PAPER P-4

Business Mathematics and Statistics Fundamentals

SECTION – IV

CO-ORDINATE GEOMETRY

June 2010 Examination

Question:

7. Answer *any two* of the following:

   Choose the correct option showing necessary reasons/calculations.

   (a) If the three points (1,2), (2,4) and (x,6) are collinear then the value of x is

   (i) 1 (ii) 2 (iii) 3 (iv) 4

   (b) If the line joining the points (2,–2) and (6,4) are parallel to the line joining the point (10/3, 9) and (x, 7) then x is

   (i) 1 (ii) 2 (iii) –3 (iv) 1/2

   (c) The three points A (a,0), B(–a,0), C(c,0) and P is a point such that $PB^2 + PC^2 = 2PA^2$ then the locus of P is

   (i) $(a - c)\ x + a^2 = c^2$ (ii) $(3a–c) = a^2 - c^2$ (iii) $(6a - 2c) + a^2 - c^2 = O$

   (iv) $(6a - 2c)x + c^2 - a = 0$

   (d) The centre of a circle $3(x^2 + y^2) = 6x + 6y - 5$ is

   (i) (2,2) (ii) (1,1) (iii) (3,3) (iv) none of these

Answer to Question No. 7(a):

$$\Delta = \frac{1}{2}\left[1(4-6) + 2(6-2) + x(4-6)\right] = \frac{1}{2}(2x-6) = x-3$$

By question $x - 3 = 0 \Rightarrow x = 3$ Ans (iii)

Answer to Question No. 7(b):

Gradient of the line joining the points (2,-2) and (6,4) $= \frac{4+2}{6-2} = \frac{3}{2}$

Gradient of the line joining the points $\left(\frac{10}{3}, 9\right)$ and $(x,7) = \frac{7-9}{x-\frac{10}{3}} = -\frac{2}{x-\frac{10}{3}}$

$$\frac{2}{\frac{10}{3}-x}=\frac{3}{2}\Rightarrow x=2 \qquad \text{Ans (ii)}$$

Answer to Question No. 7(c):

Let the co-ordinates of P be (x, y)

A = (a,0), B = (–a,0), C = (c,0)

$PB^2 + PC^2 = 2PA^2$

$\Rightarrow (x + a)^2 + y^2 + (x - c)^2 + y^2 = 2(x - a)^2 + 2y^2$

$\Rightarrow (6a - 2c)\,x + c^2 - a^2 = 0$ Ans (iv)

Answer to Question No. 7(d):

$3x^2 + 3y^2 - 6x - 6y + 5 = 0$

$\Rightarrow x^2 + y^2 - 2x - 2y + \frac{5}{3} = 0$

$\therefore\ 2g = -2 \Rightarrow g = -1,\ 2f = -2 \Rightarrow f = -1$

$\therefore$ Centre is (1,1) Ans (ii)

Question:

8. Answer *any one* of the following: 4 x 1

   (a) Find the equation of the parabola having vertex (3,1) and focus (1,1).

   (b) For the hyperbola $9x^2 - 16y^2 - 36x - 108 = 0$ find the coordinates of the centre and its latus recturm.

Answer to Question No. 8(a):

$a = \sqrt{(3-1)^2 + (1-1)^2} = 2$

The equation of the parabola is $(y-1)^2 = -4.2(x-3)$

$\Rightarrow y^2-2y+1 = -8x + 24$

$\Rightarrow y^2-2y + 8x - 23 = 0$

Answer to Question No. 8(b):

$9x^2 - 16y^2 - 36x - 108 = 0$

$\Rightarrow 9(x^2 - 4x + 4) - 16(y - 0)^2 = 108 + 36$

$\Rightarrow 9(x - 2)^2 - 16(y - 0)^2 = 144$

$\Rightarrow \frac{(x-2)^2}{16} - \frac{(y-0)^2}{9} = 1$

$\therefore$ $a^2 = 16 \Rightarrow$ $a = 4, b^2 = 9$

Centre = (2,0)

Latus rectum = $\frac{2b^2}{a} = \frac{2\times9}{4} = 4.5$

December 2009 Examination

Question : 7.

Answer *any two* of the following:

Choose the correct option showing necessary reasons/calculations.

(a) The area of the triangle formed by points (0, 0), (5, 0) and (0, 6) is

(i) 11 sq. unit, (ii) 30 sq. unit (iii) 15 sq. unit, (iv) 7.5 sq.unit

(b) The equation of a straight line passing through the point (5, 5) and is perpendicular to the line x = y is

(i) x - y = 5 (ii) x + y = 5 (iii) x + y = 10 (iv) none of these

(c) A point P having coordinate (x, y) moves such that its distance from the points (1, 3) and (2, 3) are equal. Then locus of P is

(i) 2x = 12y + 3, (ii) 2x = 6y – 3, (iii) 6x = y + 3, (iv) none of these

(d) The directrix of the parabola $x^2 = 4x + 3y + 5$ is

(i) 4y = 15 (ii) 4y + 15 = 0 (iii) 4x + 15 = 0 (iv) none of these

Answer to Question no. 7.

(a) $\Delta = \frac{1}{2}\times6\times5 = 15$ sq. unit Ans (iii)

(b) Gradient (m_1) of the line y = x is $m_1 = 1$

The gradient m_2 of a line perpendicular to y = x is such that

$m_1m_2 = -1$ or, $m_2 = -1$. The required line passes through (5, 5)

So required line is

$y - 5 = m_2(x - 5) \Rightarrow y - 5 = (-1)(x-5) \Rightarrow x+y=10$ Ans(iii)

(c) Distances of the point P(x, y) from (1, 3) and (2, –3) are

$\sqrt{(x-1)^2+(y-3)^2}$ and $\sqrt{(x-2)^2+(y+3)^2}$ But they are equal.

So equating them $(x - 1)^2 + (y - 3)^2 = (x - 2)^2 + (y + 3)^2$

or, $x^2 - 2x + 1 + y^2 - 6x + 9 = x^2 - 4x + 4 + y^2 + 6y + 9$

or, 2x = 12y + 3 Ans (i)

(d) Equation of the parabola

$x^2 = 4x + 3y + 5$

or, $x^2 - 4x = 3y + 5$

or, $x^2 - 4x + 4 = 3y + 5 + 4$

or, $(x - 2)^2 = 3(y + 3)$

Comparing the above equation with $(x - \alpha)^2 = 4a(y - \beta)$

We get $\alpha = 2, \quad a = \frac{3}{4}, \quad \beta = -3$

Equation of directrix : $y + a = \beta$

or, $y + \frac{3}{4} = -3$

or, $4y + 3 = -12$

or, $4y + 15 = 0$ Ans (ii)

Question : 8.

Answer *any one* of the following:

(a) Find the equation of a circle which touches both the axes in the first quadrant at a distance of 5 units from the origin.

(b) Find the coordinate of the centre and eccentricity of the ellipse.

Answer to Question no. 8(a).

As the circle touches x and y axes at (5, 0) and (0, 5) then perpendiculars at (0, 5) and at (5, 0) will intersect at the centre i.e., coordinate of centre is (5, 5) and radius is 5 unit.

Thus equation of the said circle is $(x - 5)^2 + (y - 5)^2 = 5^2$

or, $x^2 - 10x + 25 + y^2 - 10y + 25 = 25$ or, $x^2 + y^2 - 10x - 10y + 25 = 0$

Answer to Question no. 8 (b).

$4x^2 - 16x + 9y^2 + 18y = 11$

or, $4(x^2 - 4x + 4) + 9(y^2 + 2y + 1) = 11 + 16 + 9$

or, $4(x - 2)^2 + 9(y + 1)^2 = 36$

or, $\frac{(x-2)^2}{9} + \frac{(y+1)^2}{4} = 1$ or, $\frac{x^2}{9} + \frac{y^2}{4} = 1$ is the equation of the ellipse where $X = x - 2$ and $Y = y + 1$

Coirdinate of centre is (2, –1)

Eccentricity (e) satisfies $4 = 9(1 - e^2) \Rightarrow 1 - e^2 = \frac{4}{9} \Rightarrow e^2 = \frac{5}{9} \Rightarrow e = \frac{\sqrt{5}}{3}$

June 2009 Examination

Question: 7.

Answer *any two* of the following:

Choose the correct option showing necessary reasons / calculations.

(a) The equation of a straight line passing through origin and perpendicular to the line $2x + 3y+1 = 0$ is

(i) $3y+2x = 0$, (ii) $2y+3x = 0$, (iii) $2y=3x$, (iv) $3y=2x$.

(b) Eccentricity of the hyperbola is $9x^2-16y^2 = 36$

(i) $\frac{5}{4}$, (ii) $\frac{4}{5}$, (iii) $\frac{25}{16}$, (iv) $\frac{16}{25}$

(c) The equation of a parabola whose vertex and axis are (0,0) and y = 0 respectively, passing through (5,4), is

(i) $4y^2 = 5x$, (ii) $5x^2 = 16y$ (iii) $5y^2 = 4x$, (iv) $5y^2 = 16x$

Answer to Question 7:

(a) Gradient of the line $2x + 3y + 1 = 0$ or, $y = -\frac{2}{3}\times\frac{1}{3}\ is -\frac{2}{3}$

A line perpendicular to this line has gradient m so that $m\left(-\frac{2}{3}\right) = -1$

i.e., $m = \frac{3}{2}$, So, the required line is the line passing through origin (0,0) and having gradient $\frac{3}{2}$ is

$(y-0) = \frac{3}{2}(x-0)$ or $2y = 3x$ Ans (iii)

(b) Equation of hyperbola is $\frac{x^2}{4} - \frac{y^2}{9/4} = 1$ which is of the form $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ where $a^2=4$, $b^2=\frac{9}{4}$.

e = Eccentricity of the hyperbola $= \sqrt{1+\frac{b^2}{a^2}} = \sqrt{1+\frac{9/4}{4}} = \sqrt{1+\frac{9}{16}} = \sqrt{\frac{25}{16}} = \frac{5}{4}$ **Ans. (i)**

(c) Equation of parabola having (0,0) as vertex and x- axis as the axis is $y^2 = 4ax$. As it passes through (5,4), $16= 4a\times5 \Rightarrow a = \frac{4}{5}$

Required equation of parabola is $y^2 = 4\frac{4}{5}x \Rightarrow 5y^2 = 16x$ Ans (iv)

Question: 8

Answer *anyone* of the following:

(a) Prove that the points (x,0) (0, y) and (1,1) are collinear if $\frac{1}{x}+\frac{1}{y}=1$

(b) Find the equation of the circle which is concentric to the circle $x^2+y^2+8x-10y+5=0$ and passes through the point (4,11).

Answer to Questions 8(a):

Since the points are collinear, area of the triangle with these points as vertices is zero, we get

$$xy-0+0-y+0-x=0 \Rightarrow x+y=xy \Rightarrow \frac{1}{x}+\frac{1}{y}=1$$

Answer to Question 8(b):

The given equation of the circle can be written as

$(x+4)^2+(y-5)^2=16+25-5=36 \Rightarrow (x+4)^2+(y-5)^2=6^2$

This circle has centre at (-4, 5)

Let another circle with centre at (-4, 5) is $(x+4)^2+(y-5)^2=r^2$

As second circle passes through (4,11), $(4+4)^2+(11-5)^2=r^2 \Rightarrow r^2=100$

Thus required equation of the circle is $(x+4)^2+(y-5)^2=100$

$$\Rightarrow x^2+y^2+8x-10y-59$$

December 2008 Examination

Question: 7

Answer *any three* of the following:

Choose the correct option showing necesary reasons/calculation.

(a) Equation of a line passing through (2, 4) and having y - intercept 2 (on the positive side) is

(i) $y=x-2$ (ii) $y=x+2$ (iii) $y+x=2$ (iv) none of them

(b) The coordinaets of the point which divide the line joining (3, 6) and (12, 9) internally in the ratio 1:2 is

(i) (7,6) (ii) (9,3) (iii) (6,7) (iv) none of them

(c) Perpendicular distance of the line $3x+4y=1$ from the point (4,1) is

(i) 3 units (ii) 4 units (iii) $\frac{17}{5}$ units (iv) none of them

(d) The radius of the circle $2x^2 + 2y^2 + 12y = 8x + 6$ is

(i) 3 units (ii) 4 units (iii) 5 units (iv) none of them

(e) Eccentricity of the ellipse $5x^2 + 9y^2 = 405$ is

(i) 2 (ii) $\frac{1}{3}$ (iii) $\frac{4}{9}$ (iv) none of them

Answer to Question No. 7:

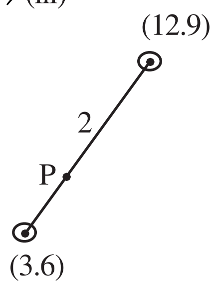
(a) $\rightarrow$ (ii)

Straight line with gradient m and y intercept 2 is

$y = mx + 2$. As it passes through (2,4), $4 = 2m + 2$. or, $m = 1$.

So, equation of the line is $y = x + 2$

(b) $\rightarrow$ (iii)



$$\text{x coordinate of } P = \frac{1\times12+2\times3}{1+2} = 6,$$

$$\text{y coordinate of } P = \frac{1\times9+2\times6}{1+2} = 7.$$

(c) $\rightarrow$ (i)

Perpendicular distance of the $3x + 4y = 1$ from the point (4, 1) is

$$\left|\frac{3\times4+4\times1-1}{\sqrt{3^2+4^2}}\right| = \frac{15}{5} = 3 \text{ units}$$

(d) $\rightarrow$ (ii)

The equation of the circle is $2x^2 + 2y^2 + 12y = 8x + 6$

or, $x^2 + y^2 + 6y - 4x = 3$

or, $(x - 2)^2 + (y + 3)^2 = 3 + 4 + 9$

or, $(x - 2)^2 + (y - 3)^2 = 4^2$

So, radius is of length 4 units.

(e) $\rightarrow$ (iv) (none of them)

$$\text{Ellipse}: \frac{x^2}{81} + \frac{y^2}{45} = 1,\ a^2 = 81,$$

$b^2 = 45$,

e = eccentrity, $b^2 = a^2 (1 - e^2)$ or, $45 = 81 (1 - e^2)$

or, $1 - e^2 = \frac{45}{81} = \frac{5}{9}$ or, $e^2 = \frac{4}{9}$ or, $e = \frac{2}{3}$.

Question: 8

Answer any one of the following:

(a) Find the equation of a straight line passing through the point (2,3) and perpendicular to the line $x + 2y = 5$.

(b) Find the equation of the parabola whose focus is (1,1) and directrix is $x + y = 1$. Find also length of the latus rectum.

Answer to Question No. 8(a):

Gradient of the line $x + 2y = 5$ or, $y = -\frac{1}{2}x + \frac{5}{2}$ is $-\frac{1}{2}$..

Let gardient of the line perpendiular to the above line be m.

Then $m\left(-\frac{1}{2}\right) = -1$ or, $\frac{m}{2} = 1$ or, $m = 2$

The required line passes through (2,3)

So, required line is $(y - 3) = 2(x - 2)$ or, $y - 3 = 2x - 4$.

or, $y + 1 = 2x$.

Answer to Question No. 8(b):

For a parabola distance of any point on the parabola from the focus is same as distance of it from directrix.

Let this general point be (x, y)

Thus

$$\sqrt{(x-1)^2 + (y-1)^2} = \left|\frac{x + y - 1}{\sqrt{1^2 + 1^2}}\right|$$

$$\text{Squaring, } (x-1)^2 + (y-1)^2 = \left(\frac{x + y - 1}{\sqrt{2}}\right)^2$$

$$\text{or, } 2\left[x^2 - 2x + 1 + y^2 - 2y + 1\right] = x^2 + y^2 + 2xy - 2x - 2y + 1$$

$$\text{or, } 2x^2 + 2y^2 - 4x - 4y + 4 = x^2 + y^2 + 2xy - 2x - 2y + 1$$

$$\text{or, } x^2 + y^2 + 3 = 2x + 2y + 2xy.$$

This is equation of the parabola.

Length of the latus rectum

= 2 × distrance of directrix from focus

$$=2\times\frac{1+1-1}{\sqrt{1^2+1^2}}=\frac{2}{\sqrt{2}}=\sqrt{2}$$

June 2008 Examination

Question: 5

(a) If the point P divides the line segment joining (4,5) and (7,2) internally in the ratio 1:2 then the co-orinates of P are

(A) (5.5, 3.5) (B) (6,3) (C) (5,4) (D) none of these

(b) The equation of a line passing through the point (4,5) and having gradient 2 is

(A) $y + 3 = 2x$ (B) $x + 6 = 2y$ (C) $x + 2y = 14$ (D) none of these

(c) The equation of a circle touching the x-axis at orgin and having radius unity on the positive side of y-axis, is

(A) $x^2 + y^2 = 2y + 2$ (B) $x^2 + y^2 = 2y$ (C) $x^2 + y^2 = 2x$ (D) none of these

(d) A parabola passing through (4, -2) has the directrix

(A) $x + 1 = 0$ (B) $x = \frac{1}{4}$ (C) $x + \frac{1}{4} = 0$ (D) none of these

(e) The (eccenricity, length of latus rectum) of a hyperbola $9x^2 - 16y^2 = 144$ is

(A) $\left[\frac{4}{5},\frac{16}{3}\right]$ (B) $\left[\frac{5}{4},\frac{18}{4}\right]$ (C) $\left[\frac{5}{3},\frac{3}{8}\right]$ (D) none of these

Answer to Question No. 5:

(a) $x-\text{coordinate of P is } \frac{2\times4+1\times7}{2+1}=\frac{15}{3}=5$

$y-\text{coordinate of P is } \frac{2\times5+1\times2}{2+1}=\frac{12}{3}=4$

So, the point P is (5,4)

So, correct answer is (C).

(b) Equation of the line passing through (4,5) and having gradient 2 is $(y-5) = 2(x-4)$ or, $y-5 = 2x - 8$ or, $y + 3 = 2x$

So, correct answer is (A).

(c) Centre is (0,1)

Radius is 1

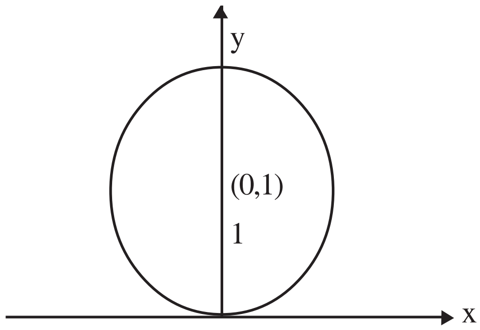
Equation of circle is

$(x - o)^2 + (y - 1)^2 = 1$

or, $x^2 + y^2 - 2y + 1 = 1$

or, $x^2 + y^2 = 2y$

So, correct answer is (B).



(d) As, $y^2 = px$ passes through (4, –2),
$(-2)^2 = p.\ 4$ or, $4 = 4p$ or, $p = 1$.

Then $y^2 = x$ is the parabola, i.e., $y^2 = 4.\frac{1}{4}.x$

Directrix is $x + \frac{1}{4} = 0$

So, correct answer is (C).

(e) Hyperbola is $\frac{x^2}{16} - \frac{y^2}{9} = 1 \quad a^2 = 16, b^2 = 9$

Eccentricity $e = \sqrt{1 + \frac{b^2}{a^2}} = \sqrt{1 + \frac{9}{16}} = \sqrt{\frac{25}{16}} = \frac{5}{4}$

Length of Latus rectum $= \frac{2b^2}{a} = \frac{2 \times 9}{4} = \frac{18}{4}$

So, correct answer is $\left[\frac{5}{4}, \frac{18}{4}\right]$ **i.e., (B).**

Questions: 6

(a) Find the equation of a straight line passing through the point (3,5) and perpendicular to $3x + 5y = 7$.

(b) Show that the two circles $x^2 + y^2 - 2x - 2y = 2$ and $x^2 + y^2 - 10x + 4y + 20 = 0$ touch each other externally. Find the point of contact.

(c) Find the vertex, focus and length of latus rectum of the curve $y^2 - 6y - 4x = 3$.

(d) Find the equation of the ellipse passing through (2,1) and having eccentricity and centre $\sqrt{\frac{2}{3}}\,(0,0)$.

Answer to Question No. 6(a):

Gradient of the line $3x + 5y = 7$ or, $y = -\frac{3}{5}x + \frac{7}{5}$ is $m_1 = -\frac{3}{5}$

Gradient of a line perpendicular to it has grandient so that $m_1\ m_2 = -1$

$$\text{or}, -\frac{3}{5}m_2 = -1 \text{ or}, m_2 = \frac{5}{3}$$

The required line is the line pasing through (3,5) and having gradient $\frac{5}{3}$

i.e. it is $(y-5) = \frac{5}{3}\ (x-3)$

or, $3(y-5) = 5(x-3)$ or, $3y - 15 = 5x - 15$ or, $3y = 5x$

Answer to Question No. 6(b):

Circle $x^2 + y^2 - 2x - 2y = 2$ or, $(x-1)^2 + (y-1)^2 = 4$

has centre (1,1) and radius 2.

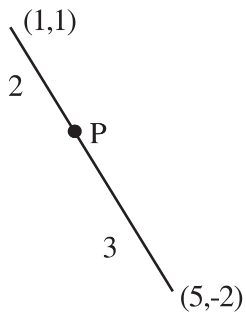
Circle $x^2 + y^2 - 10x + 4y + 20 = 0$ or, $(x-5)^2 + (y+2)^2 = 9$

has centre (5,–2) and radius 3.

Distrance between two centres = $\sqrt{(5-1)^2 + (-2-1)^2} = \sqrt{16+9} = 5$

Sum of length of 2 radii = 2 + 3 = 5

So, the circle touches externally.



Point of contact P of the two circles divide the line segment between 2 centres (1,1) and (5,–2) in the ratio 2:3 internally i.e., the point has the coordinates.

$$\left(\frac{2\times5+1\times3}{2+3}, \frac{2(-2)+1\times3}{2+3}\right) = \left(\frac{13}{5}, \frac{-1}{5}\right)$$

Answer to Question No. 6(c):

$y^2 - 6y - 4x - 3$ or $(y-3)^2 = 4x + 12$ or, $(y-3)^2 = 4(x+3)$

Let $y-3 = y$ and $x + 3 = x$ Then the equation of the curve i.e., parabola would be $y^2 = 4x$

Vertex is $x = 0$, $Y = 0$ i.e., $x + 3 = 0$, $y - 3 = 0$ or, -3, $y = 3$

i.e., vertex is (-3,3).

Focus is $x = 0$, $Y = 1$ or, $x + 3 = 0$, $y - 3 = 1$ or, $x = -3$, $y = 4$

So, focus is (–3, 4)

Length of the latus rectum = 4 $\times$ 1 = 4 units.

Answer to Question No. 6(d):

Ellipse with centre (0,0) is, say $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$

As it parses through (2,1), $\frac{4}{a^2}+\frac{1}{b^2}=1$

$$b^2=a^2(1-e^2)=a^2\left(1-\frac{2}{3}\right)=\frac{a^2}{3}$$

$$\text{i.e.,} \frac{4}{a^2}+\frac{1}{a^2/3}=1 \text{ or, } \frac{4}{a^2}+\frac{3}{a^2}=1 \text{ or, } \frac{7}{a^2}=1 \text{ or, } a^2=7$$

Then $b^2=\frac{7}{3}$

So equation of the ellipse is $\frac{x^2}{7}+\frac{y^2}{7/3}=1$

or, $x^2+3y^2=7$

December 2007 Examination

Question: 5

(a) The distance between two points (1, 3) and (4, 7) is

(i) 7 units (ii) 5 units (iii) 25 units (iv) none of these

(b) The equation of the line passing through the points (3, –4) and (1, 2) is

(i) $x + 3y - 5 = 0$ (ii) $3x + y - 5 = 0$ (iii) $x + y - 5 = 0$ (iv) none of these

(c) The radius of the circle $x^2 + y^2 + 16x + 14y - 8 = 0$ is

(i) 10, (ii) 11, (iii) 21 (iv) none of these

(d) The coordinates of vertex and the length of the latus rectum of the parabola $y2 - 4x - 2y - 7 = 0$ are given in pairwise as

(i) (–2, 1); 2units (ii) (–2, 1); 8 units (iii) (1, –2); 4 units (iv) (-2, 1); 4 units,

(e) The eccentricity and foci of the ellipse $\frac{x^2}{100}+\frac{y^2}{36}=1$ are

(i) $\frac{4}{5}$; $(0, \pm 8)$, (ii) $\frac{16}{25}$; $(\pm\frac{32}{5}, 0)$, (iii) $\frac{4}{5}$; $(\pm 8, 0)$, (iv) none of these

Answer to Question No. 5:

(a) $\rightarrow$(ii): $d = \sqrt{(1-4)^2 + (3-7)^2} = \sqrt{9+16} = \sqrt{25} = 5$ units.

(b) $\rightarrow$(ii): $m = \frac{2+4}{1-3} = \frac{6}{-2} = -3$. The equation is $y + 4 = -3(x-3)$

$$\Rightarrow 3x + y - 5 = 0$$

(c) $\rightarrow$(ii): $g = 8$, $f = 7$, $c = -8$, radius $= \sqrt{g^2 + f^2 - c} = \sqrt{8^2 + 7^2 + 8}$

$$= \sqrt{121} = 11$$

(d) $\rightarrow$(iv): $y^2 - 4x - 2y - 7 = 0$

$\Rightarrow y^2 - 2y + 1 = 4x + 8$

$\Rightarrow (y-1)^2 = 4(x+2)$

vertex $(-2, 1)$, Latus rectum $= 4$ units

(e) $\rightarrow$(iii): $a^2 = 100$, $b^2 = 36$ $\therefore e = \sqrt{\frac{100-36}{100}} = \frac{4}{5}$

Foci $\left(\pm 10.\frac{4}{5}, 0\right) = (\pm 8, 0)$

Question: 6

(a) Find the locus of a point (x, y) equidistant from the point (a, b) and (b, a)

(b) Find the equation of the circle which passes through the points (3, 4) and (–1, 6) and the centre of it lies on the line $3x + 5y = 28$

(c) If the point R (6, 3) divides the line segment joining P(4, 5) and Q (x, y) in the ratio 2 : 5, find Q (x, y).

Answer to Question No. 6 (a):

As the point (x, y) is equidistant from (a, b) and (b, a)

$\therefore (x-b)^2 + (y-a)^2 = (x-a)^2 + (y-b)^2$

$\Rightarrow x^2 - 2bx + b^2 + y^2 - 2ay + a^2 = x^2 - 2ax + a^2 + y^2 - 2by + b^2$

$\Rightarrow -bx - ay = -ax - by$

$\Rightarrow x(a-b) = y(a-b)$

$x = y$ which is the required locus.

Answer to Question No. 6 (e):

Let (h, k) be the centre of the circle. As (3,4) and (–1,6) are the points on the circle.

radius = $\sqrt{(h-3)^2+(k-4)^2} = \sqrt{(h-(-1)^2+(k-6)^2}$

Squaring, we get,

$(h-3)^2+(k-4)^2 = (h+1)^2+(k-6)^2$

$\Rightarrow h^2-6h+9+k^2-8k+16 = h^2+2h+1+k^2-12k+36$

$\Rightarrow 8h-4k+37-25=0 \Rightarrow 8h-4k+12=0$

$\Rightarrow 2h-k+3=0 \quad \Rightarrow k=2h+3$ ....(1)

As centre lies on 3x + 5y = 28, 3h + 5k = 28................(2)

Solving (1) and (2) we get h = 1 and k = 5

So centre is (1,5) and radius = $\sqrt{(1-3)^2+(5-4)^2} = \sqrt{4+1)} = \sqrt{5}$

The equation of the circle is $(x-1)^2+(y-5)^2 = \left(\sqrt{5}\right)^2$

$\Rightarrow x^2-2x+1+y^2-10y+25=5 \Rightarrow x^2+y^2-2x-10y+21=0$

Answer to Question No. 6 (f):

Here $6 = \dfrac{2x+20}{2+5} \Rightarrow 42 = 2x+20 \quad \therefore x = 11$

and $3 = \dfrac{2y+25}{2+5} \Rightarrow 21 = 2y+25 \quad \therefore y = -2$

$\therefore$ Q (x, y) = (11 – 2)

June 2007 Examination

Question: 8

(a) If the x-intercept of a line is 2 and it passes through the point (1,3) , find the line.

(b) Find the centre and radius of the circle $x^2 + y^2 - 6y = 0$.

(c) Find the focus and length of the latus rectum of the parabola $y^2=4ax$ which passes through the point (–1,3.)

(d) Determine the eccentricity of the hyperbola $\dfrac{x^2}{25}-\dfrac{y^2}{16}=1$

Answer to Question 8:

(a) If a and b the x-intercept and y-intercept respectively then the eqn. of the line

Given $a = 2$. So, $\frac{x}{2} + \frac{y}{b} = 1$

The line passes through the point (1,3).

$$\text{So, } \frac{1}{2} + \frac{3}{b} = 1 \Rightarrow \frac{3}{b} = \frac{1}{2} \Rightarrow b = 6$$

$$\text{So, the required line is } \frac{x}{2} + \frac{y}{6} = 1 \Rightarrow 3x + y = 6$$

Aleter, $y = mx + c$

The line passes through the point (2, 0) and (1,3). So, we have

$0 = 2m + c$.................(i)

$3 = m + c$...................(ii)

Solving (i) & (ii) we get, m=-3 and hence c = 6

So, eqn. of the line $y = -3x + 6$ or $y + 3x = 6$

(b) The equation of the circle is $x^2 + y^2 - 6y = 0 \Rightarrow x^2 + (y^2 - 6y + 9) = 9$

or, $x^2 + (y-3)^2 = 3^2$

Hence the centre of the circle : (0, 3);

Radius = a = 3 unit

(c) The parabola $y^2 = 4ax$ passes through the point (–1,3).

$$\text{So, } 3^2 = 4a(-1) \Rightarrow a = -9/4$$

Hence the equation of the parabola $y^2 = 4ax = 4\left(-\frac{9}{4}\right)x = -9x$

Focus $\left(-\frac{9}{4}, 0\right)$. Latus rectum = 9 units

(d) For hyperbola, $b^2 = a^2\left(e^2 - 1\right) \Rightarrow 16 = 25\left(e^2 - 1\right) \Rightarrow 25e^2 = 41$

$$\Rightarrow e = \frac{\sqrt{41}}{\sqrt{25}} = \frac{\sqrt{41}}{5}$$

$$\text{Eccentricity} = \frac{\sqrt{41}}{5}$$

Question : 9

(a) A (1,2) and B(5,–2) are two given points on the xy-plane on which C is moving point such that the value of area of the triangle ABC is 12 sq. units. Find the equation to the locus of C. 5

Answer to Question 9:

(a) Let the co-ordinate of the point C is (h, k). Given A (1,2) and B(5,–2).

The area of the triangle ABC is

$$\Delta ABC = \frac{1}{2}\left[1(-2-k)+5)(k-2)+h(2+2)\right] = 12$$

or, $-2 - k + 5k - 10 + 4h = 24$

or, $4k + 4h = 36$

or, $k + h = 9$

So the equation of the moving point is $x + y = 9$

Question:10.

(a) Find the equation of an ellipse whose principle axes are along the coordinate axes, whose eccentricity is $\sqrt{\frac{2}{5}}$ and length of latus rectum is $\frac{8}{5}\sqrt{6}$

Answer to Question 10:

(a) Let major axis be x-axis and minor axis be *y*-axis of length 2*a* and 2*b* respectively.

Then the equation of the ellipse is $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

Length of Latus rectum $= \frac{2b^2}{a} = \frac{8}{5}\sqrt{6}$ or, $b^2 = a\frac{4}{5}\sqrt{6}$ ....(i)

Also, $b^2 = a^2\left(1-e^2\right)$, *e* being the eccentricity

$$\text{or, } b^2 = a^2\left[1-\left(\sqrt{2/5}\right)^2\right] = a^2\left(1-\frac{2}{5}\right) = \frac{3}{5}a^2 \text{ ....(ii)}$$

$$\text{from (i) and (ii), } a\frac{4}{5}\sqrt{6} = \frac{3}{5}a^2 \Rightarrow a = \frac{4\sqrt{6}}{3} \Rightarrow a^2 = \frac{4^2 6}{3^2} = \frac{32}{3}$$

$$\text{and } b^2 = \frac{3}{5}\times\frac{32}{3} = \frac{32}{5}$$

$$\text{So, the equation of the ellipse } \frac{3x^2}{32} + \frac{5y^2}{32} = 1 \text{ or, } 3x^2 + 5y^2 = 32$$

Question:11

(a) Find the equation of the straight line passing through the point of intersection of the lines 2x + y = 4 and x-y+1 = 0 and is perpendicular to the line 3x-5y+7=0

Answer to Question 11:

(a) A line passing through the point of intersection of two lines

$2x + y = 4$ and x-y+1 = 0 is

$$(2x + y - 4) + k(x-y+1) = 0 \text{ ....(i)}$$

$$\text{or, } (2+k)\, x = (k-1)\, y + (4-k)$$

where k is so determined that the line (1) and 3x - 5y+7 = 0 are perpendicular to each other.

Then gradient of (i) is $\frac{2+k}{k-1}$ and that of $3x + 7 = 5y$ is $\frac{3}{5}$

By the property of perpendicularity

$$\frac{3}{5}\left(\frac{2+k}{k-1}\right) = -1 \text{ or, } \frac{6+3k}{5k-5} = -1$$

or, $6+3k = -5k + 5$ or, $8k = -1$ or, $k = -\frac{1}{8}$

So, equation of the required line is

$$(2x + y - 4) - \frac{1}{8}(x - y + 1) = 0$$

or, $16x + 8y - 32 - x + y - 1 = 0$

or, $15x + 9y - 33 = 0$

December 2006 Examination

Question: 8

(a) If the point (5, y) divides the line joining points (4, 2) and (7, 5) internally, find y.

(b) Find the gradient of the line having equal intercepts of the axes in the first quadrant of the co-ordinate system.

(c) Determine the equation of a circle with (2, 3) and (4, 5) as the extremeties of its diameter.

(d) Find the eccentries of the ellipse whose minor axis is half of its major axis.

Answer to Question No. 8:

(a) Let, the point (5, y) divides the line (4, 2) and (7, 5) in the ratio m : n internally

Then $5 = \frac{4n+7m}{n+m}$ and $y = \frac{2n+5m}{n+m}$

So, 5m + 5n = 4n + 7m $\Rightarrow n-2m \Rightarrow \frac{m}{n}=\frac{1}{2}$

Thus, $y=\dfrac{2+5\frac{m}{n}}{\frac{m}{n}+1}=\dfrac{2+\frac{5}{2}}{\frac{1}{2}+1}=\dfrac{9/2}{3/2}=3$

(b) If the line has equal x intercept and y intercept each equal to a (say) then equation of the line is

$\frac{x}{a}+\frac{y}{a}=1$. Then y = -$x$ + a

So, then gradient of the line is -1

(c) The equation of a circle with (x_1, y_1) and (x_2, y_2) as the extremities of a diameter is
$(x-x_1)(x-x_2)+(y-y_1)(y-y_2)=0$
So, the required equation $(x-2)(x-4)+(y-3)(y-5)=0$
$\Rightarrow x^2+y^2-6x-8y+23=0$

(d) $b^2 = a^2(1-e^2)$

$\Rightarrow \left(\frac{a}{2}\right)^2 = a^2\left(1-e^2\right)$

$\Rightarrow \left(1-e^2\right)=\frac{1}{4} \Rightarrow e^2=\frac{3}{4} \Rightarrow e=\frac{\sqrt{3}}{2}$

Question: 9

(a) Find the equations of the sides of a triangle ABC whose vertices are A (–1, 8), B(4, –2) and C (–5, –3).

Answer to Question No. 9:

(a) The equation of side AB of a triangle ABC is:

$\frac{y-8}{x+1}=\frac{-2-8}{4+1} \Rightarrow 2x+y-6=0$

The equation of side BC is:

$\frac{y+8}{x-4}=\frac{-3+2}{-5-5} \Rightarrow x+9y-22=0$

The equation of side CA is:

$\frac{y-8}{x+1}=\frac{-3-8}{-5+1} \Rightarrow 11x-4y+43=0$

Question: 10

(a) Show that the two circles $x^2 + y^2 + 4x + 10y = 20$ and $x^2 + y^2 - 8x - 6y + 16 = 0$ touch each other externally. Also determine the co-ordinates of the point of contact

Answer to Question No. 10:

(a) $x^2 + y^2 + 4x + 10y = 20$

$\Rightarrow (x - 4)^2 + (y - 3)^2 = 49$

This circle has co-ordinates of the centre as (–2, –5) and radius = 7

$x^2 + y^2 - 8x - 6y + 16 = 0$

$\Rightarrow (x - y)^2 + (y - 3)^2 = 9$

This circle has co-ordinates of the centre as (4, 3) and radius = 3

Distance between two centres $= \sqrt{(4-(-2))^2 + (3-(-5))^2}$

$= \sqrt{6^2 + 8^2} = \sqrt{36+64} = 10$

Again sum of two radii = 7 + 3 = 10. they are equal

So, the circles touch externally.

They point of contact divides the line joining cantres i.e., (–2, –5) and (4, 3) in the ratio 7:3 internally.

The co-ordinates of the point of contact $= \left(\frac{7\times4+3\times(-2)}{7+3}, \frac{7\times3+3\times(-5)}{7+3}\right)$

$= \left(\frac{28-6}{10}, \frac{21+15}{10}\right) = \left(\frac{11}{5}, \frac{3}{5}\right)$

Question: 11

(a) Find the equation of the hyperbola whose focus is at (1, 2), directrux 2x + y = 1 and eccentricity $\sqrt{3}$.

Answer to Question No. 11:

(a) By definition,

$$\sqrt{(x-1)^2 + (y-2)^2} = \sqrt{3}\frac{2x+y-1}{\sqrt{2^2+1^2}}$$

$$\Rightarrow (x-1)^2 + (y-2)^2 = \frac{3}{5}(2x+y-1)^2$$

$$\Rightarrow 5(x^2 + y^2 - 2x - 4y + 5) = 3(4x^2 + y^2 + 1 + 4xy - 4x - 2y)$$

$$\Rightarrow 7x^2 + 12xy - 2y^2 - 2x + 14y - 22 = 0$$

This is the required eduation of the hyperbola.

June 2006 Examination

Q8. Answer *any five* of the following:

(a) Find the point which divides the joining of the points (3,5) and (6,8) by a straight line in the ration 2:1 internally.

(b) Find the equation of the Circle whose centre is (3,7) and radius is 5 units.

(c) Find the equation of the straight line which passes through the points (3,4) and making equal intercepts on the two axes.

(d) Find the vertex and the length of latus rectum of the parabola $(y+3)^2 = 2(x+2)$

Answer to Question 8:

(a) Let (x, y) be the point

$$x = \frac{3\times1+6\times2}{1+2} = 5;\ \ y = \frac{5\times1+8\times2}{1+2} = 7$$

Reqd. point (5, 7)

(b) Equation of the circle = $(x-\alpha)^2 + (y-\beta)^2 = a^2$

$$\Rightarrow (x-3)^2 + (y-7)^2 = 5^2$$

$$\Rightarrow x^2 + y^2 - 6x - 14y + 33 = 0$$

(d) Equation of the straight line : $\frac{x}{a} + \frac{x}{a} = 1 \Rightarrow x + y = a \Rightarrow (3+4) = a \Rightarrow a = 7$

Therefore, the regd. straight line is $x + y = 7$

(d) Equation of the parabola $(y + 3)^2 = 2(x + 2)$. vertex is $(-2, -3)$

Latus rectum = 2 units

Q 9. (a) Find vertex, axis, focus and length of the latus rectum of the curve $x^2-6x-3y=3$

Answer to Question 9 :

(a) The eqn. of the curve is

$(x-3)^2 = 3y + 3 + 9$

$\Rightarrow (x-3)^2 = 4.\left(\frac{3}{4}\right)(y+4)$

$\Rightarrow x^2 = 4.\left(\frac{3}{4}\right) Y$

Where $X = x - 3$ and $Y = y + 4$

Thi is an equation of ellipse with vertex (0, 0). ax is $x = 0$, focus $\left(0, \frac{3}{4}\right)$.

and length of the Latus rectum $= 4.\frac{3}{4} = 3$ units since x = x + 3 and y = Y – 4

Vertex of the curve (0+3, 0–4) = (3, –4)

Axis x = (0+3) = 3

focus $(0 + 3, \frac{3}{4} - 4) = (3, \frac{-13}{4})$

length of the Latus rectum = 3 units

Q 10.(a) *A*(–5.3) and *B* (2. 4) are two fixed points. If a *P* moves in the (x–y) plane such that *PA* : *PB* = 3:2 find the equation to the locus of P.

Answer to Question 10 :

(a) Let P (h, k) be the moving point

By question, $\frac{PA}{PB} = \frac{3}{2} \Rightarrow 4PA^2 = 9PB^2$

$\Rightarrow 4\left[(h+5)^2 + (K-3)^2\right] = 9\left[(h-2)^2 + (K-4)^2\right]$

$\Rightarrow 4\left[h^2 + K^2 + 10h - 6K + 34\right] = 9\left[h^2 + K^2 - 4h - 8K + 20\right]$

$\Rightarrow 5h^2 + 5K^2 - 76h - 48K + 44 = 0$

Hence the required of locus = $5x^2 + 5y^2 = 76x - 48y + 44 = 0$

Q11. (a) Find the equation of the straight line passing through (2, 3) and is perpendicular to 3x –5y + 7 = 0

(b) The major and minor axes of an ellipse are the x and y axes respectively. Its eccenricity is 1/ $\sqrt{2}$ and the length of the latus rectum is 3 units. Find the equation of the ellipse.

Answer to Question 11 :

(a) The gradient of the straight line 3x – 5y + 7 = 0 is $m_1 = \frac{3}{5}$

If m_2 be the gradient of the required straight line then by question

$m_1 m_2 = -1 \Rightarrow m_2 = -\frac{5}{3}$

so, the equation of line perpendicular to 3x – 5y + 7 = 0 is

$y = -\frac{5}{3}x + C$

The line passes through (2, 3), So, $3 = -\frac{5}{3} \times 2 + C \Rightarrow C = 3 + \frac{10}{3} = \frac{19}{3}$

The required line is $y = -\frac{5}{3}x + \frac{19}{3} \Rightarrow C$ i.e., $3y + 5x = 19$

(b) Equation of the ellipse is $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

Here, $\frac{2b^2}{a} = 3$..........(i)

$e^2 = \frac{1}{2} \Rightarrow 1 - \frac{b^2}{a^2} = \frac{1}{2} \Rightarrow \frac{b^2}{a^2} = \frac{1}{2}$......(ii)

From (i) and (ii), $\frac{b^2}{a^2} = \frac{3}{2a} = \frac{1}{2} \Rightarrow a = 3$

and $b^2 = \frac{9}{2}$

Equation of the ellipse $\frac{x^2}{9} + \frac{y^2}{9/2} = 1 \Rightarrow x^2 + 2y^2 = 9$

December 2005 Examination

Question 8:

Answer any five of the following:

(a) Find the equation of the straight line making an intercept 3 on the x-axis and passing through the point (1, 2).

(b) For the equation of the circle $x^2 + y^2 + 2x - 4y = 11$, find the coordinates of its centre and also its radius.

(c) Find the vertex of the parabola $y^2 - 2y - 8x = 23$

(d) Find the eccentricity of the ellipse $8x^2 + 9y^2 = 288$

Answer to Q. No. 8:

(a) The equation of the straight line having x-intercept 3 is

$\frac{x}{3} + \frac{y}{b} = 1,$ b is unknown. The line passes through (1, 2)

Thus $\frac{1}{3}+\frac{2}{b}=1$ or, $\frac{2}{b}=\frac{2}{3}$ or, b = 3.

Thus the equation of the straight line is $\frac{x}{3}+\frac{y}{3}=1$, or $x+y=3$.

(b) The equation of the circle is

$x^2 + y^2 + 2x - 4y = 11$

or, $x^2 + 2x + 1 + y^2 - 4y + 4 = 16$

or, $(x + 1)^2 + (y-2)^2 = 4^2$

Its centre is at (–1, 2) and radius = 4

(c) $y^2 - 2y - 8x = 23$

or, $(y - 1)^2 = 8x + 24$

or, $(y - 1)^2 = 8 (x + 3)$

or, $y^2 = 4{\cdot}2{\cdot}X$ where $Y = y - 1$, $X= x+3$

Vertex is at (X, Y) = (0, 0) i.e. (x, y) = (–3, 1)

i.e. vertex is at (–3, 1)

(d) $8x^2 + 9y^2 = 288$

or, $\frac{8x^2}{288}+\frac{9y^2}{288}=1$ or, $\frac{x^2}{36}+\frac{y^2}{32}=1$

Then in ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$, eccentricity (e) is obtained

from $b^2= a(a - e^2)$, $0<e<1$.

Thus $32 = 36 (1-e^2)$ or, $36e^2 = 36 - 32 = 4$ or $e^2 = \frac{1}{9}$ or, $e=\frac{1}{3}$

i.e., Eccentricity $=\frac{1}{3}$

Question 10

(a) If the three points (x, y), (5–2) and (3,– 4) are collinear, prove that 3x + 4y –7=0

Answer to Q. No. 10:

(a) Slope of the line segment joining (x, y) and (5 –2)

= slope of the line segment joining (5, –2) and (–3, 4)

$$\Rightarrow \frac{y+2}{x-5}=\frac{-2-4}{5+3}\Rightarrow 8y+16=-6x+30 \Rightarrow 3x+4y-7=0$$

Question 11

(a) Find the centres and radii of two circles $x^2 + y^2 + 6x + 14y + 9 = 0$ and $x^2 + y^2$ $4x - 10y - 7 = 0$ and hence show that they touch each other externally. $1\frac{1}{2} + 1\frac{1}{2} + 2$

(b) Find the coordinates of centre, length of latus rectum, eccentricity and the coordinates of foci of the hyperbola $3x^2 - 4y^2 - 12x - 8y - 4 = 0$. 2+1+1

Answer to Q. 11:

(a) $x^2 + y^2 + 6x + 14y + 9 = 0$

so the centre is c, (–3, –7) and radius $(r_1) = = \sqrt{3^2 + 7^2 - 9} = 7$

Thus, the centre and radius of the second circle = $x^2 + y^2 - 4x - 10y - 7$

= 0 are C_2 (2, 5), $r_2 = \sqrt{2^2 + 5^2 + 7} = 6$

Distance between the centres C_1 and C_2 is

$C_1 C_2 = \sqrt{(-3-2)^2 + (-5-7)^2} = \sqrt{25+144} = \sqrt{169} = 13$

$r_1 + r_2 = 7 + 6 = 13$

Since $C_1 C_2 = r_1 + r_2 \Rightarrow$ the circles touch each other externally.

(b) $3x^2 - 4y^2 - 12x - 8y - 4 = 0$

$\Rightarrow 3x^2 - 12x + 12 - 4y^2 - 8y - 4 = 12$

$\Rightarrow 3(x^2 - 4x + 4) - 4(y^2 + 2y + 1) = 12$

$$\Rightarrow \frac{(x-2)^2}{4} - \frac{(y+1)^2}{3} = 1$$

Co-ordinates of the centre is (α, β) for the hyperbola $\frac{(x-\alpha)^2}{a^2} - \frac{(y-\beta)^2}{b^2} = 1$

Here $\alpha = 2$, $\beta = -1$, so the centre is (2, –1)

Length of the latus rectum $= \frac{2b^2}{a} = \frac{2 \times 3}{2} = 3\,\text{cm} \left[\because\ a^2 = 4, b^2 = 3\right]$

$$e = \sqrt{1 + \frac{b^2}{a^2}} = \frac{\sqrt{7}}{2}$$

Co-ordinates of the foci are

$\left(\alpha \pm ae, \beta\right) = \left(2 \pm \frac{2\sqrt{7}}{2}, -1\right)$

$= (2+\sqrt{7}, -1), (2-\sqrt{7}, -1)$

June 2005 Examination

Question 8

Answer any five of the following:

(a) Find the slope of the line perpendicular to the line joining the points (4, –3), (2, 1).

(b) Find the coordinates of the point which divide the line segment joining the points (5, 8) and (6, 3) in the ratio 2 : 3 externally.

(c) Find the centre and radius of the circle $x^2 + y^2 - 4x - 6y - 12 = 0$.

Answer to Q. No. 8:

(a) Slope (m) of the line joining the points (4, –3) and (2, 1) is:

$\frac{1-(-3)}{2-4} = \frac{1+3}{-2} = \frac{4}{-2} = -2$ Now, if slope of the perpendicular to the line joining the given points be m_1 then $m \times m_1 = -1$ or, $m_1 = \frac{-1}{m} = \frac{-1}{-2} = \frac{1}{2}$

(b) let (x, y) be the coordinates of the point that divides externally the line joining the points (5, 8) and (6, 3) in the ratio 2: 3.

Now $x = \frac{2\times 6 - 3\times 5}{2-3} = \frac{12-15}{-1} = 3$

and, $y = \frac{2\times 3 - 3\times 8}{2-3} = \frac{6-24}{-1} = 18$

Hence, required co-ordinates are (3, 18)

(c) $x^2 + y^2 - 4x - 6y - 12 = 0 \Rightarrow x^2 - 2 \times 2x + 2^2 + y^2 - 2 \times 3y + 3^2 = 12 + 4 + 9 = 25$

or, $(x - y)^2 + (y - 3)^2 = 5^2$ so centre is at (2, 3); radius = 5 units

Alternative way:

2g = –4, i.e. g = –2, 2f = –6, f = –3, Here c = –12

Radius = $\sqrt{g^2 + f^2 - c} = \sqrt{4+9+12} = \sqrt{25 = 5,}$ Centre (–g, –f), i.e. (2, 3)

Hence, co-ordinates of centre are (2,3) and radius = 5 units.

Question: 9

(a) For the equation of the parabola $y^2 - 6y - 12x - 3 = 0$, find the focus, directrix and the length of latus rectum.

Answer to Q No. 9:

(a) $y^2 - 6y - 12x - 3 = 0 \Rightarrow y^2 - 6y + 9 = 12x + 3 + 9$

$(y - 3)^2 = 12x + 12 = 12(x + 1) \Rightarrow Y^2 = 12x$.......(i)

Where, $Y = y - 3$, $X = x + 1$, and $a = 3$

Co-ordinates of focus: (a, 0) or, (3, 0)

i.e., $X = 3$, $Y = 0$ or, $x + 1 = 3$, $y - 3 = 0$, or, $x = 2$, $y = 3$

Hence, co-ordinates of focus are (2, 3)

Equation of directrix is $X + a = 0$ or, $x + 1 + 3 = 0$, or, $x + 4 = 0$

Lenght of Latus rectum $= 4 \times a = 4 \times 3 = 12$ units

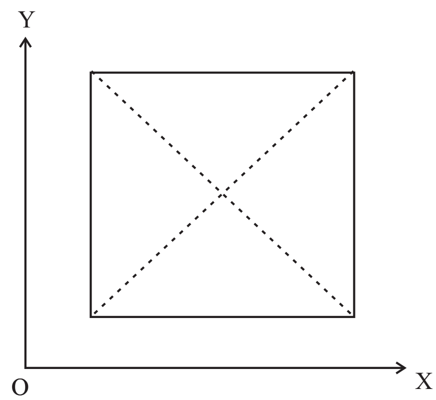
Question: 10.

(a) Find the equations of the diagonals of the rectangle formed by the lines $x = 2$, $x = 5$, $y = 2$ and $y = 5$.

Answer to Q. No. 10:

(a) Co-ordinates of the vertices of the rectangle ABCD will be as follows:

A (2, 2), B(5, 2), C(5, 5) and D(2, 5) *[refer the fig.]*



Equation of the diagonal AC is

$$\frac{x-2}{2-5} = \frac{y-2}{2-5} \Rightarrow \frac{x-2}{-3} = \frac{y-2}{-3}$$

Or, $x - 2 = y - 2$ or, $x = y$

And equation of the diagonal BD is

$$\frac{x-5}{5-2}=\frac{y-2}{2-5}\Rightarrow\frac{x-5}{3}=\frac{y-2}{-3}$$ D (2,5) $\quad$ y = 5 $\quad$ C (5,5)

$\Rightarrow x-5=-(y-2) \Rightarrow x+y-7=0$

Question: 11.

(a) A point P (x, y) moves in such a way that the differences of list distances from A (1, 4) and B (1, –4) is always equal to 6. Find the equation of the locus of P. x = 2 x = 5

(b) The ellipse $px^2 + 4y^2 = 1$ passes through the points ($\pm$ 1,0). Find the value of p and hence find the lenghts of its major and minor axes.

Answer to Q. No. 11:

(a) Let the co-ordinates of the moving point P be (x, y) so that PA – PB = $\pm$ 6.

Now, $\sqrt{(x-1)^2+(y-4)^2}-\sqrt{(x-1)^2+(y+4)^2}=\pm6$

$\Rightarrow\sqrt{(x-1)^2+(y-4)^2}=\pm6+\sqrt{(x-1)^2+(y+4)^2}$, squaring both sides

$(x-1)^2+(y-4)^2=36+(x-1)^2+(y+4)^2\pm12\sqrt{(x-1)^2+(y+4)^2}$

$\Rightarrow y^2-8y+16=36+y^2+8y+16y+12\sqrt{(x-1)^2+(y+4)^2}$

$\Rightarrow -16y-36=12\sqrt{(x-1)^2+(y+4)^2}$

$\Rightarrow -(4y+9)=3\sqrt{(x-1)^2+(y+4)^2}$, again squaring

$\Rightarrow 16y^2+72y+81=9\left\{(x-1)^2+(y+4)^2\right\}=9\left\{x^2-2x+1+y^2+8y+16\right\}$

$\Rightarrow 9x^2-7y^2-18x+72=0$

(b) Since the ellipse $px^2 + 4y^2 = 1$ passes through the points ($\pm$ 1,0), so we have

$p(\pm 1)^2 + 4 \times 0 = 1$

or, $p + 0 = 1$, $\quad$ or, $p = 1$

The equation of the ellipse becomes $x^2+4y^2=1\Rightarrow\frac{x^2}{1}+\frac{y^2}{\frac{1}{4}}=1$

i.e., $a^2=1,\ b^2=\frac{1}{4}$; or, $a=1,\ b=\frac{1}{2}$

Lenght of major axis = 2a = 2 × 1 = 2

Length of minor axis = 2b = 2 × 1/2 = 1

December 2004 Examination

Question 8.

Answer *any five* of the following:

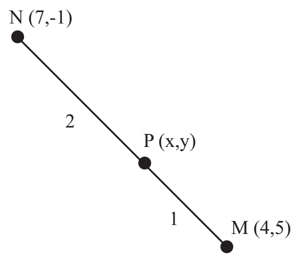
(a) The point *P* divides the line joining the points *M* (4, 5) and *N*(7, –1) internally in the ratio 1:2. Find the co-ordinates of *P*.

(b) Find the eccentricity of the ellipse $\frac{x^2}{36}+\frac{y^2}{20}=1.$

(c) Find the distance of the line x – 2y = 4 from the point (3, –5).

Answer to Question No. 8 (a):

$$\text{Co-ordinates of P}=\left(\frac{mx_2+nx_1}{m+n}.\frac{my_2+ny_1}{m+n}\right)=\left(\frac{7+8}{3},\frac{-1+10}{3}\right)=\left(\frac{15}{3},\frac{9}{3}\right)=(5,3)$$



Answer to Question No. 8 (b):

Here $a^2 = 36$, $b^2 = 20$, $(a > b)$, we have $b^2 = a^2 (1 – e^2)$ or, $20 = 36 (1 – e^2)$

$$\text{or, } 1-e^2=\frac{20}{36}\text{ or, } e^2=1-\frac{20}{36}=\frac{16}{36}=\left(\frac{4}{6}\right)^2\text{ or, } e=\frac{4}{6}=\frac{2}{3}.$$

Answer to Question No. 8 (c):

$$\text{distance}=\frac{3-2.(-5)-4}{\sqrt{1^2+(-2)^2}}=\frac{3+10-4}{\sqrt{1+4}}=\frac{9}{\sqrt{5}}=\frac{9\sqrt{5}}{5}\text{ units}$$

Question :

10. (a) Find the value of the constant *m* such that the three lines 2x – 3y + m = 0, 3x – 4y = 1 and 4x –5y = 2 are concurrent.

Answer to Question No. 10 (a):

3x – 4y = 1 .... (i) × 4 $\Rightarrow$ 12x – 16y = 4

4x – 5y = 2 .... (ii) × 3 $\Rightarrow$ 12x – 15y = 6

Subtracting, –y = –2 or, y = 2

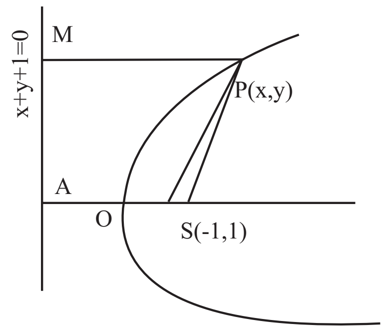
From (i), 3x – 4.2 = 1 or, 3x = 9 or, x =3

Since the given three lines are concurrent so the line 2x – 3y + m = 0 must pass through the point (3, 2). So we get 2 × 3 – 3 × 2 + m or, m = 0.

Question :

11. (a) Find the equation of parabola whose focus is (–1, 1) and equation of directrix is x + y + 1 = 0. Also find the length of the latus rectum and the equation of axis.

Answer to Question No. 11 (a) :



Refer the given figure. We have PS = PM

or, $\sqrt{(x+1)^2+(y-1)^2} = \dfrac{x+y+1}{\sqrt{1+1}}$ now squaring

or, $(x+1)^2+(y-1)^2 = \dfrac{(x+y+1)^2}{2}$

or, $2\{(x + 1)^2 + (y - 1)^2\} = (x + y + 1)^2$

or, $x^2 + y^2 + 2x - 6y - 2xy + 3 = 0$ (on reduction) which is the equation of the parabola.

Perpendicular distance from S(–1, 1) on directrix $x + y + 1 = 0$ is $\dfrac{-1+1+1}{\sqrt{1^2+1^2}} = \dfrac{1}{\sqrt{2}}$

$\therefore$ Latus rectum $= 2\times\dfrac{1}{\sqrt{2}} = \sqrt{2}$ units.

Equation of axis is the line perpendicular to directrix $x + y + 1 = 0$ and passing through the point (–1, 1).

Any line perpendicular to x + y + 1 = 0 is x – y + k = 0. again it passes through the point (–1, 1)

so we get –1 –1 + k = 0 or k = 2

$\therefore$ x + y + 2 = 0

SECTION - V

CALCULUS

FUNCTION
LIMITS & CONTINUITY
DERIVATIVE
INTEGRATION

PAPER P-4

Business Mathematics and Statistics Fundamentals

SECTION – V

CALCULUS

June 2010 Examination

Question:

9. Answer *any three* of the following:

   Choose the correct option showing proper reasons/calculations.

   (a) $f(x) = \log_e (x-3)(x-5)$ is undefined in the region

   (i) $x < 3$, (ii) $x > 5$ (iii) $3 \le x \le 5$ (iv) none of the above

   (b) The value of $\lim \dfrac{2x^2 - 10x + 12}{x \to 2 \; 2x^2 + 4x - 16}$ is

   (i) 1/2 (ii) 1 (iii) 1/6 (iv) none of these

   (c) If $y = x + 1/x$ then $x^2 \; dy/dx - xy$ is

   (i) 0 (ii) 1 (iii) 2 (iv) –2

   (d) The differentiation of x^4 with respect to x^3 is

   (i) 4/3 (ii) 4x/3 (iii) 3x/4 (iv) none of these

   (e) The value of $+\int_0^1 \dfrac{e^x}{e^x + 3} dx$ is

   (i) $\log_e \dfrac{(e+3)}{4}$ (ii) $\log_e \dfrac{(e+3)}{3}$ (iii) $\dfrac{e}{e+3} - 1/3$ (iv) none of these

Answer to Question No. 9(a):

$\log_e (x-3)(x-5)$ is undefined when $(x-3)(x-5) \le 0$

i.e. when $x - 3 \ge 0$ and $x - 5 \le 0$ i.e. $3 \le x \le 5$

or when $x - 3 \le 0$ and $x - 5 \ge 0$ i.e. $x \le 3$ and $x \ge 5$ which is absurd. Ans (iii)

Answer to Question No. 9(b):

$$\frac{2x^2 - 10x + 12}{2x^2 + 4x - 16} = \frac{x^2 - 5x + 6}{x^2 + 2x - 8} = \frac{2(x-2)(x-3)}{2(x-2)(x+4)} = \frac{x-3}{x+4} = -\frac{1}{6} (\text{as } x \to 2) \qquad \text{Ans (iii)}$$

Answer to Question No. 9(c):

$$y=x+\frac{1}{x}\therefore\frac{dy}{dx}=1-\frac{1}{x^2}=\frac{x^2-1}{x^2}$$

$$x^2\frac{dy}{dx}-xy=(x^2-1)-x(x+\frac{1}{x})=x^2-1-x^2-1=-2 \quad \text{Ans (iv)}$$

Answer to Question No. 9(d):

Let $u = x^4$ and $v = x^3$

$$\frac{du}{dx}=4x^3,\frac{dv}{dx}=3x^2$$

$$\therefore\frac{du}{dv}=\frac{du}{dx}\times\frac{dx}{dv}=4x^3\times\frac{1}{3x^2}=\frac{4x}{3} \quad \text{Ans (ii)}$$

Answer to Question No. 9(e):

$$\int_0^1\frac{e^x}{e^x+3}dx=\left[\log_e(e^x+3)\right]_0^1=\log_e\left(\frac{e+3}{4}\right) \quad \text{Ans (i)}$$

Question:

10. Answer *any two* of the following :

(a) If $y = \log\,(x + \sqrt{x^2 + a^2})$ then prove that $(a^2 + x^2)\,y_2 + xy_1 = 0$.

(b) If $f(u,v) = u^3 - v^3 + 3u^2v - 3vu^2$, then verify that $u\,\frac{\delta f}{\delta u}+\frac{\delta f}{\delta v}=3f(u,v)$

(c) Find the area of the region bounded by curves $y^2 = x$ and $y = x$.

Answer to Question No. 10(a):

$$y=\log\left(x+\sqrt{x^2+a^2}\right)$$

$$y_1=\frac{1+\frac{1}{2}(x^2+a^2)^{-\frac{1}{2}}.2x}{x+\sqrt{x^2+a^2}}=\frac{1+\frac{x}{\sqrt{x^2+a^2}}}{x+\sqrt{x^2+a^2}}=\frac{1}{\sqrt{x^2+a^2}}$$

$$y_2=-\frac{1}{2}(x^2+a^2)^{-\frac{3}{2}}.2x=-\frac{x}{(x^2+a^2)\sqrt{x^2+a^2}}$$

$$\therefore\left(x^2+a^2\right)y_2+xy_1=0$$

Answer to Question No. 10(b):

$f(u, v) = u^3 - v^3 + 3u^2 v - 3uv^2$

$$\frac{\delta f}{\delta u} = 3u^2 + 6uv - 3v^2, \frac{\delta f}{\delta v} = -3v^2 + 3u^2 - 6uv$$

$$u\frac{\delta f}{\delta u} + v\frac{\delta f}{\delta v} = 3u^3 + 6u^2 v - 3uv^2 - 3v^3 + 3u^2 v - 6uv^2$$

$= 3(u^3 - v^3 + 3u^2 v - 3uv^2) = 3 f(u, v)$

Answer to Question No. 10(c):

$y^2 = x$ and $y = x$ cut at (0,0) and (1,1)

Required area $= \int_0^1 \sqrt{x dx} - \int_0^1 x dx$

$$\left[\frac{x^{3/2}}{3/2}\right]_0^1 - \left[\frac{x^2}{2}\right]_0^1 = \frac{2}{3}(1-0) - \frac{1}{2}(1-0) = \frac{1}{6} \text{ sq. unit}$$

December 2009 Examination

Question : 9

Answer any *three* of the following:

Choose the correct option showing proper reasons/calculations.

(a) If, $h = g(x) = \dfrac{px + q}{rx - p}$ then *g(h)* is equal to

(i) q, (ii) x, (iii) p, (iv) none of these.

(b) $\lim\limits_{x \to 0} \dfrac{e^{px} - e^{qx}}{x}$ is evaluated as

(i) $q - p$, (ii) $\dfrac{p}{q}$ (iii) $p - q$, (iv) none of these.

(c) If $y = x\sqrt{1 + x^2}$ then $\dfrac{dy}{dx}$ at $x = \sqrt{3}$ is

(i) $\dfrac{1}{2}$, (ii) $\dfrac{7}{2}$, (iii) 5, (iv) none of these.

(d) $\int_0^1 \frac{dx}{\sqrt{x+1}-\sqrt{x}}$ is evaluated as

(i) $\frac{2\sqrt{2}}{3}$, (ii) $\frac{4\sqrt{2}}{3}$, (iii) $\frac{2}{3}\left(2\sqrt{2}+1\right)$, (iv) none of these.

(e) If $f(x, y) = 3x^3 - 5x^2y + 2y^3$ then $x\frac{\partial f}{\partial x} + y\frac{\partial f}{\partial y}$ is

(i) $f(x, y)$, (ii) 3, (iii) $3f(x, y)$, (iv) none of these.

Answer to Question no. 9:

(a) $hrx - hp = px + q$

$\Rightarrow (hr - p)\times = ph + q$

$\Rightarrow x = \frac{ph+q}{rh-p} = g(h)$ Ans(iii)

(b) $\lim_{x\to 0} \frac{e^{px} - e^{qx}}{x} = \lim_{x\to 0} \frac{e^{px} - 1 + 1 - e^{qx}}{x}$

$= p \lim_{x\to 0} \frac{e^{px} - 1}{px} - q \lim_{x\to 0} \frac{e^{qx} - 1}{qx}$

$p \times 1 - q \times 1 = p - q$ Ans (iii)

(c) $\frac{dy}{dx} = \sqrt{1+x^2} + x.\frac{2x}{2\sqrt{1+x^2}} = \sqrt{1+x^2} + \frac{x^2}{\sqrt{1+x^2}} = \frac{1+2x^2}{\sqrt{1+x^2}}$

Then at $x = \sqrt{3}$, $\frac{dy}{dx} = \frac{1+2\left(\sqrt{3}\right)^2}{\sqrt{1+\left(\sqrt{3}\right)^2}} = \frac{1+2x3}{\sqrt{1+3}} = \frac{7}{2}$ Ans(ii)

(d) $\int_0^1 \frac{dx}{\sqrt{x+1}-\sqrt{x}} = \int_0^1 \frac{\left(\sqrt{x+1}+\sqrt{x}\right)dx}{\left(\sqrt{x+1}-\sqrt{x}\right)\left(\sqrt{x+1}+\sqrt{x}\right)}$

$= \int_0^1 \frac{\sqrt{x+1}+\sqrt{x}}{(x+1)-x}dx = \int_0^1 \left(\sqrt{x+1}+\sqrt{x}\right)dx$

$$= \left[\frac{(x+1)^{3/2}}{\frac{3}{2}} + \frac{x^{3/2}}{\frac{3}{2}}\right]_0^1 = \frac{2}{3}\left[2^{\frac{3}{2}} + 1^{\frac{3}{2}} - 1^{\frac{3}{2}} - 0\right] = \frac{2}{3}\left(2\sqrt{2}\right) = \frac{4\sqrt{2}}{3} \text{ Ans(ii)}$$

(e) $f(x, y) = 3x^3 - 5x^2y + 2y^3$

$$\frac{\partial f}{\partial x} = 9x^2 - 10xy, \quad \frac{\partial f}{\partial y} = -5x^2 + 6y^2$$

$$x\frac{\partial f}{\partial x} + y\frac{\partial f}{\partial y} = x(9x^2 - 10xy) + y\left(-5x^2 + 6y^2\right)$$

$$= 9x^3 - 10x^2y - 5x^2y + 6y^3$$

$$= 3(3x^3 - 5x^2y + 2y^3)$$

$$= 3f(x, y) \qquad \text{Ans (iii)}$$

Question : 10

Answer *any two* of the following:

(a) If $x^a y^b = (x + y)^{a+b}$ show that $\frac{dy}{dx} = \frac{y}{x}$ where *a* and *b* are independent of *x* and *y*.

(b) If $y = Ae^{mx} Be^{-mx}$ show that $y_2 - m^2y = 0$.

(c) Find the area of the region lying in the first quadrant bounded by the parabola $y^2 = 4x$, the *x*-axis and the ordinate $x = 4$.

Answer to Question no. 10(a).

$a\log x + b\log y = (a + b) \log (x + y)$

Differentiating w.r.t. x

$$\frac{a}{x} + \frac{b}{y}.\frac{dy}{dx} = \frac{a+b}{x+y}\left(1 + \frac{dy}{dx}\right)$$

or, $$\left(\frac{b}{y} - \frac{a+b}{x+y}\right)\frac{dy}{dx} = \frac{a+b}{x+y} - \frac{a}{x}$$

or, $$\frac{b(x+y) - (a+b)y}{y(x+y)}.\frac{dy}{dx} = \frac{x(a+b) - a(x+y)}{x(x+y)}$$

or, $\frac{dy}{dx} = \frac{bx - ay}{x(x + y)} \text{ x } \frac{y(x + y)}{(bx - ay)} = \frac{y}{x}$

Answer to Question no. 10(b).

$y = Ae^{mx} + Be^{-mx}$

$y_1 = Ame^{mx} - Bme^{-mx}$

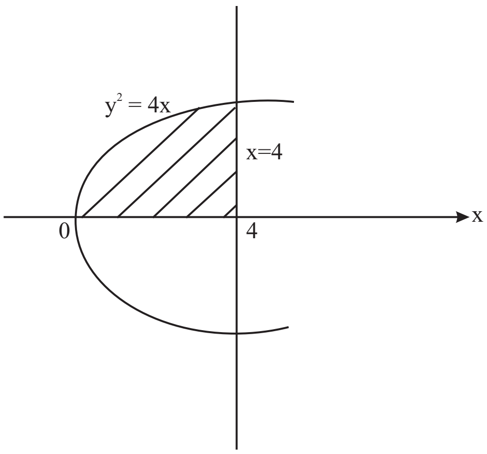
$= m(Ae^{mx} - Be^{-mx})$

$y_2 = m(Ame^{mx} + Bme^{-mx})$

$= m^2(Ae^{mx} + Be^{-mx}) = m^2y$

$\therefore\ y_2 - m^2y = 0$

Answer to Question no. 10(c).



Required area $= \int_0^4 y\,dx = \int_0^4 \sqrt{4x dx} = 2\left[\frac{x^{\frac{3}{2}}}{\frac{3}{2}}\right]_0^4$

$= 2x\frac{2}{3}x\left(2^2\right)^{\frac{3}{2}} = \frac{2x2x8}{3} = \frac{32}{3}$ sq.unit

June 2009 Examination

Question: 9.

Answer *any three* of the following:

Choose the correct option showing necessary reasons / calculations.

(a) if $f(x) = \frac{x+1}{x-1}$, f(f(x)) for $x \neq 1$ is

(i) 1, (ii) 2, (iii) x, (iv) $\frac{x+1}{x-1}$

(b) $\lim\limits_{x \to 1} \frac{(x^2 - 1)2^x}{2x^2 - 3x + 1}$ is evaluated as

(i) 1, (ii) 2, (iii) 3, (iv) 4.

(c) if $y = (x^2+5)^2$ then $\frac{dy}{dx}$ at $x = 2$ is

(i) 18, (ii) 72, (iii) 81, (iv) 36

(d) if $f(x,y) = x^3 + y^3$ then $x\frac{\partial f}{\partial x} + y\frac{\partial f}{\partial y}$ is

(i) f(x,y), (ii) $3f(x,y)$, (iii) 3, (iv) none of these.

(e) $\int_1^2 \frac{dx}{1\sqrt{x-1}}$ is evaluated as

(i) 2, (ii) $2\sqrt{2}$, (iii) -2, (iv) $-2\sqrt{2}$

Answer to Question 9:

(a) $$f(f(x)) = f\left(\frac{x+1}{x-1}\right) = \frac{\frac{x+1}{x-1}+1}{\frac{x+1}{x-1}-1} = \frac{\frac{x+1+x-1}{x-1}}{\frac{x+1-x+1}{x-1}} = \frac{2x}{2} \text{ for } x \neq 1$$

$= x$ Ans (iii)

(b) $$\lim_{x\to 1} \frac{(x^2-1)2^x}{2x^2-3x+1} = \lim_{x\to 1} \frac{(x-1)(x+1)2^x}{(2x-1)(x-1)} = \lim_{x\to 1} \frac{(x+1)2^x}{(2x-1)}$$

(as $x \to 1$ but not $x = 1$)

$$= \frac{(1+1)2^1}{(2\times1-1)} = \frac{2.2}{2-1} = \frac{4}{1} = 4 \quad \text{Ans (iv)}$$

(c) $\frac{dy}{dx} = 2(x^2+5)\,2x = 4x(x^2+5)$

so at $x = 2$, $\frac{dy}{dx} = 4\times2\,(2^2+5) = 8\times9 = 72$ Ans (ii)

(d) $f(x,y) = x^3 + y^3 \quad \therefore \frac{\partial f}{\partial x} = 3x^2, \frac{\partial f}{\partial y} = 3y^2$

$$\times\frac{\partial f}{\partial x} + y\frac{\partial f}{\partial y} = 3x^3 + 3y^3 = 3(x^3+y^3) = 3f(x,y) \quad \text{Ans (ii)}$$

(e) $\int_1^2 \frac{dx}{\sqrt{x-1}} = \left[\frac{(x-1)^{\frac{1}{2}+1}}{-\frac{1}{2}+1}\right]_1^2 = \left[\frac{\sqrt{x-1}}{\frac{1}{2}}\right]_1^2 = \left[2\sqrt{x-1}\right]_1^2$

$$= 2\sqrt{2-1} - 2\sqrt{1-1} = 2\sqrt{1} - 2\times0 = 2\times1-0 = 2 \quad \text{Ans (i)}$$

Question:10

Answer *any two* of the following:

(a) if $y = \frac{x}{\sqrt{1-x^2}}$ show that $x\frac{dy}{dx} = y(y^2+1)$

(b) if $y = \left(x+\sqrt{1+x^2}\right)^m$ show that $(1+x^2)\,y_2 + xy_1 = m^2y$

(c) Evaluate $\int_1^2 x\log_e x\,dx$

Answer to Question 10(a):

$$\frac{dy}{dx} = \frac{1}{\sqrt{1-x^2}} + x\frac{(-2x)\left(-\frac{1}{2}\right)}{(1-x^2)^{3/2}} \Rightarrow x\frac{dy}{dx} = \frac{x}{\sqrt{1-x^2}} + \frac{x^3}{(1-x^2)^{3/2}}$$

$$\Rightarrow x\frac{dy}{dx} = \left(\frac{x}{\sqrt{1-x^2}}\right)^3 + \frac{x}{\sqrt{1-x^2}} \Rightarrow x\frac{dy}{dx} = y^3 + y = y(y^2+1)$$

Answer to Question 10(b):

$$y=\left(x+\sqrt{1+x^2}\right)^m \Rightarrow y_1 = m\left(x+\sqrt{1+x^2}\right)^{m-1}\left\{1+\frac{1\times 2x}{2\sqrt{1+x^2}}\right\}$$

$$\Rightarrow y_1 = m\left(x+\sqrt{1+x^2}\right)^{m-1}\left(\frac{x+\sqrt{1+x^2}}{\sqrt{1+x^2}}\right)$$

$$\left[\Rightarrow y_1=\frac{my}{\sqrt{1+x^2}}\Rightarrow y_1^2(1+x^2)=m^2y^2\right]$$

$$\therefore 2y_1y_2\left(1+x^2\right)+2xy_1^2 = 2m^2yy_1$$

$$\Rightarrow y_2\left(1+x^2\right)+xy_1 = m^2y$$

$$\left(1+x^2\right)y_2+xy_1 = m^2y \text{ Proved}$$

Answer to Question 10(c):

$$\int_1^2 x\log_e x dx = \left[\frac{x^2}{2}\log_e x - \int \frac{x^2}{2}\frac{1}{x}dx\right]_1^2 = \left[\frac{x^2}{2}\log_e x\right]_1^2 - \frac{1}{2}\int_1^2 xdx$$

$$= \frac{4}{2}\log_e 2 - 0 - \frac{1}{2}\left[\frac{x^2}{2}\right]_1^2 = 2\log_e 2 - \frac{1}{4}(4-1) = 2\log_e 2 - \frac{3}{4}$$

December 2008 Examination

Question: 9.

(a) Answer *any three* of the following:

Choose the correct option showing proper reasons/calculations.

(i) If $f(x)=e^{2x-3}$ then $\frac{f(x+y)}{f(x)\,f(y)}$ is

(A) e^3, (B) e^{-3}, (C) 1, (D) none of them.

(ii) The value of $\lim_{x\to 0}\frac{3^x-2^x}{x}$ is

(A) $\log_e\left(\frac{3}{2}\right)$, (B) $\log_{10}\left(\frac{3}{2}\right)$, (C) 1, (D) none of them.

(iii) If y = 4^x then $\frac{d^2y}{dx^2}$ is

(A) 4^x, (B) $4^x \log_e 4$, (B) $\log_e 4$, (D) none of them.

(iv) The value of x for which $x(12 - X^2)$ is maximum is

(A) 0,(B) -2 (C) (D) none of them.

(v) The value of $\int_0^1 \frac{e^x dx}{1+e^x}$ is

(A) $\log_e (1 + e)$, (B) $\log_e \left(\frac{1+e}{2}\right)$, (C) 2, (D) none of them.

(vi) If the total cost function $C = X^3 - 2x^2 + 5x$, then marginal cost is equal to

(A) $x^2 - 4x + 5$, (B) $3x^2 - 4x + 5$, (C) $3x^2 - 4x$ (D) none of them.

(b) Answer *any three* of the following: 1×3

(i) Find the domain of definition of the function $\frac{2x-5}{\sqrt{x^2-9}}$.

(ii) If $y = f(x) = \frac{x+1}{x-1}$ prove that $f(y) = x$.

(iii) Evaluate $\lim_{x \to 2} \frac{x^2-6x+8}{x^2-5x+6}$.

(iv) If $y = 10^x + x^{10}$ find $\frac{dy}{dx}$.

(v) If $\frac{dy}{dx} = x^2 - 3x + 1$ and $y = 2$ when $x = 1$ then show that

Answer to Question No. 9 (a):

(i) $\rightarrow$ (A)

$f(x) = e^{2x-3}$, $f(y) = e^{2y-3}$ and $f(x + y) = e^2(x+y)^{-3}$

So, $\frac{f(x+y)}{f(x)\,f(y)} = \frac{e^{2(x+y)-3}}{e^{2x-3}.e^{2y-3}} = \frac{e^{2(x+y)-3}}{e^{2(x+y)-6}} = e^{2(x+y)-3-2(x+y)+6} = e^3$

(ii) $\rightarrow$ (A)

$$\lim_{x\to 0}\frac{3^x-2^x}{x}=\lim_{x\to 0}\frac{e^{x\log e^3}-e^{x\log e^2}}{x}$$

$$=\lim_{x\to 0}\frac{e^{x\log e^3}-1}{x\log e^3}\left(\log e^3\right)-\lim_{x\to 0}\frac{e^{x\log e^2}-1}{x\log e^2}\left(\log e^2\right)$$

$$=1.\log_e 3-1.\log_e 2=\log_e\left(\frac{3}{2}\right)$$

(iii) $\rightarrow$ (D)

$$y=4^x=e^{x\log_e 4}\text{ So, }\frac{dy}{dx}=(\log_e 4)e^{x\log_e 4}$$

$$\text{and }\frac{d^2y}{dx^2}=\log_e 4\frac{d}{dx}\left(e^{x\log_e 4}\right)=(\log_e 4)^2 e^{x\log_e 4}$$

$$=4^x(\log_e 4)^2$$

(iv) $\rightarrow$ (C)

$y = x(12 - x^2) = 12x - x^3$

$$\frac{dy}{dx}=0\Rightarrow 12-3x^2=0\Rightarrow x^2=4\Rightarrow x\pm 2$$

$$\frac{d^2y}{dx^2}=-6x\text{ which is negative for }x=2\text{ and is positive for }x=-2$$

So, $x(12 - x^2)$ is maximum at x = 2.

(v) $\rightarrow$ (B)

Let $1 + e^x = y$ then $dy = (0 + e^x)\, dx = e^x\, dx$

When x = 0, y = 2,

When x = 1, y = 1 + e

$$\text{So, }\int_0^1\frac{e^x dx}{1+e^x}=\int_2^{1+e}\frac{dy}{y}=\left[\log_e y\right]_2^{1+e}=\log_e(1+e)-\log_e 2$$

$$=\log_e\left(\frac{1+e}{2}\right)$$

(vi) $\to$ (B)

$C = x^3 - 2x^2 + 5x$ where C = Total Cost function.

$$MC = \frac{dc}{dx} = 3x^2 - 4x + 5$$

Answer to Question No. 9(b):

(i) Denominator will be zero when $x = \pm 3$.

$x^2 - 9$ is negative for $|x| < 3$ and is positive for $|x| > 3$

So, $\frac{2x-5}{\sqrt{x^2-9}}$ is undefined in $|x| \le 3$

Hence $\frac{2x-5}{\sqrt{x^2-9}}$ is defined in the domain $|x| > 3$.

(ii) $$f(y) = \frac{y+1}{y-1} = \frac{\frac{x+1}{x-1}+1}{\frac{x+1}{x-1}-1} = \frac{\frac{x+1+x-1}{x-1}}{\frac{x+1-x+1}{x-1}} = \frac{2x}{2} = x$$

(iii) $$\lim_{x\to 2} \frac{x^2-6x+8}{x^2-5x+6} = \lim_{x\to 2} \frac{(x-2)(x-4)}{(x-2)(x-3)} = \lim_{x\to 2} \frac{(x-4)}{(x-3)} = \frac{2-4}{2-3} = \frac{-2}{-1} = 2$$

(iv) $y = 10^x + x^{10}$. Then $\frac{dy}{dx} = 10^x \log_e 10 + 10x^9$

since $10^x = e^{x\log_e 10}$

(v) $$y = \int (x^2 - 3x + 1)\,dx = \frac{x^3}{3} - \frac{3}{2}x^2 + x + C$$

$$\text{Now } 2 = \frac{1}{3} - \frac{3}{2} + 1 + C \Rightarrow C = \frac{13}{6}$$

$$y = \frac{1}{3}x^3 - \frac{3}{2}x^2 + x + \frac{13}{6}$$

Question: 10.

Answer *any two* the following:

(i) A function $f(x)$ is defined as

$$f(x) = \begin{cases} x+1 \text{ when } x \le 1 \\ 3 - x^2 \text{ when } x > 1 \end{cases}$$

Examine whether $f(x)$ is continuous at $x = 1$.

(ii) Verify Euler's theorem for the function $ax^2 + 2bxy + cy^2$.

(iii) Evaluate $\int_3^5 \frac{dx}{3x^2 - 3x - 6}$.

Answer to Question No. 10:

(i) $f(1-0) = \lim_{x \to 1-} f(x) = \lim_{x \to 1} (x + 1) = 2.$

$f(1) = 1 + 1 = 2$

$f(1+0) = \lim_{x \to 1+} f(x) = \lim_{x \to 1} (3 + x^2) = 2$

Thus $f(1-0) = f(1+0) = f(1)$

So, is continuous at $x = 1$.

(ii) $u = f(x,y) = ax^2 + 2bxy + cy^2$ which is homogenous function of x and y of degree 2.

$$\frac{\partial u}{\partial x} = 2ax + 2by, \quad \frac{\partial u}{\partial x} = 2bx + 2cy$$

Now, $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = x(2ax + 2by) + y(2bx + 2cy)$

$$= 2(ax^2 + 2bxy + cy^2) = 2f(x, y) .$$

(iii) $\int_3^5 \frac{dx}{3(x^2 - x - 2)} = \int_3^5 \frac{dx}{3(x-2)(x+1)} = \frac{1}{9}\int_3^5 \left[\frac{1}{x-2} - \frac{1}{x+1}\right]dx$

$$= \frac{1}{9}\left[\int_3^5 \frac{dx}{x-2} - \int_3^5 \frac{dx}{x+1} - \right] = \frac{1}{9}\left\{\left[\log_e(x-2)\right]_3^5 - \left[\log_e(x+1)\right]_3^5\right\}$$

$$= \frac{1}{9}\left\{(\log_e 3 - \log_e 1) - \log_e 6 - \log_e 4)\right\}$$

$$= \frac{1}{9}\left\{(\log_e 3 - 0 - \log_e 6 + \log_e 4\right\} = \frac{1}{9}\log_e\left(\frac{3 \times 4}{6}\right)$$

$$= \frac{1}{9}\log_e 2$$

SECTION - VI

STATISTICAL METHODS

DATA TABULATION & PRESENTATION
FREQUENCY DISTRIBUTION
MEASURES OF CENTRAL TENDENCY
MEASURES OF DISPERSION
MEASURES OF SKEWNESS, KURTOSIS

PAPER P-4

Business Mathematics and Statistics Fundamentals

SECTION – VI

STATISTICAL METHODS

June 2010 Examination

Question:

11. Answer *any seven* of the following :

Choose the correct option showing proper reasons/calculations.

(a) The arithmetic mean of the 4 observations is 8 and that of 10 observations including those 4 is 11. Then arithmetic mean of remaining 6 observations is

(i) 13 (ii) 11 (iii) 10 (iv) none of these

(b) Geometrices mean of 10 observations 2,2,4,4,8,8,16,16,32,32 is

(i) 12 (ii) 4 (iii) 8 (iv) none of these

(c) Harmonic mean of 1,2,2,3,3,3,4,4,4,4,5,5,5,5,5 is

(i) 1/5 (ii) 4 (iii) 5 (iv) none of these

(d) Number of peas of 50 peapods are as follows:

No. of peas:	0	1	2	3	4	5	Total
No. of peapods:	2	10	12	15	10	1	50

Median of number of peas is

(i) 2.5 (ii) 3 (iii) 3.5 (iv) none of these

(e) The number of members in 30 families are as follows:

1,3,1,3,4,5,3,3,1,3,3,4,5,4,2,3,3,2,2,5,2,4,2,2,3,2,4,2,4,4

Then mode of number of members in a family is

(i) 2.2 (ii) 4 (iii) 3.5 (iv) none of these

(f) If arithmetic mean and harmonic mean of two positive numbers are 3 and 8/3 then the two numbers are

(i) 1 and 5 (ii) 2 and 4 (iii) 3 and 3 (iv) none of these

(g) The mean deviation about 12 of the following distribution

x :	10	11	12	13	14	Total
frequency :	5	9	20	13	3	50

is

(i) 0.66 (ii) 0.7 (iii) 0.76 (iv) none of these

(h) If variance of 10 values is 9 and sum of deviation of those ten values about 3 is 60 then mean of squares of deviations of those 10 values about 5 is

(i) 25 (ii) 16 (iii) 9 (iv) none of these

(i) If runs of two players A and B in 10 cricket matches are such that player A has mean 50 and variance 36 and player B has mean 60 and variance 81 of runs then the player more consistent in runs is

(i) A (ii) B (iii) both are equally consistent (iv) none of these

(j) For a distribution with A.M = 50, coefficient of skewness – 0.4 and s.d 20, value of mode is

(i) 66 (ii) 42 (iii) 9 (iv) none of these

Answer to Question No. 11:

(a) Mean of 10 observations $= \frac{4\times 8+6\times m}{4+6} = \frac{32+6m}{10}$ where m = a.m of remaining 6 observations.

Thus $6m = 110 - 32 = 78$ or, $m = 13$ Ans (i)

(b) G.M of 10 observations $= \left(2^2\times 4^2\times 8^2\times 16^2\times 32^2\right)^{\frac{1}{10}} = \left(2^{2+4+6+8+10}\right)^{\frac{1}{10}} = \left(2^{30}\right)^{\frac{1}{10}}$

$= 2^3 = 8$ Ans (iii)

(c)

Values :	1	2	3	4	5	Total
freq :	1	2	3	4	5	15

H.M of values $= \frac{15}{\frac{1}{1}+\frac{2}{2}+\frac{3}{3}+\frac{4}{4}+\frac{5}{5}} = \frac{15}{1+1+1+1+1} = \frac{15}{5} = 3$ Ans (iv)

(d)

No. of peas (x) :	0	1	2	3	4	5
freq = no. of peapods (f) :	2	10	12	15	10	1
Cumulative freq (C.F less than type)	2	12	24	39	49	50

Now 50/2 = 25. So there are two middlemost values in the 25th and 26th position when arranged in non decreasing order. So median = average of those two middlemost values = (3+3)/2 =3, since 24<25, 26<39 Ans (ii)

(e) No. of members (x) : 1 2 3 4 5 Total

No. of members (x) :	1	2	3	4	5	Total
No. of families (f) :	3	8	9	7	3	30

$x = 3$ occurs most frequently i.e. 9 times. So 3 is mode. Ans (iv)

(f) Let two numbers be a and b. Then $3 = \frac{a+b}{2}$ or, $a + b = 6$ (1)

and $\frac{8}{3} = \frac{2}{\frac{1}{a}+\frac{1}{b}}$ or, $ab = 8$ (2)

Then $a(6 - a) = 8$ or, $a^2 - 6a + 8 = 0$ or, $(a - 4)(a - 2) = 0$

$a = 4$ or 2. Then $b = 2$ or 4. Thus two numbers are 2 and 4 Ans (ii)

(g)

x :	10	11	12	13	14	Total
f :	5	9	20	13	3	50
$\lvert x - 12 \rvert$:	2	1	0	1	2	
$f\lvert x - 12 \rvert$:	10	9	0	13	6	38

Then M.D about 12 = $\frac{1}{50}\sum f|x-12| = \frac{38}{50} = 0.76$ Ans (iii)

(h) $\sum_{i=1}^{10}(x_i - 3) = 60 \Rightarrow \frac{1}{10}\sum_{i=1}^{10} x_i - 3 = 6 \Rightarrow \bar{x} = 9 \Rightarrow \bar{x} - 5 = 4 \Rightarrow (\bar{x}-5)^2 = 16$

$$\frac{1}{10}\sum_{i=1}^{10}(x_i - \bar{x})^2 = 9 \Rightarrow \frac{1}{10}\sum_{i=1}^{10}\{(x_i - 5) - (\bar{x} - 5)\}^2 = 9 \Rightarrow \frac{1}{10}\sum_{i=1}^{10}(x_i - 5)^2 - 16 = 9$$

$$\Rightarrow \frac{1}{10}\sum_{i-1}^{10}(x_i - 5)^2 = 25$$ Ans (i)

(i) C.V of runs of player A = (s.d/mean) x 100 = (6/50) 100 = 12%

C.V of runs of player B = (s.d/mean) 100 = (9/60) 100 = 15%

Since 12% < 15%, player A is more consistent in runs than B Ans (i)

(j) For a distribution, coeff. of skewness = (mean – mode)/s.d.

i.e. – 0.4 = (50 – mode)/20

i.e. mode = 58 Ans (iii)

Question :

12. (a) Answer *any two* of the following:

(i) Find mean and standard deviation of following frequency distribution of ages:

Class of age (yrs)	:	0-10	10-20	20-30	30-40	40-50	Total
No. of persons	:	2	4	9	3	2	20

(ii) Find median and mode of the following distribution:

Weekly wages (Rs.)	:	50-59	60-69	70-79	80-89	90-99	Total
No. of workers	:	6	14	16	13	3	52

(iii) If the fast of two samples has 100 items with mean 15 and variance 9 and the second has 150 items with mean 16 and variance 16, find the mean and variance of the combined sample.

(b) Write short note on *any one* of the following:

(i) Central tendency of data;

(ii) Ogive less than type.

Answer to Question No. 12(a):

(i)

Mid value (x)	:	5	15	25	35	45	Total
u = (x – 25)/10	:	–2	–1	0	1	2	
freq (f)	:	2	4	9	3	2	20
fu	:	–4	–4	0	3	4	–1
fu^2	:	8	4	0	3	8	23

$$\text{mean} = 25 + 10 \times \frac{\sum fu}{\sum f} = 25 + \frac{10x(-1)}{20} = 25 - 0.5 = 24.5 \text{ yrs.}$$

$$\text{s.d.} = \sqrt{\left[\frac{\sum fu^2}{\sum f} - \left(\frac{\sum fu}{\sum f}\right)^2\right]} \times 10 = \sqrt{\left[\frac{23}{20} - \left(\frac{-1}{20}\right)^2\right]} \times 10 = \sqrt{114.75} = 10.71 \text{ yrs.}$$

(ii)

Class of weekly wages (in boundaries)	49.5-59.5	59.5- 69.5	69.5-79.5	79.5-89.5	89.5-99.5
Frequency (f)	6	14	16	13	3
Cum. freq (<type)	6	20	36	49	52

N/2 = 26, 20<26<36; Median class is 69.5 – 79.5

$$\text{Median} = 69.5 + \frac{26-20}{16} \times 10 = 69.5 + 3.75 = 73.25 \text{ Rs.}$$

Modal class is 69.5 – 79.5 as maximum freq = 16 is in that class

$$\text{Mode} = 69.5 + \frac{16-14}{(16-14)+(16-13)} \times 10 = 69.5 + 4 = 73.5 \text{ Rs.}$$

(iii) For the combined sample

$$\text{mean} = \frac{100\times15+150\times16}{100+150} = \frac{3900}{250} = 15.6 \text{ and}$$

$$\text{variance} = \frac{100\times9+150\times16+100\times(15.6-15)^2+150\times(15.6-16)^2}{100+150}$$

$$= \frac{900+2400+36+24}{250} = 13.44$$

Answer to Question No. 12(b):

SHORT NOTE:

(i) **Central tendency of data**

These are statistical constants which give an idea about the concentration of the values in the central part of the distribution. It can be thought of as the value of the variable which is representative of the entire distribution.

Properties :
(a) It should be rigidly defined.
(b) It should be understood and calculated easily.
(c) It should be based on all observations.
(d) It should be amenable to algebraic treatment.
(e) It should be least affected by extreme values.
(f) It should be least affected by sampling fluctuations.

A frequency distribution can be taken in the form

$x \rightarrow x_1 \quad x_2 \quad \ldots\ldots \quad x_n$

$f \rightarrow f_1 \quad f_2 \quad \ldots\ldots \quad f_n$

where $x_1, x_2, \ldots\ldots x_n$ are the values of the variable x with frequencies $f_1, f_2, \ldots\ldots f_n$ Let $f_1 + f_2 + \ldots\ldots + f_n = N$. Then N is total frequency

1. Arithmetic Mean = $\dfrac{x_1 f_1 + x_2 f_2 + \ldots\ldots + x_n f_n}{N}$

2. Geometric Mean = $(x_1^{f_1} . x_2^{f_2} \ ...... \ xn^{f_n})^{1/N}$

3. Harmonic Mean = $\dfrac{N}{f_1\dfrac{1}{x_1}+f_2\dfrac{1}{x_2}+......+f_n\dfrac{1}{x_n}}$

4. Median: Median is defined to be the middle most variate value when the variate values are arranged in increasing or decreasing order of magnitude.

   For continuous variable arranged in a grouped frequency distribution, median is defined as

$$\text{Median} = L_1 + \frac{\frac{N}{2} - F_1}{f_{me}} \times h$$

   L_1 = lower class boundary of the median class

   N = total frequency

   F_1 = cumulative frequency of the class preceding the median class

   f_{me} = frequency of the median class

   h = width of the median class

5. Mode : It is that value of the variate for which frequency is maximum. For grouped distribution it is given by

$$\text{Mode} = L + \frac{f_m - f_1}{2f_m - f_1 - f_2} \times h$$

   L = lower limit of modal class

   f_m = frequency of modal class

   f_1 = frequency of the class preceding the modal class

   f_2 = frequency of the class following the modal class

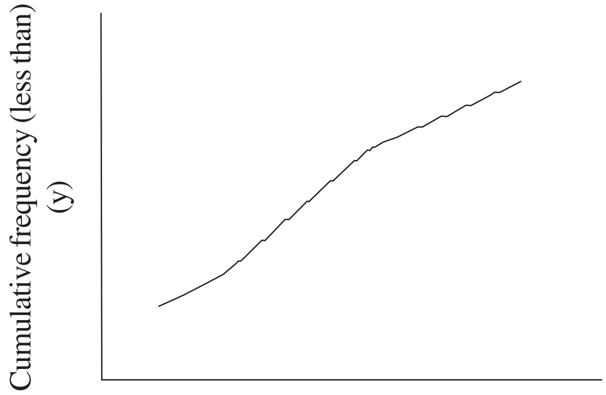
(ii) **Ogive less than type**

Cumulative frequency corresponding to a given variate value of a distribution is defined to be the sum total of frequencies up to and including that variate value. This is known as cumulative frequency of less than type. In case of a grouped frequency distribution, cumulative frequency (less than) of a class corresponds to the upper class boundary of that class and it is the sum total of frequencies of classes up to and including that class.

For grouped frequency distribution of a continuous variable, cumulative frequency distribution of less than type can be represented graphically by means of a cumulative frequency polygon also known as ogive less than type. To draw a cumulative frequency polygon, boundary values of each class are located in the *X* axis. The cumulative frequency table provides the cumulative frequency (less than type) corresponding to upper class boundary of a class along y axis. For each pair of values (U_i,

CF_i), a point is plotted in the graph paper. Joining all these points by straight lines, we get a cumulative frequency polygon of 'less than type'.

Fig: Ogive less than type



December 2009 Examination

Question: 11.

Answer *any seven* of the following:

Choose the correct option showing proper reasons/calculations.

(a) First 10 odd counting numbers each occurring twice has arithmetic mean

(i) 40, (ii) 10, (iii) 20, (iv) none of these.

(b) Geometric mean (G.M.) of six numbers is 16. If G.M. of first four of them is 8 then G.M. of other two is

(i) 8, (ii) 16, (iii) 32, (iv) none of these.

(c) Two positive observations have arithmetic mean 3 and geometric mean $2\sqrt{2}$. If each observation is multiplied by 2 then harmonic mean will be

(i) $\frac{16}{3}$, (ii) $\frac{8}{3}$, (iii) 12, (iv) none of these.

(d) If the sum of deviations of a number of observations about 4 and that about 3 are 40 and 50 respectively then arithmetic mean of the observation is

(i) 4, (ii) 6, (iii) 10, (iv) none of these.

(e) If the relation between 2 variables x and y is xy = 2 and arithmetic mean of variable x is 10, then harmonic mean of variable y is

(i) $\frac{1}{5}$ (ii) $\frac{1}{10}$, (iii) $\frac{2}{5}$, (iv) none of these.

(f) If the variable x takes 20 values $x_1, x_2, ..., x_{10}, -x_1, -x_2, ..., -x_{10}$ such that $\sum_{i=1}^{10} x_i^2 = 40$ then standard deviation of x is

(i) 1, (ii) 2, (iii) 4, (iv) none of these.

(g) If relation between 2 variables x and y is 2x + 3y = 5 and mean deviation of x values about mean is 9 for 10 observations, then sum of absolute deviations of corresponding 10 y-values about means is

(i) 90, (ii) 30, (iii) 6, (iv) none of these.

(h) If for 10 values of x sum of deviation about 5 is 10 and sum of squares of deviations about 4 is 100 then variance of x is

(i) 4, (ii) 6, (iii) 10, (iv) none of these.

(i) If two samples of sizes 4 and 5 have same mean but different standard deviations 1 and 3 respectively then the standard deviation of the combined sample is

(i) $\sqrt{5}$, (ii) $\frac{\sqrt{51}}{3}$, (iii) $\frac{7}{3}$, (iv) none of these.

(j) If the mode, variance and coefficient of skewness of a frequency distribution are 100, 16 and 6 respectively then mean of the distribution is

(i) 124, (ii) 76, (iii) 108, (iv) none of these.

Answer to Question no. 11.

(a) $$AM = \frac{\sum fx}{\sum f} = \frac{2[1+3+5+.......+19]}{2x10} = \frac{10x20}{2x10} = 10 \quad \text{Ans (ii)}$$

(b) Let y_1 and y_2 be other two observations and x_1, x_2, x_3, x_4 are first 4 observations. Then

$$(x_1x_2x_3x_4y_1y_2)^{\frac{1}{6}} = 16 \text{ or, } x_1x_2x_3x_4y_1y_2 = 16^6 \text{ and } (x_1x_2x_3x_4)^{\frac{1}{4}} = 8 \text{ or, } x_1x_2x_3x_4 = 8^4$$

$$\text{So} \left(8^4 y_1y_2\right) = 16^6 \text{ or, } y_1y_2 = \frac{16^6}{8^4} = \frac{\left(2^4\right)^6}{\left(2^3\right)^4} = 2^{24-12} = 2^{12}$$

$$\text{So GM of other two} = \left(y_1y_2\right)^{\frac{1}{2}} = \left(2^{12}\right)^{\frac{1}{2}} = 2^6 = 64 \quad \text{Ans (iv)}$$

(c) Let *a, b* be two positive observations.

$$\frac{a+b}{2} = 3 \text{ and } \sqrt{ab} = 2\sqrt{2} \text{ i.e., } a + b = 6, ab = 8$$

$$\text{Required harmonic mean} = \frac{2}{\frac{1}{2a}+\frac{1}{2b}} = \frac{4ab}{a+b} = \frac{4x8}{6} = \frac{16}{3} \text{ Ans(i)}$$

(d) $\sum(x-4) = 40$ i.e., $\sum x = 40+4n$

and $\sum(x-3) = 50$ i.e., $\sum x = 50+3n$, n = no.of observations

Thus $40 + 4n = 50 + 3n$ or, $n = 10$. So $\sum x = 40+4x10 = 80$

$$\text{So arithmetic mean of observations} = \frac{1}{10}\sum x = \frac{80}{10} = 8 \quad \text{Ans (iv)}$$

(e) $y = \frac{2}{x}$ Let x_1, x_2, ..., xn be n values of x and y_1, y_2, ... y_n be the corresponding n values of y, i.e., $y_1 = \frac{2}{x_1}, y_2 = \frac{2}{x_2}, \ldots\ldots, y_n = \frac{2}{x_n}$

$$\text{H.M.of } y = \frac{n}{\frac{1}{y_2}+\frac{1}{y_2}+\ldots\ldots..+\frac{1}{y_n}} = \frac{n}{\frac{x_1}{2}+\frac{x_2}{2}+\ldots\ldots+\frac{x_n}{2}}$$

$$= \frac{2}{\frac{x_1+x_2+\ldots\ldots..+x_n}{n}} = \frac{2}{10} = \frac{1}{5} \quad \text{Ans (i)}$$

(f) $$\text{Mean of 20 observations} = \frac{(x_1+x_2+\ldots\ldots..+x_{10})-(x_1+x_2+\ldots\ldots..+x_{10})}{20} = 0$$

$$\text{Then s.d.} = \sqrt{\frac{1}{20}\left[\sum_{i=1}^{10}(x_i-0)^2+\sum_{i=1}^{10}(x_i-0)^2\right]} = \sqrt{\frac{1}{20}\sum_{i=1}^{10}x_i^2}$$

$$= \sqrt{\frac{2x40}{20}} = \sqrt{4} = 2 \text{ Ans(ii)}$$

(g) $y_1 = -\frac{2}{3}x_1 + \frac{5}{3}$ for $i = 1, 2, \ldots\ldots\ldots.., n$

$$\sum_{i=1}^{n} y_i = -\frac{2}{3}\sum_{i=1}^{n} x_i + \frac{5}{3}n \Rightarrow \bar{y} = -\frac{2}{3}\bar{x} + \frac{5}{3}$$

$$\text{So}\left|y_i-\bar{y}\right|=\left|-\frac{2}{3}\left(x_i-\bar{x}\right)\right|=\frac{2}{3}\left|x_i-\bar{x}\right|$$

$$\therefore \sum_{i-1}^{n}\left|y_i-\bar{y}\right|=\frac{2}{3}\sum_{i-1}^{n}\left|x_i-\bar{x}\right| \text{ Now } n=10$$

$$=\frac{2x10}{3}x\frac{1}{10}\sum_{i=1}^{n}\left|x_i-\bar{x}\right|=\frac{20}{3}x9=60 \text{ Ans(iv)}$$

(h) Let 10 values of x be $x_1, x_2, \ldots\ldots, x_{10}$

$$\text{Then }\sum_{i=1}^{10}\left(x_i-5\right)=10 \text{ and } \sum_{i=1}^{10}\left(x_i-4\right)^2=100$$

$$\text{So }\sum_{i=1}^{10}\left(x_i-4-1\right)=10 \Rightarrow \sum_{i=1}^{10}\left(x_i-4\right)=10+10=20$$

$$\text{Variance of } x=\frac{1}{10}\sum_{i=1}^{10}\left(x_i-4\right)^2-\left\{\frac{1}{10}\sum_{i=1}^{10}\left(x_i-4\right)\right\}^2$$

$$\left[\text{Since }\frac{1}{10}\sum_{i=1}^{10}\left(x_i-\bar{x}\right)^2=\frac{1}{10}\sum_{i=1}^{10}\left\{\left(x_i-4\right)-\left(\bar{x}-4\right)\right\}^2\right]$$

$$=\frac{100}{10}-\left(\frac{20}{10}\right)^2=10-4=6 \text{ Ans(ii)}$$

(i) Combined mean $\bar{x}=\frac{4\bar{x}_1+5\bar{x}_2}{4+5}=\frac{4\bar{x}_1+5\bar{x}_1}{9}=\frac{9\bar{x}_1}{9}=\bar{x}_1=\bar{x}_2$ (given)

$$\text{s.d. of combined sample}=\sqrt{\frac{4x1^2+5x3^2+4x\left(\bar{x}_1-\bar{x}_1\right)^2+5x\left(\bar{x}_2-\bar{x}_2\right)^2}{4+5}}$$

$$=\sqrt{\frac{4+45+0+0}{9}}=\sqrt{\frac{49}{9}}=\frac{7}{3}\text{Ans(iii)}$$

(j) Coefficient of skewness $=\frac{\text{mean}-\text{mode}}{\text{s.d.}}$, s.d. $=\sqrt{16}=4$ as s.d. can not be negative. Thus $6=\frac{\text{mean}-100}{4}$ or, mean $=100+24=124$ Ans (i)

Question : 12.

(a) Answer *any two* of the following:

(i) Find the mean and the mean deviation about mean of the following frequency distribution:

Weight (in kg)	:	50	55	60	65	70	Total
No. of persons	:	1	4	2	2	1	10

(ii) Show that the combined arithmetic mean two groups lies between the arithmetic means and of the two groups.

(iii) The arithmetic mean and geometric mean of two observations are 20 and 12 respectively. Find the observations and harmonic mean of them.

(b) Write short note on *any one* of the following:

(iii) Histogram,

(iv) Skewness and its two important measures.

Answer to question no. 12(a).

(i)

(x) Weight (in Kg)	:	50	55	60	65	70	Total
(f) No. of persons	:	1	4	2	2	1	10
f, x	:	50	220	120	130	70	590, mean $= \frac{590}{10} = 59$
$x - \bar{x} = x - 59$	:	–9	–4	1	6	11	
$f\lvert x - \bar{x}\rvert$	:	9	16	2	12	11	50

Mean deviation about mean $= \frac{1}{\sum f} = \sum f\lvert x - \bar{x}\rvert = \frac{50}{10} = 5\,\text{kg}$

(ii) $\bar{x} = \frac{n_1\overline{x_1} + n_2\overline{x_2}}{n_1 + n_1}$ where and are no. of observations of two groups.

Let $\overline{x_1} < \overline{x_2}$. Then $\bar{x} - \bar{x} = \frac{n_1\overline{x_1} + n_2\overline{x_2}}{n_1 + n_2} - \overline{x_1}$

$$= \frac{n_1\overline{x_1} + n_2\overline{x_2} - n_1\overline{x_1} - n_2\overline{x_1}}{n_1 + n_2} - \frac{n_1\left(\overline{x_2} - \overline{x_1}\right)}{n_1 + n_2} > 0 \text{ or, } \bar{x} > \overline{x_1} \ldots(1)$$

$$\text{Also} = \overline{x_2} - \bar{x} = \overline{x_2} - \frac{n_1\overline{x_1} + n_2\overline{x_2}}{n_1 + n_2} = \frac{(n_1 + n_2)\overline{x_2} - \left(n_1\overline{x_1} + n_2\overline{x_2}\right)}{n_1 + n_2}$$

$$\frac{n_1\left(\overline{x_2} - \overline{x_1}\right)}{n_1 + n_2} > 0 \text{ or, } \overline{x_2} > \bar{x} \ldots(2)$$

From (1) and (2) $\overline{x_2} > \overline{x} > \overline{x_1}$

Similarly for $\overline{x_2} < \overline{x_1}$, $\overline{x_2} > \overline{x} > \overline{x_1}$

(iii) Let x and y be two observations

Then $\frac{x+y}{2} = 20$, $\sqrt{xy} = 12$. So $x + y = 40$, $xy = 144$

Thus $x(40 - x) = 144 \Rightarrow x^2 - 40x + 144 = 0$

$\Rightarrow x^2 - 36x - 4x + 144 = 0 \Rightarrow (x - 36)(x - 4) = 0$

So $x = 36$ or 4

If $x = 36$ then $y = 40 - 36 = 4$

If $x = 4$ then $y = 40 - 4 = 36$

Thus two observations are 36 and 4.

Harmonic mean $= \dfrac{2}{\frac{1}{4} + \frac{1}{36}} = \dfrac{2x36}{10} = 7.2$

Answer to Question no. 12(b).

(i) <u>HISTOGRAM:</u>

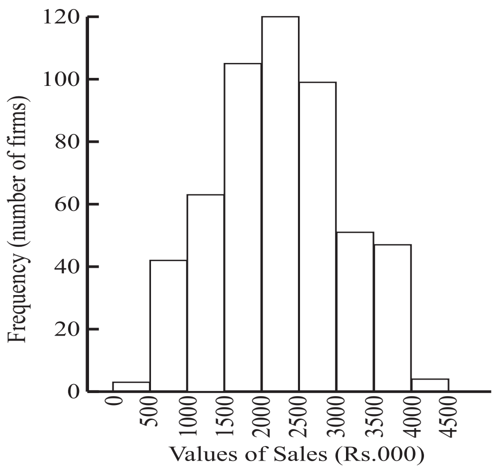
Histogram is the most common form of diagrammatic representation of a group frequency distribution. It consists of a set of adjoining rectangles drawn on a horizontal base line with areas proportional to the class frequencies. The width of rectangles, one for each class, extends over the class boundaries (not class limits) shown on the horizontal scale. When all classes have equal width, the heights of rectangles will be proportional to the class frequencies and it is then customary to take the heights numerically equal to the class frequency. For Example:

The following is an analysis of sales of 534 firms in an industry.

Values of Sales (Rs.000)		Number of Firms
0 – 500	...	3
500 – 1000	...	42
1000 – 1500	...	63
1500 – 2000	...	105
2000 – 2500	...	120
2500 – 3000	...	99
3000 – 3500	...	51
3500 – 4000	...	47
4000 – 4500	...	4

On the basis above parameter, a HISTOGRAM is drawn and exhibited in

FIGURE–A



Uses :- The series of rectangles in a histogram give a visual representation of the relative sizes of the various groups and the entire distribution of total frequency among the different classes becomes at once visible.

The histogram may be used to find the mode graphically.

(ii) <u>SKEWNESS AND ITS TWO IMPORTANT MEASURES:</u>

Skewness is a measure that refers to the extent to the extent of symmetry or asymmetry in a distribution. In other words, it describes the shape of a distribution.

Skewness is lack of symmetry. When a frequency distribution is plotted on a chart Skewness present in the items tends to be dispresed more on one side of the mean on the other.

MEASURES OF SKEWNESS:

Measures of skewness tell us the direction and extent of asymmetry in a series, and permit us to compare two or more series with regard to these. They may either be absolute or relative.

Absolute Measures of Skewness

Skewness can be measured in absolute terms by taking the difference between mean and mode. Symbolically,

Absolute $Sk^* = X -$ Mode

If the value of mean is greater than mode skewness will be positive, i.e., we shall get a plus sign in the above formula. Conversely, if the value of mode is greater than mean, we shall get a minus sign meaning thereby that the distribution is negatively skewed.

When skewness is based on quartiles, absolute skewness is given by the formula Absolute Sk = $Q_3 + Q_1 - 2$ Median.

Relative Measures of Skewness

There are four important measures of relative skewness, namely,

1. TheKarl Pearson's coefficient of skewness.
2. The Kelly's coefficient of skewness.
3. Measure of skewness based on moments.

These measures of skewness should mainly be used for making comparison between two or more distributions. As a description of one distribution alone, the interpretation of the measure of skewness is necessarily vague as "slight skewness", "marked skewness", or "moderate skewness:.

June 2009 Examination

Question: 11

Answer *any seven* of the following:

Choose the correct option showing proper reasons/calculations

(a) Arithmetic mean of 5 observations is 8. After calculation it was noted that observations 10 and 20 have been wrongly taken place of correct values 15 and 25 respectively. The correct mean is

(i) 18, (ii) 9, (iii) 13, (iv) none of these.

(b) Two groups of 10 and 15 observations have means 10 and 20 respectively. Then grouped mean is

(i) 15, (ii) 16, (iii) 14, (iv) none of these.

(c) Geometric mean of first group of five observations is 8 and that of second group of 4 observations is 128 $\sqrt{2}$. Then grouped geometric mean is

(i) 64, (ii) 32 $\sqrt{2}$, (iii) 32, (iv) none of these.

(d) If two groups with 2 and 3 observations have harmonic means $\frac{2}{5}$ and $\frac{1}{5}$ respectively then combined harmonic mean of 5 observation is

(i) $\frac{1}{2}$ (ii) $\frac{1}{4}$ (iii) $\frac{1}{3}$, (iv) none of these

(e) if the two observations have harmonic mean and geometric mean 9 and 15 respectively, then arithmetic mean of the two observations

(i) 12, (ii) 25, (iii) $\sqrt{135}$ (iv) none of these

(f) If the two variables x and y are related by 2x + 3y=12 and standard deviation of x is 6 then standard deviation of y is

(i) 2, (ii) 10, (iii) 4 (iv) none of these

(g) For, 10 values of variable x it is given that , $\sum x = 13, \sum x^2 = 400$, and $u = \frac{x - 5}{2}$. Then $\sum u^2$ is

(i) 100, (ii) 520 (iii) 260, (iv) none of these.

(h) Mean deviation about mean is 5.8. Co-efficient of mean deviation about mean is 0.2. Then mean is

(i) 1.16, (ii) 2.9 (iii) 29, (iv) none of these.

(i) For a group of 10 observations, $\sum x = 452$, $\sum x^2 = 24270$ and mode 43.7 the coefficient of skewness is

(i) 0.8, (ii) 0.08, (iii) 8, (iv) none of these.

(j) The mean and coefficient of variation of runs made by a batsman in 10 innings are 40 and 125% respectively. The s.d. of the runs made by the batsman is

(i) 50, (ii) 40, (iii) 20, (iv) none of these

Answer to Question 11:

(a) $\bar{x} = \frac{\sum x}{n} = \frac{\text{sum of observation}}{\text{no. of observation}} \Rightarrow 8 = \frac{\sum x}{5}$ i.e. $\sum x = 40$

This is wrong total

Correct total = correct $\sum x = 40 - (10 + 20) + (15 + 25)$

$= 40 - 30 + 40 = 50$

Correct mean = $\frac{\text{correct } \Sigma x}{5} = \frac{50}{5} = 10$ Ans (iv) None of these

(b) $\bar{x}$ = grouped mean $= \frac{n_1 \bar{x}_1 + n_2 \bar{x}_2}{n_1 + n_2}$

$n_1 = 10, n_2 = 15, \bar{x}_1 = 10\ \bar{x}_2 = 20$

$\therefore \bar{x} = \frac{10 \times 10 + 15 \times 20}{10 + 15} = \frac{400}{25} = 16$ Ans (ii)

(c) Let the observations be $x_1, x_2, \ldots\ldots\ldots\ldots, x_5$ in Group 1 and $G_1 = GM$, the observations be x_6, x_7, x_8, x_9, in Group 2 and $G_2 = GM$

Thus grouped mean $G = (x_1\ x_2 \ldots\ldots x_5\ x_6.\ \ x_7 \ldots x_9,)^{1/9}$

where $G_1 = (x_1 x_2 \ldots\ldots x_5)^{\frac{1}{5}}, G_2 = (x_6 x_7 x_8 x_9)^{\frac{1}{4}}$

$$\left(G_1^5\, G_2^4\right)^{\frac{1}{9}} = \left(8^5 \left(128\sqrt{2}\right)^4\right)^{\frac{1}{9}} = \left[\left(2^3\right)^5 \left(2^{7+\frac{1}{2}}\right)^4\right]^{\frac{1}{9}}$$

$$= \left(2^{15}\, 2^{30}\right)^{\frac{1}{9}} = \left(2^{45}\right)^{\frac{1}{9}} = 2^5 = 32 \text{ Ans (iii)}$$

(d) $H = \text{Combined H.M.} = \dfrac{5}{\left(\dfrac{1}{x_1} + \dfrac{1}{x_2}\right) + \left(\dfrac{1}{x_3} + \dfrac{1}{x_4} + \dfrac{1}{x_5}\right)}, x_1, x_2$ are observations in Gr1 and x_3, x_4, x_5 are in Gr2.

Their H.M. = H_1 and H_2.

$$H_1 = \frac{2}{\frac{1}{x_1}+\frac{1}{x_2}}, H_2 = \frac{3}{\frac{1}{x3}+\frac{1}{x4}+\frac{1}{x5}}.\text{Thus } H = \frac{5}{\frac{2}{H_1}+\frac{3}{H_2}} = \frac{5}{\frac{2}{2/5}+\frac{3}{1/5}}$$

$$= \frac{5}{5+15} = \frac{5}{20} = \frac{1}{4} \text{ Ans(ii)}$$

(e) For two observations,

$$\text{A.M} \times \text{H.M.} = (\text{GM})^2 \Rightarrow \text{A.M.} = \frac{(\text{G.M.})^2}{\text{H.M.}} = \frac{15^2}{9}$$

(f) $y = -\dfrac{2}{3}x + 4$. So $y_1 = -\dfrac{2}{3}x_1 + 4$ for $i = 1, 2, \ldots\ldots\ldots, n$. So $\bar{y} = -\dfrac{2}{3}\bar{x} + 4$

$$\text{Then s.d. } (y) = \sqrt{\frac{\sum_i (y_i - \bar{y})^2}{n}} = \sqrt{\frac{1}{n}\sum_i \left[\left(-\frac{2}{3}x_i + 4\right) - \frac{-2}{3}\bar{x} + 4\right]^2}$$

$$= \sqrt{\frac{4}{9n}\sum_i (x_i - \bar{x})^2} = \frac{2}{3}\text{s.d.(x)} = \frac{2}{3} \times 6 = 4 \text{ Ans (iii)}$$

(g) $\sum u^2 = \sum \frac{(x-5)^2}{4} = \frac{1}{4}(x^2 - 10x + 25) = \frac{1}{4}\left[\sum x^2 - 10\sum x + 25n\right]$

$= \frac{1}{4}[400 - 10\times13 + 25\times10] = \frac{520}{4} = 130$ Ans (iv) None of these

(h) Coefficient of mean deviation about mean =

$\frac{\text{Mean deviation about mean}}{\text{mean}} \Rightarrow 0.2 = \frac{5.8}{\text{mean}} \Rightarrow \text{mean} = \frac{5.8}{0.2} = 29$ Ans(iii)

(i) $\text{S.D.} = \sqrt{\frac{\sum x^2}{n} - \left(\frac{\sum x}{n}\right)^2} = \sqrt{\frac{24270}{10} - \left(\frac{452}{10}\right)^2}$

$= \sqrt{2427 - 2043.04} = \sqrt{383.96} = 19.6$

$\text{Mean} = \frac{\sum x}{n} = \frac{452}{10} = 45.2$

$\text{Coefficient of skewness} = \frac{\text{Mean-Mode}}{\text{SD}} = \frac{45.2-43.7}{19.6} = \frac{1.5}{19.6} = 0.08$ Ans(ii)

(j) Coefficient of variation $= \frac{\text{SD}}{\text{Mean}} \times 100\% \Rightarrow 125 = \frac{\text{SD}}{40} \times 100$

So S.D.= $\frac{125}{2.5} = 50$ Ans (i)

Question: 12

(a) Answer *any two* of the following:

(i) Find the median and mode of the following grouped frequency distribution:

Salaries (in Rs.) per hour	5-9	10-14	15-19	20-24	25-29	Total
No. of persons	10	20	30	25	15	100

(ii) Prove that for any two positive real quantities AM $\geq$ GM $\geq$ HM.

(iii) The following are the sizes of 50 families in a village:

2, 3, 4, 5, 4, 3, 2, 6, 1, 3, 5, 3, 5, 5, 4, 2, 4, 4, 3, 3, 3, 2, 4, 4, 3

3, 2, 3, 4, 3, 5, 4, 2, 3, 4, 4, 2, 4, 4, 2, 6, 4, 3, 5, 4, 3, 2, 3, 3, 1.

Obtain the frequency distribution of the family size and calculate the mean deviation about mean of the family size.

(b) Write short note on *any one* of the following: 4×1

(i) Pie Chart,

(ii) Primary data.

Answer to Question 12(a)(i):

COMPUTATION OF MEAN AND MODE:

Class (Rs.)	Frequency	Cumulative frequency (< type)
4.5 – 9.5	10	10
9.5 – 14.5	20	30
14.5 – 19.5	30	60
19.5 – 24.5	25	85
24.5 – 29.5	15	100

$\frac{100}{2} = 50$. So median class is 14.5 – 19.5 since value corresponding to 50(C.F.) lies in that class.

$$\text{Median} = 14.5 + \frac{\frac{100}{2} - 30}{60 - 30} \times 5 = 14.5 + \frac{20}{30} \times 5 = 14.5 + \frac{10}{3}$$
$$= 14.5 + 3.33 = 17.83 \text{ Rs.}$$

Modal class is 14.5 - 19.5 since maximum frequency 30lies in that class

$$\text{Mode} = 14.5 + \frac{30 - 20}{(30-20)+(30-25)} \times 5 = 14.5 + \frac{10}{10+5} \times 5 = 14.5 + \frac{10}{3}$$
$$= 14.5 + 3.33 = 17.83 \text{ Rs.}$$

Answer to Question 12(a) (ii):

Let x_1 and x_2 be any two positive real quantities.

$$\text{Now } (x_1 - x_2)^2 = (x_1 + x_2)^2 - 4x_1x_2$$

$$\Rightarrow (x_1 + x_2)^2 - 4x_1x_2 \geq 0$$

$$\Rightarrow \left(\frac{x_1 + x_2}{2}\right)^2 \geq x_1x_2 \Rightarrow \frac{x_1 + x_2}{2} \geq \sqrt{x_1x_2} \Rightarrow \text{AM} \geq \text{GM} \text{.......(I)}$$

$$\text{Next } \frac{\frac{x_1 + x_2}{x_1x_2}}{2} \geq \frac{\sqrt{x_1x_2}}{x_1x_2} \Rightarrow \frac{\frac{1}{x_1} + \frac{1}{x_2}}{2} \geq \frac{1}{\sqrt{x_1x_2}}$$

$$\Rightarrow \frac{2}{\frac{1}{x_1} + \frac{1}{x_2}} \leq \sqrt{x_1x_2} \Rightarrow \text{HM} \leq \text{GM} \text{........(II)}$$

Combining(I) & (II)

AM ≥ GM ≥ HM.

Answer to Question 12(a) (iii):

Size of family (X)	Tally mark of No. of family	No. of families (f)	f.x	$\lvert x-\bar{x}\rvert$	$f\lvert x-\bar{x}\rvert$
1	||	2	2	2.4	4.8
2	$\cancel{\|\|\|\|}$ ||||	9	18	1.4	12.6
3	$\cancel{\|\|\|\|}$ $\cancel{\|\|\|\|}$ $\cancel{\|\|\|\|}$ |	16	48	0.4	6.4
4	$\cancel{\|\|\|\|}$ $\cancel{\|\|\|\|}$ $\cancel{\|\|\|\|}$	15	60	0.6	9.0
5	$\cancel{\|\|\|\|}$ |	6	30	1.6	9.6
6	||	2	12	2.6	5.2
	Total	50	170		47.6

$$\text{Mean} = \frac{170}{50} = 3.4$$

Mean deviation about mean $= \frac{1}{N}\sum f\,|-\bar{x}\,| = \frac{47.6}{50} = 0.952$

Frequency distribution is

Size of family :	1	2	3	4	5	6	Total
No. of families :	2	9	16	15	6	2	50

Mean deviation about mean = 0.952

Answer to Question 12(b):

(i) **PIE CHART:**

Pie Chart/Pie Diagram is a circle whose area is divided proportionately among the different components by straight lines drawn from the centre to the circumference of the circle. The pie chart is so called because the entire graph looks like a pie and the components slices cut from pie. When statistical data are given for a number of categories, and we are interested in the comparison of the various categories or between a part and the whole, such a diagram is very helpful in effectively displaying data. For example: A PIE CHART is drawn and exhibited in Figure-B for the following data.

Principal Exporting Countries of Cotton (1000 bales) in a year.

USA	India	Egypt	Brazil	Argentina
6367	2999	1688	650	202

PIE Chart

Showing Principal Exporting Countries of Cotton in a year

For drawing a Pie chart, it is necessary to express the value of each category as a percentage of the total. Since the full angle 360^0 around the centre of the circle represents the whole i.e.

100%, the percentage Figure of each component is multiplied by 3.6 degrees to find the angle of the corresponding sector at the centre of the circle.

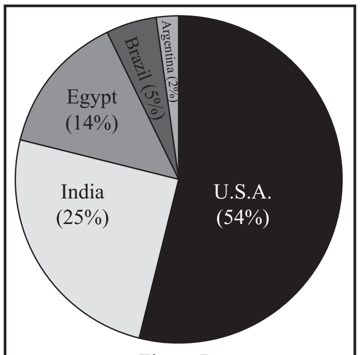


Figure B

(ii) Primary data.

Primary data are those statistical data which are collected for the first time and are original in nature. Primary data are collected originally by the Authorities who are required to collect them. The source from which primary data are collected is called Primary source. The method of collection of Primary data is also known as the Primary Method of collection of data. Primary data are collected for the first time by the authorities who require the data for their own use and treatment.

Data collected by the Field-works, Investigators and Enumerators are Primary Data. The Census of India published by the Government, the Reserve Bank of India Bulletin published by the Reserve Bank of India, the Indian Textile Bulletin published by the Textile Commissioner of India, etc. are the source of Primary Data.

The various methods of Primary Collection of data are:

(i) Direct Personal Investigations

(ii) Indirect Oral Investigation

(iii) Information from local agents and correspondence

(iv) Mailed questionnaires and Schedules

(v) Schedules (questionnaires) to be filled in by enumerators etc.

December 2008 Examination

Question: 11

(a) Answer *any nine* of the following:

Choose the correct option showing proper reasons/calculations.

(i) The arithmetic mean of first 9 counting numbers occurring with same frequency has its value
(A) 45, (B) 190, (C) 5, (D) none of them.

(ii) If 2 occurs 4 times, 4 occurs 3 times, 8 occurs twice and 16 occurs once then the geometric mean of them is
(A) 4, (B) 8, (C) 2, (D) none of them.

(iii) If a person travels first 2 km @ 2 km/hr., next 3 km. 3 km/hr and another 5 km @ 5 km/hr, his average speed during this journey is

(A) 3 km/hr, (B) $\frac{38}{10}$ km/hr, (C) $\frac{10}{3}$ km/hr, (D) none of them.

(iv) The median of marks 55, 60, 50, 40, 57, 45, 58, 65, 57, 48 of 10 students is

(A) 55, (B) 57, (C)52.5, (D) none of them.

(v) If the relation between two variables x and y is $3x - 2y = 5$ and mode of x is 5 then mode of y is

(A) 5, (B) 7.5 (C) 10 (D) none of them.

(vi) If the two variables x and y are related by the equation $3x = 2y + 4$ and mean deviation of x about its mean is 4 then mean deviation of y about its mean is

(A) $\frac{8}{3}$ (B) 4 (C) 6 (D) none of them.

(vii) If $\sum_{i=1}^{10}(x_i - 3) = 10$ and $\sum_{i=1}^{10}(x_i - 3)^2 = 100$ then standard deviation of 10 observations x_1, x_2,........., x_{10} is

(A) 9,(B) 3, (C) 10, (D) none of them.

(viii) If the relation between two variables x and y is $2x + 3y = 5$ and standard deviation of y is 10 then the standard deviation of x is

(A) 15, (B) 10, (C) $\frac{25}{2}$ (D) none of them.

(ix) If mean, mode and standard deviation of 10 observations are 65, 80 and 25 respectively then type of skewness of the data is

(A) Symmetric, (B) Positively skewed, (C) Negatively skewed,
(D) none of them.

(x) If the mean of 50 observations is 50 and one observation 94 is wrongly recorded there as 49 then correct mean will be

(A) 49.1, (B) 50, (C) 50.9 (D) none of them.

(xi) If for two observations arithmetic mean is 80 and harmonic mean is 5 then geometric mean of them is

(A) 20, (B) 400, (C) 16, (D) none of them.

(xii) For moderately skewed distribution A.M. = 110, Mode = 104, then median is

(A) 112, (B) 108, (C) 104, (D) none of them.

(xiii) If the maximum and minimum values of 10 observations are 40 and 10 then coefficient of range is

(A) $\frac{5}{3}$, (B) $\frac{3}{5}$, (C) 30, (D) none of them.

(xiv) The standard deviation (SD) of a variable x is 10, then the SD of the variable $2x + 10$ is

(A) 10, (B) –10, (C) 20, (D) none of these.

(b) Answer *any three* of the following:

(i) 12 observations are 2, 4,6, 3, 3, 5, 6, 8, 4, 3, 5, 4. If 1 is subtracted from each of them find the range.

(ii) If $\sum_{i=1}^{n}(x_i-4)=10\sum_{i=1}^{n}(x_i-3)=15$ and, find *n*.

(iii) If the means of two groups of m and n observations are 50 and 60 respectively and combined mean is 54. find the ratio m:n.

(iv) The mean deviation about mean 40 is 20, find the coefficient of mean deviation about mean.

(v) If mean and standard deviation of runs scored by a batsman in 10 tests are 50 and 4 respectively, find the coefficient of variation of runs.

(vi) Show that the standard deviation of x for its two value x_1 and x_2 is $\frac{1}{2}|x_1-x_2|$.

Answer to Question No. 11(a):

(i) $\rightarrow$ (D) (none of them):

$$\text{mean} = \frac{1^2+2^2+..........+9^2}{2}=\frac{9(9+1)(2\times9+1)}{9\times6}=\frac{10\times19}{6}=\frac{95}{3}$$

(ii) $\rightarrow$ (A):

$$\text{G.M.} = \left(2^4.4^3.8^2.16\right)^{\frac{1}{4+3+2+1}}=\left(2^4(2^2)^3(2^3)^2\,2^4.\right)^{\frac{1}{10}}$$

$$= \left(2^{4+6+6+4}\right)^{\frac{1}{10}}=\left(2^{20}\right)^{\frac{1}{10}}=20^{20/10}=2^2=4$$

(iii) $\rightarrow$ (C):

$$\text{Average Speed} = \frac{\text{Total distance}}{\text{Total time}}=\frac{2+3+5\text{km}}{\frac{2}{2}+\frac{3}{3}+\frac{5}{5}\text{hr}}$$

$$=\frac{10}{1+1+1}=\frac{10}{3}\text{km / hr.}$$

(iv) $\rightarrow$ (D):

When marks are arranged in non-decreasing order they are 40,45,48,50,55,57,57,58,60, 65

Then two middle most values of the arrangement are 55 and 57

$$\text{So, median}=\frac{55+57}{2}=56\,.$$

(v) $\rightarrow$ (A):

$y = \frac{3}{2}x - \frac{5}{2}$. Then Mode ($y$) = $\frac{3}{2}$ Mode (x) $- \frac{5}{2}$ since each value of y will be related to corresponding value of x by relation $y = \frac{3}{2}x - \frac{5}{2}$.

Thus Mode (y) $\frac{3}{2} \times 5 - \frac{5}{2} = \frac{10}{2} = 5$.

(vi) $\rightarrow$ (C):

$y = \frac{3}{2}x - 2$. So, $y_i = \frac{3}{2}x_i - 2$ for different i = 1, 2,..., n and $\bar{y} = \frac{3}{2}\bar{x} - 2$

$$\frac{1}{n}\sum_i |y_i - \bar{y}| = \frac{1}{n}\sum_i \left|\left(\frac{3}{2}x_i - 2\right) - \left(\frac{3}{2}\bar{x} - 2\right)\right|$$

$$\frac{1}{n}.\frac{3}{2}\sum_i |x_i - \bar{x}| = \frac{3}{2} = \text{M.D. of } x \text{ about}$$

Thus MD of y about is

(vii) $\rightarrow$ (B)

$$\text{s.d.} = \sqrt{\frac{1}{10}\sum_{i=1}^{10}(x_i - \bar{x})^2} = \sqrt{\frac{1}{10}\sum_{i=1}^{10}\{(x_i - 3) - (\bar{x} - 3)\}^2}$$

$$= \sqrt{\frac{1}{10}\sum_{i=1}^{10}(y_i - \bar{y})^2} \text{ (where yi} = \text{xi} - 3, i = 1, 2, ..., 10, \text{i.e.,)} \bar{y} = \bar{x} - 3$$

$$= \sqrt{\frac{1}{10}\sum_{i=1}^{10} yi^2 - (\bar{y})^2} = \sqrt{\frac{100}{10} - \left(\frac{10}{10}\right)^2} = \sqrt{10 - 1} = \sqrt{9} = 3$$

(viii) $\rightarrow$ (A)

$$x = -\frac{3}{2}y + \frac{5}{2}. \ \bar{x} = \frac{1}{n}\sum_{i=1}^{n}\left(-\frac{3}{2}y_i + \frac{5}{2}\right) = -\frac{3}{n}\sum_i y_i + \frac{\left(5/2\right)^n}{n}$$

$$= -3\bar{y} + \frac{5}{2} \text{ for n values } x_1 .....x_n . y_1, y_2 ....y_n$$

$$\text{Thus } x_i - \bar{x} = \left(-\frac{3}{2}y_i + 5\right) - \left(-\frac{3}{2}\bar{y} + \frac{5}{2}\right)$$

$$= -\frac{3}{2}(y_i + \bar{y}) \text{i.e.}$$

$$\text{So, S.D.}(x) = \sqrt{\frac{1}{n}\sum_i \left(x_i - \bar{x}\right)^2} = \frac{1}{n}\sqrt{\frac{1}{n}\sum_i \left(-\frac{3}{2}\right)(y_1 - \bar{y})^2}$$

$$= \sqrt{\left(\frac{3}{2}\right)^2 . \frac{1}{n}\sum_i (y_1 - \bar{y})^2} = \frac{3}{2}\text{s.d.}(y).$$

$$= \frac{3}{2} \times 10 = 15$$

(ix) $\rightarrow$ (C)

$$\text{Coefficient of skewness} = \frac{\text{mean - mode}}{\text{s.d.}} = \frac{65-80}{25} = \frac{-15}{25} = -\frac{3}{5} < 0$$

So they are negatively skewed.

(x) $\rightarrow$ (C)

$$\text{Correct mean} = \frac{50 \times 50 - 49 + 94}{50} = \frac{2500 + 45}{50} = \frac{2545}{50}$$

$$= \frac{5090}{100} = 50.9$$

(xi) $\rightarrow$ (A)

$$\text{For two observations, H.M.} = \frac{2}{\frac{1}{x_1} + \frac{1}{x_2}} = \frac{2x_1x_2}{x_2 + x_1} = \frac{(\text{G.M.})^2}{\text{A.M.}}$$

Thus $(\text{G.M.})^2 = \text{H.M.} \times \text{A.M.} = 5 \times 80 = 400$ i.e., $\text{G.M.} = 20$

(xii) $\rightarrow$ (B)

For moderately skewed distribution mean - mode = 3(mean -median)

Thus 110–104 = 3(110 – median) or 2 = 110 - median

So median = 110 – 2 = 108.

(xiii) $\rightarrow$ (B)

$$\text{Coefficient of rage} = \frac{\text{max value} - \text{min value}}{\text{max value} + \text{min value}}$$

$$= \frac{40-10}{40+10} = \frac{30}{50} = \frac{3}{5}$$

(xiv) $\rightarrow$ (C)

SD (2x + 10) = 2SD(x) = 20

Answer to Question No. 11(b):

(i) If 1 is substracetd from each of them, then

Maximum observation = 2 – 1 = 1

Minimum observation = 8 –1 = 7

Then range = 7 – 1 = 6

(ii) $\sum_{i=1}^{n} x_i - 4n = 10$ and $\sum_{i=1}^{n} x_i - 3n = 15$

i.e., $\sum_{i=1}^{n} x_i = 10 + 4n = 15 + 3n$ or $15 + 3n = 10 + 4n$

or, $5 = n$. i.e., $n = 5$

(iii) $\frac{50m + 60n}{m + n} = \text{combined mean} = 54$

or, $50m + 60n = 54m + 54n$

or, $(60\text{-}54)n = (54 - 50)m$

or, $6n = 4m$

or, $\frac{m}{n} = \frac{6}{4} = \frac{3}{2}$

ratio of m : n = 3 : 2

(iv) Coefficient of mean diviation about mean

$$= \frac{\text{mean deviation about mean}}{\text{mean}} = \frac{20}{40} = 0.5$$

(v) Coefficient of variation $= \frac{\text{s.d.}}{\text{mean}} = \frac{4}{50} = \frac{8}{100} = 8\%$

(vi) $SD = +\sqrt{\frac{1}{2}\left(x_1^2 + x_2^2\right) - \frac{1}{2}\left(x_1^2 + x_2^2\right)^2} = \frac{1}{2}\left|x_1 - x_2\right|$

Question: 12.

(a) Answer *any two* of the following:

(i) Calculate mean an variance of the following grouped frequency distribution:

Score :	0-10	10-20	20-30	30-40	40-50	50-60	Total
No. of Sudents:	10	30	60	60	30	10	200

(ii) Prove that the standard deviation of n observations is independent of change of origin but is dependent on the change of scale.

(iii) Two samples of sizes 100 and 150 hae means 45 and 55 and standard deviations 7 and 12 respectively. Find the mean and standard deviation of the combined sample.

(iv) Find median and mode from the following grouped frequency distribution:

Class	:	0-5	5-10	10-15	15-20	20-25	25-30	Total
Frequency	:	5	15	25	30	20	5	100

(b) Write short note on any one of the following:

(i) Histogram (ii) Pie Chart

Answer to Question No. 12(a):

(i) Calculation of Mean and Variance

Score	Freq(f)	Mid pt(x)	$d=\dfrac{x-30}{5}$	fd	fd2
0–10	10	5	–5	–50	250
10–20	30	15	–3	–90	270
20–30	60	25	–1	–60	60
30–40	60	35	1	60	60
40–50	30	45	3	90	270
50–60	10	55	5	50	250
Total	200		0	0	1160

$$\text{mean}=30+5\times\frac{\sum fd}{\sum f}=30+5\times\frac{0}{200}=30.\ \text{Hear}\ \bar{d}\frac{\sum fd}{\sum f}=\frac{0}{100}$$

$$\text{Variance}=5^2\times\frac{\sum f\left(d-\bar{d}\right)^2}{N}=25\times\frac{\sum fd^2}{200}=\frac{1160}{8}=145$$

(ii) Let $x^1, x^2, ...x_n$ be n observations of x. Let $y=\dfrac{x-a}{b}$

Then $yi=\dfrac{xi-a}{b}$ for $i=1,2,...n.$

$$\bar{y}=\frac{1}{n}\sum_1^n yi=\frac{1}{n}\sum_1^n\left(\frac{x_i-a}{b}\right)=\frac{1}{b}\left(\frac{1}{n}\sum x_i-\frac{na}{n}\right)=\frac{1}{b}\left(\bar{x}-a\right)$$

$$(SD(Y))^2 = \frac{1}{n}\sum_{1}^{n}(y_i - \bar{y})^2 = \frac{1}{n}\sum\left[\frac{x_i - a}{b} - \frac{\bar{x} - a}{b}\right]^2$$

$$= \frac{1}{b^2}.\frac{1}{n}\sum\left(x_i - \bar{x}\right)^2 = \frac{1}{b^2}(SD(x))^2$$

$SD(y) = \frac{1}{|b|}SD(x)$ i.e., independent of a (i.e., origin) but is dependent on b (i.e., scale)

(iii) Mean of combined sample $= \frac{100 \times 45 \times 150 \times 55}{250}$

$$= \frac{4500 \times 8250}{250} = \frac{12750}{250} = 51$$

Variance of combined sample $= \frac{1}{250}\left[100\left\{49 + (45-51)^2\right\} + 150\left\{144 + (55-51)^2\right\}\right]$

$$= \frac{1}{250}[100(49+36) + 150(144+16)] = \frac{1}{250}[100 \times 85 \times 150 \times 160]$$

$$= \frac{1}{250}[8500 + 24000] = \frac{32500}{250} = 130$$

Standard Deviation of combined sample $= \sqrt{130} = 11.402$

(iv) Calculation of Median and Mode

Class	Frequency	Cumulative frequency (<type)
0–5	5	5
5–15	15	20
10–15	25	45
15–20	30	75
20–25	20	95
25–30	5	100 = N

$\frac{N}{2} = 50$ and $\frac{N}{4} = 25$.

As $45 < 50 < 75$ median class is 15 20.

As max class frequency is 30, So 15–20 is model class,

$$\text{Median} = 15 + \frac{50-45}{75-45} \times 5 = 15 + \frac{5}{30} \times 5 = 15 + \frac{5}{6} = \frac{95}{6} = 15.83$$

$$\text{Mode} = 15 + \frac{(30-25)}{(30-25)+(30-20)} \times 5 = 15 \frac{5}{5+10} \times 5$$

$$= 15 + \frac{5}{3} = \frac{50}{3} = 16.67$$

Answer to Question No. 12(b):

(i) Histogram:

Histogram is the most common form of diagrammatic representation of a group frequency distribution. It consists of a set of adjoining rectangles drawn on a horizontal base line with areas proportional to the class frequencies. The width of rectangles, one for each class, extends over the class boundaries (not class limits) shown on the horizontal scale. When all classes have equal width, the heights of rectangles will be proportional to the class frequencies and it is then customary to take the heights numerically equal to the class frequency. For Example:

The following is an analysis of sales of 534 firms in an indutry.

Values of Sales (Rs. 000)		Number of Firms
0–500	...	3
500–1000	...	42
1000–1500	...	63
1500–2000	...	105
2000–2500	...	120
2500–3000	...	99
3000–3500	...	51
3500–4000	...	47
4000–4500	...	4

On the basis of above parameter, a Histogram is drawn and exhibited in Figure - A

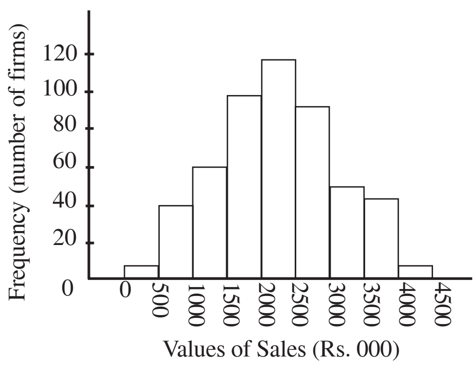


Figure A - Histogram

Uses : The series of rectangles in a histogram give a visual representation of the relative sizes of the various groups and the entire distribution of total frequency among the different classes becomes at once visible.

(ii) Pie Chart:

Pie Chart/Pie Diagram is a circle whose area is divided proportionately amont the different components by straight lines drawn from the centre to the circumference of the circle. The pie chart is so called because the entire graph looks like a pie and the components slices cut from pie. When statistical data are given for a number of categories, and we are interested in the comparison of the various categories or between a part and the whole, such a chart is very helpful in effectively displaying data. For example: A PIE CHART is drawn and exhibited in Figure - B for the following data.

Principal Exporting Countries of Cotton (1000 bales) in a year.

USA	India	Egypt	Brazil	Argentina
6367	2999	1688	650	202

PIE CHART

Showing Principal Exporting Countries of Cotton in a year

For drawing a Pie Chart, it is necessary to express the value of each category as a percentage of the total. Since the full angle 360^0 around the centre of the circle represents the whole i.e., 100%, the percentage figure of each component is multiplied by 3.6 degrees to find the angle of the corresponding sector at the centre of the circle.

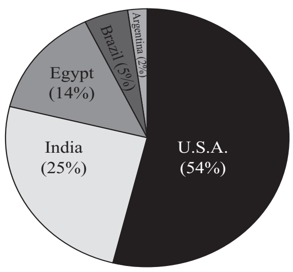


Figure B

June 2008 Examination

Question: 7

Answer *any nine* of the following:

(a) The mean of the first 10 odd numbers

(A) 11 (B) 10 (C) 12 (D) none of these

(b) The geometric mean of 3, 6, 24 and 48 is

(A) 16 (B) 12 (C) 10 (D) none of these

(c) A.M. and H.M. of two observations are 25 and 9 respectively. Their G.M. is

(A) 15 (B) 17 (C) 8 (D) none of these

(d) The mean of 20 observations is 16.5. If by mistake one observation was copied 12 instead of 21. Then the correct mean is

(A) 16 (B) 17 (C) 16.94 (D) 17.80

(e) If the number of obesrvations of the two groups G_1 and G_2 are in the ratio 1:2 and their arithmetic means (a.m.) are 16 and 10 respectively then a.m. of the combined group is

(A) 13 (B) 12 (C) 14 (D) none of these

(f) Let *h* be the harmonic mean (h.m.) of n positive observations. If each of the observations are repeated once more then h.m. of those 2n observations is

(A) h (B) $2h$ (C) $\frac{1}{2}h$ (D) none of these

(g) For 10 values of x it is given that $\sum u = 4, \sum u^2 = 20$ where $u = \frac{x-6}{5}$, then $\sum x^2$ is

(A) 360 (B) 600 (C) 1100 (D) none of these

(h) If the relation between two variables x and y is $5x + 2y = 6$ and the mean deviation (M.D.) of x about its mean is 6 then the M.D. of y about its mean is

(A) 6 (B) 15 (C) 18 (D) none of these

(i) If two variables x and y are such that $2y + 5 = 3x$ and quartile deviation (q.d.) of x is 8, then (q.d.) of y is

(A) 2 (B) 8 (C) 4 (D) none of these

(j) If for 20 observations $x_1, x_2, \ldots x_{10}$, $\sum_{i=1}^{10}(x_i - 4) = 30$ and $\sum_{i=1}^{10} (x_1 - 4)^2 = 100$ variance of the observations is

(A) 1 (B) 5 (C) 10 (D) none of these

(k) For two observations *a* and *b* standard deviation is

(A) $\frac{a-b}{2}$ (B) $\frac{|a-b|}{2}$ (C) $\sqrt{\frac{(a-b)^2}{2}}$ (D) none of these

(l) If the coefficient of variation and the variance of a series of values are 80% and 256 respectively then mean of the series is

(A) 12.8 (B) 320 (C) 3.2 (D) none of these

Answer to Question No. 7

(a) Mean $= \dfrac{1+3+5+7+9+11+13+15+17+19}{10} = 10$ (B)

(b) G.M. $= \sqrt[4]{3.6.24.48} = \sqrt[4]{3^4.2^4.2^4} = 12$ (B)

(c) G.M. $= \sqrt{(AM)\times(HM)} = \sqrt{25\times 9} = 15$ (A)

(d) Correct mean $= \dfrac{20\times 16.5 - 12 + 21}{20} = 16.94$ (C)

(e) No. of observations and mean of G_1 and G_2 are n_1 and n_2, $\bar{x}_1$ and $\bar{x}_2$. Mean of combined group = $\bar{x}$. Given $n_1 = m$, $n_2 = 2m$, $\bar{x}_1 = 16$, $\bar{x}_2 = 10$

$$\bar{x} = \frac{n_1\bar{x}_1 + n_2\bar{x}_2}{n_1 + n_2} = \frac{16m + 20m}{m + 2m} = \frac{36m}{3m} = 12 \text{ (B)}$$

(f) $x_1, x_2 ......xn$ are n observations. $h = \dfrac{n}{\dfrac{1}{x_1} + \dfrac{1}{x_2} + ... + \dfrac{1}{x_n}}$

$2n$ observation are $x_1, x_1, x_2, x_2 ..... x_n, x_n$

$$\text{H.M.} = \frac{2n}{2\left(\dfrac{1}{x_1} + \dfrac{1}{x_2} + ... + \dfrac{1}{x_n}\right)} = \frac{n}{\dfrac{1}{x_1} + \dfrac{1}{x_2} + ... + \dfrac{1}{x_n}} = h \text{ (A)}$$

(g) $x = 6 + 5u$, $x^2 = 36 + 60u + 25u^2$ i.e. for 10 values

$$\sum x^2 = 36\times 10 + 60\sum u + 25\sum u^2 = 360 + 60\times 4 + 25\times 20 = 1100 \text{ (C)}$$

(h) n = no. of observations, $y = -\dfrac{5x}{2} + 3$, $\bar{y} = -\dfrac{5}{2}\bar{x} + 3$

$$\frac{1}{2}\sum|y - \bar{y}| = \frac{1}{n}\sum \left|\left(-\frac{5x}{2} + 3\right) - \left(-\frac{5}{2}\bar{x} + 3\right)\right| = \frac{1}{3}\sum\left|-\frac{5}{2}(x - \bar{x})\right|$$

$$= \left|-\frac{5}{2}\right|\sum|x - \bar{x}|/n$$

So MD (y) about mean $= \dfrac{5}{2}$ MD (x) about mean $= \dfrac{5}{2} \times 6 = 15$ (B)

(i) $y = \frac{3}{2}\times - \frac{5}{2}$, Q_{1x}, Q_{3x} and Q_{1y}, Q_{3y} are first quartile, 3rd quartile of x and y respectively.

q.d. of $y = \frac{Q_{3y} - Q_{1y}}{2}$. Now $Q_{1y} = \frac{3}{2} Q_{1x} - \frac{5}{2}$ and $Q_{3y} = \frac{3}{2} Q_{3x} - \frac{5}{2}$

So.q.d.of $y = \frac{\frac{3}{2}(Q_{3x} - Q_{1x})}{2} = \frac{3}{2}\times 8 = 12$ **Answer is (D) (none of these)**

(j) $x_i - 4 = y_1$, i = 1, 2,...... 10 then $\overline{x} - 4 = \overline{y}$

Variance $= \frac{1}{10}\sum_{i=1}^{10}(xi - \overline{x})^2 = \frac{1}{10}\sum_{i=1}^{10}\left\{(x_i - 4) - (x - 4)\right\}^2$

$\frac{1}{10}\sum_{i=1}^{10}(y_i - \overline{y})^2 = \frac{1}{10}\sum_{i=1}^{10} y_i^2 - (\overline{y})^2 = \frac{100}{10} - \left(\frac{30}{10}\right)^2 = 10 - 9 = 1 =$ (A)

(k) S.d. $= \sqrt{\frac{\left\{a - \left(\frac{a+b}{2}\right)\right\}^2 + \left\{b - \left(\frac{a+b}{2}\right)\right\}^2}{2}} \left(\because \text{ mean} = \frac{a+b}{2}\right)$

$\sqrt{\frac{(a-b)^2}{8} + \frac{(b-a)^2}{8}} = \sqrt{\frac{2(a-b)^2}{8}} = \sqrt{\frac{(a-b)^2}{4}} = \frac{|a-b|}{2} =$ (B)

(l) Coefficient of variation $= \frac{\text{s.d.}}{\text{mean}}$

s.d. $= \sqrt{\text{variance}} = \sqrt{256} = 16$, It is always positive.

$\frac{80}{100} = \frac{16}{\text{mean}}$.So mean $= \frac{1600}{80} = 20$ **Answer is (D) (none of these)**

Question: 8

Answer a*ny three* of the following:

(a) Write short note on ogive.

(b) What is centreal tendency of data? Write most important measure of it. Describe why it is most important.

(c) Find the mean deviation about mean of the arithmetic progression:

a, a + d, a + 2d, ..., a + 2nd

(d) Show that for the two groups of observations the grouped mean always lies beween two group means.

(e) The means, of two samples of sizes 10 and 20 are 8 and 5, and variances are 4, 9 respectively. Obtain mean and variance of he combined sample

(f) Calculate mean and standard deviation of the following distribution:

Class interval :	0-10	10-20	20-30	30-40	40-50	Total
Frequency :	1	4	6	4	1	16

Answer to Question No. 8(a):

SHORT NOTE:

OGIVE:

Ogive is the graphical representation of a cumulative frequency distribution and hence is also called Cumulative Frequency Polygon. When cumulative frequencies are plotted against the corresponding class boundaries and the successive points are joined by straight line, the line diagram obtained is known as Ogive or Cumulative Frequency Polygon.

There are two methods of constructing "OGIVE" namely:

(i) The "less than" method (ii) The "more than" method.

In the "less than Ogive" looks like an elongated S which start from the lowest class boundary on the horizontal axis and gradually rising upward and ending at the highest class boundary corresponding to the total frequecy of the distribution. Similarly the "more than Ogive" has the appearance of an elongated S turned upside down.

For instance, Ogives (both "less than and more than" types) drawn from the following distribution of wages of 500 workers of a big manufacturing concern as follows:

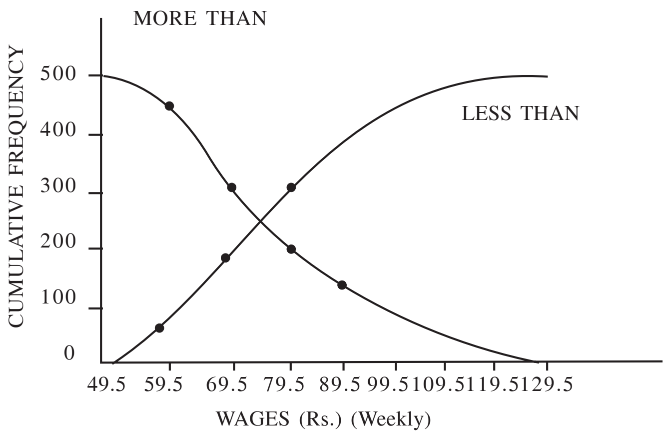
Wages (Rs.) : (Weekly)	50-59	60-69	70-79	80-89	90-99	100-109	110-119	120-129
No. of Workers :	20	60	100	150	75	50	25	20

Calculations for Drawing Ogives

Class boundary	Cumulative frequency	
	'less than'	**'more than'**
49.5	0	500 = N
59.5	20	480
69.5	80	420
79.5	180	320
89.5	330	170
99.5	405	95
109.5	455	45
119.5	480	20
129.5	500 = N	0

Fig. A

Ogives for Wages Distribution



Uses

(i) Ogive is used for estimating the cumulative frequencies of values falling within the range of values represented on graph.

(ii) It is used to find the median, quartiles, deciles, percentiles or the value of the variable such that its cumulative frequency is a specified number.

(iii) It is used to find the number of observations which are expecetd to lie between two given values.

(iv) If the class frequencies are large, they can be expressed as percentages of total frequency.

Answer to Question No. 8(b):

One of the most important objectives of statistical analysis is to get one single value that decribes the characteristic of entire mass of unwidely data. Such a value is called the central value or an average or the expected value of the variable. It is generally observed that the observations (data) on a variable tend to luster around some central value. This tendency of clustering around some Central Value is known as Central tendency.

Since the single value has a tendency to be somewhere at the centre and within the range of all values it is also known as the measure of Central Tendency.

There are three measures of Central tendency:

(i) Mean (2) Median (3) Mode

Mean is the most important measure which is of three types (i) Arithmetic Mean (ii) Geometric Mean (iii) Harmonic Mean.

Mean of a series (usually denoted by $\overline{x}$) is the value obtained by dividing the sum of the values of various items, in a series ($\sum x$) divided by the number of items (N) constituting the series.

Mean is most important measure because of the following reasons.

(i) It is rigidly defined so that different interpretations by different persons are not possible.

(ii) It is easy to understand and easy to calculate.

(iii) Since it takes all values into consideration it is considered to be more representative of the distribution.

(iv) Mean also helps in obtaining an idea of a complete universe by means of sample data.

(v) It reduces a complex mass of data into a single typical figure to enable one to get a bird's eye view about the characteristics of the phenomenon under study.

(vi) It provides a good basis for comparison.

(vii) It is possible to Calculate Mean if some of the details of the data are lacking.

(viii) Mean (average) are valuable in setting standards for managerial control.

Answer to Question No. 8(c):

$$\text{Mean} = \frac{a + (a+d) + ... + (a+2nd)}{2n+1}$$

$$= \frac{(2n+1)a + d(1+2+......+2n)}{2n+1} = a + \frac{2n(2n+1)}{2n(2n+1)}\ d$$

$$= a + nd$$

Mean deviation about mean

$$= \frac{1}{(2n+1)}\Big[|a-(a+nd)| + |(a+d)-(a+nd)| + ...$$
$$+ |(a+(n-1)d)-(a+nd)| + |(a+nd)-(a+nd)|$$
$$+ |(a+(n+1)d)-(a+nd)| + ... + |(a+2nd)-(a+nd)|\ \Big]$$

$$= \frac{1}{(2n+1)}\Big[|-nd| + |-(n-1)|d| + ... + |-d| + 0 + |d| + ... + |nd|\Big]$$

$$= \frac{2|d|[1+2+......+n]}{(2n+1)} = \frac{2|d|\frac{n(n+1)}{2}}{(2n+1)}$$

$$= \frac{n(n+1)|d|}{(2n+1)}$$

Answer to Question No. 8(d):

Let no. of observations and means of two groups be n_1 and n_2,

$\overline{x}_1$ and $\overline{x}_2$ respectively. Grouped mean = $\overline{x}$

$$\text{Then } \bar{x} = \frac{n_1\bar{x}_1 + n_2\bar{x}_2}{n_1 + n_2}$$

$$\text{If } \bar{x}_1 \geq \bar{x}_2,\ \bar{x}_1 - \bar{x} = \bar{x}_1 - \frac{n_1\bar{x}_1 + n_2\bar{x}_2}{n_1 + n_2} = \frac{(n_1 + n_2)\bar{x}_1 - n_1\bar{x}_1 - n_2\bar{x}_2}{n_1 + n_2}$$

$$= \frac{n_2(\bar{x}_1 - \bar{x}_2)}{n_1 + n_2} \geq 0 \text{ i.e., } \bar{x}_1 \geq \bar{x}$$

$$\text{and } \bar{x} - \bar{x}_2 = \frac{n_1\bar{x}_1 + n_2\bar{x}_2}{n_1 + n_2} = \frac{n_1\bar{x}_1 + n_2\bar{x}^2 - (n_1 + n_2)\bar{x}_2}{n_1 + n_2}$$

$$= \frac{n_1(\bar{x}_1 - \bar{x}_2)}{n_1 + n_2} \geq 0 \text{ i.e., } \bar{x} \geq \bar{x}_2$$

Thus if $\bar{x}_1 \geq \bar{x}_2$, $\bar{x}_1 \geq \bar{x} \geq \bar{x}_2$

Similarly, it can be shown that if $\bar{x}_1 \leq \bar{x}_2$, $\bar{x}_1 \leq \bar{x} \leq \bar{x}_2$

ie. $\bar{x}$ lines between $\bar{x}_1$ and $\bar{x}_2$

Answer to Question No. 8(e):

No. of observations, means and variances of

1st sample are $n_1 = 10,\ \bar{x}_1 = 8, S_1^2 = 4$

2nd sample are $n_2 = 20,\ \bar{x}_2 = 5,\ S_2^2 = 9$

$$x = \text{mean of combined sample} = \frac{n_1\bar{x}_2 + n_2\bar{x}_2}{n_1 + n_2} = \frac{80 + 100}{10 + 20} = \frac{180}{30} = 6$$

$$\text{variance of the combined sample} = \frac{n_1s_1^2 + n_2s_2^2 + n_1(\bar{x}_1 - \bar{x})^2 + n_2(\bar{x}_2 - \bar{x})^2}{n_1 + n_2}$$

$$= \frac{10 \times 4 + 20 \times 9 + 10(8-6)^2 + 20(5-6)^2}{10 + 20} = \frac{40 + 180 + 40 + 20}{30} = \frac{280}{30} = \frac{28}{3}$$

Answer to Question No. 8(f):

Calculation of mean and standard deviation

Class	Mid Value (x)	(f) freq	$u \frac{x-25}{10}$	fu	fu^2
0-10	5	1	–2	–2	4
10-20	15	4	–1	–4	4
20-30	25	6	0	0	0
30-40	35	4	1	4	4
40-50	45	1	2	2	4
Total		16 = N		0	16

$$\text{Mean} = 25 + \frac{\sum fu}{N} \times 10 = 25 + \frac{10}{16} \times 10 = 25 + 0 = 25$$

$$\text{Standard deviation} = \sqrt{\left[\frac{1}{N}\sum fu^2 - \left(\frac{\sum fu}{N}\right)^2\right]}$$

$$= \sqrt{\left[\frac{16}{16} - \left(\frac{0}{16}\right)^2\right]} \times 10 \quad = \sqrt{1-0} \times 10 = 1 \times 10 = 10$$

December 2007 Examination

Question: 7

Answer *any nine* of the following

Choose the correct option showing necessary reason/ calculations.

(a) The mean of first 10 even number is

(i) 5.5, (ii) 55, (iii) 11, (iv) non of these

(b) The mean of the marks of 4,12, 7,9, 14, 17, 16, 21, of eight student

(i) 12, (ii) 14, (iii) 13, (iv) none of these

(c) If for a distribution mean = 22, median = 24 and s.d. = 10, then coefficient of skewness is

(i) 0.6, (ii) – 0.6 (iii) – 6 (iv) none of these

(d) For two positive observations x_1 and x_2 which one of the following is true?

(i) $(AM)(HM) = (GM)^2$ (ii) $(AM)(GM) = (HM)^2$

(iii) $(GM)(HM) = (AM)^2$ (iv) none of the above

(e) Given $\sum_{i=1}^{n}(x_i - 4) = 72$ and $\sum_{i=1}^{n}(x_i - 7) = 3.$ Then arithmetic mean of *x is*

(i) 68.8, (ii) 6.88, (iii) 0.688 (iv) none of these

(f) The geometric mean of 4, x and 8 is 8. Then x is

(i) 4, (ii) 8, (iii) 16, (iv) 32

(g) For a variable the mean is 10 and the coefficient of variation is 50%. Then the variance is

(i) 5 (ii) 20, (iii) 400 (iv) 25.

(h) For a set of 10 observations, mean is 50. One observation is wrongly recorded as 50 instead of 60. Then the correct mean is

(i) 49, (ii) 51, (iii) 60, (iv) none of these

(i) The harmonic mean $\frac{1}{2}, \frac{1}{4}, \frac{1}{5}, \frac{1}{6}$ and $\frac{1}{8}$ is

(i) $\frac{1}{4}$, (ii) $\frac{1}{5}$, (iii) $\frac{1}{6}$ (iv) 5.

(j) Means and standard deviations of runs of 10 innings of two players Mr. X and Mr. Y are as follows:

X : mean = 60 std. dev. = 8

Y : mean = 50 std. dev. = 5

Which player is more consistent?

(i) X (ii) Y, (iii) equally consistent (iv) no conclusion is possible

(k) $\sum_{i=1}^{10}(x_i - 4) = 50$, then the value of $\sum_{i=1}^{10}\left(\frac{x_i - 1}{2}\right)$ is

(i) 80, (ii) 75 (iii) 65, (iv) 40.

(l) The mode of the observations 40, 50, 30,20, 25, 35, 30, 30, 20, 30, 30, 20, 25, 20 is

i) 20, (ii) 30, (iii) 35, (iv) 40

Answer to Question No. 7:

(a) $\rightarrow$(iii) : Mean $= \frac{2 + 4 + 6 + 8 + 10 + 12 + 14 + 16 + 18 + 20}{10}$

$$= \frac{2 \times 10 \times 11}{2 \times 10} = 11$$

(b) $\rightarrow$(iii) : Rearranging in ascending order of magnitude we get

$4, 7, 9, 12, 14, 16, 17, 21$ Median $= \dfrac{12+14}{2} = 13$

(c) $\rightarrow$(ii) : Co-efficient of skewness $= \dfrac{3(\text{Mean} - \text{Median})}{\text{s.d.}}$

$$= \frac{3(22-24)}{10} = -0.6$$

(d) $\rightarrow$(i) : $(AM)\,(HM) = (GM)^2$ (A)

(e) $\rightarrow$(i) : $\sum\limits_{i=1}^{n} x_i = 72 + 4n$ and $\sum\limits_{i=1}^{n} x_i = -3 + 7n$

$\therefore 72 + 4n = -3 + 7n$

$\Rightarrow n = 25$

$\sum\limits_{i=1}^{n} x_i = 72 + 100 = 172$, $\bar{x} = \dfrac{172}{25} = 6.88$

(f) $\rightarrow$(iii) : $(4.x.8)^{\frac{1}{3}} = 8 \Rightarrow (4x)(8) = 8^3 \Rightarrow 4x = 8^2 \Rightarrow x = 16$

(g) $\rightarrow$(iv) :C.V. $= \dfrac{\text{s.d}}{\text{mean}} \times 100\%$

$\Rightarrow 50 = \dfrac{\text{s.d}}{\text{mean}} \times 100\% \Rightarrow 50 = \dfrac{\text{s.d}}{10} \times 100 \Rightarrow \text{s.d.} = 5$

$\therefore$ variance $= 25$

(h) $\rightarrow$(ii) : Correct total = wrong total $- 50 + 60 = 10 \times 50 + 10 = 510$

So, correct mean $= \dfrac{1}{10} \times$ correct total $= \dfrac{510}{10} = 51$

(i) $\rightarrow$(ii) : H.M. $= \dfrac{5}{2+4+5+6+8} = \dfrac{5}{25} = \dfrac{1}{5}$

(j) $\rightarrow$(ii) : $(CV)_x \; \dfrac{8}{60} \times 100 = 13.33\%$

$$(CV)_Y = \frac{5}{50} \times 100 = 10\%$$

$$(CV)_Y < (CV)_X$$

$\therefore$ Y is more consistent.

(k) $\rightarrow$(iv) : $\sum_{i=1}^{10}\left(\frac{x_i - 1}{2}\right) = \sum_{i=1}^{10}\left(\frac{x_i - 1}{2}\right) = \frac{1}{2}\sum_{i=1}^{10}(xi - 4 + 3)$

$$= \frac{1}{2}\sum_{i=1}^{10}(x_i - 4) + \frac{30}{2} = \frac{50}{2} + 15 = 25 + 15 = 40$$

(l) $\rightarrow$(ii) : 30 (Frequency of 30 is maximum)

Question : 8

Answer *any three* of the following:

(a) What is a pie diagram? Describe how it can be drawn.

(b) The following are the sizes of 40 families in a village:

4, 3, 2, 6, 1, 3, 3, 5, 5, 4, 2, 2, 4, 3, 3, 3, 6, 4, 5, 4

3, 4, 3, 5, 4, 2, 3, 4, 4, 2, 6, 4, 3, 5, 4, 3, 2, 3, 5, 1

Obtain the frequency ditribution of family size and calculate the variance.

(c) Prove that standard deviation calculated from two values is half of their difference and hence compute the variance of 37 and 67.

(d) Find the mean deviation about the mean of the following observations:

x	:	10	11	12	13	14
Frequencey	:	3	12	18	12	3

(e) Write short note *(any one)*:

(i) Bar diagram, (ii) Histogram

Answer to Question No. 8(a):

PIE DIAGRAM:

Pie Diagram is a circle whose area is divided proportionately among the different components by straight lines drawn from the centre to the circumference of the circle. When statistical data are given for a number of categories, and we are interested in the comparison of the various categories or between a part and the whole, such a diagram is very helpful in effectively displaying data. For example: A PIE Diagram is drawn and exhibited in Figure-A for the following data.

Principal Exporting Countries of Cotton (1000 bales) in a year.

USA	India	Egypt	Brazil	Argentina
6367	2999	1688	650	202

PIE DIAGRAM

Showing Principal Exporting Countries of Cotton in a year

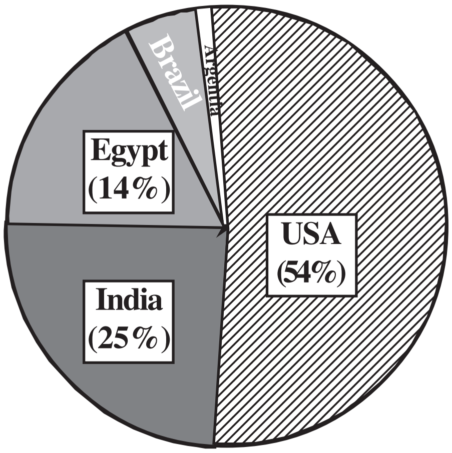


Figure A

For drawing a Pie diagram, it is necessary to express the value of each category as a percentage of the total. Since the full angle 360^0 around the centre of the circle represents the whole i.e. 100%, the percentage Figure of each component is multipled by 3.6 degree to find the angle of the corresponding sector at the centre of the circle.

Answer to Question No. 8 (b):

FREQUENCY DISTRIBUTION OF SIZE OF THE FAMILY IN A VILLAGE (WITH TALLY MARKS)

Size of Family	Tall mark	Frequency
1	||	2
2	𝍸 |	6
3	𝍸 𝍸 ||	12
4	𝍸 𝍸 |	11
5	𝍸 |	6
6	|||	3
Total		**40**

x	y = x – 4	f	fy	fy^2	C.F. (type)
1	– 3	2	– 6	18	2
2	– 2	6	– 12	24	8
3	– 1	12	– 12	12	20
4	0	11	0	0	31
5	1	6	6	6	37
6	2	3	6	12	40
Total		**40**	**– 18**	**72**	

Here x = family size

$$\text{mean} = 4 + \frac{\sum fy}{\sum f} = 4 - \frac{18}{40} = 3.55$$

$$\text{Variance} = \frac{\sum fy^2}{\sum f} - \left(\frac{\sum fy}{\sum f}\right)^2 = \frac{72}{40} - \left(\frac{-18}{40}\right)^2 = 1.5975$$

Answer to Question No. 8(c):

Let x_1 and x_2 be two observations of variable x

$$(\text{s.d.})^2 = \frac{1}{n}\sum_{i=1}^{n}(x_1 - x)^2 = \frac{\left(x_1 - \frac{x_1 + x_2}{2}\right)^2 + \left(x_2 - \frac{x_1 + x_2}{2}\right)^2}{2}$$

$$= \frac{(x_1 - x_2)^2 + (x_2 - x_1)^2}{8} = \frac{(x_1 - x_2)^2}{4}$$

$$\text{sd} = = \frac{1}{2}|x_1 - x_2|$$

$$\text{var iance of 37 and 67} = \left\{\frac{1}{2}|37 - 67|\right\}^2 = 225$$

Answer to Question No. 8(d):

x	u = x – 12	f	fu	f|u|
10	– 2	3	– 6	6
11	– 1	12	– 12	12
12	– 0	18	0	0
13	1	12	12	12
14	2	3	6	6
		48 = N	0	36

$$\bar{u} = \frac{\sum fu}{\sum f} = 0$$

$$\text{Mean deviation about mean} = \sum \frac{f\left|u - \bar{u}\right|}{N} = \frac{1}{48}\sum f\left|u\right| = \frac{36}{48} = 0.75$$

Answer to Question No. 8(e):

SHORTE NOTES:

(i) BAR DIAGRAM

Bar diagram consists of a group of equispaced rectangular bars, one for each category (or class) ofgiven statistical data. The bars, starting from a common base line, must be of equal width and their lengths represent the values of statistical data. They are shaded or coloured suitably.

There are two types of bar diagrams – vertical bar diagram and horizontal bar diagram. Vertical bars are used to represent time series data or data clasified by the values of a variable. Horizontal bars are used to depict data classified by attributes only.

The following points must be kept in view while constructing a BAR DIAGRAM.

1. Since the width of the bars does not indicate anything, all bars drawn should be of the same width.
2. The length of the bar should be proportional to the magnitude of the variable they represent.
3. The gap between one bar and the other must be kept uniform throughout.
4. The scale should be adjusted in relation to the magnitude of the highest magnitude.
5. All bars should rest on the same line called the base.

6. It is desirable to write a representative figure on the bar to enable the reader grasp the magnitude at a glance.

For example

The following data relating to lending operation is represented by a horizontal bar diagram: (Fig B).

Year	No. of operations ('000) in Braces	Amount (Rs. crores)
1986	8.2	256
1987	15.6	479
1988	21.0	745
1989	18.0	675
1990	26.7	1189

LENDING OPERATIONS

(Rs. Crores)

NO. OF OPERATIONS (000) IN BRACES

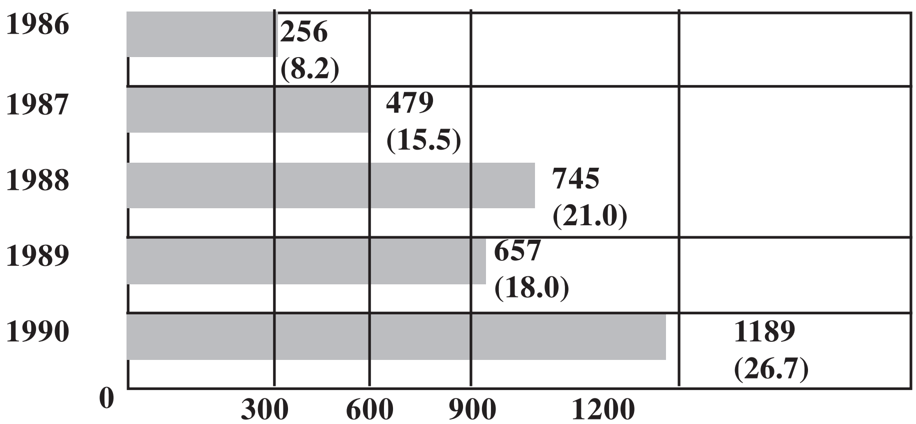


Figure B

(ii) HISTOGRAM:

Histogram is the most common form of diagrammatic representation of a group frequency distribution. It consists of a set of adjoining rectangles drawn on a horizontal base line with areas proportional to the class frequencies. The width of rectangles, one for each class, extends over the class boundaries (not class limits) shown on the horizontal scale. When all classes have equal width, the heights of rectangles will be proportional to the class frequenceis and it is then customary to take the heights numerically equal to the class frequency.

For Example:

The following is an analysis of sales of 534 firms in an industry.

Values of Sales (Rs. 000)		Number of Firms
0 – 500	....	3
500 – 1000	....	42
1000 – 1500	....	63
1500 – 2000	....	105
2000 – 2500	....	120
2500 – 3000	....	99
3000 – 3500	....	51
3500 – 4000	....	47
4000 – 4500	....	4

On the basis above parameter, a HISTOGRAM is drawn and exhibited in FIGURE-C

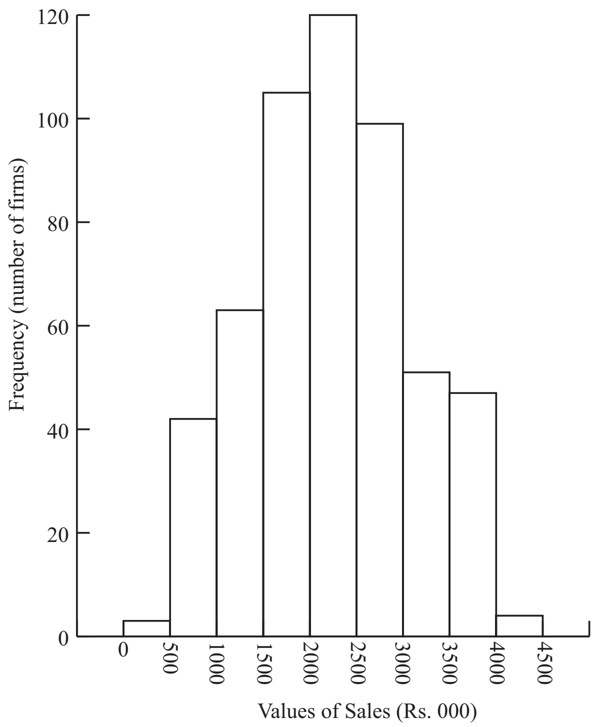


FIGURE C : HISTOGRAM

Uses : The series of rectangles in a histogram give a visual representation of the relative sizes of the various groups and the entire distribution of total frequency among the different classes becomes at once visible.

The histogram may be used to find the mode graphically.

June 2007 Examination

Question: 12

Answer any five of the following:

(a) Find the geometric mean of 8 observations: 2 occurring 4 times, 4 occurring twice, 8 and 32 occurring once each.

(b) Find the harmonic mean of the observations $\frac{1}{2}, \frac{1}{4}, \frac{1}{5}, \frac{1}{6}$ and $\frac{1}{8}$

(c) If the means of two groups of m and n observations are 40 and 50 respectively and the combined group mean is 42, find the ratio m: n.

(d) For a group of 5 items $\sum_{i=1}^{5}(x_i - 4) = 40$ and $\sum_{i=1}^{5}(x_i^2 = 1600$ Find the variance of the group.

(e) Compute the standard deviation of 6 numbers 5,5,5,7,7,7.

(f) Determine the Mean deviation about mean of six observations: 4,4,4,6,6,6.

(g) Mean and standard deviations of runs of 10 innings of two players are as follows:

First player : Mean = 50, s.d. = 4

Second player : Mean = 40, s.d.= 5

Player who is more consistent in scoring runs?

(h) Which group is more skewed?

Group I : a.m. = 20, mode = 25, s.d. = 8;

Group II : a.m. = 18, mode = 27, s.d. = 9.

Answer to Question 12:

(a) $GM = \left(2^4.4^2.8.32\right)^{1/8} = \left(2^{16}\right)^{1/8} = 2^2 = 4$

(b) $$HM = \frac{5}{\frac{1}{\left(\frac{1}{2}\right)} + \frac{1}{\left(\frac{1}{4}\right)} + \frac{1}{\left(\frac{1}{5}\right)} + \frac{1}{\left(\frac{1}{6}\right)} + \frac{1}{\left(\frac{1}{8}\right)}}$$

$$= \frac{5}{2+4+5+6+8} = \frac{5}{25} = \frac{1}{5} = 0.2$$

(c) $$\overline{x} = \frac{n_1\overline{x}_1 + n_2\overline{x}_2}{n_1 + n_2}$$

So, $\frac{m \times 40 + n \times 50}{m + n} = 42$

$\Rightarrow 40m + 50n = 42m + 42n$

$\Rightarrow 2m = 8n$

$\therefore m : n = 4:1$ Thus, $m : n = 4:1$

(d) $\sum_{i=1}^{5} X_i = 40 + 4 \times 5 = 60 \therefore \bar{x} = \frac{60}{5} = 12$

Variance $= \frac{1}{5}\sum x_i = (\bar{x})^2 = \frac{1}{5} \times 1600 - 12^2 = 320 - 144 = 176$

(e) $\text{mean}(\bar{x}) = \frac{3 \times 5 + 3 \times 7}{6} = \frac{3 \times 12}{6} = 6$

Standard Deviation $= \sqrt{\frac{1}{6}3\left[(5-6)^2 + 3(7-6)^2\right]} = \sqrt{\frac{1}{2}(1+1)} = 1$

(f) $\text{mean}(\bar{x}) = \frac{3 \times 4 + 3 \times 6}{6} = 5$

Mean Deviation about mean $= \frac{1}{6}\left[3.|4-5| + 3\lfloor 6-5 \rfloor\right] = \frac{1}{2}[1+1] = 1$

(g) $C.V = {}^{S.D.}\!/_{mean} \times 100$

1st player $\quad {}^{4}\!/_{50} \times 100 = 8\%$

2nd player $\quad {}^{5}\!/_{40} \times 100 = 12.5\%$

1st player is more consistent in scoring runs since its CV is lesser than CV of 2nd

(h) $\text{Skewness} = \frac{\text{Mean} - \text{mode}}{\text{s.d.}}$

For Group1, $S_k = \frac{20 - 25}{8} = -\frac{5}{8}$

For Group2, $S_k = \frac{18 - 27}{9} = -1$

So, Group2 is more (negatively) skewed than Group1

Question: 13

(a) Prepare a frequency distribution table with the help of tally marks for the words in the expression given below taking number of letters in the words as variable.

"Business Mathematics and Statistics Fundamentals in the Institute of Cost and Works Accounts of India."

Also calculate the mean, median and mode of the distribution.

(b) Find he mean and variance of first 10 natural numbers.

Answer to Question 13:

(a) Frequency Distribution of number of Letters in 15 words

Number of Letters (x)	Tally marks	Frequency (f)	fx	Cumulative frequency
2	|||	3	6	3
3	|||	3	9	6
4	|	1	4	7
5	||	2	10	9
8	|	1	8	10
9	|	1	9	11
10	|	1	10	12
11	||	2	22	14
12	|	1	12	15

Average number of letters per word $\bar{x} = \text{Mean} = \frac{90}{15} = 6$

Median = value correspondents to $\frac{(15+1)}{2} = 8^{th}$ observation = 5

Mode = Value correspondent to maximum frequency = 2 or 3 (max. fr = 3)

(b) $x_i = 1\ (1)\ 10$

$$\text{Mean}\ (\bar{x}) = \sum_{1}^{10} X_i / 10 = \frac{1+2+......+10}{10} = \frac{10 \times 11}{2 \times 10} = 5.5$$

$$\frac{1}{n}\sum x_i^2 = \frac{1}{10}\left[1^2 + 2^2 + ...+10^2\right] = \frac{10 \times 11 \times 21}{10 \times 6} = \frac{11 \times 7}{2} = \frac{77}{2} = 38.5$$

$$\text{Variance}\,(x) = \frac{1}{n}\sum x_i^2 - (\bar{x})^2 = 38.5 - 5.5^2 = 38.5 - 30.25 = 8.25$$

Question: 14

(a) Calculate the medain of the table given below:

Class interval :	0 – 10	10 – 20	20 – 30	30 – 40	40 – 50
Frequency :	5	4	6	3	2

(a) The means of two samples of size 50 and 100 are 54.4 and 50.3 and their standard deviations are 8 and 7 respectively. Obtain the mean and standard deviation of the combined sample of size 150. 5

Answer to Question 14:

(a)

Class interval :	1 – 10	10 – 20	20 – 30	30 – 40	40 – 50
Frequency :	4	5	6	3	2
Cum. Frequency (less than) :	5	9	15	18	20

$$\text{Here, } \frac{N}{2} = 10 = {}^{*}\text{Median Class. So } L = 20, f_m = 6, \; F{=}9, \; i{=}10$$

$$\text{Median} = L + \left(\frac{N}{2} - F\right) \times \frac{i}{f_m} = 20 + (10-9) \times \frac{10}{6} = 20 + \frac{10}{6} = 20 + 1.67$$

$$= 21.67$$

(b) $n_1 = 50$ $\qquad n_2 = 100$

$\bar{x}_1 = 54.4$ $\qquad \bar{x}_2 = 50.3$

$S_1{=}8$ $\qquad S_2 = 7$

$$\text{Combined mean } (\bar{x}) = \frac{n_1\bar{x}_1 + n_2\bar{x}_2}{n_1 + n_2} = \frac{50 \times 54.4 + 100 \times 50.3}{150} = \frac{7750}{150} = 51.67$$

$$\text{Now } d_1 = \bar{x}_1 - \bar{x} = 54.4 - 51.67 = 2.73$$
$$d_2 = \bar{x}_2 - \bar{x} = 50.3 - 51.67 = -1.37$$

$$\text{Combined variance}(S^2) = \frac{n_1 s_1^2 + n_2 s_2^2}{n_1 + n_2} + \frac{n_1 d_1^2 + n_2 d_2^2}{n_1 + n_2}$$

$$= \frac{n_1(s_1^2 + d_1^2) + n_2(s_2^2 + d_2^2)}{n_1 + n_2}$$

$$= \frac{50(8^2 + 2.73^2) + 100(7^2 + (-1.37)^2}{150}$$

$$= \frac{50(64 + 7.4529) + 100(49 + 1.8769)}{150}$$

$$= \frac{3572.645 + 5087.69}{150}$$

$$= 57.7356 \quad \therefore = \sqrt{57.7356} = 7.598 = 7.60$$

$$\text{Hence, Mean}(\bar{x}) = 51.67, \text{and Std Deviation}(S) = 7.60$$

Question: 15

(a) In a distribution mean = 65, median = 70, co-efficient of skewness = - 0.6. Find the mode and co-efficient of variation.

(b) Draw the histogram of the following data and comment on the shape of the distribution:

Wages (in Rs.)	:	50 – 59	60 – 69	70 – 79	80 –89	90 – 99
No. of employees	:	8	10	16	12	7

Answer to Question 15:

(a) We know, mean - mode = 3 (mean - median)

$$\Rightarrow 65 - \text{mode} = 3\ (65\text{-}70) = -15$$

$$\Rightarrow \text{mode} = 65 + 15 = 80$$

$$\text{Again, Co-efficient skewness} = \frac{\text{mean} - \text{mode}}{\text{s.d.}}$$

$$\Rightarrow -0.6 = \frac{65-80}{\text{s.d}} = \frac{-15}{\text{s.d}}$$

$$\Rightarrow \text{s.d} = \frac{-15}{-0.6} = \frac{15\times 10}{6} = 25$$

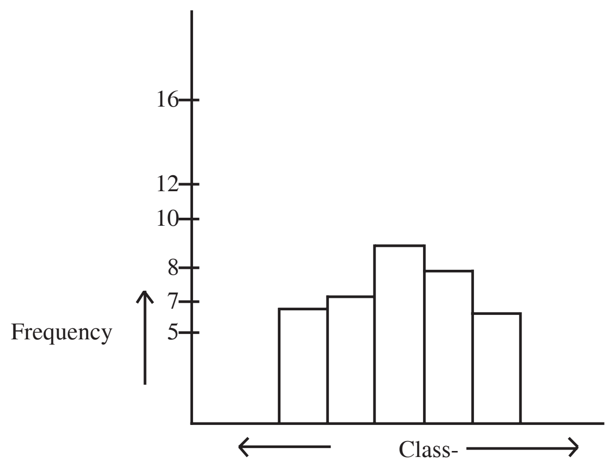
$$\text{Co-efficient of Variation} = \frac{\text{s.d}}{\text{mean}}\times 100 = \frac{25}{65}\times 100 = 38.46\%$$

(b)

Class-boundaries	:	49.5 – 59.5	59.5 – 69.5	69.5 – 79.5	79.5 –89.5	89.5 – 99.5
Frequency	:	8	10	16	12	7

HISTOGARM:

Distribution is almost symmetrical.



December 2006 Examination

Question: 12.

Answer any five folowing:

(a) Find the mean and mode of the 9 observations 9, 2, 5, 3, 5, 7, 5, 1, 8.

(b) If two groups have number of observations 10 and 5 and mean 50 and 20 respectively, find the grouped mean.

(c) Two variables x and y are related by $y = \frac{x-5}{10}$ and each of them has 5 observations. If mean of x is 45, find the mean of y.

(d) If $2x_i + 3y_i = 5$ for i = 1, 2........, n and mean deviation of $x_1, x_2, \ldots, x_n$ about their mean is 12 find the mean deviation of $y_1, y_2, \ldots, y_n$ about their mean.

(e) If the mean and variance of a variable are 8 and 4 respectively, find the co-efficient of variation of the variable in form of percentage.

(f) If the co-efficient of skewness, mean and variance of a variable are -6, 80 and 4, find the mode of that variable.

(g) If the mean of two groups of 30 and 50 observations are equal and their standard deviations are 8 and 4 respectively, find the grouped variance.

(h) For two observations *a* and *b*, show that standard deviations is half the distance between them.

Answer to Question No. 12:

(a) Mean = $\frac{9+2+5+3+5+7+5+1+8}{9} = \frac{45}{9} = 5$

Mode = value of which occurs maximum no. of times = 5

(b) Grouped Mean = $\frac{10\times50+5\times20}{10+5} = \frac{500+100}{15} = 40$

(c) Here, $x = 5 + 10y$

Then $\sum x = 5\times5+10\sum y$ as there are 5 values of each of two variables.

So, dividing by 5,

$$\frac{\sum x}{5} = 5+10\frac{\sum y}{5} \Rightarrow \overline{x} = 5+10\overline{y}$$

$\therefore 45 = 5+10\overline{y} \Rightarrow 40 = 10\overline{y} \Rightarrow \overline{y} = 4$

So, mean of y=4.

(d) As $2x_i + 3y_i = 5$, $\quad y_i = \frac{5}{3} - \frac{2}{3}x_i$

$$\therefore \overline{y} = \frac{5}{3} - \frac{2}{3}\overline{x}$$

Thus $|y_i - \bar{y}| = \left|\left(\frac{5}{3} - \frac{2}{3}x_i\right) - \left(\frac{5}{3} - \frac{2}{3}\bar{x}\right)\right|$

$$= \left|-\frac{2}{3}(x_i - \bar{x})\right|$$

$$= -\frac{2}{3}\left|(x_i - \bar{x})\right|$$

So, $\frac{1}{n}\sum_i |y_i - \bar{y}| = \frac{2}{3n}\sum_i |x_i - \bar{x}|$

i.e., $MD_{\bar{y}}(y) = \frac{2}{3}MD_{\bar{x}}(x)$, $\quad MD_{\bar{x}}(x) =$ mean deviation of x about $\bar{x}$

Thus mean deviation of y about mean $= \frac{2}{3} \times 12 = 8$

(e) Coefficient of variation $= \frac{\text{Standard deviation (S.D.)}}{\text{mean}} \times 100$

$$= \frac{2}{8} \times 100 = 25\%$$

Since, S.D. $= \sqrt{\text{variance}}$ and S.D. cannot be negative.

(f) Coefficient of skewness $= \frac{\text{mean} - \text{mode}}{\text{standard deviation}}$

Now standard deviation $= \sqrt{\text{variance}} = \sqrt{4} = 2$

Thus, $-6 = \frac{80 - \text{mode}}{2}$

$\Rightarrow$ mode $= 80 + 12 = 92$ Thus mode is 92.

(g) Let no. of observations; mean; variance of two groups G_1 and G_2 are n_1, m_1, s_1^2 and n_2, m_2, s_2^2 respectively.

Then grouped variance $= \frac{n_1 s_1^2 + n_2 s_2^2 + n_1(m_1 - m)^2 + n_2(m_2 - m)^2}{n_1 + n_2}$

But, m1 = m2, so, m = grouped mean $= \frac{n_1 m_1 + n_2 m_2}{n_1 + n_2} = m_1 = m_2$

Then, $m_1 - m = m_2 - m = 0$
Given, $n_1 = 30$, $n_2 = 50$, $s_1 = 8$, $s_2 = 4$

Then grouped mean $= \dfrac{n_1 s_1^2 + n_2 s_2^2}{n_1 + n_2} = \dfrac{30\times 64 + 50\times 16}{30+50}$

$$= \frac{1920+800}{80} = \frac{2720}{80} = 34$$

(h) For two observations a and b mean $= \dfrac{a+b}{2}$

Standard deviation (S.D.) $= \sqrt{\dfrac{1}{2}\left[\left(a-\dfrac{a+b}{2}\right)^2 + \left(b-\dfrac{a+b}{2}\right)^2\right]}$

$$= \sqrt{\frac{1}{2}\left[\frac{(a-b)^2}{4} + \frac{(b+a)^2}{4}\right]}$$

$$= \sqrt{\frac{1}{2}\left[\frac{(b-a)^2}{4} + \frac{(b-a)^2}{4}\right]}$$

$$= \sqrt{\frac{1}{2}\times\frac{2(b-a)^2}{4}} = \frac{|b-a|}{2}$$

= Half the distance between a and b

Question: 13.

(a) Draw a blank table to show the number of students sexwise admitted in each of 3 streams Arts, Science and Commerce in the year 2000 and 2001 in a college of Kolkata showing totals in each stream, sex and year.

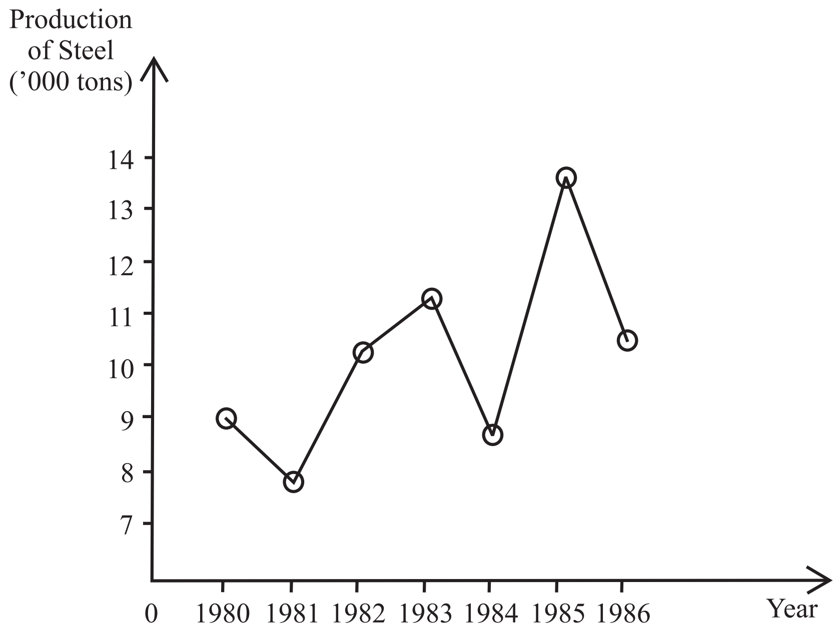
(b) Represent the following data of productions of a steel factory by a line diagram:

Year :	1980	1981	1982	1983	1984	1985	1986
Sale of steel : ('000 tons)	9.1	7.9	10.3	11.3	8.7	13.6	10.5

Answer to Question No. 13:

(a) Title: No. of students admitted sex and streamwise in 2000 and 2001

Streams	2000			2001		
	male	female	total	male	female	total
Arts						
Science						
Commerce						
Total						



Question: 14.

Marks obtained by 30 students in History of a Text Examination, 2004 of some school are as follows:

34,	36,	10,	21,	31,	32,	22,	43,	48,	36,
48,	22,	39,	26,	34,	39,	10,	17,	47,	38,
40,	51,	35,	52,	41,	32,	30,	35,	53,	23.

Construct a frequency table with class intervals 10 - 19, 20 - 29, etc. Calculate the median and mode from the frequency distribution formed. 4+3+3

Answer to Question No. 14:

Frequency distribution of marks of students in History of Test Examination 2004 of some school.

Classess of marks	Tally mark	Frequency = No. of students
10 - 19	|||	3
20 - 29	~~||||~~	5
30 - 39	~~||||~~ ~~||||~~ |||	13
40 - 49	~~||||~~ |	6
50 - 59	|||	3
Total		30

Maximum class frequency is 13 and so model class is 29.5 –39.5 in terms of class boundaries.

Mode = $L + \dfrac{f_0 - f_{-1}}{2f_0 - f_{-1} - f_1} \times c$ where L = lower boundary = 29.5

f_0 = frequency = 13, c = class length = 10 for model class

f_{-1} = frequency of the class just preceding modal class = 5

f_1 = frequency of the class justsucceeding model class = 6

$\therefore 29.5 + \dfrac{13-5}{26-5-6} \times 10 = 29.5 + \dfrac{8}{15} \times 10 = 29.5 + \dfrac{16}{3} = 29.5 + 5.33$ =34.83

Cumulative frequency distribution table:

Classes	9.5 – 19.5	19.5 – 29.5	29.5 – 39.5	39.5 – 49.5	49.5 – 59.5
C.F. (<type)	3	8	21	27	30

N = total frequency = 30, $\dfrac{N}{2}$ =15 So, median lies in class 29.5 – 39.5 . It is median class.

Median = $L + \dfrac{\frac{N}{2} - F}{f} \times c$ where L = lower boundary = 29.5,

f = frequency = 13 and, c = class length = 10 for the median class and N = total freq = 30,
F = cum freq (C.F.) less than type of just preceding class to median class = 8

Thus median = $29.5 + \dfrac{15-8}{13} \times 10 = 29.5 + \dfrac{70}{13} = 29.5 + 5.38 = 34.88$

Question: 15.

(a) Fifty students appeared in an examination. The result of the passed students are given below

Marks :	40	50	60	70	80	90
No. of students :	6	14	7	5	4	4

The average marks of all students is 52. Find the average marks of the students who failed in the examination.

(b) Find the standard deviation from the following frequency distribution:

Weights (in kg.) :	45-50	50-55	55-60	60-65	65-70	
No. of students :	10	16	32	28	14	5

Answer to Question No. 15:

(a)

Marks	f	f_x
40	6	240
50	14	700
60	7	420
70	5	350
80	4	320
90	4	360
	N = 40	$\sum f_x = 2390$

$$\bar{x}_1 = \frac{\sum f_x}{N} = \frac{2390}{40} = 59.75$$

$n_1 = 40 \quad \therefore n_2 = 50 - 40 = 10$

$$\bar{x}_1 = \frac{n_1\bar{x}_1 + n_2\bar{x}_2}{n_1 + n_2} \Rightarrow 52 = \frac{40 \times 50.75 + 10\bar{x}_2}{50} \Rightarrow \bar{x}_2 = 21$$

Average marks of students who failed in the examination is 21.

(b)

Weights (kg)	Mid value (x)	Frequency (f)	Deviation from A=57.5 (d)	fd	fd²
45-50	47.5	10	–10	–100	1000
50-55	52.5	16	–5	–80	400
55-60	57.5 = A	32	0	0	0
60-65	62.5	28	5	140	700
65-70	67.5	14	10	140	1400
		N=100			3500

$$\text{S.D.} = \sqrt{\frac{\sum fd^2}{N} - \left(\frac{\sum fd}{N}\right)^2} = \sqrt{\frac{3500}{100} - \left(\frac{100}{100}\right)^2} = \sqrt{35-1} = \sqrt{34} = 5.83 \text{ kg.}$$

Thus, Standard Deviation is 5.83 kg.

June 2006 Examination

Q12.Answer *any five* of the following:

(a) Find the median of the 10 observations : 9, 4, 6, 2, 3, 4, 4, 6, 8, 7

(b) If the relation between two variables x and y be 2x + 5y = 24 and mode of y be 4, find the mode of x.

(c) Find the harmonic mean of $\frac{1}{2}, \frac{2}{3}, \frac{3}{4} \ldots\ldots\ldots\ldots\ldots \frac{n}{n+1}$, occuring with frequencies 1, 2, 3, ..., n respectively.

(d) For 10 values x_1, x_2, ... x_{10} of a variables x, $\sum_{i=1}^{10} x_i = 110$ and $\sum_{i=1}^{10} (x_i - 5)^2 = 1000$, find variance of x,

(e) If the relation between two varibles x and y be 2x-y+3 = 0 and range of x be 10, then find the range of y.

(f) The mean of 10 observations was found to be 20. Later one observation 24 was wrongly noted as 34. Find the correct mean.

(g) Runs made by two groups G_1 and G_2 of cricketers have means 50 and 40 and variances 49 and 36 respectively. Find which group is more consistent in scoring runs.

(h) Calculate which of the following two distributions is more skewed?

(i) mean = 22, mode = 20, s.d. = 23, (ii) mean = 24, mode = 18, s.d. = 3

Answer to Question 12 :

(a) 10 Values are arranged in non-decreasing order.

2, 3, 4, 4, 4, 6, 6, 7, 8, 9

Median = $\frac{\text{5th value} + \text{6th value}}{2} = \frac{4+6}{2} = 5$

(b) $x = 12 - \frac{5}{2y}$ is the relation.

The values of (x, y) also satisfy the same relation

$\therefore$ mode of x = mode of $\left(12 - \frac{5}{2}y\right) = 12 - \frac{5}{2}$ (mode of y)

Hence mode of x = $12 - \frac{5}{2} x\, 4 = 12 - 10 = 2$

(c) $\text{H.M} = \dfrac{1+2+.........+n}{\frac{1}{1/2}+\frac{2}{2/3}+\frac{3}{3/4}+....\frac{n}{n/n+1}} = \dfrac{n(n+1)/2}{2+3+4+....+(n+1)}$

$$= \dfrac{n(n+1)/2}{\frac{(n+1)(N+2)}{2}-1} = \dfrac{n(n+1)}{2}\times\dfrac{2}{n^2+3n} = \dfrac{n(n+1)}{n(n+3)} = \dfrac{n+1}{n+3}$$

(d) $\sum_{i=1}^{10}(x_i-5) = \sum_{i=1}^{10}x_i - 50 = 110-50 = 60 \Rightarrow \bar{x}-5=6$

Var (x)

$$= \frac{1}{10}.\sum_{i-1}^{10}(xi-\bar{x})^2 = \frac{1}{10}.\sum_{i=1}^{10}\left[(xi-5)-(\bar{x}-5)\right]^2 = \frac{1}{10}.\sum_{i=1}^{10}(xi-5)^2-(\bar{x}-5)^2 = \frac{1}{10}\times 1000-36$$

$= 100 - 36 = 64$

So, required variance of $x = 64$

Aliter :

(d) $\overset{10}{\underset{i=1}{\Sigma}} xi = 110 \therefore x = 11$

$\overset{10}{\underset{i=1}{\Sigma}}(xi-5)^2 = 1000$

$\Rightarrow \sum_i xi^2 - 10\sum_i xi + 10\times 25 = 1000$

$\Rightarrow \underset{i}{\Sigma} xi^2 - 1000 - 250 + 110 = 1850$

Var (x) $= \frac{1}{n}\sum_{i=1}^{n} xi^2 - \bar{x}^2 = \frac{1}{10}\times 1850 - (11)^2 = 185 - 121 = 64]$

(e) The relationship is $y = 2x + 3$

max y = 2 (max X) + 3;

min y = 2 (min. x) + 3.

Subtracting, Range of y = max y – min y = 2 (max x - min.x) =

2 x Range of x = 2 x 10 = 20

(f) Correct Total = 10 x 20 – 34 + 24 = 190

Correct mean = $\frac{190}{10} = 19$

(g) C.V. of group $G_1 = \frac{7}{50} \times 100 = 14\%$

,, ,, ,, $G_2 = \frac{6}{40} \times 100 = 15\%$

C. V. G_1 < C. V. of G_2 . So G_1 cricketers are more consistent.

(h) Coefficient of skewness :

$$\text{for distribution (i)} = \frac{\text{A.M.-Mode}}{\text{S.d}} = \frac{22-20}{2} = 1;$$

$$\text{for distribution (ii)} = \frac{24-18}{2} = 3.$$

for distribution (ii) is more skewed.

Q13. (a) The expenditure during a year in a state is shown as below :

Particulars	Amount (in crores)
Industries	100.00
Irrigation	92.50
Agriculture	127.50
Transport & Roads	92.50
Education	68.00

Draw the pie diagram.

(b) The variance of a series of numbers 2, 3, 11 and x is $12\frac{1}{4}$.

Find the values of x.

Answer to Question - 13:

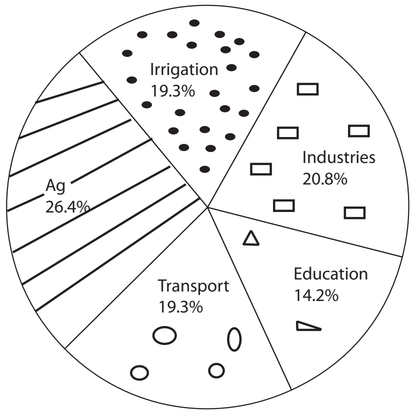
(a) Industries = 100/480.50 × 100 = 20.8 = 74.9°

Irrigation = 19.3% = 69.5°

Agriculture = 26.4% = 95° Where 1% = 3.6°

Transports and roads = 19.3% = 69.5°

Educations = 14.2% = 51.1°



(b) Var (y) = $\dfrac{\sum y_i^2}{n} - \left(\dfrac{\sum y_i}{n}\right)^2 = \dfrac{49}{4}$

$\Rightarrow \dfrac{4+9+121+x^2}{4} - \left(\dfrac{2+3+11+x}{4}\right) = \dfrac{49}{4}$

$\Rightarrow$ 4 (134 + x^2) – (16 + x)^2 = 196

$\Rightarrow$ 536 + 4x^2) – 256 – x^2 – 32x = 196

$\Rightarrow$ 3x^2 – 32x + 84 = 0

$\Rightarrow$ (x – 6) (3x – 14) = 0

$\Rightarrow$ x = 6, $\dfrac{14}{3}$ value of x = 6 and $\dfrac{14}{3}$

[Otherwise : 3x^2 – 32x + 84 = 0

$\Rightarrow x = \dfrac{32 \pm \sqrt{(-32)^2 - 4.3.84}}{2.3} = \dfrac{32 \pm 4\sqrt{64-63}}{6}$

$= \dfrac{36}{6} = 6; \dfrac{28}{6} = \dfrac{14}{3}$

Q 14. (a) Draw a histogram to represent the following distribution :

Wages/ hour (in Rs.)	:	5-10	10-15	15-20	20-25	25-30
No. of workers	:	10	25	30	20	15

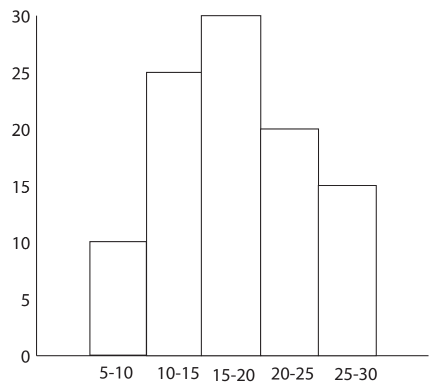
(b) Find the mean deviation about the mean of the following distribution :

x	:	10	11	12	13	14
Frequency	:	3	12	18	12	3

Answer to Question 14 :

(a)

Wages per hour	No. of workers
5-10	10
10-15	25
15-20	30
20-25	20
25-30	15



HISTOGRAM

(b)

x	f	fx	$\lvert x-\bar{x}\rvert$	$f\lvert x-\bar{x}\rvert$
10	3	30	2	6
11	12	132	1	12
12	18	216	0	0
13	12	156	1	12
14	3	42	2	6
Total	N=48	$\sum fx=576$		$\sum\lvert x-\bar{x}\rvert=36$

$$\bar{x}=\frac{576}{48}=12$$

Mean Deviation from mean $=\dfrac{36}{48}=\dfrac{3}{4}$

Q 15. (a) Calculate the median of the following frequency distribution :

Marks	:	1-20	21-40	41-60	61-80	81-100	
No. of students	:	3	5	9	3	2	5

(b) The mean and standard deviation of the marks obtained by the groups of the students consisting of 50 each are given below :

Group	Mean	S.d.
A	60	8
B	55	7

Calculate the mean and standard deviation of the marks obtained by all 100 students.

Answer to Question 15:

(a)

Class Interval	0.5 - 20.5	20.5 - 40.5	40.5 - 60.5	60.5 - 80.5	80.5- 100.5
f	3	5	9	3	2
Cu. fr. (<type)	3	8	17	20	22

Here N = 22; $\frac{N}{2}=11$

$\therefore$ Median class : 40.5 - 60.5

$$\text{Median} = L+\frac{\frac{N}{2}-CF}{f}\times h = 40.5+\frac{11-8}{9}\times 20 = 40.5+\frac{60}{9}=47.17$$

(b) Here $n_1=50, n_2=50, \bar{x}_1=60, \bar{x}_2=55$

$$\text{So } \bar{x}=\frac{n_1\bar{x}_1+n_2\bar{x}_2}{n_1+n_2}=\frac{50\times 60+50\times 55}{100}=\frac{300+2750}{100}=\frac{5750}{100}=57.50$$

Now $d_1=\bar{x}_1-\bar{x}=60-57.5=2.5, d_2=\bar{x}_2-\bar{x}=55-57.5=-2.5$

$S_1^2=64, S_2^2=49$

$$\text{Hence, } S^2=\frac{n_1(S_1^2+d_1^2)+n_2(S_2^2+d_2^2)}{n_1+n_2}$$

$$=\frac{50[64+6.25+49+6.25]}{100}$$

$$=\frac{50\times 125.50}{100}=62.75$$

S $= \sqrt{62.75}$ ₹ 7.92

Hence, Mean $(\bar{x})=57.5$ and std deviation (s) = 7.92

December 2005 Examination

Question 12:

Attempt any *five* of the following :

(a) Prove that for two numbers 2 and 4, A.M. $\times$ H.M. = $(\text{G.M.})^2$

(b) If the relation between two variables x and y is $2x + 3y = 7$ and median of y is 2, find the median of x.

(c) If the observations 2, 4 8 and 16 occur with frequency 4, 3, 2 and 1 respectively, find the geometric mean of them.

(d) If the variables x and y are related by $3x–2y + 5 = 0$ and the range of x is 8, find the range of y.

(e) Determine the mean deviation about mean of 9 observations 4, 4, 4, 6, 6, 6, 8, 8, 8.

(f) If A. M. and the coefficient of variation of a variable x are 10 and 50% respectively, find the variance of x.

(g) If two groups of 50 and 100 observations have means 4 and 2 respectively, find the mean of the combined group.

(h) For a moderately skewed distribution the mean and median are 35 and 37 respectively, find the mode.

Answer to Q. No. 12:

(a) $\text{A.M} = \frac{2+4}{2} = 3, \text{G.M.} = (2 \times 4)^{\frac{1}{2}} = \sqrt{8}, \text{H.M.} = \frac{2}{\frac{1}{2}+\frac{1}{4}} = \frac{2}{\frac{2+1}{4}} = \frac{8}{3}$

Thus $\text{A.M.} \times \text{H.M.} = 3 \times \frac{8}{3} = 8 = (\text{G.M.})^2$

(b) $x = -\frac{3}{2}y + \frac{7}{2}$

When the values of variable y are arranged in non-decreasing (non-increasing) order then the corresponding values of variable x will be arranged in non-increasing (non-decreasing) order so that then middlemost values of variables x and y would satisfy the relation.

$x = -\frac{3}{2}y + \frac{7}{2}$, Thus $\text{median}(x) = -\frac{3}{2}\,\text{median}(y) + \frac{7}{2}$

or, $\text{median}(x) = -\frac{3}{2} \times 2 + \frac{7}{2} = -3 + \frac{7}{2} = \frac{1}{2}\cdot$

(c) $\text{G.M.} = (2^4 \cdot 4^3 \cdot 8^2 \cdot 16^1)^{\frac{1}{10}} = \left[2^4 \cdot (2^2)^3 (2^3)^2 (2^4)^1\right]^{\frac{1}{10}}$

$= (2^4 \cdot 2^6 \cdot 2^6 \cdot 2^4)^{\frac{1}{10}} = \left(2^{4+6+6+4}\right)^{\frac{1}{10}} = \left(2^{20}\right)^{\frac{1}{10}}$

$= 2^{\frac{20}{10}} = 2^2 = 4.$

(d) $y = \frac{3}{2}x + \frac{5}{2}$

Thus, $\max y = \frac{3}{2}\max\ x + \frac{5}{2}$

and $\min y = \frac{3}{2}\min x + \frac{5}{2}$

Subtracting $(\max y - \min y) = \frac{3}{2}(\max x - \min x)$

i.e. $\text{Range}(y) = \frac{3}{2}\text{Range}(x)$

Thus $\text{range}(y) = \frac{3}{2} \times 8 = 12$

(e) $\text{mean} = \frac{4\times3+6\times3+8\times3}{9} = \frac{4+6+8}{3} = \frac{18}{3} = 6$

Value (.x)	frequency (f)	$f\lvert x-\bar{x}\rvert$
4	3	$3\lvert 4-6\rvert = 6$
6	3	$3\lvert 6-6\rvert = 0$
8	3	$3\lvert 8-6\rvert = 6$
Total	**9**	**12**

Mean deviation about mean

$= \frac{1}{\sum f}\sum f\lvert x-\bar{x}\rvert = \frac{12}{9} = \frac{4}{3}$

(f) $\text{Coefficient of variation} = \frac{\text{s.d.}}{\text{mean}} \times 100\%$

Thus $50 = \frac{\text{s.d.}}{10} \times 100$

Variance (x) = (s.d. (x))^2 = 5^2 = 25.

(g) $\text{Mean of Combined group} = \frac{50\times4+100\times2}{50+100} = \frac{200+200}{150} = \frac{400}{150} = \frac{8}{3}$

(h) For a moderately skewed distribution

Mean – Mode = 3 (mean–median)

or, mode = 3 median – 2 mean

= 3 × 37 – 2 × 35

= 111–70=41

Question 13:

(a) Draw a simple Bar Chart for the following productions of bicycles of a small factory in 4 consecutive years:

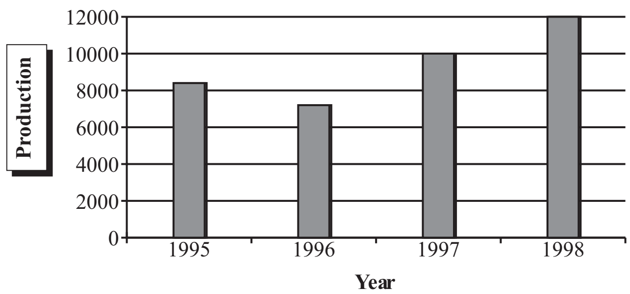
Year	:	1995	1996	1997	1998
Production	:	**8400**	**7200**	**10000**	**12000**

(b) Draw in ogive (less than type) from the following distribution:

Daily wages (Rs.)	:	0-30	30-60	60-90	90-120	120-150
No. of Workers	:	20	50	60	40	30

Answer to Q. No. 13 :

(a)

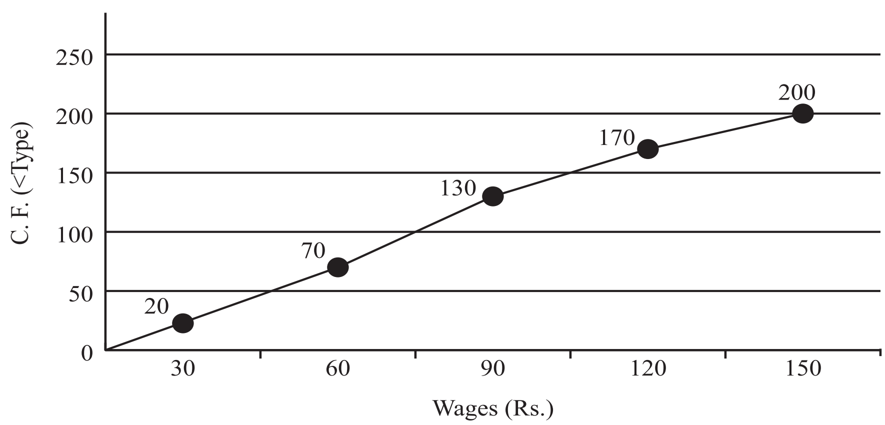


Bar diagram of production of bicycles in 4 years.

(b) Cumulative frequency Distribution :

Daily wages (Rs.)	:	0	30	60	90	120	150
C. F. (<type)	:	0	20	70	130	170	200

Ogive of Daily Wages



Question 14 :

(a) Arithmetic mean of the following frequency distribution is 8.8. Find the missing frequencies:

Wages (Rs.)	:	4 – 6	6 – 8	8 – 10	10 – 12	12–14	Total
No. of Workers	:	6	–	16	–	5	50

(b) Find the quartile deviation of the following distribution :

Marks	:	0 – 10	10 – 20	20 – 30	30 – 40	40 – 50
No. of Student	:	10	20	35	25	10

Answer to Q. No. 14 :

(a)

Wages Class mark (x)	**frequency (f)**	**fx**
5	6	30
7	x	7x
9	16	144
11	y	11y
13	5	65
	27 + x + y	**239 + 7x + 11y**

27 + x + y = 50 or x + y = 23................ (1)

$$8.8 = \frac{239 + 7x + 11y}{50} \text{ or } 440 = 239 + 7x + 11y$$

or 7x + 11y = 201 .... (2)

(2) – 7x (1) gives 4y = 201 – 161 = 40 or y = 10

Then x = 23 – y = 23 – 10 = 13

Thus, number, of workers of the classes 6 – 8 and 10 – 12 are 13 and 10 respectively.

(b)

<u>Marks</u>	<u>C. F. (<type)</u>
0	0
10	10
$Q_1 \rightarrow$	$\leftarrow \frac{N}{4} = 25$
20	30
30	65
$Q_3 \rightarrow$	$\leftarrow \frac{3N}{4} = 75$
40	90
50	100 = N

Q_1 lies in the class 10 – 20
Q_3 lies in the class 30 – 40

$$Q_1 = 10 + \frac{25-10}{30-10} \times 10 = 10 + \frac{15}{20} \times 10 = 10 + 7.5 = 17.5$$

$$Q_3 = 30 + \frac{75-65}{90-65} \times 10 = 30 + \frac{10}{25} \times 10 = 30 + 4 = 34$$

$$\text{Quartile deviation} = \frac{Q_3 - Q_1}{2} = \frac{34-17.5}{2} = \frac{16.5}{2} = 8.25 \cdot$$

Question 15:

For the following frequency distribution, determine mean, mode, standard deviation and coefficient of skewness:

Marks	:	0 – 10	10 – 20	20 – 30	30 – 40	
No. of Students	:	10	30	40	20	2 + 3+ 3+ 2

Answer to Q. No. 15:

Class mark (x)	frequency (f)	fx	fx^2
5	10	50	250
15	30	450	6750
25	40	1000	25000
35	20	700	24500
Total	**100**	**2200**	**56500**

$$\text{Mean} = \frac{\sum fx}{\sum f} = \frac{2200}{100} = 22$$

$$\text{s.d.} = \sqrt{\frac{\sum fx^2}{\sum f} - \left(\frac{\sum fx}{\sum f}\right)^2} = \sqrt{\frac{56500}{100} - \left(\frac{2200}{100}\right)^2} = \sqrt{565 - 484}$$

$$= \sqrt{81} = 9$$

Modal class is 20 – 30 since its frequency 40 is maximum

$$\text{Mode} = 20 + \frac{40-30}{2 \times 40 - 30 - 20} \times 10 = 20 + \frac{10}{80-50} \times 10$$

$$= 20 + \frac{100}{30} = 20 + \frac{10}{3} = \frac{70}{3}$$

$$\text{Coefficient of skewness} = \frac{\text{mean} - \text{mode}}{\text{s.d.}} = \frac{22 - \frac{70}{3}}{9}$$

$$= \frac{66-70}{27} = -\frac{4}{27}$$

June 2005 Examination

Question 12:

Answer any five of the following:

(a) If a variable x takes 10 values 1, 2, 3...10 with frequencies as its values in each case, then find the arithmetic mean of x.

(b) If 2u = 5x is the relation between two variables x and u and harmonic mean of x is 0.4. find the harmonic mean of u.

(c) For a group of 10 items $\sum_{i=1}^{10}(X_i - 2) = 40$ and $\sum_{i=1}^{10} x_i^2 = 495$. Then find the variance of this group.

(d) If first of two groups has 100 items and mean 45 and combined group has 250 items and mean 51, find the mean of the second group.

(e) Find the median of the following distribution:

Weight (in kg.):	65	66	67	68
No. of students:	5	15	17	4

(f) Calculate the mode of the following distribution:

7, 4, 3, 5, 6, 3, 3, 2, 4, 3, 3, 4, 4, 2, 3.

(g) if the relation between two variables *x* and *u* is *x – 10 = 2u* and mean deviation of x about its mean is 10, find the mean deviation of u about its mean.

(h) Which group is more skewed?

(i) Group I: A.M. = 22, mode = 27, s. d. = 10,

(ii) Group II: A.M. = 20, mode = 26, s.d. = 9?

Answer to Q. No. 12:

(a) $$\text{A.M} = \frac{1\times1+2\times2+3\times3+........+9\times9+10\times10}{1+2+3....+9+10} = \frac{1^2+2^2+3^2+.....9^2+10^2}{1+2+3+.....+9+10}$$

$$\frac{10\times(10+1)(2\times10+1)}{6\times10\times(10+1)/2} = \frac{10\times11\times21}{6\times5\times11} = 7$$

(b) $$\text{HM}(u) = \frac{N}{\sum f/u}, \text{ where total frequency} = N,\ u = \frac{5}{2}\times(\text{given})$$

$$\frac{N}{\sum \frac{f}{\frac{5}{2}x}} = \frac{5}{2}.\frac{N}{\sum \frac{f}{x}} = \frac{5}{2}.\text{HM}(x) = \frac{5}{2}\times0.4 = 1.$$

(c) $\sum_{i=1}^{10}(X_i - 2) = 40$ or, $\sum_{i=1}^{10} X_i - 2\times 10 = 40$ or, $\sum_{i=1}^{10} X_i = 40 + 20 = 60$

$$\text{Variance} = (\sigma^2) = \frac{\sum x_i^2}{n} - \left(\frac{\sum x_i}{n}\right)^2 = \frac{\sum x_i^2}{10} - \left(\frac{\sum x_i}{n}\right)^2 = \frac{195}{10} - \left(\frac{60}{10}\right)^2 = 49.5 - 36$$

$= 13.5.$

(d) $\bar{x} = \dfrac{n_1\bar{x}_1 + n_2\bar{x}_2}{n_1 + n_2} \Rightarrow 51 = \dfrac{100\times 45 + 150\bar{x}_2}{250}$

$\left[n_1 = 100, \bar{x}_1 = 45, n_2 = n - n_1 = 250 - 100 = 150, \bar{x} = 51\right]$

or, $4500 + 150\bar{x}_2 = 51\times 250 = 12750$

or, $150\bar{x}_2 = 12750 - 4500 = 8250 \Rightarrow x_2 = \dfrac{8250}{150} = 55;$

Hence, required mean = 55.

(e)

Weight (kg) x	f	cf
65	5	5
66	15	20
67	17	37
68	4	41(=N)

Median = value of (N +1)/2th item = value of (41 + 1)/2th item = value of 21st item = 67 kg

From the 3rd column it is found that 21 is greater than the c.f. of 20, but less than next c.f. 37 corresponding to x = 67. All the 17 items (from 21 to 37) have the same variate 67. And 21st item is one of those 17 items.

Hence, median = 67 kg.

(f)

Numbers (x)	2	3	4	5	6	7
Frequency (f)	2	7	5	1	1	1

The frequency corresponding to the variate x = 3 is maximum.

Therefore, mode = 3

(g) $x - 10 = 2u \Rightarrow \bar{x} - 10 = 2\bar{u}$. Now, $u = \dfrac{x-10}{2}, u = \dfrac{\bar{x}-10}{2}$

Mean deviation of u about mean $= \dfrac{1}{N}\sum f\left|u - \bar{u}\right|$, where f = frequency, N = total frequency.

Now, $\frac{1}{N}\Sigma f|u-\bar{u}| = \frac{1}{N}\Sigma f\left|\frac{x-10}{2}-\frac{\bar{x}-10}{2}\right| = \frac{1}{2}\times\frac{1}{N}\Sigma f|x-\bar{x}|$

$= \frac{1}{2}\times$ mean deviation of $\times$ about mean $= \frac{1}{2}\times 10 = 5$

(h) For Group I, Coefficient of skewness $= \frac{\text{mean} - \text{mode}}{\text{s.d.}} = \frac{22-27}{10} = \frac{-5}{10} = \frac{-1}{2}$

For Group II, Coefficient of skewness $= \frac{\text{mean} - \text{mode}}{\text{s.d.}} = \frac{20-26}{9} = \frac{-6}{9} = \frac{-2}{3}$

As, $\frac{-1}{2} > \frac{-2}{3}$ so group II is more negatively skewed.

Question: 13.

(a) Draw a line diagram from the following data number of students in a class of a college:

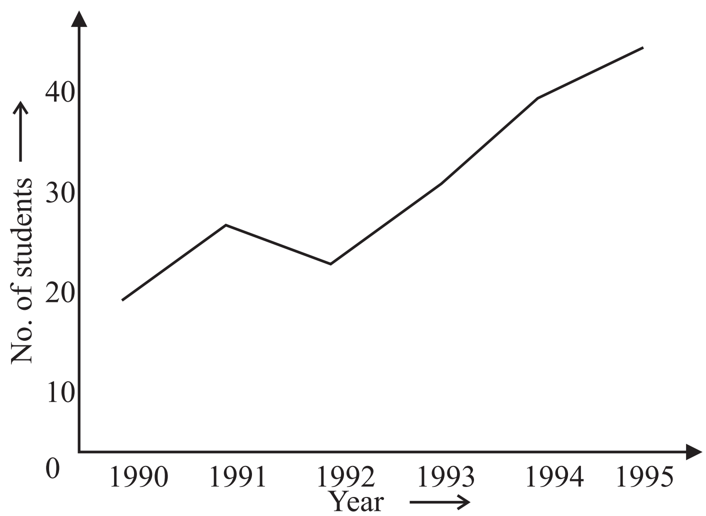
Year :	1990	1991	1992	1993	1994	1995
No. of students :	20	25	24	30	35	38

(b) Draw a histogram from the following data of a factory:

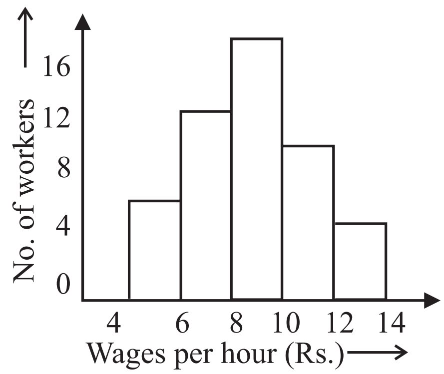
Wages per hour (Rs.) :	4–6	6–8	8–10	10–12	12–14
No. of workers :	6	12	18	10	4

Answer to Q. No. 13:

(a)



(b)



Question: 14.

(a) From the following cumulative frequency distribution of marks of 22 students in Accountancy, calculate mode:

Marks :	Below 20	Below 40	Below 60	Below 80	Below 100
No. of students :	3	8	17	20	22

(b) Prove that the standard deviation of two values of a variable is equal to half of the range.

Answer to Q. No. 14:

(a)

Class intervals (marks)	c.f.	f
0–20	3	3
20–40	8	5(=8–3)
40–60	17	9(=17–8)
60–80	20	3(=20–17)
80–100	22	2(=22–20)

Modal class 40 – 60, as it has highest frequency

$$\text{Modal } l_1 = \frac{f_1 - f_0}{2f_1 - f_0 - f_2} \times i$$

Here, $l_1 = 40$ $f_0 = 5$, $f_1 = 9$, $f_2 = 3$, $i = 20$

$$\therefore \text{Mode} = 40 + \frac{9-5}{29-5-3} \times 20 = 40 + \frac{4}{18-8} \times 20 = 40 + \frac{4}{10} \times 20$$

$$= 40 + 4 \times 4 \times 2 = 40 + 8 = 48 \text{ marks}$$

(b) Let x_1 and x_2 be the two values of a variable x.

$$\text{Now, } \sigma^2 = \frac{(x_1-\bar{x})^2+(x_2-\bar{x})^2}{2}, \text{where } x = \frac{x_1+x_2}{2}$$

$$= \frac{1}{2}\left[\left(x_1-\frac{x_1+x_2}{2}\right)^2+\left(x_2-\frac{x_1+x_2}{2}\right)^2\right] = \frac{1}{2}\left[\left(\frac{x_1-x_2}{2}\right)^2+\left(\frac{x_1-x_2}{2}\right)^2\right]$$

$$= \frac{1}{2}\left[\frac{1}{4}\left\{(x_1-x_2)^2+(x_1-x_2)^2\right\}\right] = \frac{1}{4}(x_1-x_2)^2$$

$$\therefore \sigma = \frac{1}{2}(x_1-x_2) = \frac{1}{2}\times(\text{range})\ [\textit{since } \sigma \textit{ is always positive}]$$

Question 15.

(a) Find the coefficient of variation for the following data:

Marks (x)	:	10	20	30	40	50	60
No. of students	:	8	12	20	10	7	3

(b) For a group containing 100 observations the arithmetic mean and standard deviation are 8 and $\sqrt{10.5}$ respectively. For 50 observations selected from these 100 observation the arithmetic mean and standard deviation are 10 and 2 respectively. Find the arithmetic mean and standard deviation of other 50 observations.

Answer to Q. No. 15:

(a)

Marks (s)	f	d = x–30	d´ = d/10	fd´	fd´²
10	8	–20	–2	–16	32
20	12	–10	–1	–12	12
30	20	0	0	0	0
40	10	10	1	10	10
50	7	20	2	14	28
60	3	30	3	9	27
Total	60	–	–	5	109

$$\text{Mean}(\bar{x}) = A + \frac{\sum fd'}{\sum f}\times i = 30+\frac{5}{60}\times 10 = 30+\frac{5}{6} = 30.8333$$

$$\sigma = \sqrt{\frac{\Sigma fd'}{\Sigma f} - \left(\frac{\Sigma fd'}{\Sigma f}\right)^2} \times i = \sqrt{\frac{109}{60} - \left(\frac{5}{60}\right)^2} \times 10 = \sqrt{1.8167 - (0.08333)^2} \times 10$$

$$\sqrt{1.8167 - 0.00694} \times 10 = \sqrt{1.8098} \times 10 = 1.3453 \times 10 = 13.453$$

$$\therefore \text{C.V.} = \frac{\sigma}{\text{AM}} \times 100 = \frac{13.453}{30.8333} \times 100 = 43.63\%$$

(b)

$n=100,\ \bar{x}=8,\ \ \sigma=\sqrt{105},$
$n_1=50,\ \bar{x}_1=10,\ \sigma_1=2,$
$n_2=50,\ \bar{x}_2=?,\ \ \sigma_2=?$

We have : $\bar{x}=\dfrac{n_1\bar{x}_1 + n_2\bar{x}_2}{n_1+n_2}$ or $8 = \dfrac{50\times 10 + 50 \times \bar{x}_2}{100}$ or $50\bar{x}_2 = 800 - 50$

or, $50\bar{x}_2 = 300$ or $\bar{x}_2 = 6$

$d_1 = \bar{x}_1 - \bar{x} = 10 - 8 = 2,\ d_2 = \bar{x}_2 - \bar{x} = 6 - 8 = -2,\ d_1^2 = 4,\ d_2^2 = 4$

Again,

$$\sigma^2 = \frac{n_1\sigma_1^{\,2} + n_2\sigma_2^{\,2} + n_1\sigma_1^{\,2} + n_2\sigma_2^{\,2}}{n_1\sigma_2}; \qquad 10.5 = \frac{50\times 4 + 50 \times \sigma_2^2 + 50\times 4 + 50 \times 4}{100}$$

or, $1050 = 200 + 50\sigma_2^{\,2} + 200 + 200$ or $50\sigma_2^{\,2} = 1050 - 600 = 450$

or $\sigma_2^{\,2} = 9$ or $\sigma_2 = 3$

Hence, required mean = 6, s.d. = 3.

December 2004 Examination

Question :

12. Answer *any five* of the following:
   (a) Find the geometric mean of 3, 6, 24, 48.
   (b) For a group of 10 items $\sum x = 65, \sum x^2 = 495,$ mode = 6. Comment of skewness.
   (c) For a sample mean = 112, variance = 1600; find its CV.
   (d) AM of two numbers 25 and their HM is 9; find their GM.

(e) The means of samples of sizes 50 and 75 are 60 and x respectively. If the mean of the combined group is 54, find x.

(f) Find the median of the following frequency distribution:

value (x) :	1	2	3	4
frequency (f):	7	12	18	4

(g) If each of 3, 48 and 96 occurs once and 6 occurs thrice, verify that the geometric mean is greater than harmonic mean.

Answer to Question No. 12 (a) :

$$G.M. = \sqrt[4]{3.6.24.48} = \sqrt[4]{3.(3.2).(3.2^3).(3.2^4)} = \sqrt[4]{3^4.2^8} = 3.2^2 = 3.4 = 12$$

Answer to Question No. 12 (b) :

$$\text{Skewness} = \frac{\text{mean} - \text{mode}}{\text{s.d.}} = \frac{\frac{\sum x}{n} - \text{mode}}{\text{s.d.}} = \frac{\frac{65}{10} - 6}{\text{s.d.}} = \frac{6.5 - 6}{\text{s.d.}} > 0$$ as s.d. is always positive which indicates positively skewed.

Answer to Question No. 12 (c) :

$$\text{s.d.} = \sqrt{1600} = 40, \text{C.V.} = \frac{\text{s.d.}}{\text{mean}} \times 100 = \frac{40}{112} \times 100 = 35.714\%$$

Answer to Question No. 12 (d):

$$G.M. = \sqrt{A.M. \times H.M.} = \sqrt{25 \times 9} = \sqrt{225} = 15$$

Answer to Question No. 12 (e):

$$\text{We have } \overline{x_{12}} = \frac{n_1\overline{x_1} + n_2\overline{x_2}}{n_1 + n_2} \text{ or, } 54 = \frac{50 \times 60 + 75x}{50 + 75} \text{ or, } 54 = \frac{3000 + 75x}{125}$$

or, $3000 + 75x = 6750$ or, $75x = 3750$ or, $x = 50$.

Answer to Question No. 12 (f):

x	f	c.f. (less than type)
1	7	7
2	12	19
3	18	37
4	4	41 (N)

Median $= \text{value of } \frac{N+1}{2} \text{th item}$

$= \text{value of } \frac{41+1}{2} \text{th item}$

$=$ value of 21st item

$= 3$

Answer to Question No. 12 (g):

$$G.M. = \sqrt[6]{3.48.96.6^3} = \sqrt[6]{3^6 \times 2^{12}} = 3 \times 2^2 = 3 \times 4 = 12$$

$$H.M. = \frac{6}{\frac{1}{3}+\frac{1}{48}+\frac{1}{96}+\frac{3}{6}} = \frac{6}{\frac{32+2+1+48}{96}} = \frac{6}{\frac{83}{96}} = 6 \times \frac{96}{83} = 6.94 \text{ (app)}$$

So we find that G.M. > H.M.

Question :

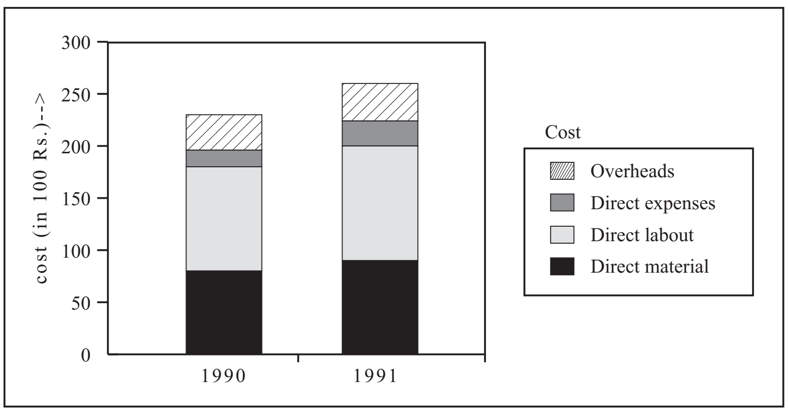
13. (a) The following table show the total cost (in 100 rupees) and its component parts in two calendar years. Draw component bar charts showing total costs and their components.

	1990	1991
Direct material	80	90
Direct labour	100	110
Direct expenses	16	24
Overheads	34	36
Total	230	260

(b) Represent the following data by Pie diagram:

Items	Wages	Materials	Taxes	Profit	Administration
Expenses (in Rs.)	125	110	180	65	20

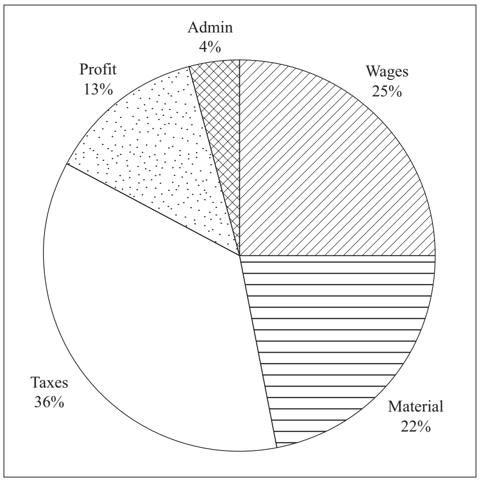
Answer to Question No. 13 (a) :



Answer to Question No. 13 (b):

Items	Expenses (in Rs.)	%	In degrees
Wages	125	$(125/500) \times 100 = 25$	$25 \times 3.6 = 90$
Materials	110	22	79.2
Taxes	180	36	129.6
Profit	65	13	46.8
Admin.	20	4	14.4
Total	500	100	360

Since 1% at the centre corresponds to 3.6°, so the angles at the centre are expressed in degrees.



Question :

14. (a) Find the mean deviation about mean of the following distribution:

Weight (in Kg)	:	45 -50	50-55	55-60	60-65	65-70
No. of persons	:	10	20	40	20	10

(b) Find the mean and standard deviation of the following distribution:

Score	:	0-10	10-20	20-30	30-40	40-50
Frequency	:	5	28	40	22	5

Answer to Question No. 14 (a) :

Weight (Kg)	f	mid-point x	d $= x - 57.5$	$d' = \frac{d}{5}$	fd'	$\|d_1\|$	$f\|d_1\|$
45-50	10	47.5	–10	–2	–20	10	100
50-55	20	52.5	–5	–1	–20	5	100
55-60	40	57.5	0	0	0	0	0
60-65	20	62.5	5	1	20	5	100
65-70	10	67.5	10	2	20	10	100
Total	100	—	—	—	0		400

$$\bar{x}(\text{mean}) = A + \frac{\sum fd'}{\sum f} \times i = 57.5 + \frac{0}{100} \times 5 = 57.5\,\text{Kg}.$$

$$\text{M.D.(about mean)} = \frac{\sum f\,|d_1|}{\sum f} = \frac{400}{100} = 4\ \text{Kg, where} \left|d_1\right| = \left|x - \bar{x}\right|$$

Answer to Question No. 14 (b) :

Score	f	mid-point x	d= x –25.5	$d' = \frac{d}{10}$	fd'	fd'^2
0-10	5	5	–20	–2	–10	20
10-20	28	15	–10	–1	–28	28
20-30	40	25	0	0	0	0
30-40	22	35	10	1	20	22
40-50	5	45	20	2	10	20
Total	100	—	—	—	(–6)	90

$$(\text{mean})\bar{x} = A + \frac{\sum fd'}{\sum f} \times i = 25 + \frac{(-6)}{100} \times 10 = 25 - 0.6 = 24.4$$

$$\text{sd}(\sigma) = \sqrt{\frac{\sum fd'^2}{\sum f} - \left(\frac{\sum fd'}{\sum f}\right)^2} \times i = \sqrt{\frac{90}{100} - \left(\frac{-6}{100}\right)^2} \times 10 = \sqrt{0.9 - 0.0036} \times 10$$

$$= 0.947 \times 10 = 9.47$$

Question :

15. (a) An analysis of the monthly wages paid to the workers in two firms A and B belonging to the same industry give the following result:

		Firm A	Firm B
No. of wage earners	:	300	700
Average monthly wages	:	Rs. 500	Rs. 400
Standard deviation of wages	:	Rs. 100	Rs. 120

(i) If in firm A average monthly wage of 100 workers is Rs. 450, what is the average monthly wage of other workers?

(ii) Which firm A or B has greater variability in individual wages?

(b) In question 15 (a), find the average monthly wage and standard deviation of the wages of all the workers in the firms A and B taken together.

Answer to Question No. 15 (a) :

(i) Let average monthly wages of remaining 200 workers be x (in Rs.)

Using $\overline{x_{12}} = \dfrac{n_1\overline{x_1} + n_2\overline{x_2}}{n_1 + n_2}$ formula we get, $500 = \dfrac{100 \times 450 + 200x}{300}$

or, $200x = 500 \times 300 - 100 \times 450$ or, $x = 525$ (in Rs.) on reduction.

(ii) Coefficient of variation for $A = \dfrac{\text{s.d.}}{\text{mean}} \times 100 = \dfrac{100}{500} \times 100 = 20\%$

Coefficient of variation for $B = \dfrac{120}{400} \times 100 = 30\%$

Since coeff. of variation of B is greater than that of A, so firm B is more variable than firm A.

Answer to Question No. 15 (b) :

Here $n_1 = 300,\ \overline{x_1} = 500,\ \sigma_1 = 100, n_2 = 700,\ \overline{x_2} = 400,\ \sigma_2 = 120$

$$\text{Now } \overline{x}_{12} = \frac{n_1\overline{x_1} + n_2\overline{x_2}}{n_1 + n_2} = \frac{300 \times 500 + 700 \times 400}{1000} = \frac{150000 + 280000}{1000} = \frac{430000}{1000} = 430 \text{ (in Rs.)}$$

Again $d_1 = \overline{x_1} - \overline{x_{12}} = 500 - 430 = 70,\ d_1^2 = 4900$

$d_2 = \overline{x_2} - \overline{x_{12}} = 400 - 430 = -30,\ d_2^2 = 900$

$$\sigma_{12} = \sqrt{\frac{n_1\sigma_1^2 + n_2\sigma_2^2 + n_1 d_1^2 + n_2 d_2^2}{n_1 + n_2}}$$

$$= \sqrt{\frac{300 \times 100^2 + 700 \times 120^2 + 300 \times 4900 + 700 \times 900}{1000}}$$

$$= \sqrt{\frac{3000000 + 10080000 + 1470000 + 630000}{1000}}$$

$$= \sqrt{\frac{15180000}{1000}} = \sqrt{15180} = 123.21(\text{Rs.})$$