

Assignment Problem

The assignment problem like transportation is another special case of LPP. In general, it is concerned with one to one basis when n jobs are to be assigned to n facilities with a view to optimizing the resource required. The emphasis in an assignment problem is on determining how assignment should be made in order that the total cost involved is minimized or alternatively, total value is maximized when pay-offs are given in terms of say profits.

Steps for solving assignment problem (Using Hungarian Assignment Method-HAM):

- 1) If problem is not balanced i.e. $m \neq n$, introduce dummy row/column to balance it.
Where m = no. of rows & n = no. of columns.
- 2) In case of problems of maximization, Convert profit matrix into loss matrix. This is done by subtracting each entry in the table from largest no. in the table.
[Important Note: Though we can also use step 2 before step 1 (i.e. we can also convert profit matrix in loss matrix before balancing the problem), but, it is recommended to apply the steps in above defined order only]
- 3) Assign M or ∞ to prohibited assignments, etc.
- 4) Subtract the minimum element of each row from all the elements in that row.
[Note: Step 4 is to calculate Men/Machine/Facility/etc. opportunity costs with regards to the respective jobs/tasks/etc. (i.e. penalty for assigning to a facility, machine, etc. in spite of least cost assignment.)]
- 5) Subtract the minimum element of each column from all the elements in that column. The resulting matrix is starting matrix.
[Note: Step 5 is to calculate total opportunity cost table.]
- 6) Draw the min. no. of lines (both horizontal & vertical) to cover all zeros.
- 7) Now check that whether n i.e. order of matrix & no. of lines drawn in Step 6 are same or not. If yes, go to step 10, otherwise go to step 8 (for improving the solution).
[Note: Order of matrix means no. of rows/column in a matrix]
- 8) Find the smallest uncovered cost element (by the lines drawn in step 7). Subtract this element from all such (uncovered) elements & add it to the elements with two lines through it (i.e. intersection of horizontal & vertical lines). Do not alter the elements through which one line passes.
[Note: Step 8 gives us revised opportunity cost table]
- 9) Repeat Steps 6-8 until we get minimum no. of lines equal to n .
- 10) Find a row with only one zero in it, & make an assignment (by encircling it) to this zero. Draw a vertical line through the column containing this zero. This eliminates any confusion of making any further assignment in that column. Repeat this step until no row with single zero is left.
- 11) Find a column with only one zero in it, & make an assignment to this zero. Draw a horizontal line through the row containing this zero. This eliminates any confusion of making any further assignment in that row. Repeat this step until no column with single zero is left.
- 12) If at this stage, there are still zeroes which are not assigned or crossed, then go to step 13 otherwise the solution is optimal.
- 13) If any row/column consists more than one zero, make an assignment to any zero arbitrarily & draw 2 lines i.e. one horizontal & one vertical line through the row & column containing this zero.
- 14) Repeat steps 10) to 13) until no further movement is possible. The resulting solution is optimal.
- 15) Determine the total cost with reference to the original cost table.

Question 1 (Minimization balanced): A Marketing Manager has 4 subordinates and 4 tasks. The subordinates differ in efficiency. The tasks also differ in their intrinsic difficulty. His estimates of the time each subordinate would take to perform each task is given in the matrix below. How should the task be allocated one to one man so that the total man-hours are minimized?
(7 Marks) Nov./ 06

	I	II	III	IV
1	16	52	34	22
2	26	56	8	52
3	76	38	36	30
4	38	52	48	20

[Ans.: Minimum time taken = 82 hours]

Question 2: Find the optimal solution for the assignment problem with the following cost matrix:

Marketing Executive	Division			
	N	E	W	S
A	14	20	11	19
B	12	10	15	9
C	16	19	18	15
D	17	13	15	14

[Ans.: Total minimum cost Rs. 49]

(10 Marks) Nov./03

Question 3 (Maximization-balanced): A manager has 4 subordinates and 4 tasks. The subordinates differ in efficiency. His estimate of the production each would do is given in the table. How the task should be allocated one to one man, so that total production is maximized.

Task	Subordinates			
	I	II	III	IV
1	8	26	17	11
2	13	28	4	26
3	38	19	18	15
4	19	26	24	10

[Ans.: Optimal Production is 114 units]

Question 4: An organization producing 4 different products viz. A, B, C, and D having 4 operators viz. P, Q, R and S, who are capable of producing any of the four products, works effectively 7 hours a day. The time (in minutes) required for each operator for producing each of the product are given in the cells of the following matrix along with profit (Rs. Per unit):

Operators	Products			
	A	B	C	D
P	6	10	14	12
Q	7	5	3	4
R	6	7	10	10
S	20	10	15	15
Profit (Rs./unit)	3	2	4	1

Find the optimal assignment of operators to products which will maximize the profit.

[Ans.:

Operator	Product	Profit
P	A	210
Q	C	560
R	B	120
S	D	28]

(10 Marks), May/96 & (10 Marks), Nov./97 - Adapted

Question 5 (Minimization-unbalanced): A has one surplus truck in each cities A, B, C, D & E and one deficit truck in each of the cities 1, 2, 3, 4, 5 & 6. The distance between the cities in kilometers is shown in the matrix below.

Cities	1	2	3	4	5	6
A	12	10	15	22	18	8
B	10	18	25	15	16	12
C	11	10	3	8	5	9
D	6	14	10	13	13	12
E	8	12	11	7	3	10

Find the assignment of trucks from the cities in surplus to cities in deficit so that the total distance covered by vehicles in minimum.

[Ans.: Optimum Distance is 34 kms]

Question 6 (Multiple optimal solutions): Solve the minimal assignment problem whose effectiveness matrix

is

Jobs	Men			
	1	2	3	4
I	2	3	4	5
II	4	5	6	7
III	7	8	9	8
IV	3	5	8	4

[Ans.: Optimal cost is Rs. 20/-]

Question 7 (Multiple Optimal Solutions): The cost matrix giving selling costs per unit of a product by salesman A, B, C and D in Regions R₁, R₂, R₃ and R₄ is given below:

- Assign one salesman to one region to minimize selling cost.
- If the selling price of the product is Rs. 200 per unit and variable cost excluding the selling cost given in the table is Rs. 100 per unit, find the assignment that would maximize the contribution.
- What other conclusion can you make from the above?

	A	B	C	D
R ₁	4	12	16	8
R ₂	20	28	32	24
R ₃	36	44	48	40
R ₄	52	60	64	56

(8 Marks) Nov./08-New Course

[Ans.: R₁->A, R₂->B, R₃ ->C and R₄->D with minimum selling cost as Rs. 136; (ii) R₁->A, R₂->B, R₃ ->C and R₄->D with maximum contribution as Rs. 264; (iii) Multiple Optimal Solutions are available for both (i) & (ii)]

Question 8 (Prohibited routes): A BPO Co. is taking bids for 4 routes in the city to ply pick-up and drop cabs. Four companies have made bids as detailed below: Bids for Routes (Rs.) :

Company/Routes	R ₁	R ₂	R ₃	R ₄
C1	4,000	5,000	---	---
C2	---	4,000	---	4,000
C3	3,000	---	2,000	---
C4	---	---	4,000	5,000

Each bidder can be assigned only one route. Determine the minimum cost.

[Ans: Rs. 15000/-]

(10 Marks) Nov./95 & (6 Marks) Nov./ 06

Question 9 (Prohibited routes): WELLDONE Company has taken the third floor of a multi-storied building for rent with a view to locate one of their zonal offices. There are five main rooms in this floor to be assigned to five managers. Each room has its own advantages and disadvantages. Some have windows; some are closer to the washrooms or to the canteen or secretarial pool. The rooms are of all different sizes and shapes. Each of the five managers was asked to rank their room preferences amongst the rooms 301, 302, 303, 304 and 305. Their preferences were recorded in a table as indicated below:

MANAGER				
M ₁	M ₂	M ₃	M ₄	M ₅
302	302	303	302	301
303	304	301	305	302
304	305	304	304	304
*	301	305	303	*
*	*	302	*	*

Most of the managers did not list all the five rooms since they were not satisfied with some of these rooms and they have left off these from the list. Assuming that their preferences can be quantified by numbers, find out as to which manager should be assigned to which room so that their total preference ranking is a minimum.

(Nov./1990)

[Ans.: M₁->302, M₂->304, M₃ ->303, M₄->305 and M₅->301 with total minimum ranking as 7]

Question 10 (Maximization-Prohibited route): Imagine yourself to be the Executive Director of a 5-star Hotel which has four banquet halls that can be used for all functions including weddings. The halls were all about the same size and the facilities in each hall differed. During a heavy marriage season, 4 parties approached you to reserve a hall for the marriage to be celebrated on the same day. These marriage parties were told that the first choice among these 4 halls would cost Rs. 10000 for the day. They were also

required to indicate the second, third and fourth preferences and the price that they would be willing to pay. Marriage party A & D indicated that they won't be interested in Halls 3 & 4. Other particulars are given in the following table:

Marriage Party	Revenue/Hall			
	Hall 1	Hall 2	Hall 3	Hall 4
A	10000	9000	X	X
B	8000	10000	8000	5000
C	7000	10000	6000	8000
D	10000	8000	X	X

Where X indicates that the party does not want that hall.

Decide on an allocation that will maximize the revenue to your hotel.

(10 Marks) May/95

[Ans.: Marriage Party

Revenue (Rs.)

A	Hall 2	9000
B	Hall 3	8000
C	Hall 4	8000
D	Hall 1	10000]

Question 11 (Airline scheduling): An airline operates seven days a week has time-table as shown below. Crews must have a minimum layover of 5 hours between flights. Obtain the pairing of flights that minimizes layover time away from home. For any given pairing the crew will be based at the city that results in smaller layover. For each pair also mention the town where the crew should be based.

Flight no.	Delhi	Jaipur	Flight no.	Jaipur	Delhi
	Departure	Arrival		Departure	Arrival
1	7.00	8.00	101	8.00	9.15
2	8.00	9.00	102	8.30	9.45
3	13.30	14.30	103	12.00	13.15
4	18.30	19.30	104	17.30	18.45

[Ans.: Minimum Layover time is 52.5 hours]

(10 Marks) May/00-Adapted

Question 12 (Data entry job allocation): A firm employs typists for piecemeal work on an hourly basis. There are five typists available and their charges and speeds are different. According to an earlier understanding, only one job is given to one typist and the typist is paid for full hours even if he works for a fraction of an hour. Find the least cost allocation for the following data:

Typist	Rate/Hour	Pages/Hour	Job	No. of Pages
A	Rs.5	12	P	299
B	Rs.6	14	Q	175
C	Rs.3	8	R	145
D	Rs.4	10	S	298
E	Rs.4	11	T	178

[Ans.:

Typist

Job

Cost(Rs.)

(10 Marks) Nov./96

A	T	75
B	R	66
C	Q	66
D	P	80
E	S	112
		<u>399]</u>

Question 13 (Traveling salesman): A travelling salesman has to visit 5 cities. He wishes to start from a particular city, visit each city once and return to his starting point. The travelling cost for each city from a particular city is given below:

	To city				
	A	B	C	D	E
A	X	4	7	3	4
B	4	X	6	3	4
C	7	6	X	7	5
D	3	3	7	X	7
E	4	4	5	7	X

From city

What is the sequence of visit of the salesman, so that the cost is minimum?

[Hint: Route A to B, B to D, D to A shall not be taken because D is not allowed to follow A, till C & E are processed. So we will follow next best solution i.e. A-E-C-B-D-A] (I.C.W.A. Final, June/97-Adapted)

Question 14: A company has four zones open and four marketing managers available for assignment. The zones are not equal in sales potentials. It is estimated that a typical marketing manager operating in each zone would bring in the following Annual sales:

Zones	Rs.
East	2,40,000
West	1,92,000
North	1,44,000
South	1,20,000

The four marketing managers are also different in ability. It is estimated that working under the same conditions, their yearly sales would be proportionately as under:

Manager M : 8
Manager N : 7
Manager O : 5
Manager P : 4

Required:

If the criterion is maximum expected total sales, find the optimum assignment and the maximum sales.

[Ans.: Assignment	Sales (Rs.)	(11Marks) Nov./07
M – East	80,000	
N – West	56,000	
O – North	30,000	
P – South	20,000	
	1,86,000]	

{Caution: Please make sure that you do the above question in accordance with answer of similar questions solved by me or provided by C.A. Institute's Suggested Answers - Please abstain from using any other method suggested by any other teacher of Delhi}

Question 15 (Replacement decisions): Average time taken by an operator on a specific machine is tabulated below. The management is considering replacing one of the old machines by a new one and the estimated time for operation by each operator on the new machine is also indicated.

Operators	Machines						New
	1	2	3	4	5	6	
A	10	12	8	10	8	12	11
B	9	10	8	7	8	9	10
C	8	7	8	8	8	6	8
D	12	13	14	14	15	14	11
E	9	9	9	8	8	10	9
F	7	8	9	9	9	8	8

Find out an allocation of operators to the old machines to achieve a minimum operation time.

(a) Reset the problem with the new machine and find out the allocation of the operators to each machine and comment on whether it is advantageous to replace an old machine to achieve a reduction in operating time only.

(b) How will the operators be reallocated to the machines after replacement?

[Ans.: Old Allocation A-3,B-4,C-6,D-1,E-5,F-2 with time = 49 New Allocation A-3,B-4,C-6,D-New,E-5,F-1 with time =47]

Question 16 (Liquidity management): X holds stock of different companies. For a certain problem he is compelled to sell off four of his holdings. Since he wants the money over the next five months, he sells not more than one stock in any month. He has estimated the sale proceeds in each of the next five months as follows:

(Rs. '000)					
	June	July	Aug	Sep	Oct

	13	16	14	19	17
	18	20	13	18	12
	17	15	10	22	18
	19	18	14	21	15

Find what will be the optimum plan of X and how much money can he realise by sales?

[Ans.: Optimum sale proceeds are Rs. 78000]

Question 17 (Market research and assignment): The market research team of the Look Forward Company requires some household data from four different cities. The team has to perform this job in two days- the next Saturday and Sunday. It plans to spend half a-day in each of the cities. The relevant data are given here:

Day and Time	Probability of a Household Contact			
	City 1	City 2	City 3	City 4
Saturday Morning	0.32	0.85	0.16	0.64
Saturday Evening	0.60	0.56	0.95	0.80
Sunday Morning	0.70	0.35	0.40	0.62
Sunday Evening	0.10	0.72	0.64	0.90
Number of households expected to interview	150	100	200	200

How should the team plan its visit to the four cities so that the expected response may be maximized? State this expected response.

[Ans.: Expected response is 560]

Question 18: The personnel manager of ABC company wants to assign Mr. X, Mr. Y and Mr. Z to regional offices. But the firm also has an opening in its Chennai office and would send one of the three to that branch if it were more economical than a move to Delhi, Mumbai or Kolkata. It will cost Rs. 2,000 to relocate Mr. X to Chennai Rs. 2,600 to relocate Mr. Y there, and Rs. 3,500 to move Mr. Z. What is the optimum assignment of personnel to offices?

Office Hires	Delhi	Mumbai	Kolkata
Mr. X	1600	2200	2400
Mr. Y	1000	3200	2600
Mr. Z	1000	2000	4600

[Ans.: X→Chennai; Y→Delhi; Z→Mumbai]

Question 19: A city corporation has decided to carry out road repairs on main four arteries of the city. The government has agreed to make a special grant of Rs. 50 lakhs towards the cost with a condition that the repairs must be done at the lowest cost and quickest time. If conditions warrant, then a supplementary token grant will also be considered favorably. The corporation has floated tenders and 5 contractors have sent in their bids. In order to expedite work, one road will be awarded to only one contractor.

Contractors/Road	Cost of Repairs (Rs. Lakhs)			
	R1	R2	R3	R4
C1	9	14	19	15
C2	7	17	20	19
C3	9	18	21	18
C4	10	12	18	19
C5	10	15	21	16

- Find the best way of assigning the repair work to the contractors and the costs.
- If it is necessary to seek supplementary grants, then what should be amount sought?
- Which of the five contractors will be unsuccessful in his bid?
- If C1 unable to accept any work, find best assignment.

[Ans.: C1-R3, C2-R1, C3-Dummy, C4-R2, C5-R4; Grant of Rs 4 Lakhs; C3]