INDEX

O.R

Learning Curve Theory

Mathematical Perceptions

$m \times n = \log m + \log n$

$m^n = n \log m$

$m/n = \log m - \log n$

Average Time p.u / lot

$Y = ax^b$

Y= Average Time p.u. /lot

a = Time Required for 1st unit / lot

x = Total no. of units/ lots for which average time is to be calculated

b = Learning curve index

Total Time

$xy = ax^{b+1}$

Question 1:

An electronics firm which has developed a new type of fire alarm system has been asked to quote for a prospective contract. The customer require separate price quotations for each of the following possibilities:

Order	No. of fire alarm System
First	100
Second	60
Third	40

The firm estimates the following cost per unit for the first order:

Direct Materials	Rs. 500
Direct Labour	
Deptt: A (Highly automatic) 20 hours at	Rs.10 per hour.
Deptt: B (Skilled Labour) 40 hours at	Rs. 15 per hour.
Variable overhead	20% of direct labour
Fixed overhead absorbed	
Deptt: A	Rs. 8 per hour
Deptt: B	Rs. 5 per hour

Determine a price per unit for each of the three orders, assuming the firm uses a markup of 25 % on total cost and allow for an 80% learning curve. Extract from 80% learning curve table:

X	1.0	1.3	1.4	1.5	1.6	1.7	1.8	1.9	2.0
Y (%)	100.0	91.7	89.5	87.6	86.1	84.4	83.0	81.5	80.0

X represents the cumulative total volume produced to date expressed as a multiple of the initial order.

Y is the learning curve factor, for a given X value, expressed as a percentage of the cost of the initial order.

Question 2:

A company has 10 direct workers, who work for 25 days a month of 8 hours per day. The estimated down time is 25 % of the total available time. The company received an order for a new product. The first unit of the new product requires 40 direct labours hours to manufacture the products. The company expects 80% (index is -0.322) learning curve for this type of work. The company uses standard absorption costing and the cost data are as under:

Direct Material	Rs. 60 per unit
Direct Labour	Rs. 6 per direct labour hour
Variable overheads	Rs. 1 per direct labour hour
Fixed overheads	Rs. 7500 per month

Required:

i. Calculate the cost per unit of the first order of 30 units.
ii. If the company receives a repeat order for 20 units, what price will be quoted to yield a profit of 25% on selling price? (Nov. 2002)

Question 3:

An electronic firm has developed a new type of fire alarm system. A first unit assembled had a material cost of Rs. 18000 and took 400 hours of direct labour to assemble labour rate is Rs. 25 per hour. This type of assembly is known to experience a learning curve effect of 80% (index of learning – 0.3219). Demonstration of this unit to potential customers resulted in an order for 20 units during the next quarter. The firm wishes to popularize this system and will therefore pass on the benefit of cost savings due to learning effect to the customers while setting the sale price.

i. Determine the price to be set for the first lot of 20 units to be sold. The initial unit will not be sold as this is required for demonstration. The firm follows a practice of inputting a fixed overhead at 125% of direct labour cost and will set the selling price to earn a 20% gross margin on sale price.
ii. Assume that a further order for a lot of 60 units was received on a contract basis from a single customer. The price was set on the basis of the contracted total. However, after delivery of 30 units against the contract,

the contract was cancelled. Determine the deferred learning cost that may have to be written – off consequent to the cancellation of contract for balance not supplied.

Question 4:

Given that a = 10 hours and learning rate is 80%, you are required to calculate:-

i. The average time for 20 units
ii. The total time for 30 units
iii. The total time for units 31 to 40

Given that Log 2 = 0.301, antilog of 0.5811 = 3.812
Log 3 = 0.4771, antilog of 0.5244 = 3.345
Log 4 = 0.6021, antilog of 0.4841 = 3.049

Question 5:

Dynamo, a manufacturer of aircraft parts, has been asked to bid for 900 units of a particular type of component. Four months ago the company had produced a first lot of 400 units for another customer. The cost details of this first lot are given below.

Repeated assemble of this type of component experiences a learning effect of 85%. The cost benefit of this will be reflected in the bid price. Dynamo follows a policy of setting the selling price to earn 30% gross margin. Tooling cost has been fully recovered from the first lot sold. Determine the selling price unit for the second lot intended.

	(Rs.)	
Direct Material	30,00,000	
Direct Labour	20,00,000	8000 hours @ Rs. 250
Tooling cost	4,80,000	
Variable overheads	9,00,000	proportional to direct labour
General overhead	12,00,000	proportional to direct labour
Total	75,80,000	

Question 6:

A customer has asked your company to prepare a bid on supplying 800 units of a new product. Production will be in batches of 100 units. You estimate that labour costs for the first batch of 100 units will average Rs. 100 a unit. You also expect that a 90% learning curve will apply to the cumulative labour cost on this contract.

Required:

i. Prepare an estimate of the labour costs of fulfilling this contract.
ii. Estimate the incremental labour cost of extending the production run to produce and additional 800 units.
iii. Estimate the incremental labour cost of extending the production run from 800 units to 900 units.

Question 7:

XYZ & Co. has given the following data:
80% Average – Time Curve

Cumulative Units (x)	Average Hours	Total Hours	Marginal Hours
1	100	100	100
2	80	160	60
3	?	?	?
4	64	256	?

Required: Fill in the blanks.

Question 8:

A company manufactures specialized equipment. Direct labour required to make the first equipment is 2000 hours. Learning curve is 80%.

Direct labour cost is Rs. 4 per hour. Direct material needed for one equipment is Rs. 10000. Fixed overheads are Rs. 32000.

Required

1. using the learning curve concept, calculate the expected average unit cost of making

   a. 4 equipments b. 8 equipments

2. After manufacturing 8 equipments, if a repeat order for manufacture of another 8 equipments are received. What lowest price can be quoted for the repeat order?

Question 9:

Illustrate the use of learning curves for calculating the expected average unit cost of making-

(a) 4 machines
(b) 8 machines

Using the data given below:-

Data:

Direct labour needed to make first machine 1000 hours
Learning curve = 80%
Direct labour cost – Rs. 3/- per hour
Direct material cost – Rs. 1800 per machine
Fixed cost for either size orders – Rs. 8000

Question 10:

A company developing a new product makes a model for testing, and then a demonstration model and then goes for regular production. The time taken to make the model is 300 hours. And from past experience of similar models, it is known that a 90% learning curve applies. The average time for each of the first two production models will be:-

1. 270 hours
2. 243 hours
3. 216 hours
4. 219 hours

Support the correct figure with calculations.

Question 11:

(a). Your company has been approached by a customer to supply four units of a new product made to the customer's individual specification. The company experiences a 90% learning curve. The estimated labour time for the first unit of this product is 150 hours and the company's direct labour cost is Rs. 5 per hour. Estimate the labour cost for this order.
(b) After receiving the first order, if the customer places a repeat order, what Will be the labour cost for the second order.
(c) If the customer had ordered all eight units at the same time, calculate the labour cost per unit for the combined order.

SIMULATION

Although simulation can be of many types, our discussion will focus on the probabilistic simulation using the Monte Carlo method. Also called Computer Simulation, it can be described as a numerical technique that involves modeling a stochastic system with the objective of predicting the systems behavior. The chance element is a very significant feature of Monte Carlo Simulation and this approach can be used when the given process has a random, a chance, and component.

In using the Monte Carlo Method, a given problem is solved by simulating the original data with random number generators. Basically, its use requires two things. First, as mentioned earlier, we must have a model, that represents an image of the reality of the situation. Here the model refers to the probability distribution of the variable in question. What is significant here is that the variable may not be known to explicitly follow any of the theoretical distribution like Poisson, normal, and so on. The distribution may be obtained by direct observation or from past record. To illustrate, suppose that a bakery keeps the record of the sale of number of cake of a certain types. Information relating to 200 days' sales is:

Demand (No. of cakes):	5	6	7	8	9	10	11	12	Total
(No. of days)	4	10	16	50	62	38	12	8	200

Assuming that this is an adequate representation of the distribution of demand for the cake, we can derive the probability distribution of demand by expressing each of the frequency in terms of proportion. This is done by dividing each one the values by 200 – the total frequency. The resultant distribution follows:

Demand (No. of cakes):	5	6	7	8	9	10	11	12
Probability	0.02	0.05	0.08	0.25	0.31	0.19	0.06	0.04

Thus, there is 0.02 or 2% chance that 5 cakes would be demanded on a day, 0.05 or 5% chance that the demand would be for 6 cakes, … and so on. This distribution would serve as the model of the situation under consideration.

The second thing requires for simulation is a mechanism to simulate the model – something to capture the random nature of the given system. Thus we should have available a procedure that would help us to select, at random, values for the variables which can be used to approximate the state of the system. Such a mechanism can be any random number generator consisting of a device or a procedure by which random number can be determined and / or selected.

There are various ways in which random numbers (or apparently random, but not truly so) may be generated this could be result of some device, coin or die; published table of random number, mid quare method, or some other sophisticated method. It may be mentioned here that the 'random' numbers generated by some method may not be really random in nature. In fact such numbers are called pseudo – random numbers. There are some test with which numbers may be tested for there randomness but we should not consider them here and consider only briefly how the number may be obtained and used.

One way to generate random number is to fix up a spinning arrow on a common clock. When the arrow is spun, the number on which it stops would be taken to be random number for that trial. Naturally, any number of spinning of the arrow would result in an equal number of random numbers. In a similar way random numbers can be generated using spinning of a roulette wheel, tossing dice…. and like that. Although simple, these are very slow methods and can not meet the practical requirements where a large number of random numbers may be needed.

A more fast and convenient method is to make use of the published table of random numbers, like the one published by the Rand Corporation (of U.S.A) : A million random digits. A random number table is very efficient way to generate random data in most situations. The numbers in this table are in random arrangements. The underlying theory is that each number has an equal opportunity of being selected.

Still on a more sophisticated level, computers are used for generating the random numbers. With computers it is typically easier to generate random numbers by an arithmetic process as needed rather than to read the number from stored table. An early, probably the earliest method proposed for use on digital computer to generate random number is a mid square. To illustrate these methods, suppose that we wish to generate four – digits integers and the last number generated was 8937. To obtain the next number, in the sequence, we square the last one and use the middle four digits of the product. In this case the product is 79869969 so that the next pseudo – number is 8699. The next few numbers in the sequence are 6726, 2390, 7121 and so on. Thus, using this method, having drawn up a suitable computer program, a four digit number may be fed into the computer and a list of pseudo random numbers obtained.

Question 1:

The director of finance for a farm cooperative is concerned about the yields per acre she can expect from this year's corn crop. The probability distribution of the yields for the current weather conditions is given below:

Yield in kg per acre	Probability
120	0.18
140	0.26
160	0.44
180	0.12

She would like to see a simulation of the yields she might expect over the next 10 years for weather conditions similar to those she is now experiencing.

(i) Simulate the average yield she might expect per acre using the following random numbers: 20, 72, 34, 54, 30, 22, 48, 74, 76, 02

She is also interested in the effect of market price fluctuations on the cooperative' s farm revenue. She makes this estimate of per - kg prices for corn.

Price per kg (Rs)	Probability
2.00	0.05
2.10	0.15
2.20	0.30
2.30	0.25
2.40	0.15
2.50	0.10

(ii) Simulate the price she might expect to observe over the next 10 years using the following random numbers:
82, 95, 18, 96, 20, 84, 56, 11, 52, 03

Question 2:

The occurrence of rain in a city on a day is dependant upon whether or not it Rained on the previous day.

If it **rained** on the previous day, the rain distribution is Given by:

Event	Probability
No rain	0.50
1 cm rain	0.25
2 cm rain	0.15
3 cm rain	0.05
4 cm rain	0.03
5 cm rain	0.02

If it **did not rain** the previous day, the rain distribution is given by:

Event	Probability
No rain	0.75
1 cm rain	0.15
2 cm rain	0.06
3 cm rain	0.04

Simulate the city 's weather for 10 days and determine by simulation the total days without rain as well as the total rainfall during the period . Use the following random numbers:

67 63 39 55 29 78 70 06 78 76

for simulation . Assume that for the first day of the simulation it had not rained the day before.

Question 3:

The output of a production line is checked by an inspector for one or more of three different types of defects, called defects A, B and C. If defect A occurs, the item is scrapped. If defect B or C occurs, the item must be reworked. The time require to rework a B defect is 15 minutes and the time required to rework a C defect is 30 minutes. The probabilities of an A, B and C defects are 15, 20 and 10 respectively. For ten items coming off the assembly line, determine the number of items without any defects, the number scrapped and the total minutes of rework time: Use the following random numbers:

RN for defect A
48 55 91 40 93 01 83 63 47 52
RN for defect B
47 36 57 04 79 55 10 13 57 09
RN for defect C
82 95 18 96 20 84 56 11 52 03

Question 4:

The management of ABC Company is considering the question of marketing a new product. The fixed cost required in the project is Rs. 4,000. Three factors are uncertain viz. the selling price, variable cost and the annual sales volume.
The product has a life of only one year. The management has the data on these three factors as under:

Selling Price Rs	Probability	Variable Cost Rs.	Probability	Sales Volume (Units)	Probability
3	0.2	1	0.3	2,000	0.3
4	0.5	2	0.6	3,000	0.3
5	0.3	3	0.1	5,000	0.4

Consider the following sequence of thirty random numbers:

81	32	60	04	46	31	67	25	24	10	40
02	39	68	08	59	66	90	12	64	79	31
86	68	82	89	25	11	98	16.			

Using the sequence (First 3 random numbers for the first trial etc.) simulate the average profit for the above project on the basis of 10 trials.

Question 5:

Dr. STRONG is a dentist who schedules all her patients for 30 minutes appointments. Some of the patients take more or less than 30 minutes depending on the type of dental work to be done. The following summary shows the various categories of work, their probabilities and the time actually needed to complete the work:

Category	Time required	Probability of Category
Filling	45 minutes	0.40
Crown	60 minutes	0.15
Cleaning	15 minutes	0.15
Extraction	45 minutes	0.10
Checkup	15 minutes	0.20

Simulate the dentist's clinic for four hours and determine the average waiting time for the patients as well as the idleness of the doctor.

Assume that all the patients show up at the clinic at exactly their scheduled arrival time starting at 8.00 a.m. Use the following random numbers for handling the above problem:

40 82 11 34 25 66 17 79

Question 6:

A book- store wishes to carry systems Analysis and Design in stock. Demand is probabilities and replenishment of stock takes 2 days (i.e. if an order is placed in March 1, it will be delivered at the end of the day on March 3). The probabilities of demand are given below:

Demand (daily)	0	1	2	3	4
Probability	0.05	0.10	0.30	0.45	0.10

Each time an order is placed, the store incurs an ordering cost of Rs. 10 per order. The store also incurs a carrying cost of Rs. 0.50 per book per day.

The inventory carrying costs is calculated on the basis of stock at the end of each day. The manager of the book-store wishes to compare two options for his inventory decision:

A. Order 5 books, when the inventory at the beginning of the day plus orders outstanding is less than 8 books.

B. Order 8 books, when the inventory at the beginning of the day plus orders outstanding is less than 8 books.

Currently (beginning of the 1st day) the store has stock of 8 books plus 6 books Ordered 2 days ago and expected to arrive next day.
Using Monte- Carlo simulation for 10 cycles recommends
Which option the manager should choose?

The two digits random numbers are given below:

89 34 78 63 61 81 39 16 13 73

Question 7:

A company manufactures 30 items per day. The sale of these items depends upon demand which has the following distribution:

Sales (Units)	Probability
27	0.10
28	0.15
29	0.20
30	0.35
31	0.15
32	0.05

The production cost and sale price of each units are Rs. 40 and Rs.50 respectively. Any unsold product is to be disposed at a loss of Rs.15 Per unit. There is a penalty of Rs.5 per unit if the demand is not met.

Using the following random numbers estimates total profit / loss for the company for the next 10 days:

10 99 64 99 95 01 79 11 16 20

If the company decides to produce 29 items per day, what is the advantage or disadvantage to the company?

Question 8:
A bakery keeps stock of popular brand of bread. Previous experience indicates the daily demand as given below:

Daily demand :	0	10	20	30	40	50
Probability :	.01	.20	.15	.50	.12	.02

Consider the following sequence of random numbers:
48, 78, 19, 51, 56, 77, 15, 14, 68, 9

Using above sequences, simulate the demand for the next 10 days.

(i) Find out the stock situation if the owner of the bakery decides to make 30 Breads everyday
(ii) Estimate the daily average demand for the bread on the basis of simulated Data.

Question 9:

A company manufactures around 200 mopeds. Depending upon the availability of raw materials and other conditions, the daily production has been varying from 196 mopeds to 204 mopeds whose probability distribution is as given below:

Production per day	Probability
196	0.05
197	0.09
198	0.12
199	0.14
200	0.20
201	0.15
202	0.11
203	0.08
204	0.06

The finished mopeds are transported in a specially designed three storey lorry That can accommodate only 200 mopeds. Using the following 15 random numbers 82, 89, 78, 24, 53, 61, 18, 45, 04, 23, 50, 77, 27, 54, 10
Simulate process find out:

(i) What will be the average numbers of mopeds, waiting in the factory?

(ii) What will be the average number of empty spaces on the lorry?

Question 10:

A retailer deals in a perishable commodity. The daily demand and supply are variables. The data for the past 500 days show the following demand and supply:

Supply		Demand	
Availability (kg)	No. of days	Demand (kg)	No. of days
10	40	10	50
20	50	20	110
30	190	30	200
40	150	40	100
50	70	50	40

The retailer buys the commodity at Rs. 20 per kg and sells it at Rs. 30 per kg. Any Commodity remains at the end of the day, has no saleable value. Moreover, the loss (Unearned profit) on any unsatisfied demands Rs. 8 per kg. Given the following pair of Random numbers simulate 6 days sales, demand and profit.

(31, 18); (63, 84); (15, 79); (07, 32); (43, 75); (81, 27)

The first random number in the pair is for supply and the second random numbers is for demand viz. in the first pair (3 1, 18). Use 31 to simulate supply and 18 to simulate demand.

Question 11:

An investment company wants to study the investment projects based on market demand. Profit and the investment required, which are independent of each other. Following probability distributions are estimated for each of these three factors:

Annual demand (Units in thousands):	25	30	35	40	45	50	55
Probability :	0.05	0.10	0.20	0.30	0.20	0.10	0.05

Profit per Unit:	3.00	5.00	7.00	9.00	10.00
Probability :	0.10	0.20	0.40	0.20	0.10

Investment Required (in thousand of Rupees):	2,750	3,000	3,500
Probability :	0.25	0.50	0.25

Using simulation process, repeat the trial 10 times, compute the investment on each trial taking these factors into trial. What is the most likely return?
Use the following random numbers:

(30, 12, 16) (59, 09, 69) (63, 94, 26) (27, 08, 74)
(64, 60, 61) (28, 28, 72) (31, 23, 57) (54, 85, 20)
(64, 68, 18) (32, 31, 87)

In the bracket above the first random numbers is for annual demand, the second one is for profit and the last one is for the investment required.

Question 12:

A town contains six wards and they contain 170, 510, 640, 75, 250, and 960 houses respectively. Make a random selection of 8 houses using the tables of random numbers. Explain the procedure adopted by you.

TRANSPORTATION

PUT THE PROBLEM IN THE STANDARD FORM

Step1 The problem should be in the MATRIX FORM and if it is not given to be in matrix form, then we must put it in matrix form.

Step2 The transportation technique is solving for minimization types of problem. This means that if the question is on maximization type, then, we should convert it in to minimization type by deducting all elements from the highest element in the matrix.

Step3 The problem should always be a balanced problem which means that the total quantity of the resources available should be exactly equal to total quantity of resource required. This is unlike the assignment problem where requirement was total no. of rows should be equal to total no. of column.

If the quantities are not same, we convert such an imbalanced problem to balanced problem by introducing dummy column or row, the quantity being the quantity which is difference between quantity available and required.

Step4 In the transportation matrix, the columns represents / requirement or demand (m) & rows represent availability or supply (n).

INITIAL BASIC FEASIBLE SOLUTION (IBFS)

There are five methods in all for developing IBFS. Regardless of the method followed for IBFS, it has to go through the optimality test, which means the final optimum solution is always the same irrespective of the method followed.

1. NORTH WEST CORNER RULE

Here we first start from the north – west corner of the matrix and we make allocation of the quantity, which is equal to the lower of availability and requirement applicable to this position. After this, we cancel the row or column depending on where the quantity gets exhausted. In the revised matrix after this cancellation, we again catch the North West Corner of this revised matrix and we make the allocation. We go on doing this as long as the entire allocation is not over.

The IBFS based on this method **ignores the cost completely** and therefore, it is **farthest form the optimum solution.**

2. COLUMN MINIMA

Here we take up each column, one by one, beginning with the first column and make the allocation of quantity in the least cost cell of the column. As long as the column quantity is not exhausted, we go to the next least cost cell in the same column and make the allocation. This way when all columns are exhausted, the IBFS is obtained.

3. ROW MINIMA

This is exactly same as the column minima method except that instead of taking each column one by one we take each row one by one.

4. MATRIX MINIMA OR LEAST COST METHOD

Here we make the allocation in the least cost cell of the entire matrix. Depending on, where the quantity gets exhausted, we cancel the row or, as the case may be column. From the revised matrix, we again identify the least cost cell and make the allocation. Once all allocations are over, the IBFS is obtained.

5. VOGEL'S APPROXIMATION METHOD (VAM)

This is the closest to the optimum solution, the next best option being the least cost method. It takes longer time for developing IBFS but saves considerable amount of time while going through the optimality test.

Following steps should be followed for getting IBFS.

1. For each row and column, we calculate the penalties by taking the difference between the least cost and the next least cost.

2. We select that column or row which has the highest penalty and we make the allocation of quantity which would be lower of availability and requirement. This allocation is to be made in the least cost cell of such row / column which has highest penalty.

3. If there is a TIE in respect of penalty.

    - We compare the least cost figures of such rows / columns and we select that row / column which has lower least cost.

    - If there is a tie between the least costs also then, we select the row or column where higher quantity allocation is possible.

- If there is a tie between the quantities also, then we make the allocation at random.

OPTIMALITY TEST

I

The number of allocation in the IBFS should be equal to (m+n-1) and further all these allocation should be independent (which means, from non of these allocation, it should be possible for us to form a loop).

All the methods discussed for IBFS always give us INDEPENDENT ALLOCATION. However, the number of such allocations would be less than m+n-1 if, while developing IBFS, row and column both get exhausted at a time.

If number of allocation is less than (m+n-1), then we introduce some artificial or zero quantity allocation known as EPSILON "e" IN THE LEAST COST INDEPENDENT CELL. If the least cost unallocated cell is not independent (which means that the loop can be formed form that cell, then we go to the next least cost unallocated cell and so on, our objective being to have "e" in UNALLOCATED BUT INDEPENDENT CELL.

For all practical purposes, epsilon is exactly zero quantity and yet we do not write zero just in order to distinguish between the unallocated cells and the cell in which we put 'e'.

It is also possible that there may be two or more 'e's required.

Sometimes, 'e' is introduced right in the beginning and it stays right up to the end in the optimum solution.

Sometimes it is introduced in the beginning but it disappears during optimality test.

Sometimes it enters afterwards and it vanishes before the end.

Sometimes it enters afterwards and stays right up o the end.

The introduction of epsilon implies degeneracy in our solution process.

- If epsilon survives in the final optimum solution, then the degeneracy is permanent.
- If epsilon does not survive in the optimum solution, then the degeneracy was temporary.

II

Calculation of values for each row (Ui) and column (Vj where j=column 1,2,3,4,-) Each cell is identified as (Cij) which is interpreted as the cost of ith row and jth column.

Value calculation: - Cij=Ui+Vj (Cost / value of row / column in respect of each allocated cell).

Against any one row or column, first we put a 'o' as value and there after we calculate values for all other rows and columns by using the equation given above. It must be remembered that the value calculation are based on ALLOCATED CELLS only

III

Calculation of PENALTIES or NET EVALUATION

- This calculation is in respect of all unallocated cells.
- Net evaluation = Cij cost – (values in respect of row + value in respect of column applicable to Cij).

1. If all net evaluation is positive or o' s, then, it means that optimality is reached.

2. If there are zeros, then, that means there is ALTERNATIVE allocation that is possible.

3. If there is negative evaluation in respect of at least one allocated cell, then, that means current allocation is not optimum.

4. If there are two or more unallocated cells, in respect of which net evaluation is negative, then we select the one with HIGHEST negative net evaluation. Beginning with this negative net evaluation cell, we form a LOOP covering alternately allocated cells in a row or column. We put alternately + and – sign against each such cell covered by the loop and beginning with + sign in the unallocated cell with the highest negative net evaluation.

We now look at the quantities of all those cells with negative signs and we select the least of such quantity. This quantity should be added to all those quantities with +sign in the loop and deducted from the quantities of all those cells in which –sign is put in the loop.

As a result, the unallocated cell with least negative net evaluation will now get some allocation. (Which could sometimes be epsilon also) and one of the previous allocated cells would vanish.

If the minimum quantity is same in respect of two or more cells against which negative sign is put, then only one new allocation comes into being but two or more allocations would vanish. This implies that the revised allocation would not satisfy m+n-1 condition and epsilon (s) will have to be introduced.

IV

If the highest negative net evaluation is same for two or more unallocated cell, then, we form loop beginning with each of such unallocated cells. We identify the minimum quantity amongst the negative sign cells in respect of each such loop and we select that loop which has HIGHEST MINIMUM QUANTITY. The other loop would now be rejected.

If the minimum quantity is also the same, then, that implies that each loop is an alternative to the other. We select one of the two at RANDOM.

V

The revised matrix is now as good as IBFS and we follow all steps written above and we keep doing so as long as the optimality is not reached.

DEVELOPING ALTERNATIVE SOLUTION /ALLOCATION

If the net evaluation of any unallocated cell is zero, then, that implies that there is an alternative allocation possible with the same minimum cost or maximum revenue.

To develop the alternative allocation, we treat the zero net evaluation as if it is negative and form the loop. We follow normal steps now and get the alternative allocation.

If should be remembered that the difference between the total cost applicable to the given allocation and the one applicable to the revised allocation is always equal to the quantity allocated to the negative net evaluation cell multiplied by the amount of negative evaluation.

In case of zero net evaluation the change in cost would be zero and that is the reason why, we say that, implies that there is an alternative solution

Q. 1 Solve the following transportation problem for the optimum cost

	To Destination				Availability
	A	B	C	D	
From 1	4	5	3	6	100
Origin 2	3	6	7	3	140
3	1	4	1	2	160
Requirement	100	80	180	40	

Solution:- Important Note:

Whenever there is at least one negative net evaluation, it implies that the current solution is not optimum. Therefore in, such cases if there is only one negative net evaluation then we form loop beginning with that position. If there are two or more negative net evaluation then we select the position where negative net evaluation is highest. If there is a tie in respect of negative net evaluation then we form loop beginning with each of such positions and we select that loop in respect of which the least negative quantity is highest. If the tie still persists then we break it arbitrarily, selecting any loop at random.

In this solution, there is tie in respect of negative net evaluation and there is further tie in respect of minimum quantity also then we select arbitrarily.

Question 2:

Solve the following transportation problem.

Origin	Profit (Rs.) / Unit Destination 1	2	3	4	Supply
A	40	25	22	33	100
B	44	35	30	30	30
C	38	38	28	30	70
Demands	40	20	60	30	

Question 3:

The following table shows all the necessary information on the available supply to each warehouse the requirement of each market and the unit transportation cost from each warehouse to each market.

		Market				
		I	II	III	IV	
Supply						
	A	5	2	4	3	22
Warehouse	B	4	8	1	6	15
	C	4	6	7	5	8
Requirement		7	12	17	9	

The shipping clerk has worked out the following schedule from his experience:

12 units from A to II
1 Unit from A to III
9 units from A to IV
15 units from B to III
7 units from C to I and
1 unit from C to III
You are required to answer the following:

(i) Check and see if the clerk has the optimal schedule:
(ii) Find the optimal schedule and minimum total shipping cost: and
(iii) If the clerk is approached by a carried of route C to II who offers to reduce His rate in the hope of getting some business by now much should the rate Be reduced before the clerk should consider giving him an order?

Question 4:
A Compressed Natural Gas (CNG) company has three plants producing gas and four outlets. The cost of transporting different production plants to the outlets, production capacity of each plant and requirement at different outlets is shown in the following cost matrix table:

Plants	Outlets A	B	C	D	Capacity of production
X	4	6	8	6	700
Y	3	5	2	5	400
Z	3	9	6	5	600
Requirement	400	450	350	500	1,700

Determine a transportation schedule so that the cost is minimized.

The cost in the cost matrix is given in thousand of rupees.

Question 5:

A company has three factories and four customers. The company furnishes the following schedule of profit per unit on transportation of its goods to the customers in rupees:

	Customers				
Factory	A	B	C	D	Supply
P	40	25	22	33	100
Q	44	35	30	30	30
R	38	38	28	30	70
Demand	40	20	60	30	

You are required solve the transportation problem to maximize the profit and determine the resultant optimal profit.

CPM / PERT

1. **Project :-** A project can be defined as a set of large number of activities or jobs that are performed in certain sequences determine logically or technologically and it has to be completed with specified cost or specified time and meeting the performance standard. E.g. construction of bridge, building lying down pipeline, introduces a new product, installation of computer system etc.

2. **Activity: -** An activity is any large portion of project which consumes time and has definable beginning and ending. It is shown by arrow in network diagram.

3. **Events**: - The beginning and ending point of an activity are called events. Event is shown as numbers circled in the net work diagram.

4. **Slack Event :-** Slack of an event is the difference between latest event time and earliest event time (LF – ES)

5. **Critical Path: -** The longest path in the network is called Critical path. All activity that lies under critical path is called Critical Activity. All events on Critical path have zero Slack.

6. **Float: -** Float means flexibility that we have in terms of commencement and completion of an activity only non critical activity have float.

7. **Total Float: -** It indicates the time or period by which we can delay any activity without delaying the project duration.

8. **Free Float: -** This is that part of the total float which can be used without affecting float of subsequent activity.

9. **Independent Float: -** This is that part of free float which can be used without affecting Float of Preceding activity.

10. **Interfering Float: -** It is that part of total float which can be used only by affecting float of subsequent activity.

PERT

To = Optimistic Time

Tp = Pessimistic Time

Tm = Most Likely Time

Te = Expected Time

Te = (Tp+ 4Tm +To)/6

Variance = $((Tp - To)/6)^2$

S.D. of project = Square root of (Total Variance of Project) (i.e. critical path only)

Question 1:

The time schedule for different activities of a project is given below

Activity	Time in Month
1-2	16
1-3	20
1-4	16
2-3	20
2-6	32
3-5	34
4-5	36
4-6	28
5-6	18

Construct the PERT network and compute:

1. Critical path and its duration.

Question 2:

The time schedule for different activities of a project is given below

Activity	Time in Days
1-2	8
1-3	10
1-4	8
2-3	10
2-6	16
3-5	17
4-5	18
4-6	14
5-6	9

Construct the PERT network and compute:

2. Critical path and its duration.
3. Total float, Free float, Independent Float and Interfering float for each activity.

Solution :

1	2	3	6	4	5	7	8	9	10	11
Activity	Days	ES	LS	EF	LF	TF	Interfering Float	FF	Slack of Tail Event	Independent Float
1-2	8	0	0	8	8	0	0	0		0
1-3	10	0	8	10	18	8	0	8		8
1-4	8	0	9	8	17	9	9	0		0
2-3	10	8	8	18	18	0	0	0		0
2-6	16	8	28	24	44	20	0	20		20
3-5	17	18	18	35	35	0	0	0		0
4-5	18	8	17	26	35	9	0	9		0
4-6	14	8	30	22	44	22	0	22		13
5-6	9	35	35	44	44	0	0	0		0

1) Activity = From Question
2) Days = From Question
3) ES = ES of Tail or Tail event of Activity
4) EF = ES + Duration
5) LF = LF of Head or Head Event of Activity
6) LS = LF- Duration
7) Total Float = LS – ES OR LF – EF
8) Interfering Float = Slack of Head Event or Total Float – Free Float
9) Free Float = TF - Interfering Float or TF – Slack of Head Event
10) Independent Float = FF – Slack of Tail Event

Question 3:

The following table gives the activities and other relevant data for a project

Activity	**Normal Time (days)**	**Crash Time (days)**	**Normal Cost (Rs.)**	**Crash Cost (Rs.)**
1-2	4	3	600	800
1-3	2	2	400	400
1-4	5	4	750	900
2-3	7	5	400	600
2-5	7	6	800	1000
3-5	2	1	500	650
4-5	5	4	600	850

Indirect cost per day for the project is Rs. 200.

1. Draw the network of the project.
2. Find the normal duration and cost of the project.
3. Find the optimum duration and cost of the project.
4. Find the minimum duration of project and cost associated with that.

Question 4:

The following information is given:

Activity	Tp	Tm	To (in weeks)
1-2	3	3	3
2-3	9	6	3
2-4	6	4	2
3-5	8	6	4
4-6	8	6	4
5-6	0	0	0
5-7	5	4	3
6-7	8	5	2

Draw the network diagram for the above and calculate:

1. Variance to each activity
2. Critical path and expected project length
3. The probability that the project will be completed in 23 weeks.
4. What duration will have 95% confidence for project completion?

Question:5

Given the following information, develop a net work

Activity	Immediate Predecessors
A	--
B	--
C	A
D	B

Question: 6

Given the following information, develop a net work

Activity	Immediate Predecessors
A	--
B	--
C	A
D	A
E	C, B

Question: 7
Given the following information, develop a net work

Activity	Immediate Predecessors
A	--
B	--
C	A
D	B
E	C, D
F	D
G	E
H	F

Question: 8
Given the following information, develop a net work

Activity	Immediate Predecessors
A	--
B	--
C	A
D	B
E	C
F	D

Question: 9
Draw an arrow diagram showing the following relationship.

Activity	Immediate Predecessors
A	--
B	--
C	A
D	A
E	B
F	C
G	D,E

Question 10:
The Madras Construction co. is bidding on a contract to install a line of microwave towers. It has identified the following activities, along with their expected times, predecessor restrictions, and worker requirements:

Activity	Duration Weeks	Predecessors	Crew size, Workers
A	4	None	4
B	7	None	2
C	3	A	2
D	3	A	4
E	2	B	6
F	2	B	3
G	2	D,E	3
H	3	F,G	4

The contract specifies that the project must be completed in 14 weeks. This company will assign a fixed number of workers to the project for its entire duration, and so it would like to ensure that the minimum number of workers is assigned and that the project will be completed in the 14 weeks. Find a schedule which will do this.

Question 11:

The following table lists the activities of maintenance project.

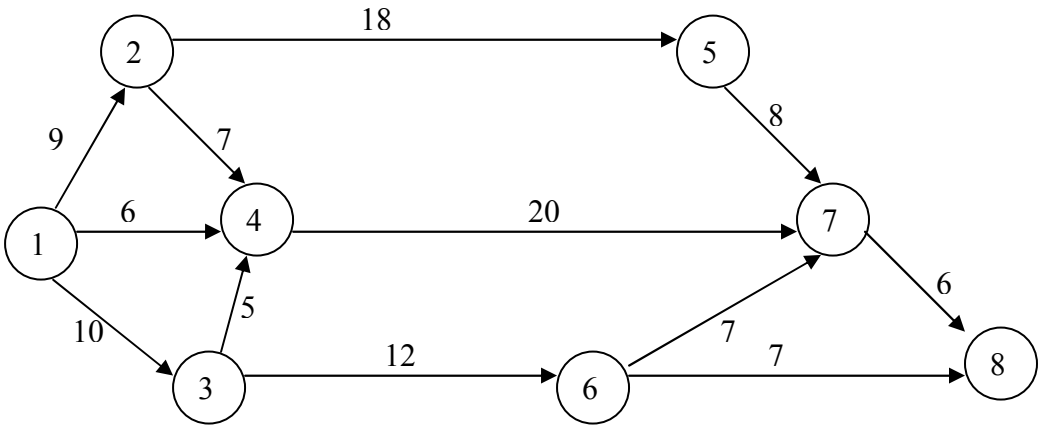
Activity	Duration (In Month)
1-2	2
1-3	2
1-4	1
2-5	4
3-6	5
3-7	8
4-7	3
5-8	1
6-8	4
7-9	5
8-9	5

(1) Draw the project network.
(2) Find the critical path & Duration of the project.
(3) Suppose we are required to employ a special piece of equipment on activities 1-3, 3-6, 2-5, 5-8 & 8-9, one at a time, will it affect the duration of the project ?

Question:12

After 15 days of working the following progress is noted for the network of an erection job
(1) Activity 1-2, 1-3, and 1-4 completed as per original schedule
(2) Activity 2-4 is in progress and will be completed in 3 more days

(3) Activity 3-6 is in progress and will need 18 days more for completion
(4) Activity 6-7 appears to present some problem and new estimated time of Completion is 12 days
(5) Activity 6-8 can be completed in 5 days instead of originally planned for 7 days



You are required to:
(1) Updated the above diagram after 15 days of the start of work based on Assumption given above
(2) Write down the critical path with total project duration.

Assignment Problem

Initial Steps

1. Ensure that the problem is in matrix form.

If it is not given in matrix form (which means it is in the form of paragraphs then we should first convert the problem to matrix form.

2. Assignment technique can be used only for minimization (of cost, time & so on) which means, we have to have minimization matrix. If the question requires maximization (of sales, profit, production & so on then we have to first convert the maximization matrix into a minimization matrix.

For this we select the highest figure from the maximization matrix & we minus all the elements in the maximization matrix form the highest element. The resultant matrix is a minimization matrix.

3. Since the assignment is always on one to one basis, it is essential that the number of rows is equal to the number of columns. When it is already so, we call it a balanced assignment problem.

If it is not so, then, we introduce a dummy row or dummy column (with zero cost) to make it a balanced problem.

SUMMARY

1. Put the question in matrix form.
2. If it is maximization problem, then convert that to minimization problem.
3. If it is imbalanced problem, then convert it to balanced problem, by introducing dummy row or column.

The above steps should be followed in that given order.

STEPS TO BE FOLLOWED TO SOLVE A BALANCED MINIMISATION TYPE PROBLEM WHICH IS IN MATRIX FORM

1. In each row which ever is the minimum element; it should be deducted from all the elements in that row (ROW MINIMA).
2. In respect of each column, whichever is the minimum element, it should be deducted from all elements in that column (COLUMN MINIMA).
3. We should now draw minimum possible number of lines so as to cover all the “zeros” in the matrix.

If the number of lines drawn is equal to the order of matrix (number of rows or columns) then that signifies that the optimality is reached.
We now start making assignment.

4. For assignment, we first select that row or column, which has only "1 zero" & we encircle that zero which means that the assignment is made & all other zeros, if any, in that column or row, as the case may be, should now be cancelled.

   If there are two or more zeros in each row or column, then, as such we can make the assignment at random & cancel the other zeros in the column or row as the case may be.
   However it is better to select that row or column which has minimum number of zeros (because it has more constraints than the others).

   While making assignment at random, if we come across an infeasible solution then that means that the random assignment is incorrect & we must change the assignment thereafter.
5. If in step no. 3 we realize that the no. of lines is not equal to the order of the matrix then that means that optimality is not reached.

   In such matrix there are three types of elements
   i. The uncovered elements which are not on any lines drawn by us.
   ii. The elements covered by the lines and are on the point of intersection of two lines.
   iii. Covered elements but not on point of intersection of two lines.

   The minimum element from the uncovered elements should be identified and it should be:
   i. Deducted form all the uncovered elements
   ii. Added to the elements on point of intersection of two lines.
   iii. The other elements should be kept as it is.

   Giving effect to these changes, we should know develop a new matrix and we should go back to step no. 3 draw minimum no. of lines and so on.

   We should keep doing this as long as the no. of lines is not equal to the order of matrix.

6. Once assignment is over we should write the final recommendation, as regards assignment and we should also calculate, the applicable cost, time, production, sales and so on as the case may be.

Question 1

Find the optimal solution for the assignment problem with the following cost matrix

Marketing Executives	Division			
	N	**E**	**W**	**S**
A	14	20	11	19
B	12	10	15	9
C	16	19	18	15
D	17	13	15	14

Question 2:

A Marketing Manager has 4 subordinates and 4 tasks. The subordinates differ in efficiency. The tasks also differ in their intrinsic difficulty. His estimates of the time each subordinate would take to perform each task are given in the matrix bellow. How should the task be allocated one to man so that the total man hour are minimized?

	I	**II**	**III**	**IV**
1	16	52	34	22
2	26	56	8	52
3	76	38	36	30
4	38	52	48	20

Question 3:

To stimulate interest and provide an atmosphere for intellectual discussion, a finance faculty in a management school decides to hold special seminars on four contemporary topics-leasing, portfolio management, private mutual funds, swaps and options. Such seminars should be held once per week in the afternoons. However, scheduling these seminars (one for each topic, and not more than one seminar per afternoon) has to be done carefully so that the number of students unable to attend is kept to a minimum. A careful study indicates that the number of students who cannot attend a particular seminar on a specific day is as follows:

	Leasing	Portfolio Management	Private Mutual funds	Swaps and Options
Monday	50	40	60	20
Tuesday	40	30	40	30
Wednesday	60	20	30	20
Thursday	30	30	20	30
Friday	10	20	10	30

Find an optimal schedule of the seminars. Also find out the total number of students who will be missing at least one seminar.

Question 4 :

Four operators O_1, O_2, O_3 and O_4 are available to a manager who has to get four jobs J_1, J_2, J_3 and J_4 done by assigning one job to each operator. Given the times needed by different operators for different jobs in the matrix below:

	J_1	J_2	J_3	J_4
O_1	12	10	10	8
O_2	14	12	15	11
O_3	6	10	16	4
O_4	8	10	9	7

(i) How should the manager assign the jobs so that the total time needed for all four jobs is minimum?

(ii) If job J2 is not to be assigned to operator O2' what should be the assignment and how much additional total time will be required?

Question 5:

XYZ airline operating 7 days a week has given the following time-table, Crews must have a minimum layover of 5 hours between flights. Obtain the pairing flights that minimum layover away from home. For any given pairing the crew will be based at the city that results in the smaller layover.

Chennai - Mumbai			Mumbai - Chennai		
Flight No	Departure	Arrive	Flight No	Departure	Arrive
A1	6 AM	8 AM	B1	8 AM	10 AM
A2	8 AM	10 AM	B2	9 AM	11 AM
A3	2 PM	4 PM	B3	2 PM	4 PM
A4	8 PM	10 PM	B4	7 PM	9 PM

Question 6:

A firm products four products. There are four operators who are capable of producing any of these four products.

The processing time varies from operator to operator. The firm records 8 hours a day and allows 30 minutes for lunch. The processing rime in minutes and the profit for each of the products are given below:

Operators	Products			
	A	B	C	D
1	15	9	10	6
2	10	6	9	6
3	25	15	15	9
4	15	9	10	10
Profit (Rs. per Unit)	8	6	5	4

Find the optimal assignment of products to operators.

FORMIULATION

Question 1:

A leading Chartered Accountant is attempting to determine a 'best' investment portfolio and is considering six alternative investment proposals. The following table indicates point estimates for the price per share, the annual growth rate in the price per share, the annual divided per share and a measure of the risk associated with each investment.

PORTFOLIO DATA

	Shares under consideration					
	A	B	C	D	E	F
Current price per share (Rs.)	80	100	160	120	150	200
Projected annual growth rate	0.08	0.07	0.10	0.12	0.09	0.15
Projected annual dividend per share (Rs.)	4.00	4.50	7.50	5.50	5.75	0.00
Projected risk in return	0.05	0.03	0.10	0.20	0.06	0.08

The total amount available for investment is Rs. 25 lakhs and the following conditions are required to be satisfied.

(i) The maximum rupee amount to be invested in alternative F is Rs. 2,50,000.

(ii) No more than Rs. 5,00,000 should be invested in alternatives A and B combined.

(iii) Total weighted risk should not be greater than 0.10, where

$$\text{Total weighted risk} = \frac{\text{(Amount invested in alternative j) (risk of alternative J)}}{\text{Total amount invested in all the alternatives}}$$

(iv) For the sake of diversity, at least 100 shares of each stock should be purchased.

(v) At least 10 percent of the total investment should be in alternatives A and B combined.

(vi) Dividends for the year should be at least Rs. 10,000.

Rupees return per share of stock is defined as price per share one year hence less current price per share PLUS dividend per share.

If the objective is to maximize total rupee return, formulate the Linear Programming model for determining the optimal number of share to be purchased in each of the shares under consideration.

You may assume that the time horizon for the investment is one year. The formulated LP problem is not required to be solved.

Question 2:

WELLTYPE Manufacturing Company produces three types of typewriters; Manual typewriters. Electronic typewriters and Deluxe Electronic typewriters. All the three models are required to be machined first and assembled. The time required for the various models are as follows:

Types	Machine Time (in hours)	Assembly Time (in hours)
Manual Typewriter	15	4
Electronic Typewriter	12	3
Deluxe Electronic Typewriter	14	5

The total available machine time and assembly time are 3,000 hours and 1,200 hours respectively. The data regarding the selling price and variable costs for the three types are :

	Manual	Electronic	Deluxe Electronic
Selling Price (Rs.)	4,100	7,500	14,600
Labour, material and other Variable costs (Rs.)	2,500	4,500	9,000

The company sells all the three types on credit basis, but will collect the amounts on the first of next month. The labour, material and other variable expenses will have to be paid in cash. This company has taken a loan of Rs. 40,000 from a co-operative bank and this company will have to repay it to the bank on 1st April,1993. The TNC Bank from whom this company has borrowed Rs. 60,000 has expressed its approval to renew the loan.

The Balance Sheet of this Company as on 31.3.93 is as follows:

Liability	Rs.	Assets	Rs.
Equity Share capital	1,50,000	Land	90,000
Capital Reserve	15,000	Building	70,000
General Reserve	1,10,000	Plant & Machinery	100,000
Profit & Loss A/c.	25,000	Furniture & Fixtures	15,000
Long term loan	1,00,000	Vehicles	30,000
Loan from TNC Bank	60,000	Inventory	5,000
Loan from Co-op. Bank	40,000	Receivables	50,000
		Cash	1,40,000
Total	5,00,000		5,00,000

The company will have to pay a sum of Rs. 10,000 towards the salary form top management executives and other fixed overheads for the month. Interest on long term loans is to be paid every month at 24% per annum. Interest on loans from TNC and Co-operative Banks may be taken to be Rs. 1,200 for the month. Also this company has promised to deliver 2 Manual typewriters and 8 Deluxe Electronic typewriters to one of its valued customers next month. Also make sure that level of operations in this company is subjected to the availability of cash next month. This company will also be able to sell all their types of typewriters in the market. The Senior Manager of this company desires to know as to how many units of each type writer must be manufactured in the factory next month so as to maximize the profits of the company. Formulate this as a linear programming problem. The formulated problem need not be solved.

Question 3:

A Mutual Fund company has Rs. 20 lakhs available for investment in Government Bonds blue chip stocks, speculative stocks and short-term bank deposits. The annual expected return and risk factor are given below:

Type of investment	Annual expected Return (%)	Risk factor (0 to 100)
Government Bonds	14	12
Blue Chip Stocks	19	24
Speculative Stocks	23	48
Short-term Deposits	12	6

Mutual Fund is required to keep at least Rs. 2 lakhs in short-term deposit and not to exceed an average risk factor of 42. Speculative stocks must be at most 20 percent of the total amount invested. How should mutual fund invest the funds so as to maximize its total expected annual return? Formulate this as a Linear Programming problem does not solve it.

Question :4

The owner of Fancy Goods Shop is interested to determine, how many advertisements to release in the selected three magazines A Band C.
His main purpose is to advertise in such a way that total exposure to principal buyers of his goods is maximized Percentages of readers for each magazine are known.
Exposure in any particular magazine is the number of advertisements released multiplied by the number of principal buyers. The following data are available:

Particulars	Magazines A	B	C
Readers	1.0 lakh	0.6 lakh	0.4 lakh
Principal buyers	20%	15%	8%
Cost per Advertisement	8,000	6,000	5,000

The budget amount is at the most Rs. 1.0 lakh for the advertisements. The owner has already decided that magazine. A should have no more than 15 advertisements and that B and C gets least 8 advertisements. Formulate a Linear Programming Model for the problem.

Question :5

Agriculture has a farm with 125 acres. He produces Radish, Mutter and Potato. Whatever he raises is fully sold in the market. He gets Rs. 5 for Radish per Kg. Rs. 4 for Mutter per Kg. and Rs. 5 for Potato per Kg. The average yield is 1,500 Kg. of Radish per acre, 1,800 Kg. of Mutter per acre and 1,200 Kg. of Potato per acre. To produce each 100 Kg. of Radish and Mutter and to produce each 80 Kg. of Potato, a sum of Rs. 12.50 has to be used for manure. Labour required for each acre to raise the crop is 6 man days for Radish and Potato each and 5 man days for Mutter. A total of 500 man days of labour at a rate Rs. 40 per man day are available.
Formulate this as a Linear Programming model to maximize the Agriculturist's total profit.

Question:6
A firm produces three products A, B and C. It uses two types of raw materials I and II of which 5,000 and 7,500 units respectively are available. The raw material requirements per unit of the products are given below.

Raw material	Requirement per unit of products A	B	C
I	3	4	5
II	5	3	5

The labour time for each unit of product A is twice that of product B and three times that of products C. the entire labour force of the firm can produce the equivalent of 3,000 units the minimum demand of the three products is 600, 650 and 500 unite respectively. Also the ratios of the number of units produced must be equal to 2: 3:4. Assuming the profits per unit of A, B, and C as Rs 50, 50 and 80 respectively.

Formulate the problem as a linear programming model in order to determine the number of units of each product which will maximize the profit.

Question:7

A company has three operational departments (weaving, processing and packing) with capacity to produce three different types of cloths namely suiting, shirting and woolens yielding the profit Rs. 2, Rs. 4 and Rs. 3 per meter respectively. One mater suiting requires 3 minutes in weaving, 2 minutes in processing and 1 minute in packing. Similarly one meter of shirting requires 4 minutes in processing and 1 minute in processing and 3 minutes in packing while one meter woolen requires 3 minutes in each department. In a week, total run time of each department is 60, 40 and 80 hours for weaving processing and packing departments respectively.

Formulate the linear programming problem to find the product mix to maximize the profit.

Question:8

A sports club is engaged in the development of their players by feeding them certain minimum amount of Vitamins (say A, B and C) in addition to their normal diet. In view of this two types of products X and Y are purchased from the market. The contents of Vitamins constituents per unit are shown in the following table.

Vitamin requirement	Vitamin Contents in Products		Minimum
Constituents	X	Y	for each player
A	36	06	108
B	03	12	36
C	20	10	100

The cost of product X is Rs. 20 and that of Y is Rs. 40

Formulate a linear programming problem for the above.

Question:9

A Company manufacturers two products namely A and B. The contribution per kg of output is Rs. 240 and Rs. 140 respectively for products A and B. The total fixed costs amount to Rs. 1,200 per week.

		Product A	Product B	Total Quantity available per week
Raw material	P	16	20	160
	Q	10	25	150
	R	4	0	32

Formulate a linear programming problem for the above.

Question:10

The budgeted data relating to two products manufactured by a company for a month are as under:

	Product A	Product B
Selling Price	300	200
Variable manufacturing costs	160	60
Sales commission	60	40

Each unit of product incurs costs in the company's two departments P and Q. The total capacity available for the month under review is budgeted to be 1,400 hours in department P and 2,000 hours in department Q. The capacity costs amount to Rs. 14,000 and Rs. 20,000 respectively per month for P and Q irrespective of the level of usage made of it. The number of hours required in each of these departments to complete one unit of output is as under:

		A	B
Department	P	2	4
Department	Q	5	4

The maximum output which the company can sell in the month is restricted to 400 units of either of the products.
You are required to formulate the Linear Programming (LP) model.

Question: 11

A manufacturing company produces two of product the SUPER and REGULAR. Resource requirements for production are given below in the table. There are 1,600 hours of assembly worker hours available per week, 700 hours of paint time and 300 hours of inspection time. Regular customers will demand at least 150 units of the REGULAR type and 90 units of the SUPER type.

Table

Product	Profit/ Contribution	Assembly time	Paint time	Inspection time
	Rs.	Hrs.	Hrs.	Hrs.
REGULAR	50	1.2	0.8	0.2
SUPER	75	1.6	0.9	0.2

Formulate the Linear Programming problem.

GRAPHICAL SOLUTION

1. Solve the LPP graphically: -
   Find the values of X1 and X2 that
   Maximize the objective function
   Z = 450X1 +550X2
   Subject to the conditions,
   6X1 + 4X2 >=120
   3X1 +6X2 <=180
   X1 >= 10
   X2 >= 10 and X1, X2 >= 0

2. Find the values of X1 and X2that
   Minimize the objective function
   Z = 30X1 + 20X2
   Subject to the conditions,
   X1 + X2 <=8
   6X1 + 4X2 >= 12
   5X1 + 8X2 >= 20 and X1 X2 >= 0

3. Minimize Z = 8X1 + 11X2
   subject to the conditions
   3X1 + 5X2 <= 15
   X1 + X2 >= 1
   2X1 – X2 = 2
   and X1, X2 >= 0
4. Find the values of X1 and X2
   that maximize the objective
   function
   Z = 2X1 + 3X2
   Subject to the conditions.
   4X1 + 3X2 <= 12
   7X1 + 3X2 >= 56
   X1, X2 >= 0

SIMPLEX

1. Solve the following L.P.P by the Simplex method.
   Maximize Z = 6X1 + 7X2
   Subject to: 2X1 +3X2 ≤ 12
   2X 1 + X2 ≤ 8
   X1 ≥ 0, X2 ≥ 0

2. Solve the following L.P.P by the simplex method.
   Maximize Z = 8X1 + 16X2
   Subject to: X1 + X2 ≤ 200
   X2 ≤ 125
   3X1 + 6X2 ≤ 900
   X1, X2 ≥ 0

3. Solve the following L.P.P by the simplex method.
   Minimize Z = 750Y2 + 200Y2 + 600Y3
   Subject to: Y1 +Y3 ≥ 3
   3/ 2Y1 + 5Y2 +Y3 ≥ 4
   Y1, Y2, Y3 ≥ 0

Dual

Q-1
Find the dual of the following problem and then obtain the solution.

Minimize $Z = 20\ X1 + 10X2$
Subject to: $X1 + X2 \geq 10$
$3X1 + 2X2 \geq 2\ 4$
$X1, X2 \geq 0$

RANDOM NUMBERS TABLE

8481	5016	0080	4376	2579	8293	5950	1048	0650	4135
0744	3447	6173	3288	6378	6704	0966	9986	5202	1728
5558	7239	2976	4836	6134	5120	1541	6514	3581	2079
9371	1463	2164	2301	3142	3866	8707	9980	2011	5111
3033	1660	6365	9054	1155	8844	4085	9589	2924	1725
1053	7320	6532	7214	8972	6466	1217	0100	1458	9416
4389	3504	4086	9434	0136	5695	6876	7937	5476	3396
2158	8854	9534	1196	4941	2697	7497	1149	1952	3482
6749	3676	4943	1406	8614	2060	6433	1660	8875	3194
2878	3447	4804	6761	5309	0636	0522	2004	3207	4684
0591	6549	2206	6185	6188	2649	2389	9483	0924	1389
1025	3438	0546	2545	1089	1280	6701	9742	3453	5573
4244	9217	1628	4524	0163	9895	9586	2083	8459	0644
1331	9032	1388	5661	0472	7128	1902	0343	7724	0528
8853	3490	2589	8744	1221	4667	8396	4779	9937	7206
5059	4192	6331	5485	5922	0982	9390	8993	3621	2602
0821	4340	3194	0118	4773	8668	1891	7989	9190	2296
5262	1746	7108	6496	2570	4243	5029	8949	4989	5008
1210	1858	9365	6562	0269	9923	1796	6626	8591	1990
3642	6629	5775	3219	8801	4047	6861	0765	2379	3494
9598	5322	3747	0363	5995	5504	6804	7033	0957	9556
3894	3173	2853	9312	2498	8878	4956	8748	6247	6673
3603	3011	6762	0848	8316	3485	6388	8925	3790	0898
1121	2978	6313	5857	8457	1395	7240	8630	3895	6348
1930	4583	4227	4120	6893	7005	2264	6067	5627	7985
6309	9158	2830	3262	9809	4606	8669	1154	5841	7695
4460	3143	5383	0327	9668	1697	8335	0860	2188	1908
8371	5095	7273	1866	4193	4163	2035	2812	4996	7142
9397	5540	9298	9076	1299	0669	0088	1809	0631	3162
9304	1468	4013	7465	0861	6787	3581	7977	8409	4708
5606	2435	8546	3209	4802	6690	8527	2210	6706	1930
6693	8333	082	7546	2910	8553	8725	1237	4423	1570
0556	7715	8994	4245	1540	8150	3889	5273	6977	2703
6973	9299	4959	7146	1426	7086	8743	6982	5547	3394
4920	1223	5208	6661	4907	1102	0501	3625	8513	3192
0132	0098	8241	0858	7627	4174	1170	3142	2455	4891
4051	3101	9854	4488	6931	3266	3147	2500	8011	8848

0267	5612	5504	7917	7928	8034	9989	4353	2675	9497
0609	9469	3149	4086	8911	8547	3518	9349	1836	0548
2593	1666	5750	5105	4287	4380	7860	7792	1625	7659
8812	9491	2602	4100	4962	1037	9778	1778	4223	3193
3540	5985	0019	7155	1471	1851	8682	9957	3772	4706
9535	5375	1239	1624	5378	6803	7177	7911	4660	5669
3174	7677	8282	6669	5879	7874	9931	6581	9784	2607
8864	4760	1129	6205	4949	4205	0222	7479	6470	8194
5245	7341	0593	5656	6799	3071	1751	4339	5630	9496
5468	6038	4511	1440	2135	5777	9903	1048	6726	8602
3951	78928	6818	4161	4840	1392	1323	5014	7538	9854
7319	4064	4024	5401	2834	7518	3978	3742	1005	4619
5892	8731	6269	5189	2071	4084	9789	3620	9819	4548

LOGARITHMS

	0	1	2	3	4	5	6	7	8	9	Mean Differences								
											1	2	3	4	5	6	7	8	9
10	0000	0043	0086	0128	0170	0212	0253	0294	0334	0374	4	8	12	17	21	25	29	33	37
11	0414	0453	0492	0531	0569	0607	0645	0682	0719	0755	4	8	11	15	19	23	26	30	34
12	0792	0828	0864	0899	0934	0969	1004	1038	1072	1106	3	7	10	14	17	21	24	28	31
13	1139	1173	1206	1239	1271	1303	1335	1367	1399	1430	3	6	10	13	16	19	23	26	29
14	1461	1492	1523	1553	1584	1614	1644	1673	1703	1732	3	6	9	12	15	18	21	24	27
15	1761	1790	1818	1847	1875	1903	1931	1959	1987	2014	3	6	8	11	14	17	20	22	25
16	2041	2068	2095	2122	2148	2175	2201	2227	2253	2279	3	5	8	11	13	16	18	21	24
17	2304	2330	2355	2380	2405	2430	2455	2480	2504	2529	2	5	7	10	12	15	17	20	22
18	2553	2577	2601	2625	2648	2672	2695	2718	2742	2765	2	5	7	9	12	14	16	19	21
19	2788	2810	2833	2856	2878	2900	2923	2945	2967	2989	2	4	7	9	11	13	16	18	20
20	3010	3032	3054	3075	3096	3118	3139	3160	3181	3201	2	4	6	8	11	13	15	17	19
21	3222	3243	3263	3284	3304	3324	3345	3365	3385	3404	2	4	6	8	10	12	14	16	18
22	3424	3444	3464	3483	3502	3522	3541	3560	3579	3598	2	4	6	8	10	12	14	15	17
23	3617	3636	3655	3674	3692	3711	3729	3747	3766	3784	2	4	6	7	9	11	13	15	17
24	3802	3820	3838	3856	3874	3892	3909	3927	3945	3962	2	4	5	7	9	11	12	14	16
25	3979	3997	4014	4031	4048	4065	4082	4099	4116	4133	2	3	5	7	9	10	12	14	15
26	4150	4166	4183	4200	4216	4232	4249	4265	4281	4298	2	3	5	7	8	10	11	13	15
27	4314	4330	4346	4362	4378	4393	4409	4425	4440	4456	2	3	5	6	8	9	11	13	14
28	4472	4487	4502	4518	4533	4548	4564	4579	4594	4609	2	3	5	6	8	9	11	12	14
29	4624	4639	4654	4669	4683	4698	4713	4728	4742	4757	1	3	4	6	7	9	10	12	13
30	4771	4786	4800	4814	4829	4843	4857	4871	4886	4900	1	3	4	6	7	9	10	11	13
31	4914	4928	4942	4955	4969	4983	4997	5011	5024	5038	1	3	4	6	7	8	10	11	12
32	5051	5065	5079	5092	5105	5119	5132	5145	5159	5172	1	3	4	5	7	8	9	11	12
33	5185	5198	5211	5224	5237	5250	5263	5276	5289	5302	1	3	4	5	6	8	9	10	12
34	5315	5328	5340	5353	5366	5378	5391	5403	5416	5428	1	3	4	5	6	8	9	10	11
35	5441	5453	5465	5478	5490	5502	5514	5527	5539	5551	1	2	4	5	6	7	9	10	11
36	5563	5575	5587	5599	5611	5623	5635	5647	5658	5670	1	2	4	5	6	7	8	10	11
37	5682	5694	5705	5717	5729	5740	5752	5763	5775	5786	1	2	3	5	6	7	8	9	10
38	5798	5809	5821	5832	5843	5855	5866	5877	5888	5899	1	2	3	5	6	7	8	9	10
39	5911	5922	5933	5944	5955	5966	5977	5988	5999	6010	1	2	3	4	5	7	8	9	10
40	6021	6031	6042	6053	6064	6075	6085	6096	6107	6117	1	2	3	4	5	6	8	9	10
41	6128	6138	6149	6160	6170	6180	6191	6201	6212	6222	1	2	3	4	5	6	7	8	9
42	6232	6243	6253	6263	6274	6284	6294	6304	6314	6325	1	2	3	4	5	6	7	8	9
43	6335	6345	6355	6365	6375	6385	6395	6405	6415	6425	1	2	3	4	5	6	7	8	9
44	6435	6444	6454	6464	6474	6484	6493	6503	6513	6522	1	2	3	4	5	6	7	8	9
45	6532	6542	6551	6561	6571	6580	6590	6599	6609	6618	1	2	3	4	5	6	7	8	9
46	6628	6637	6646	6656	6665	6675	6684	6693	6702	6712	1	2	3	4	5	6	7	7	8
47	6721	6730	6739	6749	6758	6767	6776	6785	6794	6803	1	2	3	4	5	5	6	7	8

48	6812	6821	6830	6839	6848	6857	6866	6875	6884	6893	1	2	3	4	4	5	6	7	8
49	6902	6911	6920	6928	6937	6946	6955	6964	6972	6981	1	2	3	4	4	5	6	7	8
50	6990	6998	7007	7016	7024	7033	7042	7050	7059	7067	1	2	3	3	4	5	6	7	8
51	7076	7084	7093	7101	7110	7118	7126	7135	7143	7152	1	2	3	3	4	5	6	7	8
52	7160	7168	7177	7185	7193	7202	7210	7218	7226	7235	1	2	3	3	4	5	6	7	7
53	7243	7251	7259	7267	7275	7284	7292	7300	7308	7316	1	2	3	3	4	5	6	6	7
54	7324	7332	7340	7348	7356	7364	7372	7380	7388	7396	1	2	3	3	4	5	6	6	7

LOGARITHMS

	0	1	2	3	4	5	6	7	8	9	Mean Differences								
											1	2	3	4	5	6	7	8	9
55	7404	7412	7419	7427	7435	7443	7451	7459	7466	7474	1	2	2	3	4	5	5	6	7
56	7482	7490	7497	7505	7513	7520	7528	7536	7543	7551	1	2	2	3	4	5	5	6	7
57	7559	7566	7574	7582	7589	7597	7604	7612	7619	7627	1	2	2	3	4	5	5	6	7
58	7634	7642	7649	7657	7664	7672	7679	7686	7694	7701	1	1	2	3	4	4	5	6	7
59	7709	7716	7723	7731	7738	7745	7752	7760	7767	7774	1	1	2	3	4	4	5	6	7
60	7782	7789	7796	7803	7810	7818	7825	7832	7839	7846	1	1	2	3	4	4	5	6	6
61	7853	7860	7868	7875	7882	7889	7896	7903	7910	7917	1	1	2	3	4	4	5	6	6
62	7924	7931	7938	7945	7952	7959	7966	7973	7980	7987	1	1	2	3	3	3	5	6	6
63	7993	8000	8007	8014	8021	8028	8035	8041	8048	8055	1	1	2	3	3	4	5	5	6
64	8062	8069	8075	8082	8089	8096	8102	8109	8116	8122	1	1	2	3	3	4	5	5	6
65	8129	8136	8142	8149	8156	8162	8169	8176	8182	8189	1	1	2	3	3	4	5	5	6
66	8195	8202	8209	8215	8222	8228	8235	8241	8248	8254	1	1	2	3	3	4	5	5	6
67	8261	8267	8274	8280	8287	8293	8299	8306	8312	8319	1	1	2	3	3	4	5	5	6
68	8325	8331	8338	8344	8351	8357	8363	8370	8376	8382	1	1	2	3	3	4	4	5	6
69	8388	8395	8401	8407	8414	8420	8426	8432	8439	8445	1	1	2	2	3	4	4	5	6
70	8451	8457	8463	8470	8476	8482	8488	8494	8500	8506	1	1	2	2	3	4	4	5	6
71	8513	8519	8525	8531	8537	8543	8549	8555	8561	8567	1	1	2	2	3	4	4	5	5
72	8573	8579	8585	8591	8597	8603	8609	8615	8621	8627	1	1	2	2	3	4	4	5	5
73	8633	8639	8645	8651	8657	8663	8669	8675	8681	8686	1	1	2	2	3	4	4	5	5
74	8692	8698	8704	8710	8716	8722	8727	8733	8739	8745	1	1	2	2	3	4	4	5	5
75	8751	8756	8762	8768	8774	8779	8785	8791	8797	8802	1	1	2	2	3	3	4	5	5
76	8808	8814	8820	8825	8831	8837	8842	8848	8854	8859	1	1	2	2	3	3	4	5	5
77	8865	8871	8876	8882	8887	8893	8899	8904	8910	8915	1	1	2	2	3	3	4	4	5
78	8921	8927	8932	8938	8943	8949	8954	8960	8965	8971	1	1	2	2	3	3	4	4	5
79	8976	8982	8987	8993	8998	9004	9009	9015	9020	9025	1	1	2	2	3	3	4	4	5
80	9031	9036	9042	9047	9053	9058	9063	9069	9074	9079	1	1	2	2	3	3	4	4	5
81	9085	9090	9096	9101	9106	9112	9117	9122	9128	9133	1	1	2	2	3	3	4	4	5
82	9138	9143	9149	9154	9159	9165	9170	9175	9180	9186	1	1	2	2	3	3	4	4	5
83	9191	9196	9201	9206	9212	9217	9222	9227	9232	9238	1	1	2	2	3	3	4	4	5
84	9243	9248	9253	9258	9263	9269	9274	9279	9284	9289	1	1	2	2	3	3	4	4	5
85	9294	9299	9304	9309	9315	9320	9325	9330	9335	9340	1	1	2	2	3	3	4	4	5
86	9345	9350	9355	9360	9365	9370	9375	9380	9385	9390	1	1	2	2	3	3	4	4	5
87	9395	9400	9405	9410	9415	9420	9425	9430	9435	9440	0	1	1	2	2	3	3	4	4
88	9445	9450	9455	9460	9465	9469	9474	9479	9484	9489	0	1	1	2	2	3	3	4	4
89	9494	9499	9504	9509	9513	9518	9523	9528	9533	9538	0	1	1	2	2	3	3	4	4
90	9542	9547	9552	9557	9562	9566	9571	9576	9581	9586	0	1	1	2	2	3	3	4	4
91	9590	9595	9600	9605	9609	9614	9619	9624	9628	9633	0	1	1	2	2	3	3	4	4
92	9638	9643	9647	9652	9657	9661	9666	9671	9675	9680	0	1	1	2	2	3	3	4	4

93	9685	9689	9694	9699	9703	9708	9713	9717	9722	9727	0	1	1	2	2	3	3	4	4
94	9731	9736	9741	9745	9750	9754	9759	9763	9768	9773	0	1	1	2	2	3	3	4	4
95	9777	9782	9786	9791	9795	9800	9805	9809	9814	9818	0	1	1	2	2	3	3	4	4
96	9823	9827	9832	9836	9841	9845	9850	9854	9859	9863	0	1	1	2	2	3	3	4	4
97	9868	9872	9877	9881	9886	9890	9894	9899	9903	9908	0	1	1	2	2	3	3	4	4
98	9912	9917	9921	9926	9930	9934	9939	9943	9948	9952	0	1	1	2	2	3	3	4	4
99	9956	9961	9965	9969	9974	9978	9983	9987	9991	9996	0	1	1	2	2	3	3	3	4

ANTILOGARITHMS

	0	1	2	3	4	5	6	7	8	9	Mean Differences								
											1	2	3	4	5	6	7	8	9
00	1000	1002	1005	1007	1009	1012	1014	1016	1019	1021	0	0	1	1	1	1	2	2	2
01	1023	1026	1028	1030	1033	1035	1038	1040	1042	1045	0	0	1	1	1	1	2	2	2
02	1047	1050	1052	1054	1057	1059	1062	1064	1067	1069	0	0	1	1	1	1	2	2	2
03	1072	1074	1076	1079	1081	1084	1086	1089	1091	1094	0	0	1	1	1	1	2	2	2
04	1096	1099	1102	1104	1107	1109	1112	1114	1117	1119	0	1	1	1	1	2	2	2	2
05	1122	1125	1127	1130	1132	1135	1138	1140	1143	1146	0	1	1	1	1	2	2	2	2
06	1148	1151	1153	1156	1159	1161	1164	1167	1169	1172	0	1	1	1	1	2	2	2	2
07	1175	1178	1180	1183	1186	1189	1191	1194	1197	1199	0	1	1	1	1	2	2	2	2
08	1202	1205	1208	1211	1213	1216	1219	1222	1225	1227	0	1	1	1	1	2	2	2	3
09	1230	1233	1236	1239	1242	1245	1247	1250	1253	1256	0	1	1	1	1	2	2	2	3
10	1259	1262	1265	1268	1271	1274	1276	1279	1282	1285	0	1	1	1	1	2	2	2	3
11	1288	1291	1294	1297	1300	1303	1306	1309	1312	1315	0	1	1	1	2	2	2	2	3
12	1318	1321	1324	1327	1330	1334	1337	1340	1343	1346	0	1	1	1	2	2	2	2	3
13	1349	1352	1355	1358	1361	1365	1368	1371	1374	1377	0	1	1	1	2	2	2	3	3
14	1380	1384	1387	1390	1393	1396	1400	1403	1406	1409	0	1	1	1	2	2	2	3	3
15	1413	1416	1419	1422	1426	1429	1432	1435	1439	1442	0	1	1	1	2	2	2	3	3
16	1445	1449	1452	1455	1459	1462	1466	1469	1472	1476	0	1	1	1	2	2	2	3	3
17	1479	1483	1486	1489	1493	1496	1500	1503	1507	1510	0	1	1	1	2	2	2	3	3
18	1514	1517	1521	1524	1528	1531	1535	1538	1542	1545	0	1	1	1	2	2	2	3	3
19	1549	1552	1556	1560	1563	1567	1570	1574	1578	1581	0	1	1	1	2	2	3	3	3
20	1585	1589	1592	1596	1600	1603	1607	1611	1614	1618	0	1	1	1	2	2	3	3	3
21	1622	1626	1629	1633	1637	1641	1644	1648	1652	1656	0	1	1	2	2	2	3	3	3
22	1660	1663	1667	1671	1675	1679	1683	1687	1690	1694	0	1	1	2	2	2	3	3	3
23	1698	1702	1706	1710	1714	1718	1722	1726	1730	1734	0	1	1	2	2	2	3	3	4
24	1738	1742	1746	1750	1754	1758	1762	1766	1770	1774	0	1	1	2	2	2	3	3	4
25	1778	1782	1786	1791	1795	1799	1803	1807	1811	1816	0	1	1	2	2	2	3	3	4
26	1820	1824	1828	1832	1837	1841	1845	1849	1854	1858	0	1	1	2	2	3	3	3	4
27	1862	1866	1871	1875	1879	1884	1888	1892	1897	1901	0	1	1	2	2	3	3	3	4
28	1905	1910	1914	1919	1923	1928	1932	1936	1941	1945	0	1	1	2	2	3	3	4	4
29	1950	1954	1959	1963	1968	1972	1977	1982	1986	1991	0	1	1	2	2	3	3	4	4
30	1995	2000	2004	2009	2014	2018	2023	2028	2032	2037	0	1	1	2	2	3	3	4	4
31	2042	2046	2051	2056	2061	2065	2070	2075	2080	2084	0	1	1	2	2	3	3	4	4
32	2089	2094	2099	2104	2109	2113	2118	2123	2128	2133	0	1	1	2	2	3	3	4	4
33	2138	2143	2148	2153	2158	2163	2168	2173	2178	2183	0	1	1	2	2	3	3	4	4
34	2188	2193	2198	2203	2208	2213	2218	2223	2228	2234	1	1	2	2	3	3	4	4	5
35	2239	2244	2249	2254	2259	2265	2270	2275	2280	2286	1	1	2	2	3	3	4	4	5
36	2291	2296	2301	2307	2312	2317	2323	2328	2333	2339	1	1	2	2	3	3	4	4	5
37	2344	2350	2355	2360	2366	2371	2377	2382	2388	2393	1	1	2	2	3	3	4	4	5

38	2399	2404	2410	2415	2421	2427	2432	2438	2443	2449	1	1	2	2	3	3	4	4	5
39	2455	2460	2466	2472	2477	2483	2489	2495	2500	2506	1	1	2	2	3	3	4	5	5
40	2512	2518	2523	2529	2535	2541	2547	2553	2559	2564	1	1	2	2	3	4	4	5	5
41	2570	2576	2582	2588	2594	2600	2606	2612	2618	2624	1	1	2	2	3	4	4	5	5
42	2630	2636	2642	2649	2655	2661	2667	2673	2679	2685	1	1	2	2	3	4	4	5	6
43	2692	2698	2704	2710	2716	2723	2729	2735	2742	2748	1	1	2	3	3	4	4	5	6
44	2754	2761	2767	2773	2780	2786	2793	2799	2805	2812	1	1	2	3	3	4	4	5	6
45	2818	2825	2831	2838	2844	2851	2858	2864	2871	2877	1	1	2	3	3	4	5	5	6
46	2884	2891	2897	2904	2911	2917	2924	2931	2938	2944	1	1	2	3	3	4	5	5	6
47	2951	2958	2965	2972	2979	2985	2992	2999	3006	3013	1	1	2	3	3	4	5	5	6
48	3020	3027	3034	3041	3048	3055	3062	3069	3076	3083	1	1	2	3	4	4	5	6	6
49	3090	3097	3105	3112	3119	3126	3133	3141	3148	3155	1	1	2	3	4	4	5	6	6

ANTILOGARITHMS

	0	1	2	3	4	5	6	7	8	9	Mean Differences								
											1	2	3	4	5	6	7	8	9
50	3162	3170	3177	3184	3192	3199	3206	3214	3221	3228	1	1	2	3	4	4	5	6	7
51	3236	3243	3251	3258	3266	3273	3281	3289	3296	3304	1	2	2	3	4	5	5	6	7
52	3311	3319	3327	3334	3342	3350	3357	3365	3373	3381	1	2	2	3	4	5	5	6	7
53	3388	3396	3404	3412	3420	3428	3436	3443	3451	3459	1	2	2	3	4	5	6	6	7
54	3467	3475	3483	3491	3499	3508	3516	3524	3532	3540	1	2	2	3	4	5	6	6	7
55	3548	3556	3565	3573	3581	3589	3597	3606	3614	3622	1	2	2	3	4	5	6	7	7
56	3631	3639	3648	3656	3664	3673	3681	3690	3698	3707	1	2	3	3	4	5	6	7	8
57	3751	3724	3733	3741	3750	3758	3767	3776	3784	3793	1	2	3	3	4	5	6	7	8
58	3802	3811	3819	3828	3837	3846	3855	3864	3873	3882	1	2	3	4	4	5	6	7	8
59	3890	3899	3908	3917	3926	3936	3945	3954	3963	3972	1	2	3	4	5	5	6	7	8
60	3981	3990	3999	4009	4018	4027	4036	4046	4055	4064	1	2	3	4	5	6	6	7	8
61	4074	4083	4093	4102	4111	4121	4130	4140	4150	4159	1	2	3	4	5	6	7	8	9
62	4169	4178	4188	4198	4207	4217	4227	4236	4246	4256	1	2	3	4	5	6	7	8	9
63	4266	4276	4285	4295	4305	4315	4325	4335	4345	4355	1	2	3	4	5	6	7	8	9
64	4365	4375	4385	4395	4406	4416	4426	4436	4446	4457	1	2	3	4	5	6	7	8	9
65	4467	4477	4487	4498	4508	4519	4529	4539	4550	4560	1	2	3	4	5	6	7	8	9
66	4571	4581	4592	4603	4613	4624	4634	4645	4656	4667	1	2	3	4	5	6	7	9	10
67	4677	4688	4699	4710	4721	4732	4742	4753	4764	4775	1	2	3	4	5	7	8	9	10
68	4786	4797	4808	4819	4831	4842	4853	4864	4875	4887	1	2	3	4	6	7	8	9	10
69	4898	4909	4920	4932	4943	4955	4966	4977	4989	5000	1	2	3	5	6	7	8	9	10
70	5012	5023	5035	5047	5058	5070	5082	5093	5105	5117	1	2	4	5	6	7	8	9	11
71	5129	5140	5152	5164	5176	5188	5200	5212	5224	5236	1	2	4	5	6	7	8	10	11
72	5248	5260	5272	5284	5297	539	5321	5333	5346	5358	1	2	4	5	6	7	9	10	11
73	5370	5383	5395	5408	5420	5433	5445	5458	5470	5483	1	3	4	5	6	8	9	10	11
74	5495	5508	5521	5534	5546	5559	5572	5585	5598	5610	1	3	4	5	6	8	9	10	12
75	5623	5636	5649	5662	5675	5689	5702	5715	5728	5741	1	3	4	5	7	8	9	10	12
76	5754	5768	5781	5794	5808	5821	5834	5848	5861	5875	1	3	4	5	7	8	9	11	12
77	5888	5902	5916	5929	5943	5957	5970	5984	5998	6012	1	3	4	5	7	8	10	11	12
78	6026	6039	6053	6067	6081	6095	6109	6124	6138	6152	1	3	4	6	7	8	10	11	13
79	6166	6180	6194	6209	6223	6237	6252	6266	6281	6295	1	3	4	6	7	9	10	11	13
80	6310	6324	6339	6353	6368	6383	6397	6412	6427	6442	1	3	4	6	7	9	10	12	13
81	6457	6471	6486	6501	6516	6531	6546	6561	6577	6592	2	3	5	6	8	9	11	12	14
82	6607	6622	6637	6653	6668	6683	6699	6714	6730	6745	2	3	5	6	8	9	11	12	14
83	6761	6776	6792	6808	6823	6839	6855	6871	6887	6902	2	3	5	6	8	9	11	13	14
84	6918	6934	6950	6966	6982	6998	7015	7031	7047	7063	2	3	5	6	8	10	11	13	15

85	7079	7096	7112	7129	7145	7161	7178	7194	7211	7228	2	3	5	7	8	10	12	13	15
86	7244	7261	7278	7295	7311	7328	7345	7362	7379	7396	2	3	5	7	8	10	12	13	15
87	7413	7430	7447	7464	7482	7499	7516	7534	7551	7568	2	3	5	7	9	10	12	14	16
88	7586	7603	7621	7638	7656	7674	7691	7709	7727	7745	2	4	5	7	9	11	12	14	16
89	7762	7780	7798	7816	7834	7852	7870	7889	7907	7925	2	4	5	7	9	11	13	14	16
90	7943	7962	7980	7998	8017	8035	8054	8072	8091	8110	2	4	6	7	9	11	13	15	17
91	8128	8147	8166	8185	8204	8222	8241	8260	8279	8299	2	4	6	8	9	11	13	15	17
92	8318	8337	8356	8375	8395	8414	8433	8453	8472	8492	2	4	6	8	10	12	14	15	17
93	8511	8531	8551	8570	8590	8610	8630	8650	8670	8690	2	4	6	8	10	12	14	16	18
94	8710	8730	8750	8770	8790	8810	8831	8851	8872	8892	2	4	6	8	10	12	14	16	18
95	8913	8933	8954	8974	8995	9016	9036	9057	9078	9099	2	4	6	8	10	12	15	17	19
96	9120	9141	9162	9183	9204	9226	9247	9268	9290	9311	2	4	6	8	11	13	15	17	19
97	9333	9354	9376	9397	9419	9441	9462	9484	9506	9528	2	4	7	9	11	13	15	17	20
98	9550	9572	9594	9616	9638	9661	9683	9705	9727	9750	2	4	7	9	11	13	16	18	20
99	9772	9795	9817	9840	9863	9886	9908	9931	9954	9977	2	5	7	9	11	14	16	18	20

NORMAL DISTRIBUTION TABLE

X	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.00000	0.00399	0.00798	0.01197	0.01595	0.01994	0.02392	0.02790	0.03188	0.03586
0.1	0.03983	0.04380	0.04776	0.05172	0.05567	0.05962	0.06356	0.06749	0.07142	0.07535
0.2	0.07926	0.08317	0.08706	0.09095	0.09483	0.09871	0.10257	0.10642	0.11026	0.11409
0.3	0.11791	0.12172	0.12552	0.12930	0.13307	0.13683	0.14058	0.14431	0.14803	0.15173
0.4	0.15542	0.15910	0.16276	0.16640	0.17003	0.17364	0.17724	0.18082	0.18439	0.18793
0.5	0.19146	0.19497	0.19847	0.20194	0.20540	0.20884	0.21226	0.21566	0.21904	0.22240
0.6	0.22575	0.22907	0.23237	0.23565	0.23891	0.24215	0.24537	0.24857	0.25175	0.25490
0.7	0.25804	0.26115	0.26424	0.26730	0.27035	0.27337	0.27637	0.27935	0.28230	0.28524
0.8	0.28814	0.29103	0.29389	0.29673	0.29955	0.30234	0.30511	0.30785	0.31057	0.31327
0.9	0.31594	0.31859	0.32121	0.32381	0.32639	0.32894	0.33147	0.33398	0.33646	0.33891
1.0	0.34134	0.34375	0.34614	0.34849	0.35083	0.35314	0.35543	0.35769	0.35993	0.36214
1.1	0.36433	0.36650	0.36864	0.37076	0.37286	0.37493	0.37698	0.37900	0.38100	0.38298
1.2	0.38493	0.38686	0.38877	0.39065	0.39251	0.39435	0.39617	0.39796	0.39973	0.40147
1.3	0.40320	0.40490	0.40658	0.40824	0.40988	0.41149	0.41308	0.41466	0.41621	0.41774
1.4	0.41924	0.42073	0.42220	0.42364	0.42507	0.42647	0.42785	0.42922	0.43056	0.43189
1.5	0.43319	0.43448	0.43574	0.43699	0.43822	0.43943	0.44062	0.44179	0.44295	0.44408
1.6	0.44520	0.44630	0.44738	0.44845	0.44950	0.45053	0.45154	0.45254	0.45352	0.45449
1.7	0.45543	0.45637	0.45728	0.45818	0.45907	0.45994	0.46080	0.46164	0.46246	0.46327
1.8	0.46407	0.46485	0.46562	0.46638	0.46712	0.46784	0.46856	0.46926	0.46995	0.47062
1.9	0.47128	0.47193	0.47257	0.47320	0.47381	0.47441	0.47500	0.47558	0.47615	0.47670
2.0	0.47725	0.47778	0.47831	0.47882	0.47932	0.47982	0.48030	0.48077	0.48124	0.48169
2.1	0.48214	0.48257	0.48300	0.48341	0.48382	0.48422	0.48461	0.48500	0.48537	0.48574
2.2	0.48610	0.48645	0.48679	0.48713	0.48745	0.48778	0.48809	0.48840	0.48870	0.48899
2.3	0.48928	0.48956	0.48983	0.49010	0.49036	0.49061	0.49086	0.49111	0.49134	0.49158
2.4	0.49180	0.49202	0.49224	0.49245	0.49266	0.49286	0.49305	0.49324	0.49343	0.49361
2.5	0.49379	0.49396	0.49413	0.49430	0.49446	0.49461	0.49477	0.49492	0.49506	0.49520
2.6	0.49534	0.49547	0.49560	0.49573	0.49585	0.49598	0.49609	0.49621	0.49632	0.49643
2.7	0.49653	0.49664	0.49674	0.49683	0.49693	0.49702	0.49711	0.49720	0.49728	0.49736
2.8	0.49744	0.49752	0.49760	0.49767	0.49774	0.49781	0.49788	0.49795	0.49801	0.49807
2.9	0.49813	0.49819	0.49825	0.49831	0.49836	0.79841	0.49846	0.49851	0.49856	0.49861
3.0	0.49865	0.49869	0.49874	0.49878	0.49882	0.49886	0.49889	0.49893	0.49896	0.49900
3.1	0.49903	0.49906	0.49910	0.49913	0.49916	0.49918	0.49921	0.49924	0.49926	0.49929
3.2	0.49931	0.49934	0.49936	0.49938	0.49940	0.49942	0.49944	0.49946	0.49948	0.49950
3.3	0.49952	0.49953	0.49955	0.49957	0.49958	0.49960	0.49961	0.49962	0.49964	0.49965
3.4	0.49966	0.49968	0.49969	0.49970	0.49971	0.49972	0.49973	0.49974	0.49975	0.49976
3.5	0.49977	0.49978	0.49978	0.49979	0.49980	0.49981	0.49981	0.49982	0.49983	0.49983
3.6	0.49984	0.49985	0.49985	0.49986	0.49986	0.49987	0.49987	0.49988	0.49988	0.49989
3.7	0.49989	0.49990	0.49990	0.49990	0.49991	0.49991	0.49992	0.49992	0.49992	0.49992
3.8	0.49993	0.49993	0.49993	0.49994	0.49994	0.49994	0.49994	0.49995	0.49995	0.49995
3.9	0.49995	0.49995	0.49996	0.49996	0.49996	0.49996	0.49996	0.49996	0.4997	0.49997
4.0	0.49997	0.49997	0.49997	0.49997	0.49997	0.49997	0.49998	0.49998	0.49998	0.49998

NOTES