

Question Paper

Quantitative Methods (CFA530): January 2008

- Answer all 83 questions.
- Marks are indicated against each question.

Total Marks : 100

1. A company manufactures two types of printed circuits. The requirements of transistors, resistors and capacitors for each type of printed circuits along with other data are given below:

[<Answer>](#)

	Circuit		Stock (available) (units)
	A	B	
Transistor	15	10	180
Resistor	10	20	200
Capacitor	15	20	210
Profit	Rs.5	Rs.8	

Find the maximum profit of the company by using graphical method.

- (a) 35
(b) 48
(c) 69
(d) 73
(e) 82.

(3marks)

[<Answer>](#)

2. The annual income of the 70 employees of a company is as given below:

Income (Rs. in '000)	10 – 20	20 – 30	30 – 40	40 – 50	50 – 60	60 – 70
No. of persons	4	7	16	20	15	8

The average annual income is

- (a) Rs.15,180
(b) Rs.22,850
(c) Rs.34,250
(d) Rs.43,430
(e) Rs.51,920.

(2marks)

[<Answer>](#)

3. Find the combined mean of the following data:

Particulars	Firm I	Firm II	Firm III
Mean wage (Rs.)	12	15	18
No. of workers	20	40	30

- (a) Rs.10.32
(b) Rs.12.48
(c) Rs.15.33
(d) Rs.17.19
(e) Rs.19.74.

(1 mark)

[<Answer>](#)

4. Find the median of the following data.

17	19	21	13	16	18	24	22	20
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- (a) 13
(b) 16
(c) 18
(d) 19
(e) 21.

(1 mark)

5. If $Q_1 = 19$ and $Q_3 = 33$, then the coefficient of quartile deviation is

[<Answer>](#)

- (a) 0.1345
- (b) 0.2692
- (c) 0.3456
- (d) 0.4215
- (e) 0.5714.

(1 mark)

6. If mean = 39.51 and median = 39.69, then the value of empirical mode is

[<Answer>](#)

- (a) 38.75
- (b) 39.31
- (c) 40.05
- (d) 41.12
- (e) 42.35.

(1 mark)

7. The market value of an investment has undergone the following growth rates in different years:

[<Answer>](#)

Year	1	2	3	4	5	6	7
Growth rate (%)	5	8	-10	-4	6	10	7

The average growth rate over the seven year period is

- (a) 2.91%
- (b) 4.55%
- (c) 6.13%
- (d) 8.46%
- (e) 9.26%.

(1 mark)

8. If a couple travels 900 km by train at a speed of 60 kmph, 300 km by ship at an average speed of 25 kmph, 400 km by an aeroplane at 350 kmph and finally 15 km by car at 25 kmph, then the average speed of their entire journey is

[<Answer>](#)

- (a) 32.65 kmph
- (b) 41.45 kmph
- (c) 50.95 kmph
- (d) 56.19 kmph
- (e) 58.47 kmph.

(1 mark)

9. If the mean of 40 terms is 25 and standard deviation is 4, then the sum and sum of the squares of all terms respectively are

[<Answer>](#)

- (a) 1,000 and 25,640
- (b) 1,012 and 12,645
- (c) 852 and 14,156
- (d) 457 and 18,789
- (e) 360 and 23,645.

(2marks)

10. If $f(x) = 4x^3 - 5x^2 + 8$ for all $x \geq 0$, then $f(3) - f(2)$ is

[<Answer>](#)

- (a) 71
- (b) 20
- (c) 91
- (d) 51
- (e) 99.

(1 mark)

[<Answer>](#)

11. $\frac{\log_a x + \log_b x}{\log_a x - \log_b x}$ is equal to

- (a) $\frac{\log_a a}{2 \frac{1}{b}}$
- (b) $\frac{\log_b a}{2 \frac{1}{a}}$
- (c) $\log_a ab$
- (d) $\log_b ab$
- (e) $\log_b a$

(1 mark)

[<Answer>](#)

12. In an examination a candidate is required to answer 6 out of 10 questions, which are divided in two groups each containing 5 questions. If the candidate is not permitted to answer more than four questions from each group, in how many ways can he make his choice?

- (a) 120
- (b) 200
- (c) 330
- (d) 460
- (e) 680.

(2marks)

[<Answer>](#)

13. A man borrows Rs.1,200 and agrees to pay back with a total interest of Rs.110 in 10 installments, each installment being less than the immediately preceding one by Rs.15. What should be his first installment?

- (a) Rs.158.25
- (b) Rs.198.50
- (c) Rs.245.75
- (d) Rs.278.00
- (e) Rs.302.50.

(1 mark)

[<Answer>](#)

14. $\sigma = 18$, $n = 36$ and $\bar{x} = 25$

From the above information, the upper limit of 95% confidence interval is

- (a) 19.12
- (b) 23.56
- (c) 30.88
- (d) 35.73
- (e) 42.15.

(1 mark)

[<Answer>](#)

15. The value of $\log_3 \frac{81}{27}$ is

- (a) 1
- (b) 0
- (c) 3
- (d) 9
- (e) 7.

(1 mark)

[<Answer>](#)

16. The inverse function of $y = f(x) = 7x + 9$ is

- (a) $y + 9$
- (b) $\frac{y - 9}{7}$
- (c) $\frac{y + 9}{7}$
- (d) $\frac{-y + 9}{7}$
- (e) $\frac{7}{y} + \frac{1}{9}$.

(1 mark)

[<Answer>](#)

17. The roots of the equation $x^2 - 8x + 15 = 0$ are

- (a) 5, -3
- (b) -5, -3
- (c) -5, 3
- (d) 5, 3
- (e) 5, 1.

(1 mark)

[<Answer>](#)

18. Which of the following relationships is/are **true** between Arithmetic Mean (A.M), Geometric Mean (G.M) and Harmonic Mean (H.M)?

- I. $A.M \leq G.M \geq H.M$.
 - II. $A.M \geq G.M \geq H.M$.
 - III. $G.M \geq A.M \geq H.M$.
- (a) Only (I) above
 - (b) Only (II) above
 - (c) Both (I) and (II) above
 - (d) Both (I) and (III) above
 - (e) Both (II) and (III) above.

(1 mark)

[<Answer>](#)

19. The best measure of central tendency is

- (a) Median
- (b) Mode
- (c) Geometric mean
- (d) Arithmetic mean
- (e) Harmonic mean.

(1 mark)

[<Answer>](#)

20. Which of following statements is/are **false** regarding interpolation?

- I. It is a statistical technique.
 - II. It is a method of statistical estimation.
 - III. The figures obtained by interpolation are fairly correct as long as underlying assumptions hold good.
 - IV. It allows us to make insertions.
 - V. It allows us to forecast a value for some future date.
- (a) Only (II) above
 - (b) Only (IV) above
 - (c) Only (V) above
 - (d) Both (I) and (III) above
 - (e) Both (II) and (V) above.

(1 mark)

21. At $K = 19\%$, present value of cash flow of a project is equal to Rs.33.42 and at $K = 20\%$ the present value of cash flow is Rs.29.82. At what percent the present value of cash flow is Rs.30?

[<Answer>](#)

- (a) 19.18%
- (b) 19.24%
- (c) 19.32%
- (d) 19.81%
- (e) 19.95%.

(1 mark)

22. Which of the following statements is **true**?

[<Answer>](#)

- (a) The tallest rectangle in a histogram represents the modal class of the distribution
- (b) In a symmetrical distribution mean, median and mode are unequal
- (c) The medians of two sets of data can be combined mathematically
- (d) The median cannot be determined graphically
- (e) The mode is always uniquely defined.

(1 mark)

23. A sum of Rs.1,800 deposited today in a bank gets tripled in a period of 6 years. Find the annual rate of interest?

[<Answer>](#)

- (a) 13.21%
- (b) 15.41%
- (c) 18.36%
- (d) 20.09%
- (e) 24.50%.

(1 mark)

24. If by mistake you have added 35 to each of the biggest 200 numbers in a set of 399 numbers, how the median of the numbers is affected?

[<Answer>](#)

- (a) The median will increase by 35
- (b) The median will increase by 200
- (c) The median will increase by 35×200
- (d) The median will increase by $\frac{35}{399}$
- (e) The median will increase by $\frac{35 \times 200}{399}$.

(1 mark)

25. Which of the following **cannot** be determined by graphical method?

[<Answer>](#)

- (a) Mean
- (b) Median
- (c) Mode
- (d) Quartiles
- (e) Deciles.

(1 mark)

26. Which of the following is/are the major elements in every computer solution?

[<Answer>](#)

- I. Data.
- II. Hardware.
- III. Software.
- (a) Only (I) above
- (b) Only (II) above
- (c) Only (III) above
- (d) Both (I) and (III) above
- (e) All (I), (II) and (III) above.

(1 mark)

27. Which of the following statement(s) is/are **false** regarding a linear programming problem?

[<Answer>](#)

- I. Every primal has one and only one dual.
 - II. The solution of dual provides all the essential information about the solution of the primal problem.
 - III. It is always easier to solve the dual rather than a primal.
- (a) Only (I) above
 - (b) Only (II) above
 - (c) Only (III) above
 - (d) Both (I) and (II) above
 - (e) All (I), (II) and (III) above.

(1 mark)

28. To determine the appropriate width of each class interval in a grouped frequency distribution, we

[<Answer>](#)

- (a) Take the square of the number of observations
- (b) Divide the number of desired class intervals by the range of the data
- (c) Take the square root of the number of observations
- (d) Divide the range of the data by the number of desired class intervals
- (e) Divide the range of data by 2.

(1 mark)

29. If events A and B are mutually exclusive, then which of the following is **true**?

[<Answer>](#)

- (a) $P(A \text{ and } B) = 0$
- (b) $P(A) = 0$
- (c) $P(B) = 0$
- (d) $P(A \text{ or } B) = 0$
- (e) $P(A \text{ or } B) = 1$.

(1 mark)

30. A, B and C are mutually exclusive and collectively exhaustive events, resulting from an experiment. B is twice as likely as A, and C is 2.5 times as likely as B. Which of the following is **true**?

[<Answer>](#)

- (a) $P(A) = 3\%$, $P(B) = 6\%$ and $P(C) = 15\%$
- (b) $P(A) = 5\%$, $P(B) = 10\%$ and $P(C) = 25\%$
- (c) $P(A) = 8\%$, $P(B) = 16\%$ and $P(C) = 40\%$
- (d) $P(A) = 10\%$, $P(B) = 20\%$ and $P(C) = 50\%$
- (e) $P(A) = 12.5\%$, $P(B) = 25\%$ and $P(C) = 62.5\%$.

(1 mark)

31. The expression $\left(\frac{x^a}{x^b}\right)^{a+b} \times \left(\frac{x^b}{x^c}\right)^{b+c} \times \left(\frac{x^c}{x^a}\right)^{c+a}$ is equal to

[<Answer>](#)

- (a) 1
- (b) x^{a+b+c}
- (c) x^{abc}
- (d) x
- (e) x^{a-b-c} .

(1 mark)

32. The number of permutations of the word "ENGINEERING" taking all the letters at a time is

[<Answer>](#)

- (a) 3,99,16,800
- (b) 79,83,360
- (c) 39,91,680
- (d) 66,52,800
- (e) 2,77,200.

(1 mark)

33. Which of the following is **true** regarding the function $y = 48 + 16x - 4x^2$?

[<Answer>](#)

- (a) Maximum at $x = 1$
- (b) Maximum at $x = 2$
- (c) Minimum at $x = -1$
- (d) Minimum at $x = 2$
- (e) Maximum at $x = -2$.

(2marks)

[<Answer>](#)

34. If $y = 8x^4 + 3x^2 + x + 3$, then find $\frac{d^2y}{dx^2}$ for $x = 5$.

- (a) 1245
- (b) 1358
- (c) 2406
- (d) 1760
- (e) 2885.

(2marks)

[<Answer>](#)

35. There are 30 boys and 20 girls in a class. From the class list, a name is picked up at random. What is the probability that it is a girl's name?

- (a) $1/5$
- (b) $2/5$
- (c) $4/5$
- (d) $6/5$
- (e) $7/5$.

(1 mark)

[<Answer>](#)

36. There are 240 workers in a factory. 30 are from Kerala, 20 from Tamilnadu and the rest from Punjab. If a worker is selected at random from the factory, find the probability that he is a person from either Kerala or Tamilnadu?

- (a) $5/24$
- (b) $7/23$
- (c) $1/8$
- (d) $1/12$
- (e) $1/3$.

(1 mark)

[<Answer>](#)

37. Which of the following is/are **not** the assumption(s) that made in the construction of linear Programming models?

- I. Proportionality.
 - II. Additivity.
 - III. Divisibility.
 - IV. Uncertainty.
- (a) Only (I) above
 - (b) Only (III) above
 - (c) Only (IV) above
 - (d) Both (I) and (II) above
 - (e) Both (II) and (IV) above.

(1 mark)

[<Answer>](#)

38. Which of the following is **not** the requirement of a linear programming problem?

- (a) Linear programming problem must have a well-defined single objective to achieve
- (b) There must be a alternative courses of action , one of which will achieve the objective
- (c) The decision variables must be continuous in nature
- (d) Resources must be limited in supply and the achievement of the objective function is restricted by these constraints
- (e) The objective and the constraints must be non-linear functions.

(1 mark)

39. If the laspeyres price index = 147.28 and paasches price index = 143.26, then the fisher ideal price index is

[<Answer>](#)

- (a) 113.78
- (b) 125.85
- (c) 138.37
- (d) 145.26
- (e) 150.64.

(1 mark)

[<Answer>](#)

40. A logarithmic function is the _____ function of an exponential function.

- (a) Direct
- (b) Linear
- (c) Inverse
- (d) Cosine
- (e) Non-linear.

(1 mark)

[<Answer>](#)

41. From the following data compute index number of quantity using laspeyres method

Commodity	2006		2007	
	Price (Rs.)	Quantity (Kg.)	Price (Rs.)	Quantity (Kg.)
A	8	10	10	11
B	10	9	12	9
C	16	16	20	17

- (a) 84.76
- (b) 100.92
- (c) 105.63
- (d) 107.33
- (e) 109.53.

(1 mark)

[<Answer>](#)

42. From the following data, calculate value index.

Product	Base year		Current year	
	Price (Rs.)	Quantity (units)	Price (Rs.)	Quantity (units)
A	6	50	10	56
B	2	100	2	120
C	4	60	6	60
D	10	30	12	24
E	8	40	12	36

- (a) 112.45
- (b) 138.24
- (c) 154.78
- (d) 171.11
- (e) 185.61.

(1 mark)

[<Answer>](#)

43. The number of degrees of freedom in a 3×4 contingency table is

- (a) 2
- (b) 3
- (c) 6
- (d) 8
- (e) 9.

(1 mark)

44. Which of the following patterns of $\bar{X}$ chart is generally desirable to any quality control manager? [<Answer>](#)
- (a) Increasing trend
 - (b) Decreasing trend
 - (c) Cycles
 - (d) Hugging the center line
 - (e) Hugging the control limits.
- (1 mark)
45. Which of the following statements is/are **true**? [<Answer>](#)
- (a) The secular trend allows us to describe the historical pattern of the data
 - (b) Secular trend represents the short to medium term variation in time series data
 - (c) The component of a time series that oscillates predictably above and below the trend line for a period less than one year is known as irregular variation
 - (d) Seasonal variations are periodic and repetitive in nature
 - (e) Both (a) and (d) above.
- (1 mark)
46. The slope and the intercept on x-axis of a regression line are 3 and 4 respectively. The equation of the line is [<Answer>](#)
- (a) $4y = x + 3$
 - (b) $3y = 4x + 1$
 - (c) $y = 3x + 4$
 - (d) $y = 3x - 12$
 - (e) $3y = 4x$.
- (1 mark)
47. Which of the following is a case of type II error? [<Answer>](#)
- (a) The null hypothesis is true and rejected
 - (b) The null hypothesis is false and accepted
 - (c) The null hypothesis is true and accepted
 - (d) The null hypothesis is false and rejected
 - (e) None of the above.
- (1 mark)
48. Which of the following is **false** in the context of multiple regressions? [<Answer>](#)
- (a) Multicollinearity does not reduce the accuracy of the predictive power of the model
 - (b) Multicollinearity does not reduce the effectiveness of the sensitivity analysis
 - (c) Multicollinearity reduces the reliability of the regression co-efficients
 - (d) Multicollinearity reduces the influence of the individual variables in the model
 - (e) Independent variables may together explain the dependent variable in case of a multicollinearity problem.
- (1 mark)
49. Which of the following is a discrete probability distribution? [<Answer>](#)
- (a) Normal distribution
 - (b) Chi-square distribution
 - (c) Lognormal distribution
 - (d) Binomial distribution
 - (e) t-distribution.
- (1 mark)
50. Which method of sampling is appropriate when the population consists of well defined groups such that the groups are similar to each other and there is considerable variation within the groups? [<Answer>](#)
- (a) Judgment sampling
 - (b) Cluster sampling
 - (c) Systematic sampling
 - (d) Stratified random sampling
 - (e) Simple random sampling.
- (1 mark)

51. If the standard error of estimate for a regression equation is zero, then

[<Answer>](#)

- (a) The estimated value of the dependent variable will always be less than its observed value
- (b) The estimated value of the dependent variable will always be more than its observed value
- (c) The estimated value of the dependent variable will always be equal to its observed value
- (d) The coefficient of correlation between the variables involved will be zero
- (e) The coefficient of correlation between the variables involved will be positive.

(1 mark)

52. Two variables, X and Y, are related by a regression equation such that X is the independent variable and Y is the dependent variable. The following data is available:

[<Answer>](#)

Coefficient of correlation between X and Y =	-0.80
Standard deviation of X	= 12.96
Standard deviation of Y	= 20.25

The slope of the regression equation is

- (a) -1.25
- (b) 1.25
- (c) 10.37
- (d) 16.20
- (e) 25.31.

(1 mark)

53. In a test of ANOVA, the following are found.

[<Answer>](#)

Estimated population variance based on the variance among sample means = 25.50

F- ratio = 1.25

What is the estimated population variance based on the variance within the sample means?

- (a) 16.32
- (b) 20.40
- (c) 24.25
- (d) 26.75
- (e) 31.88.

(1 mark)

54. If the sample size is 100 and proportion of defective pieces in a batch of output is 0.10, then the lower control limit of p-chart will be

[<Answer>](#)

- (a) 0.01
- (b) 0.03
- (c) 0.09
- (d) 0.10
- (e) 0.19.

(1 mark)

55. Six dice are thrown 729 times. How many times do you expect at least three dice show a 4 or 5?

[<Answer>](#)

- (a) 625
- (b) 504
- (c) 437
- (d) 233
- (e) 115.

(2marks)

56. A random sample of size 9 is selected from a symmetrical population with a unique mode. The sample mean and standard deviation are 150 and 25 respectively. What is the 95% confidence interval in which the populations mean (μ) will lie?

[<Answer>](#)

- (a) (120.78, 149.22)
- (b) (125.78, 159.22)
- (c) (130.78, 169.22)
- (d) (135.78, 179.22)
- (e) (140.78, 189.22).

(2marks)

57. Each day, the Indian Customs Service has historically intercepted about Rs.28 million in contraband goods being smuggled into the country, with a standard deviation of Rs.16 million per day. On 64 randomly chosen days in 1999, the Indian Customs Service intercepted an average of Rs.30.3 million in contraband goods. The Customs Commissioner wants to test at a significance level of 5% whether the smuggling has increased above its historic level.

[<Answer>](#)

Which of the following conclusions can be drawn on the basis of the above test?

- (a) The average daily smuggling intercepted is same as the historic level
- (b) The sample mean is incorrect
- (c) The average daily smuggling intercepted is more than the historic level
- (d) The average daily smuggling intercepted is less than the historic level
- (e) The standard deviation in the smuggling intercepted has not increased significantly.

(2marks)

58. Which of the following statements are **true** regarding Pearson's coefficient of correlation?

[<Answer>](#)

- I. It lies between $-\infty \leq \rho \leq \infty$.
- II. $\rho=0 \Leftrightarrow$ No relationship between X and Y.
- III. $\rho=1 \Leftrightarrow$ Perfect positive linear relationship between X and Y.
- IV. $\rho=-1 \Leftrightarrow$ Perfect negative linear relationship between X and Y.
- (a) Both (I) and (II) above
- (b) Both (II) and (III) above
- (c) Both (III) and (IV) above
- (d) (II), (III) and (IV) above
- (e) All (I), (II), (III) and (IV) above.

(1 mark)

59. The ranks of 5 students are given according to their marks in Statistics and Mathematics.

[<Answer>](#)

Student No.	1	2	3	4	5
Statistics	2	3	5	4	1
Mathematics	3	4	5	2	1

What is the Spearman's rank correlation coefficient?

- (a) 0.5
- (b) 0.6
- (c) 0.7
- (d) 0.8
- (e) 0.9.

(2marks)

60. In which of the following conditions will the regression sum of squares for a simple regression relationship be equal to zero?

[<Answer>](#)

- (a) Correlation coefficient = 1
- (b) Correlation coefficient = -1
- (c) $0 < \text{Correlation coefficient} < 1$
- (d) $-1 < \text{Correlation coefficient} < 0$
- (e) Correlation coefficient = 0.

(1 mark)

[<Answer>](#)

61. If $x^2 + y^2 = 11xy$, then $\log \frac{x-y}{3} =$

- (a) $\log x$
- (b) $\log y$
- (c) $\log \frac{xy}{3}$
- (d) $\log x + \log y$
- (e) $\frac{1}{2}(\log x + \log y)$.

(1 mark)

[<Answer>](#)

62. Fit a straight line trend by the method of least squares to the following data and estimate the sales (in '000 units) for the year 2004.

Year	1995	1996	1997	1998	1999	2000
Sales of air conditioners (in '000 units)	10	13	16	21	24	30

- (a) 12,485
- (b) 27,328
- (c) 32,456
- (d) 38,175
- (e) 44,628.

(2marks)

[<Answer>](#)

63. A financial analyst has collected the following data on sales and net profits for the year 2006-07, of five companies manufacturing electronic components:

Sales (Rs. in crores)	38	18	45	24	55
Net profit (Rs. in crores)	1.50	1.50	3.00	2.00	4.00

The coefficient of correlation between sales and net profit is

- (a) 0.9125
- (b) 0.8832
- (c) 0.8392
- (d) 0.6150
- (e) 0.5243.

(2marks)

[<Answer>](#)

64. The following details are available with regard to two groups of data, A and B:

Group	Number of observations	Mean	Standard deviation
A	15	20	4
B	25	16	2

What is the combined standard deviation for both the groups?

- (a) 2.50
- (b) 3.50
- (c) 6.25
- (d) 7.00
- (e) 12.25.

(2marks)

65. Which of the following measures is **not** affected by the presence of extreme values in a data set?

[<Answer>](#)

- (a) Range
- (b) Arithmetic mean
- (c) Standard deviation
- (d) Variance
- (e) Mode.

(1 mark)

66. Consider the following data where X represents the number of heads when tossing of three coins.

[<Answer>](#)

X	0	1	2	3
P(X = x)	$\frac{1}{8}$	$\frac{3}{8}$	$\frac{3}{8}$	$\frac{1}{8}$

The expected value E(X) is

- (a) 1
- (b) $\frac{3}{2}$
- (c) $\frac{2}{3}$
- (d) 6
- (e) $\frac{1}{3}$.

(1 mark)

67. Two random samples were drawn from two normal populations A and B, and the data are:

[<Answer>](#)

A	66	67	75	76	82	84	88	90	92		
B	64	66	74	78	82	85	87	92	93	95	97

We want to test whether the population B has larger variance than the population A.

The appropriate null hypothesis for the above test is

- (a) The variances of the two populations are equal
- (b) The variances of the two populations are not equal
- (c) The standard deviation of the two populations are not equal
- (d) The variance of the population A is larger than the variance of the population B
- (e) The variance of the population B is larger than the variance of the population A.

(1 mark)

68. Which of the following is **false** about decision tree analysis?

[<Answer>](#)

- (a) Decision tree analysis is a diagrammatic representation of a decision process
- (b) Decision tree analysis is an excellent tool for making financial or number based decisions
- (c) Alternative decisions and the implications of taking those decisions can be laid and evaluated
- (d) Decision tree analysis cannot help in assessing the risk in making decisions concerning investment
- (e) Decision tree analysis can be used with the help of computer.

(1 mark)

[<Answer>](#)

69. Allied Engines Inc. produces two models of car engines, X and Y.

The following information is available:

Particulars	X	Y
Selling Price per unit	Rs. 800	Rs. 1,000
Cost Price per unit	Rs. 560	Rs. 625
Assembly hours per engine	2 hours	5 hours
Testing hours per engine	1 hour	0.5 hours
Total assembly hours available	600 hours	
Total testing hours available	120 hours	

Allied Engines has to decide how many of the X and Y engines it should produce if it wants to maximize the profit.

The objective function of the above linear programming problem is

- (a) $800X + 1000Y$
- (b) $240X + 375Y$
- (c) $560X + 625Y$
- (d) $240X + 625Y$
- (e) $560X + 375Y$.

(2marks)

[<Answer>](#)

70. The following data provides the values of sample range, R, for samples of size 5 each.

Sample No.	1	2	3	4	5	6	7	8	9	10
Range (R)	7	4	8	5	7	4	8	4	7	9

Calculate the value for the center line of the R-chart.

- (a) 6.1
- (b) 6.2
- (c) 6.3
- (d) 6.5
- (e) 6.6.

(1 mark)

[<Answer>](#)

71. Ten samples each of size 6 are drawn at regular intervals from a manufacturing process. The sample means and ranges are given below:

Sample No.	1	2	3	4	5	6	7	8	9	10
Mean	49	45	48	53	39	47	46	39	51	45
Range	7	5	7	9	5	8	8	6	7	6

The values of upper control limit and lower control limits respectively for $\bar{x}$ chart are

- (a) 46.2000, 41.1825
- (b) 49.4844, 42.9156
- (c) 42.9156, 49.0492
- (d) 50.1236, 43.2764
- (e) 50.1862, 42.6416.

(2marks)

72. In which of the following condition(s) a process may be said to be in control, on the basis of $\bar{x}$ charts:

[<Answer>](#)

- I. The sample means are on either side of the centre line with small deviations.
 - II. The sample means follow a random path and are between the control limits.
 - III. The sample means lie beyond the control limits.
- (a) Only (I) above
 - (b) Only (II) above
 - (c) Only (III) above
 - (d) Both (I) and (II) above
 - (e) Both (I) and (III) above.

(1 mark)

73. In a sample of 8 observations taken from population A, the sum of squared deviations of the observations from the mean was 84.4. In another sample of 10 observations taken from population B, the sum of squared deviations of the observations from the mean was found to be 102.6. It is to be tested whether the variance of population A is significantly higher than the variance of population B. What is the value of the appropriate test statistic for the above test?

[<Answer>](#)

- (a) 0.56
- (b) 0.99
- (c) 1.06
- (d) 1.18
- (e) 1.30.

(1 mark)

74. Which of the following is **false** regarding a chi-square distribution?

[<Answer>](#)

- (a) Chi-square distribution has only one parameter
- (b) The distribution becomes sharply positively skewed when the number of degrees of freedom is large
- (c) For large number of degrees of freedom the distribution approaches symmetry
- (d) The distribution is continuous
- (e) The distribution is unimodal.

(1 mark)

75. Which of the following is/are step(s) in performing analysis of variance?

[<Answer>](#)

- I. Determine an estimate of population variance from within the samples.
 - II. Determine an estimate of population variance from among the sample means.
 - III. Determine the difference between expected and observed frequency for each class.
 - IV. Examining the differences among more than two sample proportions.
- (a) Only (I) above
 - (b) Only (II) above
 - (c) Only (III) above
 - (d) Only (IV) above
 - (e) Both (I) and (II) above.

(1 mark)

76. Which of the following is an underlying assumption in ANOVA?

[<Answer>](#)

- (a) The populations follow unknown distributions
- (b) The sample means are equal
- (c) The samples are dependent on each other
- (d) The population variances are equal
- (e) The population variances are not equal.

(1 mark)

77. A test of ANOVA is applied to data obtained from five samples, where each sample contains six observations. What is the number of degrees of freedom for the denominator of the F- statistic?

[<Answer>](#)

- (a) 6
- (b) 4
- (c) 5
- (d) 25
- (e) 30.

(1 mark)

78. A chi-square test of goodness of fit is used to [<Answer>](#)
- Test the hypothesis about a population mean
 - Test the hypothesis about a population proportion
 - Test the hypothesis about difference between two population means
 - Test the hypothesis about difference between two population proportions
 - Test whether the data from a population follows a hypothesized probability distribution.
- (1 mark)
79. If $\bar{X} = 501$, $n = 36$, $\mu = 500$ and $\sigma = 1$, then the value of z-statistic is [<Answer>](#)
- 3
 - 4
 - 5
 - 6
 - 7.
- (1 mark)
80. Which of the following statement(s) is/are **true** regarding decision theory assumptions? [<Answer>](#)
- The decision maker can define all decision alternatives or strategies which are being considered.
 - The decision maker can define the various states of nature for the decision setting which are not under control.
 - The decision maker can estimate quantitatively the consequences (benefits or costs) of selecting any decision alternative and having any state of nature occurrence.
- Only (I) above
 - Only (II) above
 - Only (III) above
 - Both (II) and (III) above
 - All (I), (II) and (III) above.
- (1 mark)
81. A bag contains 5 white and 3 black balls. Two balls are drawn at random one after the other without replacement, then the probability that both balls drawn are black is [<Answer>](#)
- 1/4
 - 2/7
 - 3/8
 - 2/17
 - 3/28.
- (1 mark)
82. $f(x)$ is a function of x , if $f'(x)$ is positive and $f''(x)$ is negative, then the function is [<Answer>](#)
- Increasing at an increasing rate
 - Increasing at a decreasing rate
 - Decreasing at an increasing rate
 - Decreasing at a decreasing rate
 - Minimum or maximum.
- (1 mark)
83. If a , b , and c be the sums of p , q , and r terms respectively of an arithmetic progression, the expression [<Answer>](#)
- $$\frac{a(q-r)}{p} + \frac{b(r-p)}{q} + \frac{c(p-q)}{r}$$
- is equal to
- 1
 - 0
 - 1
 - abc
 - pqr.
- (2marks)

END OF QUESTION PAPER

Suggested Answers Quantitative Methods (CFA530): January 2008

Answer

REASON

1. E Let x_1 be the number of type A circuits and x_2 be the number of type B circuits to be produced. [<TOP>](#)

From profit the Objective function, Maximize $Z = 5x_1 + 8x_2$

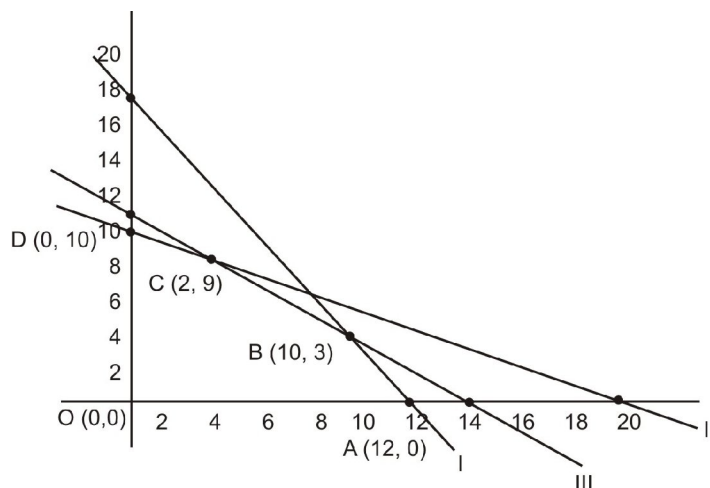
Subject to the constraints are

$$15x_1 + 10x_2 \leq 180 \text{ --- I}$$

$$10x_1 + 20x_2 \leq 200 \text{ --- II}$$

$$15x_1 + 20x_2 \leq 210 \text{ --- III}$$

And $x_1, x_2 \geq 0$



By using graphical method, the feasible region OABCD with vertices O (0,0), A(12,0) , B(10,3), C(2,9) and D(0,10)

The value of Z of these vertices are given by

Co-ordinates	Value of Z ($Z = 5x_1 + 8x_2$)
O (0,0)	0
A(12,0)	60
B(10,3)	74
C(2,9)	82
D(0,10)	80

Since the problem is of maximization type, the optimum solution is

Max $Z = 82$, $x_1 = 2$ and $x_2 = 9$.

2. D

[<TOP>](#)

Income (X)	Mid value (x)	No .of persons (f_i)	$f_i x_i$
10 – 20	15	4	60
20 – 30	25	7	175
30 – 40	35	16	560
40 – 50	45	20	900
50 – 60	55	15	825
60 – 70	65	8	520
		$N = 70$	$\Sigma f_i x_i = 3040$

$$\text{Arithmetic mean} = \frac{\sum f_i x_i}{N} = \frac{3040}{70} = \text{Rs.}43,430.$$

3. C Given [< TOP >](#)

$$\overline{X}_1 = 12 \text{ and } N_1 = 20$$

$$\overline{X}_2 = 15 \text{ and } N_2 = 40$$

$$\overline{X}_3 = 18 \text{ and } N_3 = 30$$

$$\begin{aligned} \overline{X} &= \frac{N_1 \overline{X}_1 + N_2 \overline{X}_2 + N_3 \overline{X}_3}{N_1 + N_2 + N_3} \\ \text{Combined mean,} \\ &= \frac{12 \times 20 + 15 \times 40 + 18 \times 30}{(20 + 40 + 30)} \\ &= \frac{240 + 600 + 540}{90} = \frac{1380}{90} = \text{Rs.15.33(approx.)} \end{aligned}$$

4. D Arranging the data in ascending order [< TOP >](#)

13	16	17	18	19	20	21	22	24
----	----	----	----	----	----	----	----	----

Total number of terms, $N = 9$

Here N is odd

$$\text{Median} = \frac{N+1}{2} \text{th term}$$

$$= \frac{9+1}{2} = 5 \text{th term}$$

i.e., median = 19.

5. B Given, $Q_1 = 19$ and $Q_3 = 33$ [< TOP >](#)

$$\text{Coefficient of quartile deviation} = \frac{Q_3 - Q_1}{Q_3 + Q_1}$$

$$= \frac{33 - 19}{33 + 19} = \frac{14}{52} = 0.2692$$

6. C Given median = 39.69 and mean = 39.51 [< TOP >](#)

Empirical mode = $3 \times \text{median} - 2 \times \text{mean}$

$$= 3 \times 39.69 - 2 \times 39.51$$

$$= 119.07 - 79.02 = 40.05.$$

7. A The appropriate average for quantities that change over time is the geometric mean. [< TOP >](#)

$\therefore$ The average growth rate over the seven year period is

= Geometric mean of the quantity ratios -1

$$= [1.05 \times 1.08 \times 0.90 \times 0.96 \times 1.06 \times 1.10 \times 1.07]^{1/7} - 1 = 0.0291$$

i.e. 2.91%

8. D Weighted harmonic mean would be appropriate here, because different distances are covered with varying speeds. [< TOP >](#)

X	W	W/X
60	900	15
25	300	12
350	400	1.1429
25	15	0.60
	$\Sigma W = 1615$	$\Sigma (w/x) = 28.7429$

$$\text{Weighted harmonic mean} = \frac{\sum w}{\sum \left(\frac{w}{x} \right)}$$

$$= \frac{1615}{28.7429} = 56.1878. \cong 56.19 \text{ kmph.}$$

9. A Given that mean of 40 terms = 25 [< TOP >](#)

$$\text{i.e., } 25 = \frac{\sum X}{40}$$

$$\sum X = 1000$$

$$\text{Standard deviation} = \sqrt{\frac{\sum x^2}{N} - \left(\frac{\sum X}{N} \right)^2}$$

$$4 = \sqrt{\frac{\sum x^2}{40} - \left(\frac{1000}{40} \right)^2}$$

Squaring both sides, we get

$$16 = \frac{\sum X^2}{40} - \left(\frac{1000}{40} \right)^2$$

$$\frac{\sum X^2}{40} = 625 + 16 = 641$$

$$\sum X^2 = 641 \times 40 = 25640.$$

Sum is 1000 and sum of squares is 25640.

10. D Given that, $f(x) = 4x^3 - 5x^2 + 8$ for all $x \geq 0$. [< TOP >](#)

$$f(3) = 4 \times 3^3 - 5 \times 3^2 + 8 = 71$$

$$f(2) = 4 \times 2^3 - 5 \times 2^2 + 8 = 20$$

$$f(3) - f(2) = 71 - 20 = 51.$$

11. D [< TOP >](#)

$$\frac{\frac{1}{\log_x a} + \frac{1}{\log_x b}}{\frac{1}{\log_x a} - \frac{1}{\log_x b}} = \frac{\log_x b + \log_x a}{\log_x b - \log_x a}$$

$$\frac{\log_x ab}{\log_x \frac{b}{a}} = \frac{\log_b ab}{\log_b \frac{a}{b}}.$$

12. B The candidate can take three strategies: [< TOP >](#)
 4 questions from first group and 2 questions from second group: The number of ways he can do this is ${}^5C_4 \times {}^5C_2 = 5 \times 10 = 50$.
 3 questions from first group and 3 questions from second group:
 ${}^5C_3 \times {}^5C_3 = 10 \times 10 = 100$
 2 questions from first group and 4 questions from second group:
 ${}^5C_2 \times {}^5C_4 = 10 \times 5 = 50$.
 So, total number ways by which the candidate can make his choice is $(50+100+50)$ or 200.
13. B Borrowed amount = Rs. 1,200 [< TOP >](#)
 Interest to be paid = Rs. 110
 Total sum to be paid S = Rs. 1310
 Total number of installments, $n = 10$
 Each installment is less than the preceding installment by 15
 $d = -15$
 it is a problem of sum of terms in A.P

$$S = \frac{n}{2}[2a + (n-1)d]$$

$$1310 = \frac{10}{2}[2 \times a + (10-1)(-15)]$$

$$262 = 2a - 135$$

$$2a = 397$$

$$a = 198.5$$
 Therefore, the first installment is Rs. 198.5.
14. C Upper limit of 95% confidence interval = $\bar{x} + 1.96\sigma_x^-$ [< TOP >](#)
 Where $\sigma_x^- = \frac{\sigma}{\sqrt{n}} = \frac{18}{6} = 3$
 Thus, $\bar{x} + 1.96\sigma_x^- = 25 + 1.96 \times 3 = 30.88$.
15. A $\log_3 \frac{81}{27} = \log_3 3 = 1$. [< TOP >](#)
16. B $y = f(x) = 7x + 9$ [< TOP >](#)
 here, y is a function of x
 $y - 9 = 7x$
 $\frac{y-9}{7} = x$
 $\therefore x$ is the inverse function of y.
17. D Roots of the equation $x^2 - 8x + 15 = 0$ are [< TOP >](#)

$$x = \frac{-(-8) \pm \sqrt{(-8)^2 - 4(1)(15)}}{2(1)}$$

$$x = \frac{8 \pm \sqrt{64 - 60}}{2}$$

$$x = \frac{8 \pm 2}{2}$$

$$x = 5, 3.$$

18. B For given two values a and b [< TOP >](#)
- $$A.M = \frac{a+b}{2}$$
- $$G.M = \sqrt{ab}$$
- $$H.M = \frac{2ab}{a+b}$$
- again, $(\sqrt{a} - \sqrt{b})^2 \geq 0$
- or, $a+b - 2\sqrt{ab} \geq 0$
- $$\frac{a+b}{2} \geq \sqrt{ab}$$
- $$A.M \geq G.M$$
- again, $a+b \geq 2\sqrt{ab}$
- $$(a+b)\sqrt{ab} \geq 2(\sqrt{ab})^2$$
- $$(a+b)\sqrt{ab} \geq 2ab$$
- $$\sqrt{ab} \geq \frac{2ab}{a+b}$$
- $$G.M \geq H.M$$
- $$\therefore A.M \geq G.M \geq H.M$$
19. D Arithmetic mean is the best measure of central tendency. [< TOP >](#)
- Because (i) It is mathematical Average
(ii) It can be calculated easily.
(iii) It is depending on all the data values.
20. C Statements (I), (II), (III), (IV) are true with regard to the interpolation. Statements (v) is false because Extrapolation is used for forecasting purposes. Hence (c) is the answer. [< TOP >](#)
21. E at K = 19%, present value of cash flow is equal to 33.42 [< TOP >](#)
at K = 20%, present value of cash flow is equal to 29.82
- $$K = 19 + \frac{(33.42 - 30)}{(33.42 - 29.82)} \times 1$$
- so, = 19.95%.
22. A The tallest rectangle in a histogram represents the modal class. [< TOP >](#)
- In a symmetrical distribution mean, median and mode are equal.
- The median can not be mathematically manipulated; hence medians of two sets of data can not be combined.
- The median can be determined graphically.
- The mode is not uniquely determined when more than one observation have the highest frequency.
23. D Let r be the interest. [< TOP >](#)
- $$P_n = P_0(1+r)^n$$
- i.e., $5400 = 1800(1+r)^6$
- $$\text{so, } (1+r)^6 = \frac{5400}{1800} = 3$$
- $$1+r = 3^{\frac{1}{6}} = 1.2009$$
- $$r = 0.2009 = 20.09\%$$
- The annual rate of interest per annum = 20.09%.

24. A Here, 35 is being added to each of the biggest 200 numbers in a set of 399 numbers. The median of these numbers is the 200th term. As it is being increased by 35, the median will also increase by 35. [< TOP >](#)
Hence, the option (a) is correct.
25. A Median, quartiles, deciles and mode can be determined by graphical method but mean cannot be determined by graphically. [< TOP >](#)
26. E There are three major elements in every computer solution. [< TOP >](#)
They are:
I. Data
II. Hardware
III. Software.
Hence option (e) is correct.
27. C In linear programming, there is a unique dual (primal) problem associated with every primal (dual) problem. When a primal is solved, its associated dual is also solved simultaneously. Hence, the solution for a LP problem can be determined either by solving the original problem or the dual problem. In general, a maximization problem is easier to solve when compared to the minimization problem because in the former we have addition of slack variable where as in the later we have an additional of both surplus and artificial variables. But we **cannot** conclude that it is always easier to solve the dual rather than a primal because the dual could be either maximization or minimization problem. [< TOP >](#)
28. D To determine the appropriate width of each class interval in a grouped frequency distribution, we divide the range of the data by the number of desired class intervals. [< TOP >](#)
29. A If events A and B are mutually exclusive then $P(A \text{ and } B) = 0$ because A and B can not occur at the same time. [< TOP >](#)
Mutual exclusiveness of two events A and B does not imply that both of them have zero probabilities of occurrence.
Mutual exclusiveness does not mean that the probability of any one of them occurring is either 0 or 1.
30. E $P(A) = x \quad \therefore P(B) = 2x \quad P(C) = 2.5 P(B) = 2.5(2x) = 5x$ [< TOP >](#)
Since A, B and C are mutually exclusive and collectively exhaustive,
 $P(A) + P(B) + P(C) = 1$
 $\therefore x + 2x + 5x = 1$
or $8x = 1$
 $x = \frac{1}{8} = 0.125$ i.e. 12.5%
 $\therefore P(A) = 12.5\% \quad P(B) = 2 \times 12.5 = 25\% \quad P(C) = 5 \times 12.5 = 62.5\%.$
31. A [< TOP >](#)
$$\left(\frac{x^a}{x^b}\right)^{a+b} \times \left(\frac{x^b}{x^c}\right)^{b+c} \times \left(\frac{x^c}{x^a}\right)^{c+a} = (x^{a-b})^{a+b} \times (x^{b-c})^{b+c} \times (x^{c-a})^{c+a}$$
$$= x^{a^2-b^2+b^2-c^2+c^2-a^2}$$
$$= x^0 = 1$$
32. E Since the word ENGINEERING consists of 11 letters, in which there are 3E, 3N, 2G and 2I, [< TOP >](#)
$$\frac{11!}{3!3!2!2!} = 2,77,200.$$

33. B To prove the given function minimum or maximum [< TOP >](#)
- $\frac{dy}{dx} = 0$
- Necessary condition is $\frac{dy}{dx} = 0$
- Given , $y = 48 + 16x - 4x^2$
- $\frac{dy}{dx} = 16 - 8x = 0$
- $8x = 16 \therefore x = 2.$
- $\frac{d^2y}{dx^2} = -8 < 0$
- Sufficient condition : $\frac{d^2y}{dx^2} = -8 < 0$
- Therefore, the function is maximum at $x = 2$.
34. C : $y = 8x^4 + 3x^2 + x + 3$ [< TOP >](#)
- $\frac{dy}{dx} = 8(4x^3) + 3(2x) + 1 = 32x^3 + 6x + 1$
- $\frac{d^2y}{dx^2} = 32(3x^2) + 6 = 96x^2 + 6$
- $\frac{d^2y}{dx^2}$
- for $x = 5$, $\frac{d^2y}{dx^2} = 96(5)^2 + 6 = 2406.$
35. B Number boys in a class = 30 [< TOP >](#)
- Number of girl in a class = 20
- Total number of students in a class = 50.
- One name is picked up random ,
- $\frac{{}^{20}C_1}{{}^{50}C_1} = \frac{2}{5}.$
- probability that the name is girl's name = $\frac{{}^{20}C_1}{{}^{50}C_1} = \frac{2}{5}.$
36. A Let K be the event that the selected person is from kerala and T is the event that he is from Tamilnadu. As the two events are mutually exclusive , [< TOP >](#)
- $P(K \cup T) = P(K) + P(T)$
- $\frac{m}{n} = \frac{30}{240} , \frac{m}{n} = \frac{20}{240}$
- $P(K) = \frac{30}{240} , P(T) = \frac{20}{240}$
- $\frac{30}{240} + \frac{20}{240} = \frac{50}{240} = \frac{5}{24}.$
- $P(K \cup T) = \frac{30}{240} + \frac{20}{240} = \frac{50}{240} = \frac{5}{24}.$
37. C The following are the some of the main assumptions that are made in the construction of linear programming models: [< TOP >](#)
1. Proportionality
 2. Additivity
 3. Divisibility
 4. **Certainty.**
38. E The following are the requirements of the linear programming problem. [< TOP >](#)
- a. Linear programming problem must have a well-defined single objective to achieve
 - b. There must be a alternative courses of action , one of which will achieve the objective .
 - c. The decision variables must be continuous in nature
 - d. Resources must be limited in supply and the achievement of the objective function is restricted by these constraints
 - e. The objective and the constraints must be **linear** functions.

39. D Given , laspeyres price index = 147.28 and paasches price index = 143.26 [< TOP >](#)

$$\begin{aligned}\text{Fisher ideal price index} &= \sqrt{\text{laspeyres price index} \times \text{paasches price index}} \\ &= \sqrt{147.28 \times 143.26} = 145.256 \approx 145.26.\end{aligned}$$

40. C If $x = b^y$ where both $b > 0$, $x > 0$, then we define $y = \log_b x$, and [< TOP >](#)

$\log_b x$ is the number to which b has to be raised in order to get x .

So a logarithmic function is simply the **inverse** function of an exponential function.

41. C [< TOP >](#)

Commodity	P_0	Q_0	P_1	Q_1	Q_1P_0	Q_0P_1
A	8	10	10	11	88	80
B	10	9	12	9	90	90
C	16	16	20	17	272	256
Total					450	426

$$\text{Laspeyres method} = \frac{\sum Q_1 P_0}{\sum Q_0 P_0} \times 100$$

$$= \frac{450}{426} \times 100 = 105.63.$$

42. B [< TOP >](#)

Product	P_0	Q_0	P_1	Q_1	P_0Q_0	P_1Q_1
A	6	50	10	56	300	560
B	2	100	2	120	200	240
C	4	60	6	60	240	360
D	10	30	12	24	300	288
E	8	40	12	36	320	432
					1360	1880

$$\text{Value index} = \frac{\sum P_1 Q_1}{\sum P_0 Q_0} \times 100 = \frac{1880}{1360} \times 100 = 138.24.$$

43. C Degrees of freedom in 3×4 contingency table = $(3 - 1)(4 - 1) = 2 \times 3 = 6$. [< TOP >](#)

44. D The pattern “Hugging the center line” indicates that the deviations are minor and uniform in nature. This indicates that the variation has been reduced to a great extent. So for any quality control manager this pattern is generally desirable. [< TOP >](#)

45. E Both statements (a) and (d) are true. Secular trend represents long term behaviour of a variable; hence statement (b) is false. Seasonal variations are repetitive and predictable movements above and below the trend line occurring within one year or less; hence statement (c) is false. [< TOP >](#)

46. D $y = a + bx$ (I) [< TOP >](#)

$$\text{Or } x = \frac{y-a}{b} = \frac{y}{b} - \frac{a}{b}$$

$$\text{Or } x = \left(\frac{-a}{b} \right) + \frac{y}{b} \quad \text{(II)}$$

From equation II the intercept on the X-axis = $\frac{-a}{b} = 4$ (given)
Slope, in equation I, $b = 3$ (given)

$$\begin{aligned} \therefore \frac{-a}{3} &= 4 \text{ or } a = -(4 \times 3) = -12 \\ \therefore \text{ We get : } a &= -12 \text{ and } b = 3. \\ \therefore y &= -12 + 3x = 3x - 12 \end{aligned}$$

47. B A type II error occurs when the null hypothesis is accepted though it is false. [< TOP >](#)

48. B Multicollinearity reduces the effect of individual variables on the model. So It reduces the effect of any sensitivity analysis. [< TOP >](#)

49. D In case of binomial distribution, the possible outcomes are finite in number and hence, it can be termed as a discrete probability distribution. While the other distributions may assume infinite number of possible outcomes. Hence (a), (b), (c) and (e) are continuous distributions. [< TOP >](#)

50. B In cluster sampling the population is divided into clusters or groups in such a way that the clusters are not homogeneous and each cluster is representative of the population. Hence, in the case of cluster sampling, intra-group variations are more than that of the inter-group variations. [< TOP >](#)

51. C [< TOP >](#)
The standard error of estimate for a regression equation is given by $\sqrt{\frac{\sum(y-\hat{y})^2}{n-2}}$ where y is the actual value, $\hat{y}$ is the estimated value from the regression equation and n is the number of points. The above relation implies that the standard error of estimate will be zero, if the actual values and the estimated values are equal and in such case the correlation is either perfectly positive (+1) or perfectly negative (-1).

52. A The slope ,b of a regression equation is given by [< TOP >](#)

$$b = \frac{\text{Cov}(x, y)}{\sigma_x^2} = r \frac{\sigma_y}{\sigma_x} = -0.80 \times \frac{20.25}{12.96} = -\frac{16.20}{12.96} = -1.25$$

53. B The required estimate of population variance based on the variance within the sample will be [< TOP >](#)
 $\frac{25.5}{1.25} = 20.40$

54. A [< TOP >](#)
The lower control limit = $p - 3\sqrt{\frac{pq}{n}} = 0.1 - 3 \times \sqrt{\frac{0.1 \times 0.9}{100}}$
 $= 0.1 - 3 \times 0.03 = 0.10 - 0.09 = 0.01$

55. D There will be success only when on any die 4 or 5 appear. Hence, $P = 2/6 = 1/3$ and $q = 1 - 1/3 = 2/3$. There are 6 trials in each set, total number of sets are 729. The number of successes in each set can vary from 0 to 6 and it will follow a Binomial distribution. The probability that there are at least three successes will be given by [< TOP >](#)

$${}^6C_3 \left(\frac{1}{3}\right)^3 \left(\frac{2}{3}\right)^3 + {}^6C_4 \left(\frac{1}{3}\right)^4 \left(\frac{2}{3}\right)^2 + {}^6C_5 \left(\frac{1}{3}\right)^5 \left(\frac{2}{3}\right) + {}^6C_6 \left(\frac{1}{3}\right)^6$$

$$= \frac{20 \times 8}{3^6} + \frac{15 \times 4}{3^6} + \frac{6 \times 2}{3^6} + \frac{1}{3^6} = \frac{233}{729}$$

The number of times we expect at least three successes

$$= 729 \times \frac{233}{729} = 233$$

56. C Here, $\bar{X} = 150$ [< TOP >](#)
 $s = 25$
 $n = 9$

Degrees of freedom = $n - 1 = 9 - 1 = 8$

If we refer to the t table we see that for 8 degrees of freedom, the area in both tails combined is 0.5 or 5%, when $t = 2.306$.

Hence area under the curve between $\bar{X} - t \left(\frac{s}{\sqrt{n}} \right)$ and $\bar{X} + t \left(\frac{s}{\sqrt{n}} \right)$ is 95% when $t = 2.306$.

So, the 95% confidence interval is $150 - \left[2.306 \times \frac{25}{\sqrt{9}} \right]$ to $150 + \left[2.306 \times \frac{25}{\sqrt{9}} \right]$.

Hence, we are 95% confident that the population mean lies in the interval 130.78 to 169.22.

57. A We set H_0 : The average daily smuggling intercepted is same as the historic level ($\mu = 28$) [< TOP >](#)
 We set H_1 : The average daily smuggling intercepted has increased above the historic level ($\mu > 28$)

The sample size is large and population standard deviation is known, hence we use normal distribution to test this hypothesis.

$$\frac{\bar{x} - \mu}{\sigma / \sqrt{n}} = \frac{30.3 - 28}{16 / \sqrt{64}} = \frac{2.3}{2} = 1.15$$

The test statistic $z = 1.15$

This is a right tailed test; hence the critical region falls in the right tail.

The critical value is $z = 1.645 > 1.15$

Since the test statistic falls in the acceptance region, we accept the null hypothesis and conclude at 5% level of significance that the average daily smuggling intercepted is same as the historic level.

58. D The following statements are **true** regarding Pearson's coefficient of correlation. [< TOP >](#)

II. $\rho = 0 \Leftrightarrow$ No relationship between X and Y.

III. $\rho = 1 \Leftrightarrow$ Perfect positive linear relationship between X and Y.

IV. $\rho = -1 \Leftrightarrow$ Perfect negative linear relationship between X and Y.

The Statement (I) It lies between $-\infty \leq \rho \leq \infty$ is false. Since the correct statement is $-1 \leq \rho \leq 1$

So option (d) is correct.

59. C Let rank of student in Statistics is X_i [< TOP >](#)

Let rank of student in Mathematics is Y_i

Then $D_i = X_i - Y_i$

n	Statistics	Mathematics	Difference between ranks (D_i)	D_i^2
1	2	3	-1	1
2	3	4	-1	1
3	5	5	0	0
4	4	2	2	4
5	1	1	0	0
Total			0	6

Here $\sum D_i^2 = 6$ and $n=5$.

$$R = 1 - \frac{6 \sum D_i^2}{n^3 - n} = 1 - \frac{6 \times 6}{5^3 - 5} = 1 - \frac{36}{120} = 0.7.$$

60. E [< TOP >](#)

$$r^2 = \frac{RSS}{TSS}$$

$$\Rightarrow RSS = 0 \Rightarrow r^2 = 0 \Rightarrow r = 0.$$

61. E [< TOP >](#)

$$x^2 + y^2 = 11xy$$

$$\Rightarrow x^2 + y^2 - 2xy = 9xy$$

$$\Rightarrow (x - y)^2 = 9xy$$

$$\Rightarrow \frac{x - y}{3} = (xy)^{1/2}$$

Taking logarithm of both sides, we get

$$\log\left(\frac{x-y}{3}\right) = \frac{1}{2}(\log x + \log y).$$

62. E [< TOP >](#)

The equation of the straight line trend can be calculated as below:

Year (X)	Sales (Y)	$Z = X - \bar{X}$	$x = 2Z$	xY	x^2
1995	10	-2.5	-5	-50	25
1996	13	-1.5	-3	-39	9
1997	16	-0.5	-1	-16	1
1998	21	0.5	1	21	1
1999	24	1.5	3	72	9
2000	30	2.5	5	150	25
Total	114		0	138	70

$$\bar{X} = 1997.5$$

The equation of the straight line trend is $Y = a + bx$

$$\text{Since } \sum x = 0, \quad a = \frac{\sum Y}{n} = \frac{114}{6} = 19, \quad \text{and} \quad b = \frac{\sum xY}{\sum x^2} = \frac{138}{70} = 1.9714$$

Thus the equation of the trend line is $\hat{Y} = 19 + 1.9714x$

$$\text{Therefore } \hat{Y}_{2004} = 19 + 1.9714 \times 2 (2004 - 1997.5)$$

$$= 19 + 1.9714 \times 13 = 44.6282 \text{ (in thousands)}$$

$\approx 44,628$ sales of air conditioners.

63. C Let the following notations be used.

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X : Sales (Rs. in crores)

Y : Net profit (Rs. in crores)

$$\text{Coefficient of correlation, } r = \frac{\Sigma(X - \bar{X})(Y - \bar{Y})}{\sqrt{\Sigma(X - \bar{X})^2 \Sigma(Y - \bar{Y})^2}}$$

A.	X	38.00	18.00	45.00	24.00	55.00	$\Sigma X = 180$
B.	Y	1.50	1.50	3.00	2.00	4.00	$\Sigma Y = 12$
C.	$X - \bar{X}$	2.00	-18.00	9.00	-12.00	19.00	
D.	$Y - \bar{Y}$	-0.90	-0.90	0.60	-0.40	1.60	
E.	$C \times D$	-1.80	16.20	5.40	4.80	30.40	$\Sigma(X - \bar{X})(Y - \bar{Y}) = 55$
F.	C^2	4.00	324.00	81.00	144.00	361.00	$\Sigma(X - \bar{X})^2 = 914$
G.	D^2	0.81	0.81	0.36	0.16	2.56	$\Sigma(Y - \bar{Y})^2 = 4.70$

$$\bar{X} = \frac{\Sigma X}{n} = \frac{180}{5} = 36 \quad \bar{Y} = \frac{\Sigma Y}{n} = \frac{12}{5} = 2.4$$

$$\therefore r = \frac{55}{\sqrt{914 \times 4.70}} = 0.8392 \text{ (approx.)}$$

64. B $N_A = 15, \mu_A = 20, \sigma_A = 4$
 $N_B = 25, \mu_B = 16, \sigma_B = 2$

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$$\mu = \frac{N_A \mu_A + N_B \mu_B}{N_A + N_B} = \frac{(15 \times 20) + (25 \times 16)}{15 + 25} = \frac{300 + 400}{40} = 17.5$$

$$d_A = \mu_A - \mu = 20 - 17.5 = 2.5$$

$$d_B = \mu_B - \mu = 16 - 17.5 = -1.5$$

$$\sigma_{AB} = \left[\frac{N_A \sigma_A^2 + N_B \sigma_B^2 + N_A d_A^2 + N_B d_B^2}{N_A + N_B} \right]^{1/2}$$

$$= \left[\frac{(15 \times 4^2) + (25 \times 2^2) + (15 \times 2.5^2) + [25(-1.5^2)]}{15 + 25} \right]^{1/2}$$

$$= \left[\frac{240 + 100 + 93.75 + 56.25}{40} \right]^{1/2} = 3.50$$

65. E
- Range is based on the two extreme values in the data set. Hence it is affected by the extreme values in the data set.
 - Arithmetic mean is based on all the values. Hence this is also affected by the presence of extreme of values.
 - Standard deviation is based on all the values. Hence this is also affected by the presence of extreme of values.
 - Variance is based on all the values. Hence this is also affected by the presence of extreme of values.
 - Mode is not affected by the presence of the extreme values.

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66. B Consider the following data where X represents number of heads when tossing of three coins. [< TOP >](#)

X	0	1	2	3
P(X=x)	$\frac{1}{8}$	$\frac{3}{8}$	$\frac{3}{8}$	$\frac{1}{8}$

The expected value E(X) $= 0 \times \frac{1}{8} + 1 \times \frac{3}{8} + 2 \times \frac{3}{8} + 3 \times \frac{1}{8}$
 $= 0 + \frac{3}{8} + \frac{6}{8} + \frac{3}{8} = \frac{3}{2}$.

67. A We set the null hypothesis for this test as: 'The variance of the two populations are equal' [< TOP >](#)
 We set the alternative hypothesis for this test as: 'The variance of the population B is larger than the variance of the population A'.

68. D Decision tree helps in assessing risk in decision concerning investment. All other statements are true for decision tree analysis. [< TOP >](#)

69. B [< TOP >](#)

	X	Y
Selling Price	Rs. 800	Rs. 1,000
Cost per unit	Rs. 560	Rs. 625
Profit	Rs. 240	Rs. 375

Profit function or Objective function is : $240X + 375Y$.

70. C The central line of for the R chart = Mean of the sample range ($\bar{R}$) [< TOP >](#)
 $\bar{R} = \Sigma R / n = 63 / 10 = 6.3$.

71. B The central line of the $\bar{x}$ chart is = Mean of the sample means ($\bar{\bar{x}}$) [< TOP >](#)
 Mean of the sample means or grand mean, $\bar{\bar{x}} = 462 / 10 = 46.2$.
 Mean of sample range = $\bar{R} = \Sigma R / n = 68 / 10 = 6.8$
 Upper Control Limit of $\bar{x}$ chart = $\bar{\bar{x}} + A_2 \bar{R} = 46.2 + 0.483 \times 6.8 = 49.4844$
 (The value of A_2 for sample of size 6 is 0.483 from control chart factors table)
 Lower Control Limit of $\bar{x}$ chart = $\bar{\bar{x}} - A_2 \bar{R} = 46.2 - 0.483 \times 6.8 = 42.9156$
 (The value of A_2 for sample of size 6 is 0.483 from control chart factors table)

72. D The process is in control according to the $\bar{x}$ charts, if the sample means are on either side of the center line or they follow a random path and are between the control limits. [< TOP >](#)

73. C We are given that, [< TOP >](#)

$n_1 = 8, \quad \Sigma(x_1 - \bar{x}_1)^2 = 84.4$
 $n_2 = 10, \quad \Sigma(x_2 - \bar{x}_2)^2 = 102.6$

$s_1^2 = \frac{\Sigma(x_1 - \bar{x}_1)^2}{n_1 - 1} = \frac{84.4}{7} = 12.06$

$s_2^2 = \frac{\Sigma(x_2 - \bar{x}_2)^2}{n_2 - 1} = \frac{102.6}{9} = 11.4$

$\frac{s_1^2}{s_2^2} = \frac{12.06}{11.4} = 1.06$
 The F - statistic = $\frac{s_1^2}{s_2^2} = 1.06$.

74. B The distribution is positively skewed when the number of degrees of freedom is small. [< TOP >](#)

75. E Determining an estimate of population variance from within the samples and determining an estimate of population variance from among the sample means are two of the steps in performing Analysis of Variance. [< TOP >](#)
76. D One of the assumptions in ANOVA is that the population variances are equal. [< TOP >](#)
77. D No. of degrees of freedom for the denominator of the F statistic [< TOP >](#)
 $= n_T - k = (5 \times 6) - 5 = 25.$
78. E A chi-square test of goodness of fit is used to test whether the data from a population follows a hypothesized probability distribution. [< TOP >](#)
79. D **z**-statistic is given by [< TOP >](#)

$$z = \frac{\bar{X} - \mu}{\sigma_{\bar{x}}}, \text{ where } \sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}}$$

$$\sigma_{\bar{x}} = \frac{1}{\sqrt{36}} = 0.1667. \quad \text{and} \quad z = \frac{501 - 500}{0.1667} = 5.9988 \approx 6$$
80. E The following statements are **true** regarding decision theory assumptions. [< TOP >](#)
 I. The decision maker can define all decision alternatives or strategies which are being considered.
 II. The decision maker can define the various states of nature for the decision setting which are not under control.
 III. The decision maker can estimate quantitatively the consequences (benefits or costs) of selecting any decision alternative and having any state of nature occurrence.
 So option (e) is correct.
81. E Probability of drawing a black ball in the first attempt is [< TOP >](#)

$$P(A) = \frac{{}^3C_1}{{}^8C_1} = \frac{3}{8}$$
 Probability of drawing the second black ball given that the first ball drawn is black:

$$P(B/A) = \frac{{}^2C_1}{{}^7C_1} = \frac{2}{7}$$
 The Probability that both balls drawn are black is given by

$$P(AB) = P(A) \times P(B/A) = \frac{3}{8} \times \frac{2}{7} = \frac{3}{28}.$$
82. B $f(x)$ is a function of x , $f'(x)$, $f''(x)$ are the first and second derivatives of $f(x)$ [< TOP >](#)
- | $f(x)$ | $f'(x)$ | $f(x)$ |
|----------|----------|----------------------------------|
| positive | positive | Increasing at an increasing rate |
| positive | negative | Increasing at a decreasing rate |
| negative | positive | Decreasing at an increasing rate |
| negative | negative | Decreasing at a decreasing rate |

83. B Let A denote the first term and D denote the common difference of the A.P. then

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$$a = \frac{p}{2}[2A + (p-1)D] \Rightarrow \frac{a}{p} = A + (p-1)\frac{D}{2} \dots\dots\dots (1)$$

$$b = \frac{q}{2}[2A + (q-1)D] \Rightarrow \frac{b}{q} = A + (q-1)\frac{D}{2} \dots\dots\dots (2)$$

$$c = \frac{r}{2}[2A + (r-1)D] \Rightarrow \frac{c}{r} = A + (r-1)\frac{D}{2} \dots\dots\dots (3)$$

Multiplying (1) by (q - r), (2) by (r - p) and (3) by (p - q) and adding we get,

$$\frac{a}{p}(q-r) + \frac{b}{p}(r-p) + \frac{c}{r}(p-q) =$$

$$A(q-r+r-p+p-q) + \frac{D}{2}[(p-1)(q-r) + (q-1)(r-p) + (r-1)(p-q)] = 0.$$

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