Question Paper
Quantitative Methods (CFA530): July 2008

- **Answer all 73 questions.**
- **Marks are indicated against each question.**

Total Marks : 100

1. If the letters of the word VICTORY are arranged in a row at random, the probability that the two vowels do not come together is <Answer>

(a) $\frac{2}{7}$

(b) $\frac{3}{7}$

(c) $\frac{4}{7}$

(d) $\frac{5}{7}$

(e) $\frac{6}{7}$.

(1 mark)

2. The sum of the series $\log a + \log\frac{a^2}{b} + \log\frac{a^3}{b^2} + \log\frac{a^4}{b^3} + ...$ up to n terms is <Answer>

(a) (n/2) [n log ab – log (a/b)]
(b) (n/2) [log ab – n log (a/b)]
(c) (n/2) [n log ab + log (a/b)]
(d) (n/2) [log ab + n log (a/b)]
(e) (n/2) [log (a/b) – n log ab].

(1 mark)

3. The ratio between the sum of *n* terms of two A.P.'s is (3n + 8) : (7n + 15). The ratio between their twelfth terms is <Answer>

(a) 6 : 17
(b) 7 : 15
(c) 8 : 17
(d) 7 : 16
(e) 9 : 19.

marks
(2)

4. If $x = \frac{m^{\frac{m-1}{n}}}{n^{\frac{n+1}{m}}}$, $y = \frac{n^{\frac{1-n}{m}}}{m^{\frac{m+3}{n}}}$ and $z = \frac{m^{\frac{5}{n}}}{n^{\frac{1-2n}{m}}}$, then the value of xyz is <Answer>

(a) $\frac{\sqrt[m]{m}}{\sqrt[n]{n}}$

(b) $\frac{\sqrt[n]{m}}{\sqrt[m]{n}}$

(c) $\frac{\sqrt[m]{n}}{\sqrt[n]{m}}$

(d) $\frac{\sqrt[n]{n}}{\sqrt[m]{m}}$

(e) $\sqrt[n]{m} \times \sqrt[m]{n}$.

(1 mark)

5. Consider the following simultaneous equations. <Answer>

$$2x-3y+3z=7$$
$$-x+y+z=5$$
$$2x-2y+z=2$$

The value of x in the above equations is

(a) 1
(b) 2
(c) 3
(d) 4
(e) 5.

(1 mark)

6. How many committees of five members each can be formed from 8 official and 4 non-official members when these committees would require at least 2 non-official members? <Answer>

(a) 112
(b) 336
(c) 448
(d) 456
(e) 480.

(1 mark)

7. If $f(x)=\log\frac{1+x}{1-x}$, then $f\left(\frac{2x}{1+x^2}\right)=$ <Answer>

(a) 0
(b) 1
(c) f (x)
(d) 2 f (x)
(e) 3 f (x).

(1 mark)

8. The logarithm of a number <Answer>

(a) Is always expressed with respect to base 10
(b) Is always expressed with respect to base 1
(c) Is always expressed with respect to base e
(d) Is equal to the base which must be raised to a given exponent in order to get the number
(e) Is equal to the exponent to which a given base must be raised in order to get the number.

(1 mark)

9. Differentiation of $\log(a-bx)$ with respect to x is <Answer>

(a) $\frac{a}{(a-bx)}$

(b) $\frac{b}{(bx-a)}$

(c) $\frac{-a}{(a-bx)}$

(d) $\frac{-b}{(bx-a)}$

(e) $-e^{(bx-a)}$.

(1 mark)

10. Let $f(x) = \begin{cases} 3^{-x} - 1, \text{ for } -1 \le x < 0 \\ \dfrac{x}{x^2 - 2}, \text{ for } 0 \le x \le 6 \end{cases}$

The value of f(1) is

(a) 3
(b) 2
(c) 1
(d) 0
(e) –1.

<Answer>

(1 mark)

11. An investment of Rs.1000 is earning 4% interest per annum compounded annually. If the value, V, of the investment after t years is given by $V = 1000(1.04)^t$ then t, written as a function of V, is

(a) $t = \dfrac{\log_{10} V}{3} - \log_{10}(1.04)$
(b) $t = \dfrac{\log_{10} V}{3} - 3\log_{10}(1.04)$
(c) t = [log10 V] – 3 – [log10 (1.04)]
(d) $t = \dfrac{\log_{10} V - 3}{\log_{10}(1.04)}$
(e) $t = \dfrac{\log_{10} V + 3}{\log_{10}(1.04)}$.

<Answer>

marks (2)

12. The function $f(x)=12x–x^2–52$ has

(a) Minimum at x = 6
(b) Maximum at x = 6
(c) Minimum at x = 8
(d) Maximum at x = 8
(e) Minimum at x = 4.

<Answer>

(1 mark)

13. A privately owned lake contains two types of game fish, bass and trout. The owner provides two types of food A and B, for these fish. Bass requires 2 units of food A and 4 units of food B and trout requires 5 units of food A and 2 units of food B. If the owner has 800 units of each food, the maximum number of fish that the lake can support by satisfying food constraint would be

(a) 100
(b) 150
(c) 250
(d) 300
(e) 350.

<Answer>

marks (2)

14. Consider the following linear programming problem. <Answer>

Maximize Z = 143x + 60y
Subject to the following constraints:

$x + y \leq 100$

$120x + 210y \leq 15000$

$110x + 30y \leq 4000$

and $x, y \geq 0$.

The value of x at the optimum solution would be

(a) 20
(b) 50
(c) 60
(d) 100
(e) 110.

marks
(2)

15. Jayanti Paints manufactures two varieties of dye – Kalindi and Dark Rose. The production of both the varieties primarily depends on the usage of three chemicals – A, B and C. Currently, the report of the company's central stores states that 24 litres of A, 16 litres of B and 40 litres of C are available. To produce 100 litres of Kalindi, around 6 litres of A, 2 litres of B and 8 litres of C will be required. On the other hand, to produce 100 litres of Dark Rose one needs to apply 3 litres of A, 4 litres of B and 8 litres of C. The company earns a profit of Rs. 120 from a litre of Kalindi and Rs. 80 from a litre of Dark Rose. The company is interested to maximize its profit. <Answer>

Let the company produces X_1 times of 100 litres of Kalindi and X_2 times of 100 litres of Dark Rose. In the framework of linear programming, which of the following can be represented as the objective function of the problem?

(a) Maximize $Z = 6000X_1 + 3000X_2$
(b) Maximize $Z = 12000X_1 + 3000X_2$
(c) Maximize $Z = 2000X_1 + 4000X_2$
(d) Maximize $Z = 12000X_1 + 8000X_2$
(e) Maximize $Z = 8000X_1 + 8000X_2$.

marks
(2)

16. In the graphical method of solving linear programming problems the feasible region is the set of all points <Answer>

(a) Which do not satisfy any of the constraints
(b) Which satisfy exactly one of the constraints
(c) Which satisfy all the constraints
(d) At which the objective function has the same value
(e) At which the objective function is equal to zero.

(1 mark)

17. Which of the following is an input device in a computer? <Answer>

(a) Line printer
(b) Dot matrix printer
(c) Bubble jet
(d) Laser printer
(e) Mouse.

(1 mark)

18. Which of the following indicates that the distribution curve of the data is negatively skewed? <Answer>

(a) Arithmetic Mean < Median < Mode
(b) Median < Mode < Arithmetic Mean
(c) Mode < Arithmetic Mean < Median
(d) Mode < Median < Arithmetic Mean
(e) Median < Arithmetic Mean < Mode.

(1 mark)

19. Which of the following is graph of cumulative frequency distribution? <Answer>

(a) Line Charts
(b) Bar Diagram
(c) Pie Diagram
(d) Scatter Diagram
(e) Ogive.

(1 mark)

20. Which of the following statements is **false**? <Answer>

(a) The tallest rectangle in a histogram indicates the location of the mean of the distribution
(b) For a multimodal distribution the mode is not a meaningful measure of central tendency
(c) In a symmetrical distribution the mean, median and mode are equal
(d) The median and the mode can be found out graphically
(e) The medians of two data sets cannot be combined mathematically.

(1 mark)

21. The following details are available with regard to a sample of ten observations: <Answer>
Arithmetic mean of the first four observations = 16
Arithmetic mean of the next six observations = 11

$\sum_{i=1}^{10} x_i^2 = 1726$

What is the standard deviation of this sample?

(a) 2
(b) 3
(c) 4
(d) 5
(e) 6.

(2 marks)

22. The number of categories of computer programming languages can be classified in to <Answer>

(a) 1
(b) 2
(c) 3
(d) 4
(e) 5.

(1 mark)

23. According to the Bienayme-Chebyshev rule, at least _____ percent of the observations in a sample are contained within two standard deviations around the mean, regardless of the shape of the distribution. <Answer>

(a) 88.89
(b) 93.75
(c) 68.26
(d) 75.00
(e) 50.00.

(1 mark)

24. If the mean, median and mode for a given population all equal to 25, then we can say that the distribution is <Answer>

(a) Bimodal
(b) Skewed to the right
(c) Symmetric
(d) Skewed to the left
(e) Asymmetric.

(1 mark)

25. The geometric mean of a set of five numbers is 2. The geometric mean of a set of four numbers is 4. What is the geometric mean of the numbers in both the sets considered together? <Answer>

(a) 1.7216
(b) 2.7216
(c) 3.7216
(d) 4.7216
(e) 5.7216.

(1 mark)

26. The following distribution shows the distribution of wages of 500 workers in a factory: <Answer>

Weekly wages (in Rs.)	Number of workers
250 – 300	60
300 – 350	110
350 – 400	160
400 – 450	90
450 – 500	46
500 – 550	24
550 – 600	10

What is the median wage earned by the workers in the factory?

(a) Rs. 325.7038
(b) Rs. 350.2538
(c) Rs. 374.8438
(d) Rs. 395.3038
(e) Rs. 412.5038.

(2 marks)

27. The mean marks obtained by a group of 100 students were found to be 49.96 while the mean marks obtained by another group of 200 students were 52.32. The combined mean marks is <Answer>

(a) 50.4333
(b) 50.5333
(c) 51.0333
(d) 51.5333
(e) 51.6333.

(1 mark)

28. A box contains 3 white balls and 2 black balls. If two balls are drawn, what is the probability that both balls drawn are black if balls are not replaced after being drawn? <Answer>

(a) $\frac{1}{4}$
(b) $\frac{1}{5}$
(c) $\frac{2}{5}$
(d) $\frac{3}{5}$
(e) $\frac{1}{10}$.

(1 mark)

29. Consider G_1, G_2 and G_3 denote the events that the target hits by the first, second and third guns respectively. If $P(G_1) = 0.5$, $P(G_2) = 0.6$ and $P(G_3) = 0.8$ and G_1, G_2 and G_3 are independent of each other. Then what is probability of registering exactly one hit? <Answer>

(a) 0.22
(b) 0.24
(c) 0.26
(d) 0.28
(e) 0.30.

(2 marks)

30. The probability that both S and T occur, the probability that S occurs and T does not while the probability that T occurs but S does not are all equal to p. The probability that either S or T occurs is

<Answer >

(a) p
(b) 2p
(c) 3p
(d) $3p^2$
(e) p^3.

(1 mark)

31. In a G.P. sum of n terms is 364. The first term is 1 and the common ratio is 3. The value of n is

<Answer >

(a) 5
(b) 6
(c) 7
(d) 8
(e) 9.

(1 mark)

32. A die is loaded so that probability of face x is proportional to x. When the die is rolled the probability of x being greater than two would be

<Answer >

(a) $\frac{10}{21}$
(b) $\frac{2}{7}$
(c) $\frac{3}{7}$
(d) $\frac{4}{7}$
(e) $\frac{6}{7}$.

(1 mark)

33. A small contractor undertakes various types of jobs in the construction business. From his past experience he believes that the probability that he will get a plumbing contract is $\frac{2}{5}$ and the probability that he will not get a wiring contract is $\frac{1}{4}$. He also believes that the probability that he will get at least one of the two types of contract is $\frac{17}{20}$. The two types of contracts are independent of each other.

<Answer >

What is the probability that he will get both types of contract?

(a) 0.6
(b) 0.7
(c) 0.3
(d) 0.5
(e) 0.4.

(1 mark)

34. A random variable X, has the following probability distribution: <Answer >

X	20	40	60	80	100
Probability	0.15	0.25	0.30	0.20	0.10

Five values of X have been observed. What is the probability that exactly three out of the five observations are at least 60?

(a) 0.0123
(b) 0.1234
(c) 0.2345
(d) 0.3456
(e) 0.4567.

marks (2)

35. A continuous uniform probability distribution is described by <Answer >

(a) The expected value of the distribution
(b) The variance of the distribution
(c) The upper and lower limits of interval of values of the distribution
(d) The mid-point of the range of values of the distribution
(e) Only the lower limit of the range of values of the distribution.

(1 mark)

36. If two variables X and Y are perfectly positively correlated then their covariance will be equal to <Answer >

(a) 0
(b) 1
(c) The product of the standard deviations of X and Y
(d) The sum of the standard deviations of X and Y
(e) The difference between the standard deviations of X and Y.

marks (2)

37. If a fair die is thrown ten times then, what is the probability of getting an even number greater than 2 for six times? <Answer >

(a) 0.0569
(b) 0.33
(c) 0.67
(d) 0.9431
(e) 0.60.

(1 mark)

38. A student appears for a test containing ten multiple choice type questions. Each multiple choice type question has four alternative solutions only one of which is the correct solution. The student is not prepared for the test and he is expected to answer the questions by randomly selecting one alternative from the given choices. It is assumed that each alternative is equally likely to be selected by the student. <Answer >
What is the probability that the student will answer four questions correctly?

(a) 0.1459
(b) 0.3959
(c) 0.8541
(d) 0.1789
(e) 0.9789.

marks (2)

39. The Western Delight Restaurant prepares a chicken dish, Chicken Fun every Sunday. The manager of the restaurant wants to ensure that the restaurant earns the maximum profit on this dish. From each whole chicken that is cooked, two portions of Chicken Fun are prepared. The selling price per portion of the dish is Rs.50, which is considered to be a bargain price, and this has made the dish very popular. Each portion of the dish costs Rs.32 including labour and other costs of preparation. It is observed over a long period of time that the demand for the dish is normally distributed with a mean of 210 portions and a standard deviation of 50 portions. The dish is perishable and the unsold portions of the dish cannot be preserved and sold later.

<Answer>

How many whole chickens should be ordered by the manager of the restaurant to avoid losses due to spoilage?

(a) 88
(b) 90
(c) 94
(d) 95
(e) 96.

(2 marks)

40. Which of the following probability distributions is a distribution of the ratio of two random variables each of which follows a chi-square distribution?

<Answer>

(a) Normal distribution
(b) Lognormal distribution
(c) t–distribution
(d) F–distribution
(e) Binomial distribution.

(1 mark)

41. A discrete uniform random variable M, assumes the following integer values:

<Answer>

1	2	3	4	5	6	7	8

The characteristic of the discrete uniform random variable is that all possible values of the variable have equal probability.
What is the expected value of the random variable M?

(a) 3.1
(b) 4.5
(c) 6.2
(d) 0.2
(e) 8.1.

(1 mark)

42. A medical shop sells a variety of drugs and one of the drugs sold by the shop is Drug X. This drug has a short shelf life and it is quite expensive. Each packet of Drug X costs Rs.300 and can be sold for Rs.400, The manager of the shop has observed that the demand for this drug before its expiry has the following probability distribution:

<Answer>

Demand level before expiry (number of packets)	300	400	500	600	700
Probability	0.15	0.25	0.40	0.15	0.05

Any packet of this drug not sold before expiry has no value and has to rejected. It is assumed that the demand for this drug will be one of the aforementioned values only.
How many packets of Drug X should be stocked by the shop to gain maximum profit?

(a) 300
(b) 400
(c) 500
(d) 600
(e) 700.

(2 marks)

43. A businessman has to decide on the next month's stock level of a particular product that he deals in. He has observed that the monthly demand for the product has the following probability distribution:

<Answer>

Monthly demand (number of units)	4,000	5,000	6,000	7,000	8,000
Probability	0.10	0.25	0.40	0.20	0.05

The following conditional profits (in Rs.) have been assessed by the businessman for various stock and demand levels:

Monthly stock level (in units)	Monthly demand level (in units)				
	4,000	5,000	6,000	7,000	8,000
4,000	4,00,000	4,00,000	4,00,000	4,00,000	4,00,000
5,000	1,00,000	5,00,000	5,00,000	5,00,000	5,00,000
6,000	–2,00,000	2,00,000	6,00,000	6,00,000	6,00,000
7,000	–5,00,000	–1,00,000	3,00,000	7,00,000	7,00,000
8,000	–8,00,000	–4,00,000	0	4,00,000	8,00,000

What is the expected value of perfect information for the decision on the appropriate stock level?

(a) Rs. 4,60,000
(b) Rs. 5,85,000
(c) Rs. 1,25,000
(d) Rs. 4,00,000
(e) Rs. 8,00,000.

marks (3)

44. Variable T follows a normal distribution. The standardised value for T = 35 is 2, and the standardised value for T = 40 is 3. Which of the following is **true** with regard to T?

<Answer>

(a) Mean = 5 and variance = 5
(b) Mean = 25 and variance = 25
(c) Mean = 5 and variance = 25
(d) Mean = 25 and variance = 5
(e) None of the above.

(1 mark)

45. When decisions are made under uncertain conditions, which of the following criteria for decision making is based on the opportunity costs of making particular decisions?

<Answer>

(a) Maximin criterion
(b) Maximax criterion
(c) Hurwicz criterion
(d) Regret criterion
(e) Expected value.

(1 mark)

46. Which of the following is **false** with regard to the maximin criterion of decision making?

<Answer>

(a) It is assumed that the decision maker is pessimistic
(b) It is assumed that whatever strategy is selected nature will respond in such a way as to produce the least possible gain
(c) It is used to make decisions under conditions of uncertainty
(d) It is used to make decision under conditions of risk
(e) None of the above.

(1 mark)

47. An analyst believes that the earnings per share for Company A for the next fiscal year will have the following distribution: <Answer>

Earnings/share	Probability
Re. 1.16	0.1
Re. 1.17	0.1
Re. 1.18	0.2
Re. 1.19	0.2
Re. 1.20	0.4

The standard deviation of the earnings per share estimates for company A is

(a) Re. 0.0002
(b) Re. 0.0134
(c) Re. 0.0141
(d) Re. 0.0158
(e) Re. 1.1870.

marks (2)

48. Other things remaining the same, if the sample size increases then the width of the confidence interval for the population mean will <Answer>

(a) Decrease
(b) Decrease by exactly the square root of the increase in the sample size
(c) Decrease by exactly the reciprocal of the square root of the increase in the sample size
(d) Decrease by exactly the increase in the sample size
(e) Increase.

(1 mark)

49. The manager of the quality department for a tire manufacturing company wants to study the average tensile strength of rubber used in making a certain brand of radial tire. The population is normally distributed and the population variance is not known. The manager wants to test the null hypothesis that the mean tensile strength is equal to 800 pounds per square inch against the alternative hypothesis that the mean tensile strength is less than 800 pounds per square inch. The manager has taken a sample of 25 observations and the test statistic is calculated to be –1.78. If the significance level is 0.05, then <Answer>

(a) The critical value is –1.708 and the null hypothesis is rejected
(b) The critical value is –1.708 and the null hypothesis is accepted
(c) The critical value is –1.711 and the null hypothesis is rejected
(d) The critical value is –1.711 and the null hypothesis is accepted
(e) The critical value is –2.064 and the null hypothesis is accepted.

marks (3)

50. If we accept $H_0: \mu = 20$ in favor of $H_1: \mu \neq 20$ at a given level of significance with a positive value of the test statistic, then a test with $H_0: \mu = 20$ versus $H_1: \mu > 20$ using the same sample and the same level of significance <Answer>

(a) Will always accept H_0
(b) Will always reject H_0
(c) May or may not accept H_0
(d) Will certainly lead to a Type I error
(e) Will certainly lead to a Type II error.

(1 mark)

51. The following details are available with regard to a hypothesis test on a population mean: <Answer>

$H_0 : \mu = 20$

$H_1 : \mu \neq 20$

$\sigma^2 = 81$

$n = 36$

The null hypothesis is rejected if $\bar{x} \leq 17.06$ or $\bar{x} \geq 22.94$. What is the probability of committing a type I error?

(a) 0.5
(b) 0.05
(c) 0.025
(d) 0.005
(e) 1.00.

marks (2)

52. A researcher is interested in determining the average urban household income for the state Uttar Pradesh (U.P) of the country. He divides U.P. into homogeneous sectors and then randomly chooses 3 cities from each sector to sample. From each city, he randomly chooses 1000 families to survey. This type of sampling is referred to as

<Answer>

(a) Systematic sampling
(b) Cluster sampling
(c) Simple random sampling
(d) Stratified random sampling
(e) Judgemental sampling. **(1 mark)**

53. If sample size is 8, population size is 50 and population standard deviation is 10, then standard error of sample mean is

<Answer>

(a) 2.2733
(b) 3.2733
(c) 4.2733
(d) 5.2733
(e) 6.2733. **marks (2)**

54. The following details are available with regard to a test of hypothesis for the population mean:

<Answer>

$H_0: \mu = 60$

$\bar{x} = 72$

$\sigma = 60$

Test statistic = 1.20

What is the sample size?

(a) 6
(b) 10
(c) 12
(d) 36
(e) 72. **marks (2)**

55. The estimated regression equation of the variable Y on X is : $\hat{Y} = 10 + 5X$.

<Answer>

Let Z = Y + 2.

What is the slope of the estimated regression equation of Z on X (X is the independent variable)?

(a) 2
(b) 3
(c) 7
(d) 5
(e) 10. **marks (2)**

56. The deviations of the observations in a simple random sample, from zero are given below:

<Answer>

6	11	9	11	8	15

What is the point estimate of the population mean?

(a) 6
(b) 9
(c) 10
(d) 11
(e) 60. **(1 mark)**

57. Polar Systems Ltd. produces different types of products in the field of information technology. The research and development department of the company has been witnessing a growth in the research activities over the past few years. The data on research expenditure in past few years are given below:

<Answer >

Year	2002	2003	2004	2005	2006	2007
Research expenditure (Rs. in lakh)	50	57	61	65	72	79

The estimate of research expenditure for the year 2008 is

(a) 74.2598 lakhs
(b) 83.3998 lakhs
(c) 75.2598 lakhs
(d) 85.2598 lakhs
(e) 72.1598 lakhs.

marks (2)

58. A simple regression equation is developed which relates the variables X and Y ; X is the independent variable and Y is the dependent variable. The following details are available:

<Answer >

Y	31	40	30	34	25	20

$\sum XY = 1000 \quad \sum X = 30 \quad \sum X^2 = 200$

Number of data points = 6

What percentage of the variations in Y does the regression line **not** explain?

(a) 3.26%
(b) 17.36%
(c) 20.36%
(d) 82.66%
(e) 90.66%.

marks (2)

59. A dealer of grocery-items requires an overall comparison of the prices of the commodities he deals in, between the years 2004 and 2007. The following information is provided by him:

<Answer >

Commodities	Sugar	Salt	Wheat	Rice	Pulses
$P_0 (Q_0 + Q_1)$	15750	1512.50	10500	18450	28600
$P_1 (Q_0 + Q_1)$	20650	2337.50	13125	22500	40950

The Marshall –Edgeworth price index for the year 2007 with the year 2004 as the base year is

(a) 123.2527
(b) 128.6527
(c) 133.0827
(d) 145.4527
(e) 155.6427.

marks (2)

60. The following information is regarding the actual and estimated sales of a product from the year 1997 to 2004. <Answer>

Year	1997	1998	1999	2000	2001	2002	2003	2004
Actual Sales (Rs. in lakh)	7.5	7.8	8.2	8.2	8.4	8.5	8.7	9.1
Estimated Sales (Rs. in lakh)	7.6	7.8	8.0	8.2	8.4	8.6	8.8	9.0

Under relative cyclical residual measure, in which year does the largest **fluctuation** from trend occur?

(a) 1997
(b) 1999
(c) 2002
(d) 2003
(e) 2004.

marks
(2)

61. Which of the following is **not** an advantage of chain base method of constructing index numbers? <Answer>

(a) The link relatives calculated by using the chain base method, enable comparisons over successive years
(b) It is a flexible method
(c) Whenever found necessary, weights can be adjusted in chain base method
(d) As percentages are chained together in chain index numbers, long range comparisons are very accurate
(e) Seasonal variations have minimal impact on chain index numbers. **(1 mark)**

62. Consider the following information: <Answer>

Commodity	2005 (Base year)		2007	
	Price (Rs./Kg.)	Quantity (Kg.)	Price (Rs./Kg.)	Quantity (Kg.)
Wheat	7.50	5000	10.25	4800
Rice	10.00	6000	14.00	5500
Pulses	25.00	3000	28.00	3500
Sugar	10.60	3000	14.50	2400
Salt	2.60	500	4.50	600

If $\Sigma P_0Q_0 =$ 205600 and $\Sigma P_0Q_1 =$ 205500, the Fisher's ideal price index will be

(a) 112.8272
(b) 113.5572
(c) 127.3572
(d) 128.1172
(e) 133.5872. **(1 mark)**

63. Consider the following information for a given year : <Answer>

Commodity	$\frac{P_1}{P_0} \times 100$	P_nQ_n
Rice	118.42	10,000
Wheat	130.77	4,050
Salt	160	1,300
Sugar	145	9,600
Pulses	137.78	13250

The weighted average of relatives price index for the year is

(a) 118.4294
(b) 130.7794
(c) 134.5394
(d) 137.7894
(e) 139.2094.

marks
(2)

64. The following data pertains to the consumption of materials by a bakery. <Answer>

Inputs	Units	Prices (in Rs.)		Quantities Used	
		2005	2007	2005	2007
Flour	Kilogram	15	30	500	700
Eggs	Dozen	8	14	100	70
Milk	Litres	6	17	200	120
Sugar	Kilogram	10	16	50	70

Laspeyres quantity index for the given data considering 2005 as base year is

(a) 123.4
(b) 124.8
(c) 198.2
(d) 206.0
(e) 210.1. **(1 mark)**

65. The Upper Control Limit for n = 55, $\bar{p} = 0.27$ in P-chart will be <Answer>

(a) 0.1596
(b) 0.2596
(c) 0.3596
(d) 0.4496
(e) 0.5596. **(1 mark)**

66. The Lower Control Limit in an R–chart if n = 7, $\bar{R} = 7.1$, $d_2 = 2.704$ and $d_3 = 0.833$ is <Answer>

(a) 0.3383
(b) 0.4383
(c) 0.5383
(d) 0.6583
(e) 0.7483. **(1 mark)**

67. The Upper Control Limit of $\bar{x}$–chart for a product having a mean diameter of 1.60 cm, population standard deviation of 0.15 cm and sample size of 8, will be <Answer>

(a) 1.3691 cm
(b) 1.4691 cm
(c) 1.5691 cm
(d) 1.6691 cm
(e) 1.7591 cm. **(1 mark)**

68. The quality manager of the bottling plant of a cola major wants to ensure that the quantity of soft drinks in the bottles is as printed on the label of the bottle. The manager takes a sample of ten bottles every day and plots a control chart for the mean quantity of beverage in the bottles. Towards the end of the week, the manager observes that the observations are within the control limits but there are jumps in the level around which the observations vary. Which of the following inferences is **most** appropriate? <Answer>

(a) The variations have been reduced to a great extent
(b) The means of two different populations are being observed
(c) The process mean has been shifting
(d) The process is out of control
(e) None of the above. **(1 mark)**

69. If we have large enough sample sizes, we can discard which of the following assumption(s) about ANOVA? <Answer >

I. The samples are drawn from a normal population.

II. Each population has the same variance.

III. The samples are drawn from a population which is not normal.

(a) Only (I) above
(b) Only (II) above
(c) Only (III) above
(d) Both (I) and (II) above
(e) Both (II) and (III) above. **(1 mark)**

70. Suppose you calculated the following variances for several different groups of samples, and all the groups had the same degrees of freedom. For which ratio would you be most likely to accept the null hypothesis of equal means, at a given significance level? <Answer >

(a) Between-column variance = 8, within-column variance = 3
(b) Between-column variance = 6, within-column variance = 3
(c) Between-column variance = 4, within-column variance = 3
(d) Between-column variance = 30, within-column variance = 20
(e) Between-column variance = 50, within-column variance = 25. **(1 mark)**

71. The following details are available for a test involving ANOVA: <Answer >

Number of samples = 4

Size of the first sample = 5

Size of the second sample = 5

Size of the third sample = 6

Size of the fourth sample = 4

The number of degrees of freedom in the numerator and denominator of the F ratio are

(a) 4 and 6 respectively
(b) 4 and 4 respectively
(c) 5 and 4 respectively
(d) 3 and 16 respectively
(e) 4 and 20 respectively. **(1 mark)**

72. Calculate the χ^2 statistic from the following data: <Answer >

Sample size = 30

Hypothesized value of the population standard deviation = 1

Sample standard deviation = 0.73

(a) 15.4541
(b) 16.5041
(c) 17.6541
(d) 18.6041
(e) 19.0041. **(1 mark)**

73. Which of the following statement(s) is/are **true** regarding simulation? <Answer >

I. If the mathematical model set up could always be optimized by the analytical approach, then, there would be no need for simulation.

II. Only when interrelationships are too complex or there is uncertainty regarding the values that could be assumed by the variables or both, we would have to resort to simulation.

III. Simulations cannot be used to break down complex systems into sub-systems and study the behavior of each of these subsystems.

(a) Only (I) above
(b) Only (II) above
(c) Only (III) above
(d) Both (I) and (II) above
(e) All (I), (II) and (III) above. **(1 mark)**

END OF QUESTION PAPER

Suggested Answers
Quantitative Methods (CFA530): July 2008

	ANSWE	REASON
1.	D	The total number of ways the letters of the word VICTORY can be arranged is n(S) = 7! The number of ways two vowels always come together is = 2!.6! Therefore total number of ways when the vowels do not come together is n (E) = 7! – 2!.6! = 5.6! Therefore the probability of favorable event = $\frac{n(E)}{n(S)} = \frac{5}{7}$.
2.	D	We find that the given series is in A.P. with first term as *log a* and with a common difference of *log (a/b)* Now sum of the *n* terms of the series is $S_n = (n/2)\,[2 \log a + (n-1) \log (a/b)]$ $= (n/2)\,[\log a^2 + n \log (a/b) - \log (a/b)]$ $= (n/2)\,[\log ab + n \log (a/b)]$.
3.	D	Given that $\frac{S_n'}{S_n} = \frac{(n/2)[2a+(n-1)d]}{(n/2)[2a'+(n-1)d']} = \frac{3n+8}{7n+15}$ Or, $\frac{a+[(n-1)/2]d}{a'+[(n-1)/2]d'} = \frac{3n+8}{7n+15}$ .....(i) We have to find $\frac{T_{12}}{T_{12}'} = \frac{a+11d}{a'+11d'}$ Choosing (n–1)/2 = 11 or n = 23 in relation (i), we get $\frac{T_{12}}{T_{12}'} = \frac{a+11d}{a'+11d'} = \frac{3\times23+8}{7\times23+15} = \frac{77}{176} = \frac{7}{16}$.
4.	B	$xyz = \frac{m^{\frac{m-1}{n}}}{n^{\frac{n+1}{m}}} \times \frac{n^{\frac{1-n}{m}}}{m^{\frac{m+3}{n}}} \times \frac{m^{\frac{5}{n}}}{n^{\frac{1-2n}{m}}}$ $= \frac{m^{\left(\frac{m-1}{n} - \frac{m+3}{n} + \frac{5}{n}\right)}}{n^{\left(\frac{n+1}{m} - \frac{1-n}{m} + \frac{1-2n}{m}\right)}}$ $= \frac{m^{\frac{1}{n}}}{n^{\frac{1}{m}}} = \frac{\sqrt[n]{m}}{\sqrt[m]{n}}$.
5.	B	$2x-3y+3z=7$.....(i) $-x+y+z=5$.....(ii) $2x-2y+z=2$.....(iii) By doing (ii) + 2 × (iii) – (i) $x = 5+2\times2-7 = 9-7 = 2$.

6. D As the desired number of no-official members in the committee would be at least two – there would be three types of groupings – ≤

(i) 2 non-official members and 3 official members:

This could be done in $^{4}C_{2} \times {}^{8}C_{3} = \frac{3\times 4}{1\times 2} \times \frac{6\times 7\times 8}{1\times 2\times 3} = 336$ ways

(ii) 3 non-official members and 2 official members: $^{4}C_{3} \times {}^{8}C_{2} = 4 \times \frac{7\times 8}{2} = 56 \times 2 = 112$ ways.

(iii) 4 non-official members and 1 official member. $^{4}C_{4} \times {}^{8}C_{1} = 1 \times 8 = 8$ ways.

So, total (336 + 112 + 8) or 456 committees could be formed in the give conditions.

7. D The required function, ≤

$$f\left(\frac{2x}{1+x^2}\right) = \log\left(\frac{1+\frac{2x}{1+x^2}}{1-\frac{2x}{1+x^2}}\right)$$

$$= \log\left(\frac{1+x^2+2x}{1+x^2+-2x}\right)$$

$$= \log\left(\frac{1+x}{1-x}\right)^2 = 2\log\left(\frac{1+x}{1-x}\right) = 2f(x).$$

8. E The logarithm of a number is equal to the exponent to which a given base must be raised in order to get the number. The logarithm of a number is not expressed with respect to base zero or one. The logarithm of a number may be expressed with respect to base 10 or other positive real number not equal to 1. ≤

9. B $\frac{d}{dx}\left[\log(a\text{-}bx)\right] = \frac{1}{(a-bx)}\left[-b\right] = \frac{b}{bx-a}$. ≤

10. E ≤

$$\text{Let } f(x) = \left\{\begin{matrix} 3^{-x} - 1, \text{ for } -1 \le x < 0 \\ \frac{x}{x^2-2}, \text{for } 0 \le x \le 6 \end{matrix}\right\}$$

Since $x = 1 \in 0 \le x \le 6$ interval.

$\therefore$ The value of $f(1) = \frac{1}{1^2-2} = \frac{1}{-1} = -1.$

11. D Given that ≤

$$V = 1000(1.04)^t$$

Taking both sides Log, we get

$\log_{10} V = \log_{10}[1000\ (1.04)^t]$

$= \log_{10} 1000 + \log_{10}(1.04)^t$

$= 3 + t\log_{10}(1.04)$

$t\log_{10}(1.04) = \log_{10} V - 3$

$$t = \frac{[\log_{10} V - 3]}{\log_{10}(1.04)}.$$

12. B $f(x) = 12x - x^2 - 52$ ≤

$f'(x) = 12 - 2x = 0$

$\Rightarrow x = 6.$

$f''(x) = -2 < 0$

f(x) has a maximum at x = 6.

13. C Let there are x numbers of bass and y numbers of trout. ≤

Objective is to maximize x+y.

Subject to:

$2x + 5y \leq 800$ and the terminal line is $2x + 5y = 800$ ……(i)

$4x + 2y \leq 800$ and the terminal line is $4x + 2y = 800$ ……....(ii)

Line (i) would cross x-axis at (400, 0) and line (ii) would cross x-axis at (200, 0). So, the feasibility region would contain (200, 0).

Line (i) would cross y-axis at (0, 160) and line (ii) would cross y-axis at (0, 400). So, the feasible region would contain (0, 160).

Both the lines would intersect at (150, 100).

So, the vertices and corresponding number of fishes are

(200, 0) : 200

(0, 160) : 160

(150, 100): 250

So, the optimal point is 250 and the lake can support by satisfying food constraint is at the most 250 fishes.

14. A Given that, ≤

Maximize Z = 143x + 60y

Subject to the following constraints:

$x + y \leq 100 \quad \rightarrow (1)$

$120x + 210y \leq 15000 \rightarrow (2)$

$110x + 30y \leq 4000 \rightarrow (3)$

and $x, y \geq 0.$

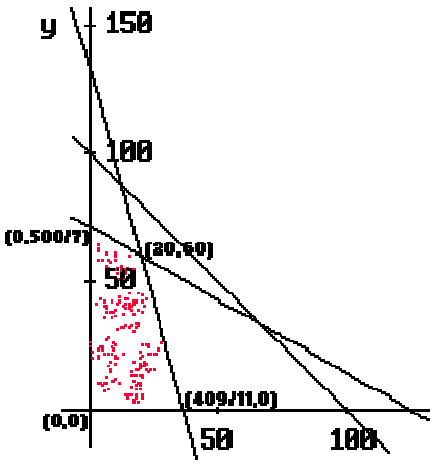
Consider the given constraints as equations. Then we will get

$x + y = 100 \quad \rightarrow (i)$

$120x + 210y = 15000 \rightarrow (ii)$

$110x + 30y = 4000 \rightarrow (iii)$

Represent above equations as straight lines on graph.



Check the vertices to find that the maximum value is 6460 **at** (20, 60).

15. D

Let the company produces X_1 times of 100 litres of Kalindi and X_2 times of 100 litres of Dark Rose. The objective of the company is to maximize profit. The company earns a profit of Rs. 120 from a litre of Kalindi and Rs. 80 from a litre of Dark Rose.

So, the objective function of the company is

Maximize $Z = 12000X_1+8000X_2$.

16. C The feasible region is the set off all points which satisfy all the constraints in the LPP.

17. E Mouse is an Input devise. Mouse is a pointing device, which is used in Graphical User Interface.

18. A In case of a negatively skewed distribution Arithmetic Mean < Median < Mode.

Hence (a) is the answer.

The relationships stated in (b), (c), (d) and (e) are not the features of negatively skewed distributions.

19. E An ogive is graph of cumulative frequency distribution.

20. A (a) The tallest rectangle in a histogram indicates the location of the mode.

(b) A multimodal distribution has more than one mode. Hence one can not say which of the values of the mode is the correct figure for central tendency.

(c) In a symmetrical distribution the mean, median and mode are all equal.

(d) The median and the mode can be found out graphically.

(e) The medians of two data sets can not be combined mathematically because mode is a positional average.

Hence from above discussion, option (a) is correct.

21. A

$$S^2 = \frac{1}{n-1}\left[\sum x^2 - n\bar{X}^2\right] = \frac{1}{n-1}\left[\sum x^2 - \frac{\left(\sum x\right)^2}{n}\right]$$

$$\sum_{i=1}^{10} x_i = \sum_{i=1}^{4} x_i + \sum_{i=5}^{10} x_i$$

$$\sum_{i=1}^{4} x_i = 16 \times 4 = 64$$

$$\sum_{i=5}^{10} x_i = 11 \times 6 = 66$$

$$\therefore \sum_{i=1}^{10} x_i = 64 + 66 = 130$$

$$\sum_{i=1}^{10} x_i^2 = 1726 \quad \text{(given)}$$

$$\therefore n = 10$$

$$\therefore s^2 = \frac{1}{9}\left[1726 - \frac{(130)^2}{10}\right] = 4$$

$$\therefore s = \sqrt{4} = 2.$$

22. C A number of programming languages are available for program writing. These languages can be classified as follows:

I. Machine Language
II. Assembly Language
III. High-level Language.

Hence Computer programming languages can be classified in to 3 categories.

23. D According to the Bienayme-Chebyshev rule, at least 75 percent of the observations in a sample are contained within two standard deviations around the mean, regardless of the shape of the distribution.

24. C If the mean, median and mode for a given population all equal 25, then we can say that its distribution is symmetric.

25. B Geometric mean of the set of five numbers = 2

$\therefore$ Product of the five numbers = $2^5 = 32$

Geometric mean of the set of four numbers = 4

$\therefore$ Product of the four numbers = $4^4 = 256$

Product of the combined set of numbers = $32 \times 256 = 8192$.

$\therefore$ Geometric mean of the combined set of numbers = $(8192)^{1/9} = 2.7216$.

26. C

Median $= L_m + \left[\frac{(N+1)/2-(F+1)}{f_m}\right]W$

The position of the median is the $\frac{N+1^{th}}{2}$ item i.e., $\frac{500+1}{2} = 250.5^{th}$ item.

Class	Frequency (f)	Cumulative Frequency (F)
250– 300	60	60
300–350	110	170
350–400	160	330
400–450	90	420
450–500	46	466
500–550	24	490
550–600	10	500

$\therefore$ Median class is 350–400.

$L_m = 350 \quad N = 500 \quad F = 170 \quad f_m = 160 \quad W = 50$

$\therefore$ Median $= 350 + \left[\frac{501/2-(170+1)}{160}\right]\times 50$

= Rs.374.8438.

27. D

Total marks obtained by the first group of students is given by = 49.96 × 100 = 4996 while the total marks of the second group of students is given by 52.32 × 200 = 10464. A total mark of the composite group is

4996 + 10464 = 15,460.

Therefore, the mean of the composite group is $\frac{15,460}{300} = 51.5333$.

28. E

Probability that the first ball drawn is black = $\frac{2}{3+2} = \frac{2}{5}$.

Probability that the second ball drawn is black = $\frac{1}{3+1} = \frac{1}{4}$.

Probability that both the balls drawn are black = $\frac{2}{5}\times\frac{1}{4} = \frac{1}{10}$.

29. C

We are given that $P(G_1) = 0.5$, $P(G_2) = 0.6$ and $P(G_3) = 0.8$ and hence their complementary events are given by:

$P(\bar{G}_1) = 0.5,\ P(\bar{G}_2) = 0.4 \ \ and \ \ P(\bar{G}_3) = 0.2$.

The event of exactly one hit may be the combinations of the following mutually exclusive events:

(i) $G_1 \ and \ \bar{G}_2 \ and \ \bar{G}_3$ happens

(ii) $\bar{G}_1 \ and \ G_2 \ and \ \bar{G}_3$ happens or

(iii) $\bar{G}_1 \ and \ \bar{G}_2 \ and \ G_3$ happens

Therefore from the additional theory of probability, the required probability may be obtained as:

$P = P(G_1 \ and \ \bar{G}_2 \ and \ \bar{G}_3) + P(\bar{G}_1 \ and \ G_2 \ and \ \bar{G}_3) + P(\bar{G}_1 \ and \ \bar{G}_2 \ and \ G_3)$

$= 0.5 \times 0.4 \times 0.2 + 0.5 \times 0.6 \times 0.2 + 0.5 \times 0.4 \times 0.8 = 0.04 + 0.06 + 0.16$

$= 0.26$.

30. C

The probability that both S and T occurs is given by $P(S \cap T)$, the occurrence of S but not T is given by $P(S \cap \overline{T})$ while the occurrence of T but not S is given by $P(\overline{S} \cap T)$.

Now, as per the terms of the problem, $P(S \cap T) = P(S \cap \overline{T}) = P(\overline{S} \cap T) = p$

Therefore, $P(S \cap T) + P(S \cap \overline{T}) + P(\overline{S} \cap T) = 3p$ or,

$P(S \cap T) + P(S) - P(S \cap T) + P(T) - P(S \cap T) = 3p$

or, $P(S) + P(T) - P(S \cap T) = 3p$ or, $P(S \cup T) = 3p$.

31. B

The sum is $\frac{3^n - 1}{3 - 1} = 364$

Or, $3^n - 1 = 728$

Or, $3^n = 729 = 3^6$

Or, $n = 6$

32. E

There are six faces in a die and they are named as 1, 2, 3, 4, 5 and 6. So, the corresponding probability would be 1k, 2k, 3k, 4k, 5k and 6k respectively. Since one of the face must appear the sum of all these probabilities should be one i.e. k+2k+3k+4k+5k+6k = 21k = 1 or k = $\frac{1}{21}$.

The probability of greater than two is 3k+4k+5k+6k = 18k = $\frac{18}{21} = \frac{6}{7}$.

33. C

Let the following notations be used:

A : Getting a plumbing contract

B : Getting a wiring contract

Given: $P(A) = \frac{2}{5}$

$P(B) = 1 - \frac{1}{4} = \frac{3}{4}$

$P(A \text{ or } B) = \frac{17}{20}$

Since the two types of contract are independent, we may consider them to be not mutually exclusive.

$\therefore$ P(A or B) = P(A) + P(B) – P(A and B)

or P(A and B) = P(A) + P(B) – P(A or B)

$= \frac{2}{5} + \frac{3}{4} - \frac{17}{20} = \frac{3}{10} = 0.3$

34. D

The event that X assumes a value at least 60 occurs when X = 60 or 80 or 100

$\therefore$ P(X assumes a value at least 60) = $P(X \geq 60)$

= P(X = 60) + (P (X= 80) + P(X = 100)

= 0.30 + 0.20 + 0.10

= 0.60

Let Y be the no. of observations of the variable X, that are at least 60. Then Y will follow a binomial distribution with number of trials = 5 and probability of success (i.e. getting at least 60) = 0.60

$\therefore$ P (Y = 3) = $^5C_3 (0.60)^3 (1 - 0.60)^2$

= 0.3456.

35. C A continuous uniform probability distribution is described by the upper and lower limits of the interval. ≤

36. C ≤

$$r = \frac{\text{Cov}(x, y)}{\sigma_x \sigma_y}$$

$r = +1 \Rightarrow \text{Cov}(x, y) = \sigma_x . \sigma_y.$

37. A ≤

Even numbers > 2 are 4 and 6

$\therefore$ P(Even numbers > 2) = P(4 or 6)

$$= \frac{2}{6} = \frac{1}{3}$$

Let X be the number of throws which give 4 or 6.

$\therefore$ X follows a binomial distribution with n = 10 and $p = \frac{1}{3}$

$\therefore$ $P(X = 6) = {}^{10}C_6 p^6 (1 - p)^{10-6}$

$$= {}^{10}C_6 \left(\frac{1}{3}\right)^6 \left(\frac{2}{3}\right)^4$$

$$= 0.0569.$$

38. A Let X be the number of questions that are correctly solved. Then X follows a binomial ≤

distribution with n = 10 and $p = \frac{1}{4} = 0.25$

$\therefore$ $P(X = 4) = {}^{n}C_r p^r (1 - p)^{n-r}$

$$= {}^{10}C_4 (0.25)^4 (1 - 0.25)^{10-4}$$

$$= 210 \times (0.25)^4 \times (0.75)^6$$

$$= 0.1459.$$

39. E

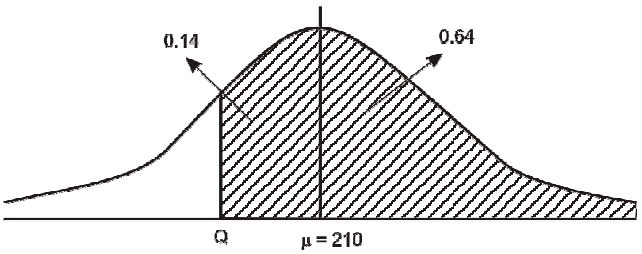
Minimum required probability of selling each portion of the dish, ≤

$$p^* = \frac{ML}{MP + ML}$$

Marginal profit, MP = 50 – 32 = Rs.18

Marginal loss, ML = Rs.32

$$\therefore p^* = \frac{32}{32+18} = 0.64$$



The Z-value for the area 0.14 to the left of the mean is –0.36.

$$\therefore -0.36 = \frac{x - 210}{50}$$

$x = 210 - (0.36 \times 50) = 192$

∴ The restaurant should prepare 192 portions at the optimal level to avoid losses due to spoilage.

$$\therefore \text{The number of whole chickens that should be ordered} = \frac{192}{2} = 96.$$

Sine from each whole chicken two portions of chicken can be prepared.

40. D

The F distribution is a distribution of the ratio of two random variables each of which follows a chi-square distribution. ≤

41. B

The random variable M is discrete uniform. So each of its 8 possible values has equal probability ≤

$$= \text{i.e., } \frac{1}{8}$$

$$E(M) = \frac{1}{8}\{1+2+3+.....8\}$$

$$= \frac{36}{8} = 4.5.$$

Or

$$E(M) = \frac{k+1}{2} \quad \text{(Where } k = 8 \text{ by the formula for expected value)}$$

$$= \frac{8+1}{2} = 4.5$$

.

42 B From given information, Maximum Profit = 400 – 300 = 100. ≤ ·

Maximum Loss = 300.

Conditional profit table for Drug X (in Rs.):

Stock	Demand					Expected
	300 (0.15)	400 (0.25)	500 (0.40)	600 (0.15)	700 (0.05)	Profit
300	30,000	30,000	30,000	30,000	30,000	30,000
400	0	40,000	40,000	40,000	40,000	34,000
500	(30,000)	10,000	50,000	50,000	50,000	28,000
600	(60,000)	(20,000)	20,000	60,000	60,000	6,000
700	(90,000)	(50,000)	(10,000)	30,000	70,000	(22,000)

Since the expected conditional profit is maximum at a stock level of 400 packets, the shop should stock 400 packets.

43 C From given information, Maximum Profit = 100. ≤ ·

Maximum Loss = 300.

Expected profit without perfect information:

Monthly stock level (units)		Expected profit
4,000	4,00,000 (0.10 + 0.25 + 0.40 + 0.20 + 0.05)	4,00,000
5,000	1,00,000 (0.10) + 5,00,000 (0.25 + 0.40 + 0.20 + 0.05)	4,60,000
6,000	(–2,00,000 × 0.10) + (2,00,000 × 0.25) + 6,00,000 (0.40 + 0.20 + 0.05)	4,20,000
7,000	(–5,00,000 × 0.10) + (–1,00,000 × 0.25) + (3,00,000 × 0.40) + 7,00,000 (0.20 + 0.05)	2,20,000
8,000	(–8,00,000 × 0.10) + (–4,00,000 × 0.25) + (0 × 0.40) + (4,00,000 × 0.20) + (8,00,000 × 0.05)	– 60,000

Highest expected profit without perfect information = Rs.4,60,000 ---- (A)

Conditional profit with perfect information:

Monthly stock level (units)	Monthly demand level in units				
	4,000 (0.10)	5,000 (0.25)	6,000 (0.40)	7,000 (0.20)	8,000 (0.05)
4,000	4,00,000				
5,000		5,00,000			
6,000			6,00,000		
7,000				7,00,000	
8,000					8,00,000

The expected profit under certainty is

= (4,00,000 × 0.10) + (5,00,000 × 0.25) + (6,00,000 × 0.40) + (7,00,000 × 0.20) + (8,00,000 × 0.05)

= Rs.5,85,000 ----- (B)

∴ Expected value of perfect information is (EVPI) = B – A = Rs.1,25,000.

44. B

For T = 35 standardised value = 2.

$$2 = \frac{35 - \mu}{\sigma} \text{ or } 2\sigma = 35 - \mu \text{ ... ... (A)}$$

For T = 40 standardised value = 3.

$$3 = \frac{40 - \mu}{\sigma} \text{ or } 3\sigma = 40 - \mu \text{ ... ... (B)}$$

Subtracting equation (A) from (B) we get $\sigma = 5$

Putting $\sigma = 5$ in (A) we get $\mu = 35 - 2\sigma = 35 - (2 \times 5)$

or $\mu = 25$

Mean = 25

Variance = $\sigma^2 = 5 \times 5 = 25$.

45. D In decision making under uncertainty the regret criterion is based on opportunity costs of decisions.

46. D The maximin criteria of decision making is useful to a pessimistic decision maker under conditions of uncertainty, not under conditions of risk.

47. B $E(x) = (1.16 \times 0.1) + (1.17 \times 0.1) + (1.18 \times 0.2) + (1.19 \times 0.2) + (1.2 \times 0.4) = 1.187$

Variance = $0.1(1.16-1.187)^2 + 0.1(1.17-1.187)^2 + 0.2(1.18-1.187)^2 + 0.2(1.19-1.187)^2 + 0.4(1.2-1.187)^2$

$= 1.81 \times 10^{-4}$

Standard deviation = $\sqrt{\text{Variance}} = 0.0134$.

48. A Other things remaining the same, if the sample size increases then the width of the confidence interval

for the population mean will decrease.

49. C The population is normally distributed and the population standard deviation is unknown.

So the t distribution with 25 –1 =24 degrees of freedom should be used. The test is left tailed and the significance level is 0.05.

So the critical value for the test is –1.711. Since the test statistic is less than the left tail critical value it falls in the critical region and we reject the null hypothesis.

50. C If we accept $H_0: \mu = 20$ in favor of $H_1: \mu \neq 20$ at a given level of significance with a positive value of the test statistic, then a test with $H_0: \mu = 20$ versus $H_1: \mu > 20$ using the same sample and the same level of significance may or may not accept H_0 because in a one tailed test the critical region is larger than a two tailed test for the same level of significance.

51. B Probability of committing a type I error = Probability of rejecting the null hypothesis when it is true ≤

By the central limit theorem $\bar{x}$ is normally distributed with mean $= \mu$ and variance $= \frac{\sigma^2}{n}$

Hence Probability of rejecting the null hypothesis when it is true =

$$P(\bar{x} \le 17.06 \text{ or } \bar{x} \ge 22.94) = P(\bar{x} \le 17.06) + P(\bar{x} \ge 22.94)$$

$$= P(\frac{17.06 - 20}{\sqrt{\frac{\sigma^2}{n}}} \le z) + P(\frac{22.94 - 20}{\sqrt{\frac{\sigma^2}{n}}} \ge z)$$

$$= P(\frac{-2.94}{\sqrt{\frac{81}{36}}} \le z) + P(\frac{2.94}{\sqrt{\frac{81}{36}}} \ge z)$$

$$= P(z \le -1.96) + P(z \ge 1.96)$$

$$= 0.025 + 0.025 = 0.05.$$

52. D This type of sampling is referred to as stratified random sampling because the entire population is divided in to classes (sectors) or strata which are homogeneous in nature and samples a fixed number of observations from each stratum. ≤

53. B Given that, sample size is 8, population size is 50 and population standard deviation is 10, then standard error of sample mean is calculated using the given formula. ≤

$\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}}\sqrt{\frac{(N-n)}{(N-1)}}$. Because the value of the sampling fraction is greater than 0.05.

$$\sigma_{\bar{x}} = \frac{10}{\sqrt{8}}\sqrt{\frac{(50-8)}{(50-1)}}$$

$$= 3.5355 \times 0.9258$$

$$= 3.2733.$$

54. D $H_o : \mu = 60$ ≤

$\bar{x} = 72$

$$\text{Test statistic} = \frac{\bar{x} - \mu}{\sigma_{\bar{x}}}$$

$$\text{Or} \quad 1.20 = \frac{72 - 60}{\left(\frac{60}{\sqrt{n}}\right)}$$

$$\text{Or} \quad \frac{60}{\sqrt{n}} = \frac{72 - 60}{1.20} = 10$$

$$\text{Or} \quad n = \left(\frac{60}{10}\right)^2 = 36.$$

55. D Slope of the estimated regression equation of Z on X = <

$$\frac{n\sum XZ-\sum X\sum Z}{n\sum X^2-(\sum X)^2}=\frac{n\sum X(Y+2)-\sum X\sum(Y+2)}{n\sum X^2-(\sum X)^2}$$

$$=\frac{n[\sum XY+2\sum X)]-\sum X(\sum Y+\sum 2)}{n\sum X^2-(\sum X)^2}$$

$$=\frac{n\sum XY+2n\sum X-\sum X\sum Y-\sum X\sum 2}{n\sum X^2-(\sum X)^2}$$

$$=\frac{n\sum XY+2n\sum X-\sum X\sum Y-2n\sum X}{n\sum X^2-(\sum X)^2}$$

$$=\frac{n\sum XY-\sum X\sum Y}{n\sum X^2-(\sum X)^2}$$

= Slope of regression equation of Y on X

= 5.

56. C The point estimate of population mean = sample mean = <

$$\frac{\sum_{i=1}^{n}(X_i-0)}{n}=\frac{6+11+9+11+8+15}{6}=\frac{60}{6}=10.$$

57. B Let the following notations be used: <

X : Year

x : Coded value of year = 2 $(X-\bar{X})$

Y : Research expenditure (in Rs.)

Year X	$x = 2(X-\bar{X})$	Y	XY	x^2
2002	–5	50	–250	25
2003	–3	57	–171	9
2004	–1	61	–61	1
2005	1	65	65	1
2006	3	72	216	9
2007	5	79	395	25
$\Sigma X = 12027$	0	$\Sigma Y = 384$	$\Sigma XY = 194$	$\Sigma x^2 = 70$

$$\bar{X}=\frac{12027}{6}=2004.5$$

$$\hat{Y}=a+bx$$

$$b=\frac{\sum xY}{\sum x^2}=\frac{194}{70}=2.7714$$

$$a=\bar{Y}=\frac{384}{6}=64$$

$\therefore$ Three linear trend equation is $\hat{Y}=64+2.7714x$

Year (X) = 2008

x = 2 (2008 – 2004.5) = 7

$\hat{Y}=64+2.7714\,(7)=83.3998.$

$\therefore$ The research and development expenditure for the year 2008 is Rs.83.39 lakh.

58. B

Proportion of variation in Y, that is not explained by the regression line = 1– Coefficient of determination (R^2)

Where, $R^2 = \frac{a\sum Y + b\sum XY - n\bar{Y}^2}{\sum Y^2 - n\bar{Y}^2}$, Given that, $\sum XY = 1000$, $\sum X = 30$, $\sum X^2 = 200$

Number of data points = 6, $\sum Y = 180$ and $\sum Y^2 = 5642$.

$$b = \frac{n\sum XY - \sum X.\sum Y}{n\sum X^2 - \left(\sum X\right)^2} = \frac{(6\times 1000) - (30\times 180)}{(6\times 200) - (30\times 30)} = 2$$

$$a = \bar{Y} - b\bar{X} = \left(\frac{180}{6}\right) - 2\times\left(\frac{30}{6}\right) = 20$$

$$\therefore R^2 = \frac{(20\times 180) + (2\times 1000) - (6\times 30^2)}{5642 - (6\times 30^2)}$$

$$= \frac{200}{242} = 0.8264$$

$\therefore$ Proportion of variation in Y that is not explained by the regression line

$= 1 - R^2$

$= 1 - 0.8264 = 0.1736$

$\Rightarrow$ Percentage of variations in Y that is not explained by the regression line = 17.36%.

59. C

Marshall –Edgeworth price index = $\frac{\sum P_1(Q_0 + Q_1)}{\sum P_0(Q_0 + Q_1)} \times 100 = \frac{99562.50}{74812.5} \times 100 = 133.0827$.

60. B

Relative Cyclical Residual Measure $= \dfrac{Y - \hat{Y}}{Y} \times 100$

$1997 = \dfrac{7.5 - 7.6}{7.6} \times 100 = -1.3158$

$1998 = \dfrac{7.8 - 7.8}{7.8} \times 100 = 0.0$

$1999 = \dfrac{8.2 - 8.0}{8.0} \times 100 = 2.5$

$2000 = \dfrac{8.2 - 8.2}{8.2} \times 100 = 0.0$

$2001 = \dfrac{8.4 - 8.4}{8.4} \times 100 = 0.0$

$2002 = \dfrac{8.5 - 8.6}{8.6} \times 100 = -1.1628$

$2003 = \dfrac{8.7 - 8.8}{8.8} \times 100 = -1.1364$

$2004 = \dfrac{9.1 - 9.0}{9.0} \times 100 = 1.111$

The largest fluctuation from trend has taken place in the year 1999, i.e., 2.5 %.

61. D

As percentages are chained together in chain index numbers, long range comparisons are **not** very accurate.

Hence, we can infer that option (d) is **not** an advantage of chain base method of constructing index numbers.

Options (a), (b), (c) and (e) are all advantages of chain base method of constructing index numbers.

62. D

Fisher's ideal price index $= \sqrt{\dfrac{\sum P_1Q_0}{\sum P_0Q_0} \times \dfrac{\sum P_1Q_1}{\sum P_0Q_1}} \times 100$

ΣP_1Q_0 $= (10.25 \times 5000) + (14.00 \times 6000) + (28.00 \times 3000) + (14.50 \times 3000) + (4.50 \times 500) = 265000$

ΣP_1Q_1 $= (10.25 \times 4800) + (14.00 \times 5500) + (28.00 \times 3500) + (14.50 \times 2400) + (4.50 \times 600) = 261700$

ΣP_0Q_0 $= 205600$ and $\Sigma P_0Q_1 = 205500$

$\therefore$ Fisher's ideal price index $\sqrt{\dfrac{265000}{205600} \times \dfrac{261700}{205500}} \times 100 = 128.1172.$

63. C Weighted average of relatives price index ≤

$$= \frac{\Sigma\left[\left(\frac{P_1}{P_0}\times 100\right)P_n\,Q_n\right]}{\Sigma P_n Q_n}$$

Commodities	$\frac{P_1}{P_0}\times 100$	$P_n\,Q_n$	$\left(\frac{P_1}{P_0}\times 100\right)P_n Q_n$
Rice	118.42	10,000	11,84,200
Wheat	130.77	4,050	5,29,618.50
Salt	160	1,300	2,08,000
Sugar	145	9,600	13,92,000
Pulses	137.78	13250	18,25,585
	Total	38,200	51,39,403.50

$\Sigma\left[\frac{P_1}{P_0}(\times 100)P_n Q_n\right]$ = 51,39,403.50

$\Sigma P_n Q_n$ = 38,200.

∴ Weighted average of relations price index for year 2001.

$= \frac{51,39,403.50}{38,200} = 134.5394.$

64. B ≤

Laspeyres Quantity Index for the given data is $\frac{\sum Q_1.P_0}{\sum Q_0.P_0}$ x 100 i.e.,

$\frac{700\times 15+70\times 8+120\times 6+70\times 10}{500\times 15+100\times 8+200\times 6+50\times 10}$ x 100 i.e., 124.8.

Hence from above discussion, we can infer that option (b) is correct.

65. D ≤

The Upper Control Limit for p-chart = $p + 3\sqrt{\frac{p.q}{n}}$

$= 0.27 + 3x\sqrt{\frac{0.27x0.73}{55}}$ (replacing p with value of $\bar{p}$)

= 0.4496.

Hence from above discussion, we can infer that option (d) is correct.

66. C ≤

The Lower Control Limit in an R Chart = $\bar{R} - \frac{3d_3\bar{R}}{d_2}$

$= 7.1 - \frac{3x0.833x7.1}{2.704}$

= 0.5383.

Hence from above discussion, we can infer that option (c) is correct.

67. E

The Upper Control Limit of $\bar{x}$ Chart $= \mu_{\bar{X}} + 3\sigma_{\bar{x}}$

$$= \mu_{\bar{X}} + 3\frac{\sigma}{\sqrt{n}}$$

$$= 1.60 + 3 \text{ x } \frac{0.15}{\sqrt{8}}$$

$= 1.7591$ cm.

Hence from above discussion, we can infer that option (e) is correct.

68. C

(a) This is a wrong answer. If we observe that the sample means are hugging the center line then the deviations are minor and uniform in nature. This indicates that the variation has been reduced to great extent.

(b) This is a wrong answer. If the observations are such that it can be described to be 'hugging the control limits', then this refers to the deviations being uniform in nature but having large magnitude. This indicates that the means of two different populations are being observed.

(c) This is the right answer. If the observations can be described as jumps in the level around which the observations vary, then this pattern indicates that the process mean has been shifting.

(d) This is a wrong answer. If some of the points lie outside the upper or lower control limits or if there is an increasing or decreasing trend in the observations, we conclude that the process is out of control.

(e) This is a wrong answer.

69. A

If we have large enough sample sizes, we can discard the assumption that the samples are drawn from a normal population.

Hence from above discussion, we can infer that option (a) is correct.

70. C

$$\text{F ratio} = \frac{\text{Between-column variance}}{\text{Within-column variance}}$$

$$= \frac{4}{3} = 1.3333.$$

The above value is the least.

Hence from above discussion, we can infer that option (c) is the answer.

71. D

In ANOVA:

Number of degrees of freedom in the numerator of F ratio = Number of samples – 1

$= 4 - 1 = 3$

Number of degree of freedom in the denominator of F ratio

= Total sample size – Number of samples

$= (5 + 5 + 6 + 4) - 4$

$= 16.$

72. A

$$\chi^2 = \frac{(n-1)s^2}{\sigma^2}$$

$$= \frac{29 \times 0.73^2}{1}$$

$$= 15.4541.$$

73. D The following statements are **true** regarding simulation. <

I. If the mathematical model set up could always be optimized by the analytical approach, then, there would be no need for simulation.

II. Only when interrelationships are too complex or there is uncertainty regarding the values that could be assumed by the variables or both, we would have to resort to simulation.

Given statement (III) is not true regarding Simulation. Since,

Simulations **can** be used to break down complex systems into sub-systems and study the behavior of each of these subsystems.

Hence option (d) is correct.

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