LPP by Simplex Method [Vogel's Approximation Method]

STEP 1: FORMULATE THE LPP

Maximisation problem used in this explanation

$Z = \text{Rs. } 4x_1 + \text{Rs. } 3x_2$

Subject to:

$2x_1 + x_2 \leq 30$ hours [Machine 1]
$X1 + 2x_2 \leq 24$ hours [Machine 2]

STEP 2: DEVELOP THE INEQUALITIES BY ADDING SLACK VARIABLES

- ★ Simplex method cannot be used unless all the variables are stated as equations.
- ★ Therefore to make the inequalities into equalities we have to add SLACK VARIABLES.
- ★ Slack Variables represents costless process whose function is to use up otherwise unemployed capacity (idle or unused resource). As it represents an unused resource it has to be positive.
- ★ Co-efficient of slack variable will be '1'.

After insertion of slack variables the variables will be as follows: (in the given example)

$2x_1 + x_2 + S_1 = 30$---[Machine 1]
$x_1 + 2x_2 + S_2 = 24$--[Machine 2]

Complete maximisation problem after insertion of Slack variables will be as follows:

$Z = \text{Rs. } 4x_1 + \text{Rs. } 3x_2 + 0S_1 + 0S_2$

Subject to:

$2x_1 + x_2 + S_1 + 0S_2 = 30$
$x_1 + 2x_2 + 0S_1 + S_2 = 24$

S_1, S_2 – Slack Variables
x_1, x_2 – Structural or Ordinary Variables

STEP 3: DEVELOP THE INITIAL SIMPLEX TABLEAU INCLUDING TRIVIAL SOLUTION

INITIAL SIMPLEX TABLEAU

C_j	Product Mix	Quantity	Rs.4 X_1	Rs.3 X_2	Rs. 0 S_1	Rs.0 S_2
Re. 0	S_1	30	2	1	1	0
Re. 0	S_2	24	1	2	0	1
	Z_j	Re. 0	Re. 0	Re. 0	Re. 0	Re. 0
	$C_j - Z_j$		4	3	0	0

STEPS FOR DEVELOPMENT OF INITIAL SIMPLEX TABLEAU

1. **C_j Row**

   Also known as objective row

   IMPORTANCE This row shows the profit made per unit of each variable produced

   In this example:

C_j	Product Mix	Quantity	Rs. 4	Rs. 3	Rs. 0	Rs. 0
			X_1	X_2	S_1	S_2

2. **Restraint Equations**

   The two restraint equations are to be shown in the tableau as follows

C_j	Product Mix	Quantity	Rs. 4	Rs. 3	Rs. 0	Rs. 0
			X_1	X_2	S_1	S_2
Re. 0	S_1	30	2	1	1	0
			Represents Coefficients of First Equation			
Re. 0	S_2	24	1	2	0	1
			Represents Coefficient of Second Equation			

Product Mix, Quantity & Variable columns

- ★ Write the two slack variables[means initially there will be no production]

IMPORTANCE:

From Graphical solution we know that all solutions for the LPP will be found amongst the corners of the feasible region. Most common and known corner point is '0' i.e., where there is no production and no profit. The same is being applied here by introducing the slack variables in the initial table. This is called **TRIVIAL SOLUTION**.

NOTE: X_1, X_2 DON'T APPEAR IN THE PRODUCT MIX COLUMN THEY ARE EQUAL TO 'ZERO'

- ★ In Quantity Column corresponding to slack cells, write the respective constraints i.e., for S_1 30 & for S_2 24
- ★ In variables columns write the co-efficient from the respective equations i.e., under X_1 2/1, X_2 ½, S_1 1/0, and under S_2 0/1

3. **C_j Column**

   This column shows the profit per for the variables S_1, S_2 i.e., always '0'

C_j	Product Mix	Quantity	Rs. 4	Rs. 3	Rs. 0	Rs. 0
			X_1	X_2	S_1	S_2
Re. 0	S_1	30	2	1	1	0
Re. 0	S_2	24	1	2	0	1

4. Z_j & C_j Rows

Z_j Row

★ Z_j Row will be inserted just below S_2 Row

★ The values in this row shall be arrived at as follows:

(Coefficient of Column corresponding to First row × C_j of that row)

+

(Coefficient of column corresponding to second row × C_j of that row)

In the given example it will be as follows:

For X_1 column	For X_2 column	For S_1 column	For S_2 column	For Quantity column
(2×0)+(1×0)=0	(1×0)+(2×0)=0	(1×0)+(0×0)=0	(0×0)+(1×0)=0	(30×0)+(24×0)=0

IMPORTANCE OF Z_j ROW

The Z_j row values represent the amount by which the profit would be reduced by introducing one unit of any of the variables (including slack variables) if they were added to the mix

EXPLANATION TO THE ABOVE IN THE GIVEN EXAMPLE

Z_j value of X_1=0 this means that by introducing one unit of X_1 we must give up 2hrs of S_1 and 1hr of S_2. By giving up S_1 and S_2 the loss will be nil as there will be no contribution from slack variables

C_j - Z_j Row [Difference between C_j and Z_j]

It is the net profit that will occur from introducing one unit of a variable to the production schedule.

By this the Initial Simplex Tableau is completed. From this table the C_j - Z_j row it is clear that for each unit of variable X_1 profit can be increased by Rs.4 and X_2 by Rs. 3. This further means that profit can further be improved and this not the final/optimal solution tableau

General interpretation of C_j - Z_j row values:

- ★ If the values are positive – profits may be further improved
- ★ If the values are negative – profits would be decreased by that amount
- ★ The tableau should be developed till there are no positive numbers in the C_j - Z_j row. This will be the optimal solution

STEP 4: FURTHER DEVELOPMENT OF THE INITIAL SIMPLEX TABLEAU

STEP 4(A) CHOOSING THE ENTERING VARIABLE

Now we have to choose the entering column which will give the highest profit. This can be done by examining the C_j - Z_j row values the highest value in this row will be entering the product mix, in the given example it is X_1 = 4.

This column is called **OPTIMUM COLUMN** or **PIVOT COLUMN**

C_j	Product Mix	Quantity	Rs. 4	Rs. 3	Rs. 0	Rs. 0
			X_1	X_2	S_1	S_2
Re. 0	S_1	30	2	1	1	0
Re. 0	S_2	24	1	2	0	1
	Z_j	Re. 0	Re. 0	Re. 0	Re. 0	Re. 0
	C_j - Z_j		4	3	0	0

OPTIMAL COLUMN

Intersectional Element

STEP 4(B) CHOOSING THE DEPARTING VARIABLE

This is done in the following manner:

1. Divide each number in the quantity column by the corresponding numbers in the first column(X_1)
   In the example
   30/2=15
   24/1=24
2. Select the row with smaller/smallest ratio as the row to be replaced
   In the example
   S_1 row 30/2 = 15 units of X_1 (as this row has smallest ratio it will be replaced by 15 units of X_1)

S_2 row 24/1 = 24 units of X_2

It is known as **PIVOT ROW** or **KEY ROW** [the row which is being replaced]

The element common to optimum column and row is called **INTERSECTIONAL ELEMENT**.

In the example 2 and 1 are intersectional elements

STEP 4(C) CALCULATION OF VALUES FOR REPLACING ROW [X_1 ROW OF THE NEW TABLEAU IS COMPUTED AS FOLLOWS]

Divide each element in the Pivot Row by the Intersectional Element

In this example
30/2 = 15
2/2 =1
1/2 =1/2
1/2 = ½
0/2 =0
Thus the new X_1 row would be (15, 1, 1/2, 1/2, 0)

STEP 4(D) COMPUTATION OF VALUES FOR REMAINING ROWS

The values for remaining rows are calculated with the help of the following formula

ELEMENTS IN OLD ROW - INTERSECTIONAL ELEMENT OF OLD ROW × CORRESPONDING ELEMENTS IN REPLACING ROW

In the given example

Element in old S_2 row	Minus	Intersectional Element of S_2 row	×	Corresponding element in replacing row	=	New S_2 row
24	-	1	×	15	=	9
1	-	1	×	1	=	0
2	-	1	×	1/2	=	1 ½
0	-	1	×	1/2	=	-½
1	-	1	×	0	=	1

NOTE: GREY REGION ONE CALCULATION

STEP 4(E) CALCULATE THE REMAINING ROWS (Z_j & C_j - Z_j rows) AS PREVIOUSLY DISCUSSED UNTIL THE VALUES IN C_j - Z_j ROW ARE NON POSITIVE