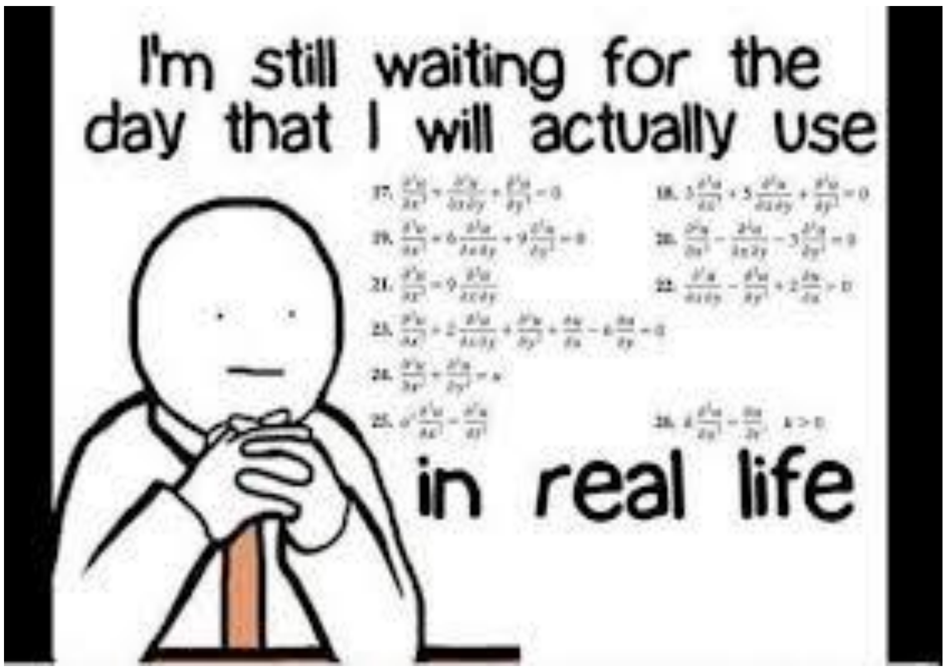
S F M n A F M Formulae



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1. Capital Budgeting

TYPES OF CAPITAL BUDGETING PROPOSALS

If more than one proposal is under consideration, then these proposals can be categorised as follows:

1.Mutually Exclusive Proposals : Two or more proposals are said to be **Mutually Exclusive Proposals** when the acceptance of one proposal implies the automatic rejection of other proposals, mutually exclusive to it.

2.Complementary Proposals: Two or more proposals are said to be **Complementary Proposals** when the acceptance of one proposal implies the acceptance of other proposal complementary to it, rejection of one implies rejection of all complementary proposals.

3.Independent Proposals: Two or more proposals are said to be **Independent Proposals** when the acceptance/rejection of one proposal does not affect the acceptance / rejection of other proposals.

Present Value (P V) = Annuity (A) / Cost of Capital (K)

O R

Annuity (A) = Present Value (P V) X Cost of Capital (K)

P V of Annuity recd. In Perpetuity = A / R

Where A is Annuity and R is rate of interest.

Discount rate inclusive of inflation is called **Money Discount Rate** and discount rate exclusive of inflation is called **Real Discount Rate.**

Risk Adj. Disc. Rate (RADR): Risk free rate (R_f)+ Risk Premium

O R

Risk Adj. Disc. Rate (RADR): Risk free rate (R_f)+ β X Risk Premium

Coefficient of Variation $= \frac{\text{S D}}{\text{Expected NPV}} = \frac{\text{S D}}{\text{Average NPV}}$

The lower coefficient of Variation, the better it is.

$$\textbf{Profitability Index (P I)} = \frac{\textbf{P V of Future cash Inflows}}{\textbf{Initial Investment}}$$

O R

$$\textbf{Profitability Index (P I)} = \frac{\textbf{P V of Future cash Inflows}}{\textbf{P V of Future cash Outflows}}$$

Accept when P I > 1. The higher the better.

$$\text{Pay Back Period} = \frac{\text{Initial Investment}}{\text{Annual Cash flows}}$$

Accept the Project with least Payback Period.

Discounted Payback period is calculated by the same formula as payback period with the exception that discounted cashflows are considered instead of actual cashflows

$$\boldsymbol{NPV = \sum_{t=1}^{n} \frac{\bar{A}_t}{(1 + R_f)^t}}$$

Where A¯t is the expected cashflow in period t and Rf is risk free rate of interest.

Accept when NPV is +ve. The higher the better.

Project NPV is the one which is calculated for the whole Project. Equity NPV is the one which is calculated to know the return of equity holders. In this case funds relating to equity holders alone are considered for computation.

I R R is the rate at which the present value cash inflows equal the present value of cash outflows i.e. NPV is zero. For **Project IRR** all cashflows of the project are considered and in case of **Equity IRR** only cashflows relating to equity holders is considered. In case IRR is greater than interest rate on loan, then, the excess rate will accrue to the benefit of equity holders.

Standard Deviation For one year period:

$$\text{Standard Deviation} = \sum \text{Probability X (Given Value - Expected Value)}^2$$

Variance = (Standard Deviation)^2

Standard Deviation is calculated by the formula:

$$\boldsymbol{\sigma} = \sqrt{\sum_{t=1}^{n} \frac{\sigma^2{}_t}{(1+R_f)^{2t}}}$$

Where, σ_t Standard deviation of possible net cashflows in period t.

When Cash flows are perfectly correlated over time:

$$\boldsymbol{\sigma} = \sum_{t=1}^{n} \frac{\sigma_t}{(1+R_f)^{t}}$$

In case cashflows are moderately correlated over time:

$$\boldsymbol{\sigma} = \sqrt{\sum_{t=1}^{n} (NPV - \overline{NPV})^2 \, P_t}$$

In case if two or more projects have same mean cashflows, then, the project with lesser standard deviation is preferred.

Certainty Equivalent approach: In this case certain cashflows are discounted instead of all project cashflows.

Joint probability: Joint Probability is the product of two or more than two dependent probabilities. Sum of Joint Probabilities will always be equal to 1.

Conditional Event: If A & B are two events, then, if event B occurs after occurrence of event A, it is called conditional event and is denoted as (B/A)

Conditional Probability: If A & B are 2 events of a sample, then the probability of B after the occurrence of event A is called conditional Probability of B given A and is denoted as P(B/A).

$$P(B/A) = \frac{P(A \cap B)}{P(A)} \Rightarrow P(A \cap B) = P(A) \text{ X } P(B/A)$$

Notation of Events:

Event A or Event B Occurs	A U B
Event A & Event B Occur	$A \cap B$
Neither A Nor B Occur	$(A \cup B)^c = A^c \cap B^c = \overline{A} \cap \overline{B}$

A Occurs but B does not Occur $A \cap B^c = A / B$

2. Leasing

Evaluation of lease for Lessee: In this case comparison is to be made between the NPV of outflows:

a. in case an asset is leased and used, or
b. the asset is bought and used.

BELR in case of lessee is the amount at which he will be indifferent between buying an asset and leasing an asset.

Evaluation of lease for Lessor: In this case comparison of P V of cashflows is to be made between:

a. buying an asset and leasing it getting lease income or
b. return expected by investing the funds elsewhere.

BELR for lessor is the amount at which he will be indifferent between buying the asset and giving it for lease and / or investing the funds elsewhere and earning desired cost of capital.

Equated yearly Installment = Loan amount / Cumulative Annuity Factor of desired period

While ascertaining Present Values, people are using different rates for discounting like interest rate, net of tax interest rate, cost of capital etc. based on respective views. Ideal one will be the one using net of tax interest rate in case of borrowing or using the cost of capital in case of using own funds.

3. Dividends

$$\text{Rate of Dividend} = \frac{\text{Dividend Per Share}}{\text{Face Value Per Share}} \times 100$$

$$\text{Dividend Yield} = \frac{\text{Dividend Per Share}}{\text{Market Price Per Share}} \times 100$$

$$\text{Dividend Payout} = \frac{\text{Dividend Per Share}}{\text{Earnings Per Share}} \times 100$$

Equity dividends are to be paid after paying preference dividends.

Traditional Theory:

$$\mathbf{P} = \frac{\mathbf{m(4D + R)}}{\mathbf{3}} = \mathbf{m(D} + \frac{\mathbf{E}}{\mathbf{3}}\mathbf{)}$$

Where, P = Mkt. Price of share; D = Dividend per share; R = Retained Earnings per share; E= EPS & m = a constant multiplier.

Walter Model:

$$\mathbf{P} = \frac{\mathbf{D} + \frac{\mathbf{r\,(E - D)}}{\mathbf{k}}}{\mathbf{K}}$$

Where, P = Mkt. Price of share; D = Dividend per share; r = Return on internal retentions; E= EPS & k = Cost of Capital.

If r > k, the share price will increase as Div. Payout decreases.
If r = k, then the price will not change with Div. Payout ratio.
If r < k, then also share price will increase because Div. Payout increases.

Optimal payout ratio will be for:
A growth co. (i.e r > k) is nil.
A normal co. (i.e. r = k) is irrelevant, and
A declining (i.e. r < k) firm is 100%

Gordon Model:

$$\mathbf{P_o} = \frac{\mathbf{D_1}}{\mathbf{K - g}} = \frac{\mathbf{D_o(1 + g)}}{\mathbf{K - g}} = \frac{\mathbf{E\,(1 - b)}}{\mathbf{K - br}}$$

Where, P_0 = Mkt. Price per share before dividend; D_1 = Dividend per share in year 1; D_0 = Dividend per share in current year; k = cost of equity; g = growth rate of dividends; E = EPS, b = % of earnings retained; and r = Return on internal retentions

g = b * r

In case the dividend amount is fixed for each year, then,

$$\mathbf{P} = \frac{\mathbf{D_1}}{\mathbf{K}}$$

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Modigliani-Miller (M M) Model:
If dividend is declared;

$$\mathbf{P_0 = \frac{P_1 + D_1}{1 + K} \Rightarrow P_1 = P_0(1 + K) - D_1}$$

If dividend is not declared;

$$\mathbf{P_0 = \frac{P_1}{1 + K} = \Rightarrow P_1 = P_0(1 + K)}$$

Where, P0 = Current Mkt. Price of share; P1 = Mkt. Price at the end of period 1; K= Cost of capital; D1 = Div. In year 1.

Lintner Model:

$$\mathbf{D_1 = D_0 + [(EPS * Target\ payout) - D_0] * Af}$$

D_1 = Div. In year 1; D_0 = Div. In year 0; Af = Adj. Factor

Dividend Discount Model:

Intrinsic value = Sum of the PVs' of future cash flows.
= PV of dividends + PV of sale value of stock.

$$\mathbf{= \frac{D_1}{(1+k)^1} + \frac{D_2}{(1+k)^2} + \frac{D_3}{(1+k)^3} + \ldots + \frac{D_n}{(1+k)^n} + \frac{P}{(1+k)^n}}$$

D_n = Dividend of year n; P = Sale price of stock; n = No. Of yrs.; K = Capitalisation rate.

Discount model has 3 sub models:

a. Zero growth model: Each year dividends are fixed:
   Then, **Intrinsic Value = D / K**
b. Constant growth (Gordon Model):
   Then, **Intrinsic Value (P) = D_1 / (K – g)**

P = Mkt. Price; D_1 = Div. In year 1; K = Cost of capital; g = growth rate of dividends.

c. Variable growth rate model: Usually the three stages of growth, the initial high rate of growth, a transition to slower growth, and lastly a sustainable steady growth are considered. For each type of stage, appropriate variables are considered and PVs' are calculated and sum of the three PVs' is the intrinsic value of share.

Holding Period Return: This is the sum of dividend yield and capital gain yield which is nothing but Total Yield.

$$\textbf{Total Yield} = \frac{D_1}{P_0} + \frac{P_1 - P_0}{P_0}$$

$$\textbf{Earning Yield} = \frac{\text{Earning Per Share}}{\text{Market Price Per Share}} \text{ X } 100$$

4. <u>Capital Markets</u>

$$\text{Today's Index Value} = \frac{\text{Today's Market Capitalisation}}{\text{Yesterday's Market Capitalisaton}} \text{ X Yesterday's Index Value.}$$

The difference between the prevailing price and futures price is known as basis.

Basis = Spot Price – Future Price

In a normal market, spot price will be less than future price as future price includes cost of carrying also. Further, apart from carrying cost the future price may also change due to dividends etc. So,

Future price = Spot price + carrying cost – returns (dividends etc.)

In the case of annual compounding, forward price is calculated by the formula:

$$A = P\,(1+r/100)^{t}$$

Where A is the terminal value for an investment of P at r rate of interest per annum for t years. In case interest is compounded n times in a year, then,

$$A = P\,(1+r/100n)^{nt}$$

In case the compounding is more than once on daily basis, then the formula stands modified as:

$$A = P * e^{rn}$$

Where e is called epsilon, a constant and its value is taken as 2.72

Alternatively,

$$\text{Future Value} = \text{Present Value} * e^{rt}$$

In case any income flows are there, then they are to be deducted and the formula will be:

$$A = (P - I) * e^{rn}$$

Where I is the present value of income inflow.

If the income accretion is in the form % yield y (like in index futures) then the formula is:

$$A = P * e^{(r-y)n}$$

Options Seller is called **Writer or Grantor** and the buyer is called simply buyer or at times as dealer / trader.

An option is said to be **in the money** when it is advantageous to exercise it.
When exercise is not advantageous it is called **out of the money**.
When option holder does not gain or lose it is called **at the money**.

In case of an option buyer, while there is no limit on the profit he can make, loss is limited to the value of premium he pays to buy the option. In problems, if premium is not given, loss is to be taken as zero. It will be opposite in case of option seller i.e. Writer or Grantor.

Intrinsic Value and Time Value of Option: Option premium has two components viz. Intrinsic Value and Time Value. Intrinsic Value is the difference between Exercise price and Market price or Zero whichever is higher. Time Value is the excess of Option price over its Intrinsic Value.

European Option can be exercised only on the due date of the option.
American Option can be exercised at any time during the period of option.

TIME IS THE ALLY OF WRITER AND ENEMY OF OPTION BUYER SINCE IN THE LONG RUN GOOD STOCKS WILL USUALLY DO BETTER.

Black-Scholes Model:

$$\mathbf{OP} = \mathbf{S\,N(d_1)} - \frac{\mathbf{X\,N(d_2)}}{\mathbf{e^{rt}}}, \text{ And}$$

$$\mathbf{d_1} = \frac{\ln\left(\frac{S}{X}\right) + \left(r + \frac{v^2}{2}\right) t}{v\,t^{1/2}}$$

$$\mathbf{d_2} = \frac{\ln\left(\frac{S}{X}\right) + \left(r - \frac{v^2}{2}\right) t}{v\,t^{1/2}} = \mathbf{d_1} - \mathbf{v t^{1/2}} \text{ where,}$$

S = Current Stock Price

X = Strike Price,
r = Continuously Compounded Risk free Interest Rate
t = Balance period of option expressed as percentage,
$N(d_1)$ = Normal distribution of d_1
$N(d_2)$ = Normal distribution of d_2
ln = Natural Logarithm
e = exponential constant with value 2.72,
v = Volatility of stock, i.e. Standard Deviation
$N(d_1)$ is the hedge ratio of stock to options, to keep the writer hedged and $N(d_2)$ / ert is Present Value of the borrowing.

The above formula can be used to find the **Value of Equity** also with little changes. In this case, in the place current stock price we need to use current value of business and in the place of strike price we need to use value of debt. Rest of all the things are same.

Binomial Model:

$$\textbf{Up tick } \mathbf{u_t} = \frac{\mathbf{uV_{t+1}}}{\mathbf{uV_t}} = \frac{\mathbf{uV_{t+2}}}{\mathbf{uV_{t+1}}}$$

$$\textbf{Down tick } \mathbf{d_t} = \frac{\mathbf{dV_{t+1}}}{\mathbf{dV_t}} = \frac{\mathbf{dV_{t+2}}}{\mathbf{dV_{t+1}}}$$

Where uV_t = up value in period $_t$; uV_{t+1} = uptick value in period $_{t+1}$ and uV_{t+2} = uptick value in period $_{t+2}$. Similarly, dV_t, dV_{t+1}, & dV_{t+2}.

In case of continuous compounding,

$$\textbf{Probability } \mathbf{P} = \frac{\mathbf{e^{rt} - d_t}}{\mathbf{u_t - d_t}}$$

In case of normal compounding,

$$\textbf{Probability } \mathbf{P} = \frac{\textbf{Spot price } (\mathbf{1 + int.rate}) - \mathbf{d_t}}{\mathbf{u_t - d_t}} = \frac{\mathbf{r - d_t}}{\mathbf{u_t - d_t}}$$

Where e is epsilon (with value 2.72) and $_t$ is time period. Please note that the $_t$ in $_{rt}$ indicates time period whereas the $_t$ in u_t and d_t indicates tick.

Risk neutral Method:

Option Value = [C_u X P + C_d (1-P)] / (1 + r)

Where C_u is option value during uptick and C_d is option value during down tick P is probability and r is rate of interest.

Put Call Parity Theory: This theory holds good only when exercise price and maturity of both put and call options are same.

Value of Call – Value of Put = Current Market Price – P V of Strike Price

In case of an All In Cost (AIC) swap, fixed interest payment is calculated by the formula:

Fixed Interest Payment = N * AIC * 180 /360

Where N denotes notional principal amount.

Floating interest payment is calculated by the formula:

Floating Interest Payment = N * LIBOR * d_t / 360

Where, d_t indicates days lapsed since the last settlement.

Generally margin of a future contract is calculated by the formula:

Initial margin = D + 3 SD

Where, D = Daily Avg. Price, and SD is Standard Deviation of the instrument.

TIME IS THE ALLY OF WRITER AND ENEMY OF OPTION BUYER SINCE IN THE LONG RUN GOOD STOCKS WILL USUALLY DO BETTER.

Long Call : buying a Call option expecting Price to Increase
Long Put : buying a Put option expecting Price to Decrease
Short Call : selling a Call option expecting Price to Decrease
Short Put : selling a Put option expecting Price to Increase

Straddle means buying a Call Option and Put Option with same exercise price and expiry date.

Strangle means buying a Call Option and buying a Put Option with same expiry date but different exercise prices.

Butterfly Strategy: In this one buys a call option at price S+a and another at price S-a. Then 2 call options are sold at price S. All calls are contracted for same maturity date. The buyer of calls makes profit when the market price lies between S+a and S-a. if the market price is outside the range then he will not make any profit and loss will be to the extent of net of premium paid and received on the four options. This strategy will give limited profits with limited risks. Profits will be maximum when market price is nearer to spot price. S means Spot price.

Caps: Caps means setting the upper limit .Usually the upper limit is set by Strike Price of a Call purchased

Floor: Floor means setting the lower limit .Usually the lower limit is set by Strike Price of Put sold.
Collar: Combination of Caps and Floor is known as Collars .

Greeks:
Delta is the sensitivity of option price due to change in the value of the underlying by a unit.
Gamma is the sensitivity of option price due to change in Delta.
Theta is the sensitivity of option price due to change in expiry date by a day.
Rho is the sensitivity of option price due to change in interest rate.
Vega is the sensitivity of option price due to volatility of the underlying asset.

Securities

Moving Averages: Moving averages of prices are plotted to make buy sell decisions. Arithmetic Moving Average (AMA) is the simple average of the available prices. In case of Exponential Moving Average (EMA) more weightage is given to current results as against older results. The reduction of weightage is done through a constant known as exponential smoothing constant or Exponent. EMA is calculated by the formula:

$$\mathbf{EMA_T = aP_t + (1 - a) * EMA_{t-1} = EMA_{t-1} + a\ (P_t - EMA_{t-1})}$$

Where, a = Exponent Constant; Pt = Price on respective day and EMAt-1 = Preceding day's EMA.

Value of a Bond: Value of a bond is sum of the discounted values of the series of interest payments (called interest strips) and principal amount (called principal strips) at maturity. Formula is:

$$V = \sum_{t=1}^{n} \frac{I}{(1+k_d)^t} + \frac{F}{(1+k_d)^n} \quad \text{where,}$$

In case interest is paid semi annually, formula is:

$$\mathbf{V} = \sum_{t=1}^{2n} \left[\frac{\frac{I}{2}}{(1+\frac{k_d}{2})^t}\right] + \left[\frac{F}{(1+\frac{k_d}{2})^{2n}}\right] \text{ Where,}$$

V = Value of bond; I = series of interest payments; I/2 = Semi Annual series of interest payments; F = Face Value incl. Prem., if any; kd = Required. Rate of return; n = maturity period; and 2n = maturity period expressed in terms of half yearly periods;

Bond Value Theorems:

a. When the required rate of return equals the coupon rate, the bond sells at par value.
b. When the required rate of return exceeds the coupon rate, the bond sells at a discount. The discount declines as maturity approaches.
c. When the required rate of return is less than the coupon rate, the bond sells at a premium. The premium declines as maturity approaches.
d. The longer the maturity of a bond, the greater is its price change with a given change in the required rate of return.
e. Price of a bond varies inversely with yield because as the required yield increases, the present value of the cash flow decreases; hence the price decreases and vice versa.
f. Value of the bond changes with duration. As the bond approaches its maturity date, the premium / discount will tend to be zero.

Yield is the payment at maturity.

Yield to maturity: The rate of return one earns is called the Yield to Maturity (YTM). The YTM is defined as that value of the discount rate ("kd") for which the Intrinsic Value of the Bond equals its Market Price (Note the similarity between YTM of a Bond and IRR of a Project). YTM is also known as COST OF DEBT or REDEMPTION YIELD or IRR or MARKET RATE OF INTEREST or MARKET RATE OF RETURN or OPPORTUNITY COST OF DEBT.

YTM and Bond value have inverse relationship. i.e. if YTM increases Bond value will decrease and Vice versa.

Duration of Bond: The term duration has a special meaning in case of Bonds. Duration is the average time taken by an investor to collect his investment. If an investor receives a part of his investment over the time on specific intervals before maturity, the investment will offer him the duration which would be lesser than the maturity of the instrument. Higher the coupon rate, lesser would be the duration.

Macaulay duration is the weighted average time until cash flows are received, and is measured in years.

Modified duration is the percentage change in price for a unit change in yield.

Macaulay Duration (in years) = $\left[\sum_{t=1}^{n}\frac{t*C}{(1+i)^{t}} + \frac{n*M}{(1+i)^{n}}\right] / P$ where,

Modified Duration = Macaulay Duration / (1 + YTM / n)

n = number of cash flows; t = time to maturity;
C = Cash flows; i = Requd. Yield;
M = Maturity Value; P = Bond Price and
YTM = Yield to Maturity

$$\textbf{Volatility or Modified Duration or Sensitivity } (\%) = \frac{\textbf{Macaulay Duration}}{\textbf{YTM}}$$

Modified duration will be always less than Macaulay Duration.

Macaulay duration and modified duration are both termed "duration" and have the same (or close to the same) numerical value, but it is important to keep in mind the conceptual distinctions between them. Macaulay duration is a time measure with units in years whereas, Modified duration is a derivative (rate of change) or price sensitivity and measures the percentage rate of change of price with respect to yield.

Zero Coupon Bond will have Macaulay Duration equal to maturity period.

Self Amortising Bonds: These are bonds which pay principal over a period of time rather than on maturity.

Inflation Bonds: These are bonds where coupon rate is adjusted according to inflation. That is investor gets inflation free interest. Suppose coupon rate is 8% and inflation 6%. Then investor will get 14.48%.

If value of Bond > Market price, then Buy and vice versa.

Bond with variable yield rates: In case a bond has different yields of Y_1, Y_2 & Y_3 in 3 years, then value of bond is calculated by the formula:

$$\mathbf{V} = \frac{\mathbf{Int.}}{(\mathbf{1}+\mathbf{Y_1})} + \frac{\mathbf{Int.}}{(\mathbf{1}+\mathbf{Y_1})(\mathbf{1}+\mathbf{Y_2})} + \frac{\mathbf{Int.}}{(\mathbf{1}+\mathbf{Y_1})(\mathbf{1}+\mathbf{Y_2})(\mathbf{1}+\mathbf{Y_3})}$$

Current Yield is expressed in annualised terms and calculated on market price.

Holding period Return = Current Interest Yield + Capital Gain Yield

If Coupon Rate = YTM, then Bond will trade at Par.

If Coupon Rate > YTM, then Bond will trade at Premium.
If Coupon Rate < YTM, then Bond will trade at Discount.
(This will be the case when Bond Value = Face Value)

$$\textbf{Convrsion parity Price} = \frac{\textbf{Mkt. Price of Convertible Bond}}{\textbf{No. of Equity Shares on Conversion}}$$

Callable Bond is the one where the issuer has an option to call back and retire the bonds before maturity date.

Puttable Bond is the one where the investor has an option to get the Bond redeemed before maturity.

5. Portfolio Management

Variance:

$$\textbf{Variance } (\textbf{Sd}^2) = \sum_{i=1}^{n}[(X_i - X)^2 * P_i(X_i)]$$ Where,

Xi = Possible returns on security; X = Expected Value of security / Portfolio; Pi(Xi) = Probability.

Covariance shows the relationship between two variables and is calculated by the formula;

$$\textbf{Cov}_{AB} = \frac{\sum(R_A - \bar{R}_A)(R_B - \bar{R}_B)}{N}$$ where

In case, if in the problem instead of just observations, if probabilities for observations are also given, then,

$$\textbf{Cov}_{AB} = \sum(R_A - \bar{R}_A)(R_B - \bar{R}_B)$$ where

R_A = Return on security A;
$\bar{R}_A$ = Expected or mean return of Security A;
R_B = Return on security B;
$\bar{R}_B$ = Expected or mean return of Security B.

Covariance between two securities may also be calculated by multiplying respective betas and the variance of market.

$$\textbf{Cov}_{AB} = \beta_A \beta_B \, sd_m^2$$

Covariance of 2 securities is +ve if the returns consistently move in same direction.

Covariance of 2 securities is -ve if the returns consistently move in opposite direction.
Covariance of 2 securities is zero, if their returns are independent of each other.

The value of covariance will be between - ∞ and + ∞

Coefficient of correlation also indicates the relationship between two variables. It is expressed by the formula:

$$r_{AB} = \frac{Cov_{AB}}{Sd_A Sd_B} \Rightarrow Cov_{AB} = r_{AB} Sd_A Sd_B \quad \text{where}$$

r_{AB} = Coefficient of correlation between A & B;
Cov_{AB} = Covariance between securities A & B
Sd_A = Standard deviation of security A.
Sd_B = Standard deviation of security B

Correlation coefficients may range from -1 to 1.
A value of +1 indicates a perfect positive correlation between the two securities returns and risk will be maximum.
A value of -1 indicates perfect negative correlation between the two securities return and risk will be minimum; and
A value of zero indicates that the returns are independent.

The Variance of a Portfolio consists of 2 components (unlike the return of a portfolio where weighted average of individual securities is taken):

a. Aggregate of the weighted variances of the respective securities and
b. Weighted covariances among different pairs of securities.

Calculation of **risk In case of only 2 securities,** A & B in portfolio, then Variance of portfolio is given by the formula

$$Sd_p^2 = [X_A^2\, Sd_A^2 + X_B^2\, Sd_B^2] + 2\, [X_A X_B\, (r_{AB}\, Sd_A\, Sd_B)]$$

Sd_p^2 = Variance of the portfolio;
X_A = Proportion of funds invested in security A;
X_B = Proportion of funds invested in security B;
Sd_A^2 = Variance of security A;
Sd_B^2 = Variance of security B;
Sd_A = Standard deviation of security A;
Sd_B = Standard deviation of security B, and
r_{AB} = Correlation coefficient between the returns of A & B securities.

In case of perfect +ve correlation, coefficient of correlation, r_{AB} = 1. So, the variance of portfolio becomes:

$$Sd_p = X_A\, Sd_A + X_B\, Sd_B$$

In case of perfect -ve correlation, coefficient of correlation, r_{AB} = -1. So, the variance of portfolio becomes:

$$Sdp = X_A\, Sd_A - X_B\, {}_S d_B$$

In case of perfect no correlation, rAB = 0. So, the variance of portfolio becomes:

$$Sd_p^{\,2} = X_A^{\,2}\, Sd_A^{\,2} + X_B^{\,2}\, Sd_B^{\,2} \text{ and}$$

$$Sd_p = [X_A^{\,2}\, Sd_A^{\,2} + X_B^{\,2}\, Sd_B^{\,2}]^{1/2}$$

Calculation of Risk of Portfolio with more than two securities:

$$Sd_p^{\,2} = \sum_{i=1}^{n} \sum_{i=1}^{n} X_i\, X_j\, Cov_{ij} = \sum_{i=1}^{n} \sum_{i=1}^{n} X_i\, X_j\, r_{ij}\, Sd_i\, Sd_j \quad \text{where,}$$

Sd_p^2 = Variance of Portfolio;
Xi = Proportion of funds invested in security i (the first of a pair of securities).
Xj = Proportion of funds invested in security j (the second of a pair of securities).
$_{Covij}$ = The Covariance between the pair of securities i and j;
r_{ij} = Correlation Coefficient between securities i and j;
Sd_i = Stabdard deviation of Security i;
Sd_j = Stabdard deviation of Security j;
n = Total number of securities in the portfolio.

Calculation of Beta: Covariance Method

$$\beta = \frac{Cov_{\cdot im}}{sd^2{}_m} = \frac{r_{im} sd_i sd_m}{Sd^2{}_m} = \frac{r_{im} sd_i}{Sd_m} = \frac{r_{pm} sd_p}{Sd_m} \quad \text{Where,}$$

r_{im} = coefficient of correlation between scrip i and the market index m;
r_{pm} = coefficient of correlation between portfolio p and the market index m
sd_i = standard deviation of returns of stock i;
sd_m= standard deviation of returns of market index,
$sd^2{}_m$= the variance of market returns; and
sd_p = standard deviation of returns of portfolio.

Regression Method: The general form of regression equation is:

$$\alpha = Y - \beta X \quad \text{Where,}$$

X = independent variable (market);
Y = dependent variable (security), and
α, β are constants. Beta is Systematic Risk and Alpha is the intercept on Y axis.

Beta is calculated by the formula:

$$\beta = \frac{n\sum XY - (\sum X)(\sum Y)}{n\sum X^2 - (\sum X)^2} = \frac{\sum XY - n\,\bar{X}\,\bar{Y}}{\sum X^2 - n\,\bar{X}^2} = \frac{\sum XY - (\sum X)\,\bar{Y}}{\sum X^2 - (\sum X)\,\bar{X}} \quad \text{where,}$$

n = number of items;
X = Independent variable (market);
Y = Dependent variable (security);
XY = product of dependent and independent variable;
$\bar{X}$ & $\bar{Y}$ are respective arithmetic means

Positive beta of security indicates that return on security is dependent on market return and will be in the same direction as that of market;
Negative beta of security indicates that return on security is dependent on market return and will be in the opposite direction as that of market;
Zero beta indicates return on security is independent of market return.

Portfolio beta, β = Σ proportion of security × beta for security.

Portfolio Beta, $\beta_p = \sum_{i=1}^{n} x_i \beta_i$

β_p = Beta of portfolio;
x_i = proportion of funds invested in each security;
β_i = Beta of respective securities; and
n = number of securities.

Since beta is the relative return of a security vis a vis market return, it can also be calculated by the formula:

$$\beta = \frac{\textbf{Change in Security Return}}{\textbf{Change in Market Return}}$$

Market Beta is taken as one unless otherwise given.

For Risk Free securities like GOI Bonds, T Bills etc. **Beta is taken as ZERO** (unless otherwise given) implying non existence of Systematic Risk.

In case of change in capital structure, the company beta will not change but only the components of debt beta and equity beta will change.

Hedge ratio is computed by the formula:

$$\textbf{Hedge Ratio} = \mathbf{r_{sf}\ X\ \frac{Sd_s}{Sd_f}}$$ Where,

r_{sf} = Correlation Coefficient between spot price and future price;
Sd_s = Standard Deviation of Changes in Spot Price; and
Sd_f = Standard Deviation of Changes in Future Price.

Formula for **Optimum proportion of investment in case of 2 securities** i & j is calculated by the formula:

$$X_i = \frac{Sd_j^2 - r_{ij}\, Sd_i Sd_j}{Sd_i^2 + Sd_j^2 - 2r_{ij}\, Sd_i Sd_j} = \frac{Sd_j^2 - Cov._{ij}}{Sd_i^2 + Sd_j^2 - 2Cov._{ij}}$$

Portfolio Return, r_p = $\sum_{i=1}^{n} x_i\, r_i$ Where

r_p = expected return of portfolio;
x_i = proportion of funds invested in each security;
r_i = expected return on securities; and
n = number of securities.

To calculate return of individual security, following CAPM formula is used:

$$R_i = \alpha + \beta R_m = R_f + \beta\,(R_m - R_f)$$ Where,

R_i = Return of individual security,
R_m = Return on market index or Risk Premium
R_f = Risk Free rate
α = Return of the security when market is stationary
β = Change in return of individual security for unit change in return of market index.

Security market line measures the relation between systematic risk and return. Formula for Security Line is:

$$y = \beta x + \alpha$$ Where,

x is independent variable and y dependant variable.

Slope of Security line is Beta

On Return Basis:
Expected Return < CAPM Return; Sell, since stock is overvalued.
Expected Return > CAPM Return; Buy, since stock is undervalued
Expected Return = CAPM Return; Hold.

On Price Basis:
Actual Market Price < CAPM price, stock is undervalued; so Buy
Actual market Price > CAPM price, stock is overvalued; so, sell.
Actual market Price = CAPM price, stock is correctly valued.;
Point of indifference.

Beta in case of Leverage:

$$\boldsymbol{\beta_l} = \boldsymbol{\beta_{ul}}\,[1 + (1 - T)\,D / E] = \boldsymbol{\beta_{ul}} + \boldsymbol{\beta_{ul}}\,(\mathbf{1} - \boldsymbol{T})\boldsymbol{D}/\boldsymbol{E}$$ Where,

β_l = Leveraged β
βul= Unleveraged β
D = Debt; E = Equity, and T = Rate of Tax

Equity Beta will be always greater than debt beta as risk of equity holders is greater than risk of debenture holders.

Risk free securities are Government Securities, T Bills, RBI Bonds etc.

Arbitrage Pricing Theory Model: Uses 4 factors Viz., Inflation and money supply, Interest Rate, Industrial Production, and personal consumption. Under this method, expected return on investment is:

$$\mathbf{E\ (R_i) = R_f + \lambda_1\beta_{i1} + \lambda_2\ \beta_{i2} + \lambda_3\ \beta_{i3} + \lambda_4\ \beta_{i4}}$$ Where,

$E(R_i)$ = Expected return on equity;
λ_1, λ_2, λ_3, λ_4 are average risk premium ($R_m - R_f$) for each of the four factors in the model and β_{i1}, β_{i2} , β_{i3} , β_{i4} are measures of sensitivity of the particular security i to each of the four factors.

Sharpe Index Model assumes that co-movement between stocks is due to change or movement in the market index. Return on security i, is calculated by the formula;

$$\mathbf{R_i = \alpha_i + \beta_i R_m + \epsilon_i}$$ where,

R_i = expected return on security i
α_i = intercept of the straight line or alpha co-efficient
β_i = slope of straight line or beta co-efficient
R_m = the rate of return on market index

$€_i$ = error term.

According to Sharpe, the return of stock can be divided into 2 components:

- Return due to market changes (systematic risk)and
- Return independent of market changes (unsystematic risk).

Total Risk = Systematic risk + Unsystematic Risk

Total variance (Sd_i^{2}) = Systematic risk ($ß_i^2$ X Sd_m^{2}) + Unsystematic risk

Formula for systematic risk is:

Systematic risk = $ß_i^2$ X Variance of market index = $ß_i^2$ X Sd_m^2

Unsystematic risk = Total Variance – Systematic risk

i.e. Unsystematic risk = $Sd_i^2 - ß_i^2 \times Sd_m^2$

(Sd_m = Standard deviation of Market index; $ß_i$ = Beta of security i; Sd_i = Standard deviation of security i)

Portfolio Variance is calculated by the formula:

$$Sd_p^2 = \left[\left(\sum_{i=1}^{n} X_i \beta_i\right)^2 Sd_m^2\right] + \left[\sum_{i=1}^{n} X_i^2 USR^2\right]$$ Where,

Sd_p^2 = Variance of portfolio;
X_i = Proportion of the Stock in portfolio;
β_i = Beta of the stock i in portfolio;
Sd_m^2 = Variance of the index;
USR = Unsystematic Risk.

(Please note the difference between the above formula and the following formula indicated elsewhere above. $Sd_p^2 = [X_A^2 Sd_A^2 + X_B^2 Sd_B^2] + 2 [X_A X_B (r_{AB} Sd_A\ Sd_B)]$)

Coefficient of Determination (r^2) gives the percentage of variation in the Security's return that is explained by the variation of the market index return.

Systematic and Unsystematic risk can also be found by the formulas:

Systematic risk = variance of security X r^2 = Sd_i^2 X r^2

Unsystematic risk = variance of security ($1 - r^2$) = Sd_i^2 ($1 - r^2$)

Where r^2 = Coefficient of Determination.

Sharpe and Treynor ratios measure the Risk Premium per unit of Risk for a security or a portfolio of securities for comparing different Securities or Portfolios. Sharpe uses Variance as a measure whereas, Treynor uses Beta as a mesure to compare the Securities or Portfolios.

Sharpe Ratio = $(R_i – R_f)/Sd_i$ and

Treynor Ratio = $(R_i – R_f)/ \beta_i$ Where,

R_i = Expected return on stock i
R_f = Return on a risk less asset
Sd_i = Standard Deviation of the rates of return for the ith Security
β_i = Expected change in the rate of return on stock i associated with one unit change in the market return

The higher the ratio, the better it is.

Jensen's Alpha: This is the difference between portfolio's return and CAPM return.

Jensen Alpha = Portfolio Return – CAPM Return.

The higher the Jensen alpha, the better it is.

Sharpe's Optimal Portfolio: Steps for finding out the stocks to be included in the optimal portfolio are as below:

a. Find out the "excess return to beta" ratio for each stock under consideration.
b. Rank them from the highest to the lowest.
c. Calculate C_i for all the stocks/portfolios according to the ranked order using the following formula:

$$C_i = \frac{Sd_m^2 \sum_{i=1}^{n} \frac{(R_i - R_f)\beta_i}{USR^2}}{1 + Sd_m^2 \sum_{i=1}^{n} \frac{\beta_i^2}{USR^2}}$$

Where,

Sd_m^2 = Variance of the index;
Ri = Expected return on stock i
R_f = Return on a risk less asset
βi = Expected change in the rate of return on stock i associated with one unit change in the market return
USR = Unsystematic Risk i.e., variance of stock movement not related to index movement.

d. Compute the cut-off point which the highest value of Ci and is taken as C*. The stock whose excess-return to risk ratio is above the cut-off ratio are selected and all whose ratios are below are rejected.
e. Calculate the percent to be invested in each security by using the following formula:

$$\textbf{\% to be invested} = \frac{Z_i}{\sum_{i=1}^{n} Z_i} \quad \text{Where,}$$

$$Z_i = \frac{\beta_i}{USR^2}\left(\frac{R_i - R_f}{\beta_i} - C^*\right)$$

Constant Proportion Portfolio Insurance Policy (CPPI), is a method where the portfolio is frequently reviewed to ensure the investments in shares is maintained as per the following formula:

Investment in shares = m(Portfolio value – Floor Value)

Floor Value is the value which market expects and m is a constant factor.

When market is raising, more amounts will be invested in Market by withdrawing from fixed income securities and vice vcersa.

Run Test: If a series of stock price changes are considered, each price change is designated + if it represents an increase and – if it represents a decrease.

A run occurs when there is no difference between the sign of two changes. When the sign of change differs, the run ends and new run begins.

Price Incr. / Decr.	**+,+,+,-,-,+,-,+,-,-,+,+,+,-,+,+,+,+**
Run	**1 2 3 4 5 6 7 8 9**

Run Test is performed as per the following procedure:

First, number of runs r is calculated.
Secondly, N+ & N- are calculated. These are the number of +ve & -ve signs in the sample.
Thirdly, N is calculated. N = N+ + N- = Total observations – 1
Fourthly, As per Null hypothesis, the number of runs in a sequence of N elements as random variable whose conditional distribution is given by observations of N+ and N- is approximately normal with Mean μ which is calculated as

$$\mu = \frac{2\,N_{+}N_{-}}{N} + 1$$

Fifthly, Standard deviation, σ is calculated by the formula:

$$\sigma = \frac{(\mu - 1)(\mu - 2)}{N - 1}$$

Sixthly, If the sample size is N, then it will have (N - 1) degrees of freedom. For this particular degrees of freedom, and the given level of significance, using the value 't' from t-table, Upper and Lower limits are found by the formula:

Upper / Lower Limit = $\mu \pm t * \sigma$

Lastly, If the value of r falls within the upper and lower limits, it is called weak form of efficiency, and if it falls outside the limits, it is called strong form of efficiency.

Efficient Market Theory is based on the Usage of Available Information by Investors to optimise their value of Holdings. The 3 forms of Efficiency are:

Weak Form of Efficiency: Market Price is reflected only by historical / past information.

Semi Strong Form of Efficiency: Market Price is reflected by both Past and Public information.

Strong Form of Efficiency: Market Price is reflected by Past , Public as well as Private Information .

6. Mutual Funds

Net Assets of the Scheme is calculated as below:

Market value of investments + Receivables + Accrued Income + Other Assets – Accrued Expenses – Payables – Other Liabilities

Valuation Rules are:

Nature of Asset	Valuation Rule
Liquid Assets e.g. cash held	As per books.
All listed and traded securities (other than those held as not for sale)	Closing Market Price
Debentures and Bonds	Closing traded price or yield
Unlisted shares or debentures	Last available price or book value whichever is lower. Estimated Market Price approach to be adopted if suitable benchmark is available.
Fixed Income Securities	Current Yield.

The asset values as obtained above are to be adjusted as follows:

Additions	Deductions
Dividends and Interest accrued	Expenses accrued
Other receivables considered good	Liabilities towards unpaid assets

Other assets (owned assets)	Other short term or long term liabilities

N A V = Net Assets of the scheme / Number of Units Outstanding

Expense ratio = Expense / Avg. Value of Portfolio O R
Expense per unit / Avg. Net Value of Assets

$$H\ P\ R = \frac{[(\textbf{Units @ end X End Price}) - (\textbf{Units @ Opg. X Opg. Price})]}{(\textbf{Units @ Opg. X Opg. Price})}\ X\ 100$$

7. Money Markets

Effective Yield (EY) is Calculated by the formula,

$$\textbf{E Y} = \frac{(\textbf{FV}-\textbf{SV}) * \textbf{365 or 12} * \textbf{100}}{\textbf{SV} * \textbf{Days or Months}}$$ Where,

FV = Face Value, SV = Sale Value

If discount is collected upfront, discount is calculated by the formula:

$$D = \frac{\textbf{FV} * \textbf{Rt. Of Int.} * \textbf{No. Of months}}{\textbf{100} * \textbf{12}}$$

Effective Discount (ED)

$$\textbf{E D} = \frac{(\textbf{FV} - \textbf{SV}) * \textbf{365 or 12} * \textbf{100}}{\textbf{SV} * \textbf{Days or Months}}$$

8. Forex

Direct quote is the one where the home currency is quoted per unit of foreign currency (eg. **USD 1 =** INR 60) and vice versa is the **indirect quote** i.e where the foreign currency is quoted per unit of home currency (eg. **INR 1 =** USD 0.01667).

$$\textbf{Direct Quote} = \frac{\textbf{1}}{\textbf{Indirect Quote}}$$

PIP is the smallest movement a price can make.

Bid rate is Buy rate and **Ask** rate is Sell rate.

Spread is the difference between Bid and Ask.

American Terms are the rates quoted in USD **per unit of foreign currency**.

European Terms are the rates quoted in foreign currency **per unit of USD.**

In a **Direct Quote** Premium / Discount is calculated by the formula:

Premium / Discount = [Forward (F) – Spot (S)] / Spot (S) X (12 / n) X 100

In an Indirect Quote it is calculated by the formula:

Premium / Discount = [Spot (S) – Forward (F)] / Forward (F) X (12 / n) X 100

Where n is number of months.

Interest Parity Equation:

$$\mathbf{1 + r_d = (F / S) * (1 + r_f)}$$ where,

r_d = Domestic rate of interest, F = one unit of foreign currency, S = Spot rate and r_f = foreign rate of interest.

In case a Forward Contract is to be extended, then existing contract is to be cancelled and new contract is to be entered.
Similarly, in case of early delivery also, original contract is to be cancelled and settlement of currency being delivered at spot rate.

9. Merger & Acquisitions

Combined Value = Value of Acquirer + Value of Target co. + Value of Synergy – Cost of Acquisition.

In case cashflows grow at a constant rate after forecast period, Terminal Value (T V) is:

$$\mathbf{T\ V = FCF * \frac{1+g}{k-g}}$$

In case cashflows are fixed every year,

$$\mathbf{T\ V = \frac{F\ C\ F}{k}}$$ Where,

F C F = Free Cashflow; k = Cost of Capital and g = growth rate.

Depending on the availability of information, Terminal Value can also be calculated by the formulae:

T V = capital at the end of forecast period * M V of share / B V of share

T V = Profit of Last year * P/ E ratio

Share exchange ratio can be either on the basis of Market Value (MV) OR Book Value (BV) OR Earning Per Share (EPS) OR any other suitable method Acquirer and Target Company.

$$\textbf{Share Exchange Ratio} = \frac{\textbf{MV or BV or EPS of Target Co.}}{\textbf{MV or BV or EPS of Acquirer}}$$

$$\textbf{Post Merger EPS} = \frac{\textbf{Post Merger EPS}}{\textbf{Total No. of Shaares Post Merger}}$$

$$\textbf{Post Merger MPS} = \textbf{Post Merger P/E Ratio X Post Merger EPS}$$
$$= \frac{\textbf{Post Merger M V}}{\textbf{Post Merger Total No. Of Shares}}$$

$$\textbf{Post Merger P/E Ratio} = \frac{\textbf{Post Merger MPS}}{\textbf{Post Merger EPS}}$$

In case of rights issue, there will not be any change in the pre issue and post issue wealth of Shareholders provided, they subscribe to the shares, or sell their rights. If they neither subscribe nor sell their rights , then they lose value.

Value of right / share = Market Price before Rights issue – Market Price after Rights issue

$$= \frac{\textbf{M P after Rights issue – Offer Price}}{\textbf{No.of Shares that are being issued for each Right Share}}$$

$$\textbf{Interest Coverage Ratio} = \frac{\textbf{EBIT}}{\textbf{Interest Charges}}$$

$$\textbf{Int.}\,\&\,\mathit{Fixed\ Div}.\,\textbf{Coverage Ratio} = \frac{\textbf{PAT + Deb. Int.}}{\textbf{Deb. Int. +Pref. Div.}}$$

$$\textbf{Capital Gearing Ratio} = \frac{\textbf{Fixed Int. bearing Funds}}{\textbf{Eq. Shareholder Funds}} = \frac{\textbf{Pref. Cap. +Debentures}}{\textbf{Eq. Capital + Reserves}}$$

Economic Value Added (EVA) = EBIT (1 – T) – WACC X Invested Capital

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