

Measure of central tendency

One Joke:

Telegraph Heading “India is invited to attend Meeting in USA from May 24 to May 28”

Santa: “I can’t come to the office from May 24 to May 28”

Boss: why

Santa: “I have to attend GS Summit in USA from May 24 to May 28”.

“See the Telegraph heading”

“India is invited to USA.” It means I, you, or we all go to attend the meeting. No, our country head (i.e. president) will go to attend the meeting. Here, The President represents the India in the meeting.

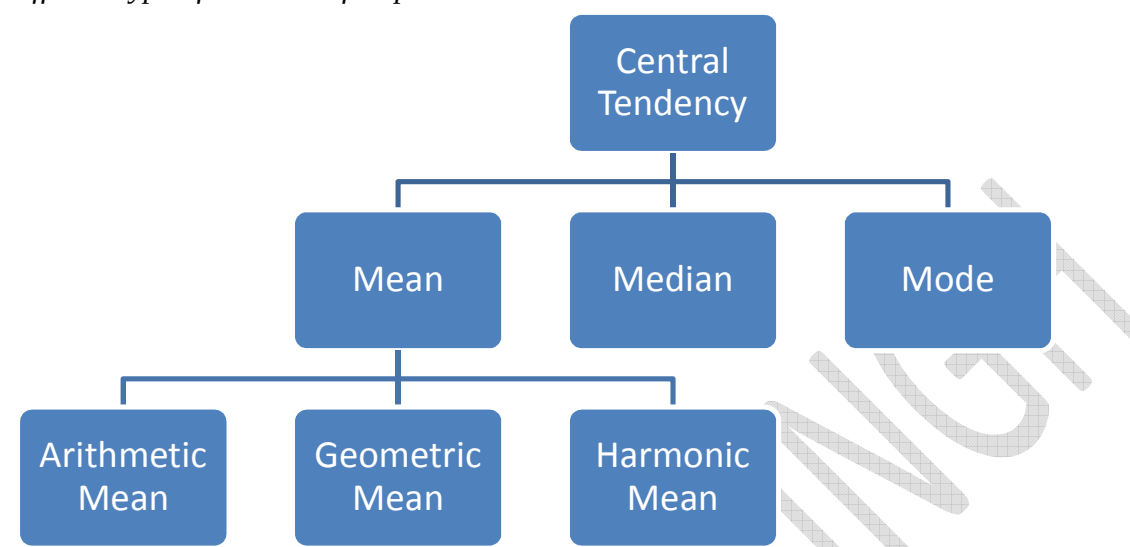
Measure of central tendency is technique or method to determine the average or central value of observations. E.g. daily income of a worker is given below for last 20 day

80, 85, 90, 95, 110, 115, 105, 95, 115, 105, 85, 95, 100, 90, 110, 80, 115, 110, 90, 95, 100

Someone ask the worker, “What is your daily income?”

Worker Reply “Sir my daily income is as follow for last 20 day **80, 85, 90, 95, 110, 115, 105, 95, 115, 105, 85, 95, 100, 90, 110, 80, 115, 110, 90, 95, 100**”

The worker never replies in this way. He tells the average of his income. Following are the different type of measure of dispersion:-



ARITHMETIC MEAN (AM)

Generally MEAN means AM and denoted by the symbol $\bar{x}$.

— For ungroup data without frequency.

— For frequency distribution

First we require to discuss about the Sigma (Σ) notations.

Suppose marks obtain by a student in different subject are as follows'

56, 59, 80, 56, 86, 76

Arithmetic Mean _____

Illustration for Frequency Distributions

- If the Sums contain class , first determine Mid Value(X)
- It means we can't calculate AM from Open End Class
- AM depends each and every observation.

Find the AM of following frequency distribution

CLASS	FREQUENCY
0-10	5
10-20	7
20-30	9
30-40	8
40-50	6

Solution:-

Table for calculation of AM

CLASS	FREQUENCY	Mid Value(X)	fx
0-10	5	5	25
10-20	7	15	105
20-30	9	25	225
30-40	8	35	280
40-50	6	45	270
Total	$N = \sum f = 35$		905

$$AM = \frac{\sum fx}{N} = \frac{905}{35} = 25.86$$

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Now Consider another Problem

CLASS	FREQUENCY
850-900	50
900-950	70
950-1000	90
1000-1050	80
1050-1100	60

[Note: in this problem mid value is a large number .so we change the origin and scale of the mid value. Please Follow Class Lecture or go to <http://www.chandansingh.co.cc> for change in origin and scale.]

Generally there is only two way to minimize or maximize a number these are

CHANGE IN SCALE: Subtract or ADD. Change in Scale: Divide or Multiply

CLASS	FREQUENCY	Mid Value(X)	$Y = \frac{x - 975}{50}$	fy
850-900	50	875	-2	-1750
900-950	70	925	-1	-925
950-1000	90	975	0	0
1000-1050	80	1025	1	1025
1050-1100	60	1075	2	2150
Total	$N = \sum f = 350$			500

$$\text{Mean of } Y = \frac{\sum fx}{N} = \frac{500}{350} = 1.43$$

Therefore, Mean of $X = 1.43 \times 50 + 975$

$$= 71 + 975$$

$$= 1046$$

Note: - Here we follow **BODMAS** (Bracket, Of, Division, Multiplication, Addition, and Subtraction) rules.

We do opposite what we have done in case of reducing the no .

COMPOSITE MEAN

Suppose that there are two cricket team viz. India and Bangladesh in Asia. They have to deposit the combined average run rate to ICC. The problem is that they have their average performance but not individual performance. In this case we use composite Mean Formula.

Suppose the case is like this.

Number	India	Bangladesh
n	n_1 (10)	n_2 (12)
Mean(x)	Mean(x_1) (50)	Mean(x_2) (60)

We know in INDIA there is n_1 player and each player makes mean(x_1) runs in average. So there total run = $n_1 \times \text{mean}(x_1)$

$$= 10 \times 50 = 500$$

Similarly Total run of Group B = $n_2 \times \text{mean}(x_2)$

$$= 12 \times 60 = 720$$

Total run of the two team = $n_1 \times \text{mean}(x_1) + n_2 \times \text{mean}(x_2)$

Then Combined Average = Total run of two team / total no of player in two team

$$\text{Therefore, Combined mean} = \frac{n_1 \times \text{mean}(x_1) + n_2 \times \text{mean}(x_2)}{n_1 + n_2} = \frac{10 \times 50 + 12 \times 60}{10 + 12}$$

$$\frac{500 + 720}{22} = \frac{1220}{22} = 55.45$$

Detection of Error and measure:

If we come to know about a error after solving the problem. We need to do problem we can correct it using following measure. Just follow the following steps.

→Determine the total ($n \times \text{mean}$)

→Subtract the wrong figure from total

→Add correct figure to last result

→Divide by number

$$\text{Correct Mean} = \frac{n * \text{mean} - \text{wrong term} + \text{correct term}}{\text{total}}$$

Example: AM of marks of 50 students in a school are 54. Later it is found that there is some mistake. 58, 87, and 65 is read as 85, 78, and 56.

$$\text{Solution} = \frac{50 * 54 - 58 - 87 - 65 + 85 + 78 + 56}{50} = \frac{2709}{50} = 54.18$$

Disadvantage of Mean:

- We cannot determine Mean in case of open end class.
- It cannot be calculated in case of open end class.

Ex:-

CLASS	FREQUENCY	X
Less than 900	50	
900 - 950	70	
950 - 1000	90	
1000 - 1050	80	
More than 1050	60	

We cannot calculate x for first and last class. So, cannot calculate mean.

MEDIAN

In the practical field there are some cases where one or two data are very large or very small. Consider the following case.

15, 18, 16, 17, 16, 17, 18, 15

Average = $(15+18+16+17+16+17+18+15)/8=16.5$

Now See the Following Data.

15, 18, 16, 17, 16, 17, 18, 15, 85

Average = $(15+18+16+17+16+17+18+15+85)/9=24.11$

Due to one data average value difference from each item is larger. So, here we apply a new technique known as median.

MEDIAN → Middle most data (data should be arranged in ascending/Descending order)

Ex: Find median of 5, 12, 13, 14, 17

Solution: First arranged it in ascending order. 5, 12, 13, 14, 17

Here, the middle one is 13. How? Because no of item on left of 13 is equals to no of item right of 13. This is third term. How it comes. See below

Median = $(n+1)/2$ th term (if data is ungroup)

$$= (5+1)/2 = 3^{\text{rd}} \text{ term} = 13$$

Ex: Find the median of following

5, 17, 14, 22, 19, 20

Arrange in ascending order, i.e., 5, 14, 17, 19, 20, 22

Median = $(6+1)/2$ th term = 3.5th term

It means third term + 0.5 of difference of 4th term and 3rd term.

$$= 3^{\text{rd}} \text{ term} + 0.5(4^{\text{th}} \text{ term} - 3^{\text{rd}} \text{ term})$$

$$= 17 + 0.5(19 - 17)$$

$$= 17 + 0.5 \times 2$$

$$= 17 + 1 = 18$$

Short Cut

In case of odd median= $(n+1)/2$ th term
In case of even median= $\{(n/2)+(n+1)/2\}/2$

Case of Grouped DATA

Here we apply interpolation formula to determine median. See a example

CLASS	FREQUENCY
0-10	5
10-20	7
20-30	9
30-40	8
40-50	6

Solution:

CLASS	FREQUENCY	CF
0-10	5	5
10-20	7	12
20-30	9	21
30-40	8	29
40-50	6	35
Total	$N = \sum f = 35$	

Me ← ————— → N/2=17.5

Note Regarding Cumulative frequency Column: 5 shows first to fifth element are lie between 0-10. 12 shows 6th to 12th elements are lie between 20 - 30. 21 shows 13th to 21st item lie between 20 - 30 and so on.

Here Median= $N/2$ th item(in case of grouped data)

i.e. Median = $35/2 = 17.5$

Now check where 17.5 th item lie. This lie in between 12 to 21 of CF. So it lie in its corresponding class, i.e. 20-30. Mark it as given.

Formula of Interpolation

$$\frac{Me - upper}{upper - lower} = \frac{N/2 - upper}{upper - lower}$$

i.e.

$$\frac{Me - 20}{30 - 20} = \frac{17.5 - 12}{21 - 12}$$

$$\text{Or, } (Me - 20) = \{5.5(10)\}/9$$

$$\text{Or, } Me - 20 = 6.11$$

$$\text{Or, } ME = 20 + 6.11 = 26.11$$

NOTE: If there is class interval instead of class boundary then convert interval into boundary or do in the same way and add the difference at end of answer (only if interval is equal in each class)

MODE

Suppose you are a shoe seller. Then which size shoe is your central attraction? The answer is shoe size which sold most in your shop. It, like in this type of cases most repeated item is the central value. This is known as mode.

MODE → item with maximum frequency

PRACTICAL APPLICATION: Suppose you are the manager of a shoe selling shop. You have to make order regarding different model of different size of shoe. Do you give the order of each size of shoe in equal amount? No, my dear you give the order of max (which sold most). This is the practical application of mode.

IN CASE OF UNGROUP DATA

Ex: find the mode of 3, 7, 8, 3, 9, 12

Mode = 3 (Since 3 comes twice)

IN CASE OF GROUP DATA

EX: find the mode of the following distribution

CLASS	FREQUENCY	
0-10	5	
10-20	7	$\rightarrow F_{-1}$
$\leftarrow L_0$ 20-30	9	$\rightarrow F_0$
30-40	8	$\rightarrow F_{+1}$
40-50	6	

Solution: Step-1:- determine highest frequency and marked as F_0 . Frequency above the F_0 as F_{-1} and just below F_0 as F_{+1} .

Here 9 is the highest frequency. Therefore 20-30 is the modal class. It means median lie between 20-30. But this is not the exact answer.

$$\text{Mode} = l_0 + \frac{f_0 - f_{-1}}{2f_0 - f_{-1} - f_{+1}}$$

$$\text{Mode} = 20 + \frac{9 - 7}{2 \times 9 - 7 - 8} = 20 + \frac{2}{18 - 15} = 20 + 0.66 = 20.66$$

Now, Consider a different case.

CLASS	FREQUENCY	
0-10	5	
10-20	7	
20-30	9	
30-40	12	$\rightarrow F_{-1}$
40-50	16	$\rightarrow F_0$

Here we cannot determine mode using the formula that we know. Because we cannot determine F_{+1} .

Here we use other technique, i.e. empirical relationship among mean, median, and mode. From the above table it is possible to determine mean and median. After that we will replace mean and median by their corresponding value in the following formula.

$$\text{Mean-Mode}=3(\text{mean-Median})$$

Note: - always use Class boundary to determine MODE.

GEOMETRIC MEAN

$$GM = (\prod x)^{\frac{1}{n}}$$

USE:-The geometric mean must be used when working with percentages (which are derived from values), whereas the standard arithmetic mean will work with the values themselves.

HARMONIC MEAN

This is explain below

Question: A man go to kolkata @ 10 km/hour and return Home @ 25 Km/hour. What is his average speed.

Soution: $(10+25)/2 = 17.5$ km/hour.

Totally WRONG

Here we have to determine Speed. Speed= Distance / Time

Distance is constant. and time is not a constant.

therefore we apply HM not AM.

Let distance between home and Kolkata is 1 km.

$$\text{Average} = (1+1) / \{(1/10) + (1/25)\}$$

It means we apply AM when Numerator is not a constant of what you are going to determine.