Stock Markets - Models and Features

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§1 Introduction

Stock market is a complex system which is a part of the much broader economic system. Stock market and other financial markets, such as foreign exchange market, play an important role in the modern economy. The rewards, for investors, which the stock market holds out are enormous. It is also equally capable of bringing calamitous ruin. People have sought methods to understand the state and trend of the market. Two different methods that have been employed are the fundamental and the technical analysis. Stock market fundamental methods comprise the following: (1) Examining the auditor's reports, (2) the profit and loss statements, (3) the quarterly balance sheets, the dividend records and policies of the companies, sales data, managerial ability, competition, production indices, bank and treasury reports, price statistics, etc. These things are used to know the state of business in general, and to study the company in particular.

The term *technical* in its application to stock markets has come to have a meaning of its own, quite different from its ordinary dictionary meaning. It refers to the study of the action of the market itself as opposed to the study of the goods in which the market deals. Technical analysis refers to the procedure of deducing the probable future trends from the past price changes, volume of transactions using graphical and pictorial tools.

It is a formidable task to understand the stock market by considering all individuals who are participating in the market and their decision strategies. Hence the focus is shifted to the study of characteristics of a stock market on an average. This exploration of trying to understand the behavior of stock market has roots that reach all the way back to 1900, when Bachelier, a young French mathematician, completed his dissertation for the degree of Doctor of Mathematical Sciences at the Sorbonne. The dissertation titled, *The theory of speculation* was the first effort ever to employ theory including mathematical techniques, to explain why the stock market behaves as it does. Since then, many approaches and frameworks have been proposed to understand the dynamics of stock market behavior. Many empirical studies have been conducted to understand the stock market dynamics like herding, limits to arbitrage, fat-tail distribution, confidence bias, volatility clustering, auto-correlation properties, scaling property, etc. In this paper, we will present various approaches used in modeling the features of a stock market.

§2 Systems Analysis

Modeling a complex system like stock market involves many assumptions and approximations. The model needs to be simple enough to facilitate analysis, but should be able to provide a reasonable approximation to the actual evolution of price movements. This is the motivation for simplistic but extremely useful models. Systems approach has been considered in the literature as an effective framework that allows inclusion of relationships among its constituents, effect of feedback, etc in a unified setup. Systems concepts include: system-environment boundary, input, output, process, state, hierarchy, goal-directedness, and information.

Dooley [KD02] has written that the emergence of order, irreversible history, and unpredictable future as the three key principles of complex systems. Different models can be constructed for the same system. Models should be able to explain the essential characteristics observed in the system. Models that generate testable results are crucial, but models that yield abstract results are also valuable if they can explain some essential structures and features of the complex system under consideration.

This leads to the importance of observables as these are the ones through which the structures and features are understood. Observables are measurable characteristics of interest. They may be associated either with individual constituent or with the system as a whole. These observables may also change over time. Any model should be able to explain the observables' characteristic it considers. Observables play a very crucial role in building models at all levels: computational, conceptual, mathematical or qualitative.

If a phenomenon is characterized by observables that can be categorized as dependent and independent, then it is considered to be oriented i.e. the phenomenon can be represented by input-output relations where input variables are independent variables and the output variables are dependent ones. These input and output variables have to satisfy a number of consistency conditions according to dynamical system theory. If the notion of state variables is considered along with input-output relations, the modern approach for modeling a process in a dynamical system framework would be complete. The identification of state variables in modeling a complex system like stock market is very difficult unlike in physical systems where fundamental laws are known. If understanding the state variables offers problems in the study of stock market, is it possible to create input-output models?

Input-output models are popular in many fields. In economics, it is widely used in building quantitative models to understand the various complex relationships among variables. Both static and dynamic models have been used in economics. The following are stated to be the advantages of using input-output models in economics: (i) The data is usually comprehensive and consistent. (ii) The nature of input-output analysis makes it possible to analyze the economy as an interconnected system of industries that directly and indirectly affect one another, tracing structural changes back through industrial interconnections. (iii) The design of input-output relations allows a decomposition of structural change which identifies the sources of change. Some limitations of the input-output approach, according to the Organization for Economic Co-operation and Development document, are: (a) The relations are assumed to be fixed. (b) It is assumed that there are no constraints on resources.

Supply is infinite and perfectly elastic. (c) It is assumed that all local resources are efficiently employed. In the case of stock market modeling, the nature of inputs is not clearly defined. The question also arises if the input should be taken as the traders' order placement or should we take the variables that motivate the traders as inputs.

The inherent difficulty in identifying the state variables and the nature of inputs in modeling the stock market dynamics suggest the output characterization as a tool to study the system. The price of an individual company share or the index of a stock market can be observed, and may be considered as an output of a dynamical system. The models proposed by Bachelier and Samuelson for stock market price process fall into the category of output (price) characterization. These models consider the price dynamics as a stochastic process that follows a particular distribution while evolving. Geometric Brownian motion has martingale property which is the key concept behind random walk hypothesis of stock market modeling. Geometric Brownian motion process is assumed for the price process in various risk analysis and derivative pricing models. The risk taken or pricing of a derivative has been studied by considering the price (observable) process without explicitly taking into account mechanisms from which this process might have arisen. Fractional Brownian motion [Man68] has been proposed in the literature to model the long-term/long-memory processes though it has been proved that it allows arbitrage possibility under certain cases. The suggestion of Mandelbrot that price time series was having scaling characteristics that obey Fractional Brownian motion also falls into the category of output characterization. The models, from which this scaling process could have come about, have been proposed much later. Understandably, people have come up with different models that exhibit the scaling behavior. Levy Processes are used to model volatility as they allow infinite variance. Hence, the characterization of observables (output) is a very important stage in the evolution of understanding any complex phenomenon. The main feature of these models is their amenability for analysis though they do not capture all empirically observed features of the market. There have been several analytical insights obtained by using these models that provided the basis for developing portfolio optimization and option pricing. These models are used in the theoretical framework because closed form expression can be obtained for various problems like portfolio optimization, pricing formula determination, etc. The basic inspiration for all these models comes from Brownian motion. Brownian motion can be understood using simple binomial models. Binomial models are developed by considering only the output characterization i.e. the price evolution. This is a very simple model and its principle forms a fundamental approach behind many numerical schemes of other complicated models.

Binomial model

The assumption here is that the stock price can take only one of two possible values at the end of each interval. Time horizon $[0, T]$ is divided into n periods of equal length Δ where $T = n\Delta$. Let $P(t)$ denote the stock price at time t, where $t = 0, \Delta, 2\Delta, ..., n\Delta$. $P(0)$ denotes the present stock price, and all future prices are uncertain. u and d refer to the proportional increase and decrease respectively in the price variation at $t + \Delta$ compared to the price at t.

$$P(t+\Delta) = \begin{cases} uP(t) & with\, probability \;\; p \\ dP(t) & with\, probability \;\; 1-p \end{cases} \tag{2.1}$$

Starting from the initial time, the price can have one of possible 2^k values at the end of k^{th} interval.

Lognormal distribution A lognormal distribution for stock price returns is the standard model used in financial economics. It can be shown that some reasonable assumptions about the random behavior of the returns lead to a lognormal distribution. Let $P(t)$ be the stock's price at time t. Let R_t be the continuously compounded return on the stock over the time interval $[t - \Delta, t]$, that is,

$$P(t) = P(t-\Delta)e^{R_t} \tag{2.2}$$

Hence, $P(t)$ can be written as,

$$P(t) = P(0)e^{R_\Delta + R_{2\Delta} + ... + R_{T-\Delta} + R_T} \tag{2.3}$$

Let $Z(T)$ represents the continuously compounded return on the stock over the horizon $[0, T]$,i.e.,

$$Z(T) = R_\Delta + R_{2\Delta} + ... + R_{T-\Delta} + R_T \tag{2.4}$$

Some conditions are imposed on the probability distributions for the continuously compounded returns R_t. Empirical evidences and analytical tractability of the model motivate these conditions. Assumption 1: The returns are independently distributed. Assumption 2: The returns are identically distributed. Assumption 3: The expected continuously compounded return can be written in the form $E[R_t] = \mu\Delta$, where μ the expected continuously compounded return per unit time. Assumption 4: The variance of the continuously compounded return can be written in the form $\text{var}[R_t] = \sigma^2\Delta$, where σ^2 is the variance of the continuously compounded return per unit time.

The expected return and the variance of the return are proportional to the length of the time interval. These assumptions imply that the distribution for the continuously compounded return, for infinitesimal time intervals, has a normal distribution. Hence, these assumptions characterize the lognormal distribution for stock returns. These assumptions allow us to derive relatively simple expressions for different types of derivative securities. For example, these are used in the Black-Scholes option model, which is the standard basic model for pricing equity options.

There is a relation between the binomial model and the lognormal distribution. Binomial model can be used to approximate the lognormal distribution if u, d and p are chosen accordingly. One choice is:

With $p = 0.5$,

$$u = e^{\mu\Delta + \sigma\sqrt{\Delta}} \tag{2.5}$$

$$d = e^{\mu\Delta - \sigma\sqrt{\Delta}} \tag{2.6}$$

Another choice is:

$$u = e^{\sigma\sqrt{\Delta}} with p = 0.5\left(1 + \left(\frac{\mu}{\sigma}\right)\sqrt{\Delta}\right) \tag{2.7}$$

$$d = e^{-\sigma\sqrt{\Delta}} with p = 0.5\left(1 - \left(\frac{\mu}{\sigma}\right)\sqrt{\Delta}\right) \tag{2.8}$$

Extensions

Changing any of the above assumptions imply a different stock price distribution. The third and the fourth assumptions are most often modified as the expected value and the variance can be made functions of the stock price. If the fourth assumption is modified as $\mathrm{var}[R_t] = \frac{\eta^2 \Delta}{P(t)}$, where η is the modified volatility term, the stochastic volatility models can be created. This condition can cause computing problems when the number of intervals is large. Moreover, successive price changes will no longer be independently distributed, which makes the estimation of the mean and volatility difficult.

Stochastic integral equation representation

The representation of log-normally distributed stock prices as a stochastic differential equation is as follows:

$$P(t) = P(t_0) + \sigma \int_{t_0}^{t} PdW + \mu \int_{t_0}^{t} Pdt \tag{2.9}$$

where dW is a Wiener process or Brownian motion, and P is the stock price.

Different assumptions about the form of the volatility give rise to different solutions to this stochastic integral equation. The standard assumption is to consider μ and η as constants as above. The solution in this case is a lognormal distribution. This is the underlying distribution for Black-Scholes option pricing model, Heath-Jarrow-Morton model, etc.

Limitations

The tails of the empirical distribution of continuously compounded returns are fatter than those expected for a normal distribution. This implies that large price changes tend to occur somewhat more frequently than would be predicted by a normal distribution of the same variance. There are also some evidences to show that the volatility of the distribution changes dynamically. Stochastic volatility models are increasingly used to model stock prices.

Efficient Market Hypothesis is related to the manner in which information is incorporated in the price formation. Binomial models actually do not explicitly take this information into account. Theoretical models, that take this information into consideration, have been proposed in the literature. We will present the concepts behind such models.

§3 Equilibrium and Disequilibrium Market Models

This section gives an overview of the research in market equilibrium and disequilibrium processes that has appeared in the finance literature. Equilibrium market models assume a stationary information process and an infinite speed of information dissemination and assimilation in the marketplace. The market prices that result from this process adjust immediately to the arrival of new information. Since by definition new information is an independent process, the usual random walk model has been developed. First, the information process is described as,

$$I(t) = \bar{I}(t) + v(t) \tag{3.10}$$

where $\bar{I}$ is the long run mean of the stationary information process. $v(t)$ is the error process. It is a stationary process and $v(t)$ is normally distributed, and has a known diagonal covariance matrix. The return on a security is a function of the information set at that time, $R(t) = f(I(t))$. Then the return process will also be independent over time (assuming an infinite speed of information dissemination):

$$R(t) = \bar{R} + e(t) \tag{3.11}$$

where $e(t)$ is also normally distributed stationary process with a zero mean and a known diagonal covariance matrix.

A problem with this random walk model is that the empirical evidence has not supported the normal distribution assumption. Information arrives continuously in small random doses is not realistic. Information arrives periodically in large doses. Therefore, the information system is not continuous and follows a sporadic jump process. Assuming an infinite speed of information dissemination, this process can be described as a jump process as,

$$I(t) = \bar{I}(t) + v(t) \tag{3.12}$$

where $v(t)$ is normally distributed and has a known diagonal covariance matrix, and $\bar{I}(t)$ is a mean process that undergoes discrete shifts that could follow a poisson-distributed jump process. Therefore, the return process can be similarly described as,

$$R(t) = \bar{R} + e(t) \tag{3.13}$$

where $\bar{R}$ is a poisson-distributed mean jump process and $e(t)$ is normally distributed with a known diagonal covariance matrix. The resulting return process is now being generated by two stable distributions and, therefore, is still a random walk process. Subordinated distributions have also been used to provide a similar explanation for non-normal distributions commonly found with security returns. Oldfield, Rogalski and Jarrow [GO77] found empirical evidence suggesting that the information process is best described as a sporadic jump process. Cohen, Hawawini, Maier, Schwartz and Whitcomb [KJCW80] discussed various empirical results that support the non-continuous information process. However, Fielitz [MTG79] argues against the market process by noting empirical studies that found persistent long-term dependence in security prices.

The assumption that markets have an infinite speed of information dissemination, however, has been questioned by an increasing number of researchers. This, in turn, leads to a developing body of literature concerning disequilibrium models of market processes. Beja and Hakansson [AB77] argued that a swift movement to a pareto-optimum price in the classical tatonnement process is unlikely in actual security prices because of institutional rigidities such as taxes and transaction costs. It is more likely that markets will trade at disequilibrium prices in a search for equilibrium. If investor preferences are constantly changing as new information arrives, then the market will not converge to equilibrium. Grossman and Stiglitz [SJG80] explored the reasons why uninformed traders do not impute equilibrium prices from the trades of the informed investors. They state that prices never fully adjust because of a noisy information system, the costs of acquiring and evaluating information, and the continuing need to adjust to new information shocks to the economy. Disequilibrium prices result from lags in the information process. Prices adjust monotonically toward the equilibrium price sequential arrival of information causing a finite speed of information dissemination. Thus disequilibrium returns are auto-correlated even though the information process is independent over time. The return process is described as

$$R(t) = \bar{R} + f(t) \tag{3.14}$$

where $f(t)$ has a covariance matrix that contains off-diagonal dependence terms. As long as the information is continuous and the speed of information dissemination is finite, then the covariance matrix should be stable.

Beja and Goldman [AB80] argued that market restrictions could cause oscillatory behavior in the rate of convergence to equilibrium. Therefore, there is not a stable monotonic movement to equilibrium. With oscillatory behavior, the amount of dependence varies and the covariance matrix is not stable over time.

In addition, the speed of information dissemination, while finite, is not constant. The speed of information dissemination varies with the amount of new in formation. This argument is based on the fact that with the arrival of new information, the greater the disparity between the equilibrium price and the actual price, the more investors want to trade. The resulting trading volume helps to increase the speed of information dissemination in the market. Because of the aforementioned restrictions affecting the speed of information dissemination, greater dependence in security returns also occurs during this period. Different securities have different levels of dependence and that some securities have varying levels of dependence during different time periods. Therefore, the autocorrelation structure of $f(t)$ is dependent on the amount of undisseminated (old as well as new) information:

$$R(t) = \bar{R} + f(I(t), t) \tag{3.15}$$

where $R(t)$ is a poisson-distributed mean jump process and $f(t)$ has a covariance matrix with autocorrelation terms that varies over time in a direct response to the amount of new information arriving into the market. The covariance matrix is non-stationary over time as a result of the sporadic jump process in the sequential information process and the varying finite speed of information dissemination.

To summarize, the market disequilibrium model that is emerging in the finance literature results from a finite speed of information dissemination with information arriving in a jump process. The finite speed of information dissemination occurs because of market friction and rigidities such as transaction costs, taxes, noisy information channels and the costs of acquiring and analyzing information.

The above information and binomial models basically form the template for the actual modelling of a complex financial system. It often happens that the actual data may contain some information about the process which is not captured by these template models. Moreover, the analysis of actual data may also yield some new insights which can then be incorporated into the basic models. Hence, time-series models occupy a very important place in modelling a complex financial market. We will describe some basic approaches that have been taken in that class of models in the next section.

§4 Time-Series Models

We review the time series models that are used to fit financial data. The main motivation for the use of time series models to analyze financial data is to represent the empirical properties often observed in real prices. The empirical properties of financial data depend significantly on the frequency of observation. There are three major classes of frequencies: Ultra-High Frequency (UHF), High Frequency (HF), and Low Frequency (LF).

Ultra-High Frequency The prices are observed at very high frequency as tick-by-tick or every hour. These are usually unequally spaced. Presence of strong daily patterns with highest volatility at the open and toward the close of the day has been observed. UHF returns are characterized by highly persistent conditionally heteroscedastic components along with discrete information arrival effects. The possibility of having multiple transactions within a single second increases the complexity further.

High Frequency Prices here are observed, say, daily or weekly. HF data is the most extensively analyzed in the empirical literature. Empirical studies have shown that HF returns are nearly non-correlated though they are not independent because significant auto-correlations have been observed for non-linear transformations. The significant auto-correlations of squared returns are often related to the presence of volatility. The slow decay of auto-correlations is usually interpreted as the presence of long-memory in the volatility. HF returns are also leptokurtic. The heavy tail property can be related to the dynamic evolution of volatility.

Low Frequency Prices are observed, at very low frequencies, every month. The monthly returns seem to have more serial correlations than daily returns.

Generally asset-pricing models concentrate mainly on UHF and HF observations. The central question in financial econometrics is whether financial prices are predictable i.e. whether future prices can be predicted using the information contained in their own past. The main hypotheses often tested are the martingale and the random walk hypothesis.

The martingale hypothesis can be expressed as follows:

$$E[P_t | P_{t-1}, P_{t-2}, ...] = P_{t-1} \tag{4.16}$$

The expected value of price at t is same as the price at the previous instant i.e. P_{t-1}. The martingale hypothesis places a restriction on expected returns. It holds for rationally determined asset prices after their returns are adjusted for risks. It is known that risk-adjusted martingale property is the basis of many financial derivatives.

The second hypothesis often tested is whether logarithmic prices are generated by a random walk plus drift model given by:

$$\log(P_t) = \mu + \log(P_{t-1}) + \varepsilon_t \tag{4.17}$$

where error ε is an independent process with zero mean and variance Σ^2, and μ is the expected price change. However, it is customary to assume the errors are uncorrelated instead of independence. This allows the model to exhibit the presence of conditional heteroscedasticity, which is observed in HF returns.

There is overwhelming evidence that asset returns are not independent due to auto-correlated squares. They can be represented by the following model assuming that returns have zero mean and serially uncorrelated:

$$r_t = \sigma_t \varepsilon_t \tag{4.18}$$

where ε_t is an independent and identically distributed process with zero mean and unit variance independent of the volatility.

There are two main proposals in the literature to represent the dynamic evolution of ε^2 Generalized Autoregressive Conditional Heteroscedasticity (GARCH) and Stochastic Volatility (SV) models. GARCH models are based on modelling the volatility as the variance of returns conditional on past observations. The original GARCH model has been extended in various ways. Two of the main extensions from the empirical point of view, are models to represent the asymmetric response of volatility to positive and negative returns and to represent the effect of the volatility on the return of a stock. The first effect is known as leverage effect. Exponential GARCH has been proposed to represent it. GARCH-in-Mean model is proposed by Enge to represent the effect of volatility on the expected return. There are many modified GARCH models that allows for regime switching when volatility persistence can take different values depending on whether returns are in a high or a low volatility regime. Long memory property of squared returns can be represented by Fractionally Integrated GARCH model. But the property of the corresponding estimators and tests are generally unknown as they are not stationary in covariance. Two components GARCH model has also been proposed; one that is nearly non-stationary and another that is much less persistent.

The variations in volatility, η_t^2, can be modeled using Stochastic Volatility (SV) models that introduce an additional noise to model the variations. Therefore, the volatility is a variable composed of a predictable component that depends on past returns plus an unexpected component. The introduction of the unobserved component in the representation of the volatility gives more flexibility to SV models to represent the empirical properties often observed in real time series of returns. The likelihood function of most of these models does not have a closed form expression and, consequently, most estimation methods proposed are based on numerical approximations of the likelihood or on transformations of the observations. As in the case of GARCH models, SV models have also been extended to represent the asymmetric response of volatility to negative and positive returns, and the response of expected returns to volatility. Many multivariate extensions for GARCH and SV have also been proposed in the literature.

The time series models depend on the frequency of the financial data and can be classified as the models for the conditional means, conditional variances, and conditional covariances. The characteristic features that can be captured by different time series models are as follows:

ARMA (Autoregressive moving average) models - for weakly stationary process i.e. mean, variance, and auto-covariance are constant

ARIMA (Autoregressive integrated moving average) - for non-stationary process; can capture mean reverting process

ARFIMA (Autoregressive fractionally integrated moving average)- long-term/long-memory processes

ARCH (Autoregressive conditional heteroskedastic) - to incorporate dynamic evolution of volatility

GARCH (Generalised autoregressive conditional heteroskedastic) - to incorporate dynamic evolution of volatility; long memory properties

EGARCH (Exponential GARCH) - can represent leverage effect i.e. volatility asymmetry

FIGARCH (fractionally integrated GARCH) - dynamic evolution of volatility; long memory properties

MGARCH (Multivariate GARCH) - time varying correlations

Stochastic Volatility Models - The introduction of unobserved component can allow the model to generate a jump term to allow for large transient variations; allow variances and covariances to evolve through positive trends.

Time-series models are developed without incorporating the individual characteristics of traders. Since stock market has been empirically observed and studied for decades, data collection has grown immensely and their processing too has become more sophisticated as computing become cheaper. These lead to the proposal of many computational models that capture some features of the stock market by simple rules, evolution and adaptation. Many models in this class are based on traders/agents who may use neural networks, fuzzy sets, and genetic algorithm for their strategies, game-theoretic models like minority game models, or one of the above traditional approaches. These models are flexible enough to incorporate individual's characteristics like overconfidence, pessimism, etc., and strategies like mean-reverting, volatility-based investment behavior, etc. These models are able to capture several features of a financial market: short range/ long-range volatility correlations, group property like herding that emerges out of interaction/competition among agents, extreme events like bubbles and crashes, and fat-tailed returns. These models can include many complex aspects of the traders. The features that have to be considered for an agent or a trader are determined by the kind of dynamics the model is expected to capture.

§5 Computational Models

Financial market is a subsystem of the economic system. The basic ideas used are inspired by the corresponding concepts in economics. We will present how these concepts are associated with individual agents. Moreover, we can also see that the dichotomy in economics as microeconomics and macroeconomics is also reflected in financial market modeling as considering individual agent's behavior and looking only at the average behavior of the market.

A key concept in economics is *value* of a property or a share. A satisfactory theory of value must answer several questions: what determines the value of a share? Is there an invariant value ascribable to a share as its intrinsic attribute? How does the intrinsic value of a share relate to its fluctuating price or exchange value? How are values measured? How are they related to the distribution of income? The answers to these questions distinguish various schools of thought.

A central microeconomic theory is that equilibrium can be achieved by a perfectly competitive market in which the desires of individual can be reconciled in costless exchange. In the approximation, individuals do not interact with one another but respond independently to a common situation which is characterized by the share prices in the case of a stock market. The theory of perfect competitive market is based on many idealistic assumptions. In recent decades, economists are making effort to relax some of these assumptions. They are developing *information economics* and *industrial organization theories*, which use game theory to study so-called market imperfections. These theories take account the direct interaction among the individuals, the asymmetry of their bargaining positions, and the 'internal structure' of economic institutions. Economists generally believe in the efficacy of the free market and its price mechanism in coordinating production efforts. However, they disagree sharply about whether market coordination is perfect, whether slight imperfection can generate significant economy-wide phenomena, whether free market by itself always generates the best possible economic results, and how information can affect the market.

All agents concerned with financial markets (traders, consultants, individuals, etc) are interested in increasing their returns. Traditional economic/financial models assume rational agents and unlimited availability of information. Prospect theory has been successful in qualitatively explaining some features of the market by considering bounded rationality of the agents. All agents in the context of financial markets are considered to be 'optimizers' who systematically survey a given 'possibility set' (set of possibilities) under given constraints, and choose the one that maximizes their returns. The possibility set open to an agent is his state space. Though this idea of state space has come from dynamical system framework, this interpretation is specific to economic theories. Economic theories limit the possibility set (state space) of an agent to those items with value tags given by him. The elements of an agent's state space and his objective are framed in terms of two variables, the 'quantity' and 'price'. An agent has 'preference order' over the elements of his state space. The preference order varies according to his taste. For any two portfolios, an agent either prefers one to the other or is indifferent to both, and his preferences are transitive, that is, if he prefers A to B and B to C, he prefers A to C. Economists represent the order of preference by a utility function, which is a rule that assigns to each portfolio a utility value according to the desirability of the portfolio. The utility function is the objective function that the agent tries to maximize. Given an agent's options (possibility set), constraint (budget) and objective (his utility function), his equilibrium state is obtained by finding the affordable portfolio with the highest expected utility value. The formulation can be expanded to include risk. Decision in risk situations must take agent's belief about the state of the world. But empirical studies have been questioning the idea that people make decisions that maximize their utility functions. In many experiments, people are found to reverse their preferences consistently in risky conditions, depending on how the questions calling for choices are framed. Discrepancies between empirical data and the fundamental behavioral postulates of micro-economics are widespread and systematic. However, it is not clear which specific axiom(s) of the utility maximization theory are violated. Behavioral finance that builds upon Prospect theory tries to provide a valid framework that captures the effect of psychological aspects of individual agents on the overall behavior.

The main motivation of using multi-agent approach in social sciences, including financial economics is to explain what is happening at the micro-level through micro-simulation. Several common examples on the use of multi-agent models in financial economic analysis (be it stock market or foreign exchange market) are given by several researchers. There is also an endeavor to pronounce macro-economy in the framework of multi-agents and investors' behavior. The development of Complexity theories has brought multi-agent analysis to be one of computational devices that can be used to study emergent properties. Multi-agent approaches will hopefully be able to explain many features of statistical mechanics models (Econophysics). A number of simulated artificial markets have been built like Santa Fe Artificial Stock Market. They are based on decision rules, and computationally intensive trading mechanism with the participation of large number of agents. Though these models are able to capture many characteristic features of stock markets like absence of auto-correlations, fat-tails, volatility clustering, herding, limits to arbitrage, confidence bias, etc., the stress is mainly on inclusion of complex rules and computation.

§6 Proposed framework

System dynamics framework is an approach to understand the characteristics of complex systems which are composed of many interacting and interrelating components. It is an effective framework that allows inclusion of interactions, effect of feedback loops, time delays at multiple time scales, hierarchal structures, etc., in a unified setup. The formulation of this framework involves formalization of concepts like state of a system and the evolution of states in time. The formalization is widely known as dynamical system framework in physical and engineering sciences. The evolution rule is explicitly given by relations for simple dynamical systems, and is implicitly given by set of equations with constraints for more complex dynamical systems. System dynamics framework also allows incorporation of decision-making entities as a part of system. Modeling decision-making entities is achieved by incorporating rules in an appropriate database.

Stock market, from the perspective of system dynamics framework, consists of traders with wide varying behavioral patterns, a set of complex interrelations among them, and a structure that creates multiple feedbacks and evolves over time under the influence of changing environment and regulatory mechanisms. Stock market exhibits both positive and negative feedback mechanisms. When stock values are rising, the belief that it would rise further provides traders an incentive to buy. More traders would take a buying position. More buyers lead to a rise in price. The traders' belief gets reinforced and provides further incentive to buy. Stock price dynamics exhibits a positive feedback mechanism. When stock values are rising, the fear that it might reach the peak soon would deter some traders to buy. They might take selling positions to move out of the market just before the peak. More sellers lead to a fall in price. The traders' belief gets reinforced and provides further incentive to sell. Stock price dynamics also exhibits a negative feedback mechanism with regard to some traders.

The type of feedback mechanism is dependent on the risk profile of a trader. Risk profile depends on past successes, failures, capacity to invest, confidence bias, etc. Moreover, difference traders could take different positions in a stock market. The interactions among them create multiple positive and negative loops in the market's dynamic evolution. The interaction among these different positive and negative loops creates a condition where herding could take place depending upon the strength of interactions.

Time delay is a crucial factor in some complex systems. Information and effect of interactions take time to affect various components. For instance, when new information is available, traders react to the new information in a cautious manner thus creating time-delay effects in system evolution. Decision regarding investment strategies and implementation of investment strategies are not instantaneous. Traders need some time, for implementation, in any regulated market. Moreover, the effects of feedback will be influenced by time-delays. These time-delays limit traders from exploiting arbitrage opportunities.

Different models, in dissimilar frameworks, have been proposed in literature to capture features such as confidence bias, herding, and limits to arbitrage. From system dynamics perspective, it can be seen from the above discussion that system dynamics framework is an ideal framework to understand the characteristics of stock market.

§7 Conclusions

The effort to understand the characteristics of stock market had been mostly through the study of the price series. It is very difficult to identify all the inputs to a stock market system and come up with a model that gives the price dynamics. In fact, technical analysis is the outcome of recognizing this, and it has proceeded in a different way. We have seen different modeling approaches to model stock market in this paper. The traditional input-output models are very limited when it comes to stock market modeling. The characterization of output is the basis for most of the market models. Agent-oriented systems are suited to developing complex, distributed systems through a choice of rules for their behavior. System theoretical framework is fairly a general one where different inter-relationships can be incorporated, and many complex behaviors of the system can be interpreted as due to the fundamental mechanisms of feedback and coupling among components. There is no single framework to model various aspects of the stock market. There are different approaches to model different characteristics of the market.

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