Risk Management and Firm's Information Policy*

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Sept 2003

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JEL classification: D82; G3

Keywords: Corporate hedging, asymmetric information

*We are grateful to M. Dewatripont and C. Gollier for helpful comments.

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Abstract

This paper deals with firm's incentives to engage in hedging policies. In our model, firms hedge to disclose their performances to external creditors. We show that, at equilibrium, a value-maximizing firm may or may not hedge depending on the nature of the risk, the observability of the hedging policy and the observability of the firm's risk exposure. The optimal hedging policy does not systematically eliminate all the risk.

1 Introduction

It is common knowledge that firms actively manage their risks. The specific area of risk management embraces risks which can generally be hedged by financial contracts (e.g. options or futures etc.) or by insurance contracts. In spite of real theoretical gains, the financial theory has still not been able to determine all the fundamental reasons of corporate risk management. Why should firms manage their risks ? What are the fundamental reasons justifying a hedging policy for different risk types? According to Modigliani Miller's conclusions, the purchase or sale of financial contracts does not modify the value of the firm because the shareholders can always either buy or sell such contracts themselves, or diversify their portfolio. Moreover, evaluation models for financial contracts assume that the negotiation of every financial contract or any financial assets are zero net present value operations. Therefore they do not affect the total value of the firm. It is certainly possible that firms, sometimes, have a higher aptitude than market average aptitude to anticipate prices correctly. However, we believe that this aptitude is not the only or even the main reason for the intervention of firms into the financial market. Even if we accept this idea as the main reason for firm's interventions into the financial market, it still doesn't explain why firms sign contract with insurance companies. This study has two main objectives:

Firstly, we will attempt to show that while managing their risks (e.g. public or private risks), firms carry out financial market information policies. We have developed an asymmetric information model in which external creditors who may finance the firm can observe the firm's current cash-flow $\widetilde{\omega} = x + \widetilde{\varepsilon}$. However, they cannot distinguish between the impact on the firm's cash flow of the fundamental quality of the firm x and of the realization of a simple transitory risk[1] $\widetilde{\varepsilon}$. In other words, the firm's current cash-flow is a noisy signal of the firm's quality. An unfavorable risk will thus lead to a certain underestimate of the firm's quality (which also represents the quality of the firm's investment opportunities), and will increase the cost of its external financing. In the same way, a favorable risk will decrease the cost of external financing. We show that, on average, the cost of external financing is not greater than that of internal financing. However, we also show that fluctuations in the cost of these external financing, distort the firm's investment policy and reduce its value. A hedging policy which acts on risk $\widetilde{\varepsilon}$ is thus likely to increase the value of the firm by moderating the asymmetric information problems between the firm and the financial market. The object of this will be to allow the financial market to better evaluate the fundamental quality of the firm by sending it a more precise signal. By hedging, the firm manages its signal broadcasts in order to becomes

[1] With null average and independent of x

more readable by the market. Guay, Haushalter and Minton (2003) find robust evidence in favour of our rational for corporate risk management. Using a broad sample of large firms between 1990 and 1999, the authors show that investors and analysts encounter difficulties incorporating the effects of interest rates, exchanges rates and commodity prices shocks into their expectations about firms' earnings. In other words, investors and analysts run into difficulties estimating the earning effects of the risk exposures that companies face. Moreover, this empirical study shows that both the absolute value of excess stock return around earnings announcements and the magnitude of analysts' earnings forecast errors increase with the firm's ex ante risk exposure. All these results lead the authors to support recent theoretical research (DeMarzo and Duffie 1995) investigating "the role of corporate hedging in increasing the flow of information to investors".

Secondly, in a more sophisticated framework, we demonstrate that the optimal risk management policy (that optimizes the market's perception of the firm's quality) does not systematically minimize the risk which affects the firm's cash flow. Indeed, this model gives a new basis to the corporate hedging policy. It places the corporate hedging policy within the framework of the firm's information policy rather than that of risk management in the strict sense. That is why partial hedging can be optimum.

The following example illustrates this conclusion. Let us consider a french firm exporting into a market where the prices are in dollars. If there is a drop in the value of the dollar, this will have two effects at the level of the firm. The first will be an exceptional loss in the value of its receivables in dollars. The second will be a decrease in its turnover, for the firm being less competitive. The first effect is considered to be an exception as it is due to the fluctuation in the dollar and it will not happen again if from then the dollar is maintained at its new rate. The second effect is definitive since the last dollar exchange rate defines the firm's new market conditions to which the firm needs to adapt its investment policy. Of the two risks resulting from random fluctuation in the dollar, only the first must be hedged.

Different types of risks such as specific risks (e.g. fire, theft, accidents etc.) and public risks (e.g. interest rate, currency, raw materials, etc.) will be studied. Moreover, different cases will be dealt with according to whether the external investors observe the realization of the risk, the exposure to risk and the hedging policy.

Our paper is mainly related to the papers of Froot, Sharfstein and Stein (1993), C. Raposo (2000), DeMarzo and Duffie (1995) and Breaden and Viswanathan (1996). As Froot, Sharfstein and Stein (1993), we link corporate hedging to investment efficiency under asymmetric information. Froot, Sharfstein and Stein (1993) consider an exogenous convex cost function representing the firm's cost of external financing which is assumed to be higher than internal financing. In Froot, Sharfstein and Stein (1993), firms hedge their random internal financing (internal cash-flow) in order to eliminate the investment fluctuations that are associated to the randomness use of complementary external financing. In this paper, hedging takes place because of the concavity of the firm's objective. C. Raposo (2000) endogenizes the firm's cost of external financing in a Myers and

Majluf (1984) context. However, contrary to our paper, in these two papers, hedging is not a signaling policy. On the other hand, in DeMarzo and Duffie (1995) hedging is viewed as a signaling instrument. The labor market discovers the ability of a firm's manager from the observation of the firm's performances. In some circumstances, this can lead the manager to undertake hedging policy to eliminate noises in the signals sent to the labor market. As in our paper, hedging is a signaling policy and the characteristics of the optimal hedging policy depends on the observability of both the firm's risk exposure and the hedging policy itself. However, we enrich the analysis by considering different types of risks such as specific risks (whose realization can be observed or not) and public risks. Lastly, although more distants from our modeling; Breaden and Viswanathan (1996) consider also managerial hedging as a signaling device when managers have career concerns.

Many other explanations for corporate hedging have been offered in the literature. For a given level of debt, hedging can reduce the probability of bankruptcy. Thus, if financial distress is costly, a risk management program reduces these expected costs. If there is an advantage to having debt in the capital structure (due to taxes or agency problems) hedging increases optimal debt level and, as a result, the value of the firm increases as well (Smith and Stulz 1985).

The tax schedules can also be an explanation of hedging as in Mayers and Smith (1982), Smith and Stulz (1985) and Stulz (1996).

In Stulz (1984) hedging results from manager's risk aversion. This managerial theory relies on the assumption that the cost of firm hedging activity is lower than the cost when managers trade in hedging contracts for their own account. In DeMarzo and Duffie (1991), the firm manages risk on behalf of risk averse shareholders because she has some information that they do not have.

Others papers postulate capital market imperfections. In Mello Parsons (2000) there is a quantitative constraint on external financing. The lack of liquidity causes a premature liquidate. Hedging relaxes the financial constraint on the firm. Froot, Sharfstein and Stein (1993) also postulate that capital markets are imperfect so that external financing is more expensive than internal financing. Firms with investment projects requiring funding, hedge their cash-flows to avoid variations in their liquidity needs and distortions in their investment policy. In Holmström and Tirole (2000), the capital market imperfection is endogenous and liquidity risk management is a way to avoid some undesirable investment liquidation decisions and some undesirable investment continuation decisions. In our paper, the capital market imperfection is also endogenous. However, in our model, rather than minimizing undesirable consequences of capital market imperfections, corporate hedging directly eliminate capital market imperfections by eliminating asymmetric information between the firm and the financial market.

The structure of the paper is as follows: Section 2 presents the basic model. Section 3 deals with the firm's investment behavior. Section 4 presents the symmetric information case. Section 5 presents a typology of corporate risks and risk management policies. Sections 6 and 7 consider different types of risk (e.g. private risk and public risk) and deal with optimal hedging policies. Section 8 shows that, under some assumptions, a partial hedging is optimal.

2 The basic model

We consider a three-date model ($t = 0, 1, 2$).

At date 0, the firm chooses its hedging policy. This consists of hedging a risk partially or totally which affects the firm's cash flow at date 1. When there is hedging, the firm's cash flow is written as follows:

$$\widetilde{\omega} = \widetilde{x} + (1 - h)\,\widetilde{\varepsilon} \tag{1}$$

As equation (1) indicates, the cash flow is the addition of two random flows:

− a positive random variable $\widetilde{x}$ that represents the firm's economic performance.

− a random variable $(1 - h)\,\widetilde{\varepsilon}$ where $\widetilde{\varepsilon}$ represents a hedgeable risk and where h is the hedging policy chosen by the firm at date 0.

When $h = 0$ the firm does not hedge.

When $h = 1$ the firm fully hedges.

When $0 < h < 1$ the firm partially hedges.

When $h \notin [0, 1]$ the firm speculates. We do not exclude this possibility a priori.

The possible characteristics of this risk $\widetilde{\varepsilon}$ will be defined in section 5.

The two random variables $\widetilde{x}$ and $\widetilde{\varepsilon}$ are taken to be independent but their distribution are such that $\widetilde{\omega}$ is always positive. The variable $\widetilde{\varepsilon}$ has a null average (indeed, only the deviations from the average are important in this model).

In our model, the realization x of the random variable $\widetilde{x}$ is a key variable because it also represents the quality of the firm's opportunity of investment at date 1. Indeed, at date 1, the firm makes an investment where they choose the amount I. They finance it by raising external cash[2]. This investment will generate in $t = 2$ a cash flow $xf\,(I)$. We assume that $f^{'}\,(I) > 0$ and $f^{"}\,(I) < 0$. As we mentioned before, the variable x is the quality of the investment.

The hedging policy h chosen by the firm at date 0 may or may not be observable by the external creditors (we will later refer to them as the financial market). We will consider the two cases. Let h^e denote the financial market belief about the firm's hedging policy. Obviously, if h is observable then we necessarily have $h^e = h$.

At date 0, the firm and the financial market have the same information on $\widetilde{x}$ and $\widetilde{\varepsilon}$: they only know the distribution of $\widetilde{x}$ and $\widetilde{\varepsilon}$.

At date 1, the firm learns the realization of $\widetilde{x}$ and the realization of $\widetilde{\varepsilon}$ while the market observes only the realization of the firm's cash flow $\widetilde{\omega} = \widetilde{x} + (1 - h)\,\widetilde{\xi}$. Therefore, at date 1, only the firm knows the quality of its investment opportunity x. Nevertheless, the cash-flow $\widetilde{\omega}$ is a noisy signal on x. This signal can

[2] We assume that $\widetilde{\omega}$ is paid back to shareholders in the form of dividends in $t = 1$. See Froot, Scharfstein and Stein (1993) for a model where hedging is motivated by the use of both a random internal cash-flow and external cash to finance the firm's investment.

be observed, without cost, by any external investor who is likely to finance the firm. The conditions for external financing of the firm will therefore depend upon $\widetilde{\omega}$. In an effort to lighten the notation, the discount rate is taken as zero. The expression $xf(I) - I$ thus represents the net present value (NPV) of the investment project.

We call Γ_t^F the information set of the firm on the variables of the model at date t and Γ_t^M the information set of the financial market at date t. The exact contents of these information sets will be presented later.

Let denote $E_t^F(.)$ the expectation conditional to the firm's information at date t, Γ_t^F, and $E_t^M(.)$ the expectation conditional to the market information at date t, Γ_t^M.

We note x_1^e the belief of the financial market on the value of x at date 1:

$$x_1^e = E\left(x/\Gamma_1^M\right)$$

In particular, x_1^e is a function of ω observed at date 1 by the financial market since $\omega \in \Gamma_1^M$. As we will see, x_1^e will also determine the firm's cost of external financing.

Let V_0 denote the wealth of the firm's shareholders at date 0 and V_1 the wealth of these shareholders at date 1. We assume that the firm maximizes the wealth of its shareholders at each date. Consequently, the hedging policy is chosen in order to maximize V_0 and the investment I is chosen in order to maximize V_1.

We consider a perfect Bayesian Nash equilibrium defined as follows:

Definition 1 *(h^*, I^*, x_1^e) is a perfect Bayesian Nash equilibrium if $x_1^e = E_1^M(\widetilde{x})$, $h^* \in \arg\max_h (V_0)$, $h^e = h^*$ and $I^* = \arg\max_I (V_1)$.*

We use a two-step resolution procedure:

Firstly, we find the optimal investment policy at $t = 1$ given the realization of the cash-flow ω.

Then, we characterize the optimal hedging policy chosen at $t = 0$.

3 Investment policy at date 1

We assume that the firm does not have an initial debt. Moreover, we assume that the firm issues equity in order to finance the investment I. In Appendix 1, we show that we could have adopted a more general approach in which the firm has an initial debt while the investment I is financed by outside funds whose composition (e.g. percentage of equity and percentage of debt) respects the firm's initial financial structure. In other words, we assume here that the firm cannot use its financial structure in order to signal its quality x. At first sight, that can seem to be a strong assumption. Indeed, since Leland and Pyle (1977) and Myers and Majluf (1984) among others, it is well-known that financial structure can be a signaling instrument. But, that does not mean that it is the only instrument. In our paper, our objective is precisely to show that the hedging policy is at least an alternative tool for signaling the firm's quality. Moreover, it seems to us that hedging instruments are more suitable for a signaling policy since they are more flexible than the use of the financial structure. In fact, modifying the financial structure has many costs. For example, the firm must move away from the optimal financial structure defined by the traditional trade-off between tax shields and bankruptcy costs. Moreover, in accordance with agency theory, modifying financial structure may increase agency costs. On the other hand, hedging policies often use portfolio of derivatives instruments that are less expensive and easy to modify.

We call q the part of the firm's cash flow that goes to the outside capital contributors at date 2. At date 1, the firm is the only one which observes the realization of the random variable $\widetilde{x}$. We must consider the possibility of a signalling game between the firm and the financial market. Indeed, we need to investigate the possibility for the firm of choosing the amount of investment I and the stake of external investors in the firm q in order to reveal the value of x. However, we show it in Appendix 2 that if a separating equilibrium existed, it would not be a separating equilibrium in the strict sense. In other words, there are no functions $I(x)$ and $q(x)$ that strictly incitate the firm to reveal its true quality x. That is why we consider only a pooling equilibrium where the choice of I and q don't reveal any information about x[3]. Consequently, the only information available to the market is the observation of the current cash flow $\widetilde{w}$ which is a noisy signal about x.

Assuming that external investors just break even, q must satisfy the following condition:

$$qx_1^e f(I) = I$$

We therefore have:

$$q = \frac{I}{x_1^e f(I)} \tag{2}$$

The cost of external funds which is measured by q is determined by x_1^e.

We assume that the firm's objective in $t = 1$ is to choose an investment I that maximizes the wealth V_1 of the initial shareholders:

$$V_1 = (1 - q)\, x f(I)$$

Taking into account equation (2), the program of the firm becomes the following:

$$\max_I \frac{x}{x_1^e} \left(x_1^e f(I) - I\right) \tag{3}$$

Since the investment I doesn't reveal any information on x, the variable x_1^e is independent of I. By deriving equation (3) with respect to I, the optimal investment level verifies:

$$x_1^e f'(I) = 1 \tag{4}$$

[3] This choice is not fundamental for what follows. It is only necessary to recognize that, in all cases (pooling or separating equilibrium), the presence of asymmetric information at $t = 1$ leads to distortions in the firm's investment policy relative to a situation of symmetric information. In the following sections, we show that the hedging policy is a way to eliminate these distortions.

This investment level is exclusively determined by the firm's quality as perceived by the market x_1^e. Moreover, we have $I'(x_1^e) > 0$. In fact, x_1^e also represents the conditions of the firm's external financing since it determines the value of q (See equation (2)). The higher x_1^e is, the less expensive the external financing is and the more the firm invests.

4 The symmetric information case

Let us compare our results with the symmetric information case where x is observable. In this case, we have $x_1^e = x$ and the firm's value at date 1, noted V_1^{SI}, is equal to the net present value of the project:

$$V_1^{SI} = xf(I) - I \tag{5}$$

The optimal investment is determined according to the following equation:

$$xf'(I) = 1 \tag{6}$$

Under symmetric information, the firm's investment policy consists of maximizing the net present value of the project. In so far as there is asymmetric information, equation (4) differs from equation (6) and the investment $I(x_1^e)$ no longer maximizes the net present value of the project.

If $x_1^e < x$, the firm is undervalued. The cost of external funds is greater than the cost of internal funds and the firm under invests: $\mathrm{I}(x_1^e) < I(x)$.

If $x_1^e > x$, the firm is overvalued and it over invests: $I(x_1^e) > I(x)$.

We can also demonstrate the following Proposition:

Proposition 1 *Under symmetric information, corporate risk management is irrelevant.*

Indeed, the firm's value at date 0, V_0^{SI}, is equal to the expectation of the shareholder's wealth at date 1, V_1^{SI} (See equation (5)):

$$V_0^{SI} = E_0^F\left[x \cdot f(I(x)) - I(x)\right] \tag{7}$$

Consequently, the firm's value at date 0 is independent of the hedging policy h.

5 Nature of the risk and observability of the hedging policy

The hedgeable risk can be of several types:

It can be a specific risk $\widetilde{\varepsilon}$ (e.g. theft, fire, etc.) which may or may not be observable by the market.

It can be a public risk $\widetilde{\xi}$ (e.g. interest rate or exchange rate risks, etc.) perfectly observable by the market at date 1. On the other hand, the exposure of the firm to this public risk is not necessarily observable by the market. If we call θ the firm's exposure, the incidence of the risk on the firm's cash-flow is $\theta\widetilde{\xi}$ and the hedged risk is $(\theta - h)\,\widetilde{\xi}$. Full hedging is thus defined as $h = \theta$.

Several cases can therefore be studied. The following table attempts to make a classification based on two criteria: the nature of the risk and the observability of the hedging policy.

risk nature	risk observability	h observable	h non observable	hedging instrument
specific risk $\widetilde{\varepsilon}$ with weak moral hazard ex: theft, fire,...	observable	section 4	Appendix 9	insurance contract
	non observable	section 6.1	section 6.2	
specific risk with strong moral hazard (non insurable risk) ex: risk turnover	observable	not treated		diversification?
	non observable	not treated		
	risk exposure observability			
public risk $\widetilde{\xi}$ ex: exchange rate risk, interest rate risk, price risk	θ observable	section 4	Appendix 9	futures markets
	θ non observable	section 7.1	section 7.2	

Risk with strong moral hazard are risk for which there is no hedging instrument. In this case, we suggest that product diversification can be used as a hedging instrument. We do not consider such a risk in our model. The crossing over of the risk observability criteria and the hedging policy observability criteria throws up a possible combination where the risk itself or the risk exposure are observable but where the hedging policy will be not. Although, a priori, the observability of the risk would seem to us to imply the observability of the hedging policy we will deal with this situation in Appendix 9.

6 Non observable specific risk

In this section the firm is exposed to a non observable specific risk $\widetilde{\varepsilon}$. The hedging policy h is itself observable (section 6.1) or non-observable (section 6.2).

Random cash-flow $\widetilde{\omega} = \widetilde{x} + (1-h)\,\widetilde{\varepsilon}$ being a noisy signal on the quality x, the firm can use its hedging policy to modify the precision of this signal (i.e. to modify the level of asymmetric information on x at date 1).

Choosing h at date 0, the firm maximizes its value V_0 which is equal to the expectation, at date 0, of the shareholders' wealth at date 1, $E_0^F\,(V_1)$. Using equation (3), we can write the firm's program as follows:

$$\max_h E_0^F \left\{ \frac{x}{x_1^e} \left[x_1^e f\left(I\left(x_1^e\right)\right) - I\left(x_1^e\right) \right] \right\} \tag{8}$$

with $I\left(x_1^e\right)$ defined by equation (4).

We can rewrite equation (8) as follows:

$$\max_h E_0^F \left\{ x f\left(I\left(x_1^e\right)\right) - I\left(x_1^e\right) \right\} + E_0^F \left\{ \left(1 - \frac{x}{x_1^e}\right) I\left(x_1^e\right) \right\} \tag{9}$$

The first term in the expression for V_0 is the net present value (NPV) of the project. The distortions in the investment policy have the effect of diminishing this value relative to a symmetric information situation. Indeed, the investment which maximizes the NPV is $I(x)$ given by equation (6). We thus have:

$$E_0^F \left\{ x f\left(I\left(x_1^e\right)\right) - I\left(x_1^e\right) \right\} < E_0^F \left\{ x f\left(I(x)\right) - I(x) \right\} = V_0^{SI} \tag{10}$$

The second term corresponds to what we call the "expected gain from external financing". It is the difference between the cost of internal financing and the expected cost of external financing. It can be positive when the firm is overvalued or negative when it is undervalued.

The hedging policy will be the result of an arbitration between the cost of the asymmetric information (due to the distortions in the investment policy) and the gain of asymmetric information (that could be obtained in certain cases when the "expected gain from external financing" is positive).

We recall that h^e represents the market belief about the hedging policy adopted by the firm. Obviously, if the hedging policy is observable then we will necessarily have $h^e = h$. But in any case, this equality need to be satisfied at equilibrium (See definition 1). Now, it is possible to show that equality between h^e and h implies a null "expected gain from external financing".

Proposition 2 *If $\widetilde{\varepsilon}$ is a non observable specific risk and if $h^e = h$, then the "expected gain from external financing" is equal to zero.*

Proof.

Noting that $x_1^e = E_1^M(x) = E\left(x/\omega, h^e\right)$ and $h^e = h$, we have $x_1^e = E\left(x/\omega, h\right)$. Moreover, we have:

$$E_0^F \left\{ \left(1 - \frac{x}{x_1^e}\right) I\left(x_1^e\right) \right\} = E_0^F \left\{ \left(1 - \frac{E\left(x/\omega, h\right)}{x_1^e}\right) I\left(x_1^e\right) \right\}$$

But we also have $E\left(x/\omega, h\right) = x_1^e$. Therefore, the "expected gain from external financing" is equal to zero.

$\therefore$

By definition, for all equilibria, the equality $h^e = h$ is verified. Therefore, any equilibrium is characterized by a zero "expected gain from external financing". We obtain the following corollary:

Corollary 1 If $\widetilde{\varepsilon}$ is a non observable specific risk, any equilibrium is characterized by $V_0 \leq V_0^{SI}$.

Indeed, $V_0^{SI} = E_0^F \{xf(I(x)) - I(x)\}$ and Proposition 2 and equations (9) and (10) allow us to conclude.

6.1 Observable hedging policy

When h is observable, the sets of information are as follows:

$$\Gamma_0^F = \Gamma_0^M = \{h\}, \Gamma_1^F = \{x, \varepsilon, h\} \text{ and } \Gamma_1^M = \{\omega, h\}$$

Hedging policy being observable, due to Proposition 2, the "expected gain from external financing" is equal to zero.

The firm's value at date 0 is equal to the expected NPV of the project (See equation (9)):

$$V_0 = E_0^F \{xf(I(x_1^e)) - I(x_1^e)\} \tag{11}$$

It can thus be shown that the firm fully hedges.

Proposition 3 *If $\widetilde{\varepsilon}$ is a non observable specific risk and if h is observable then the equilibrium is unique and it correspond to $h = 1$ (full hedging) and $V_0 = V_0^{SI}$.*

Proof. See Appendix 3

On average, external financing is no more costly than internal financing. Nevertheless, the undervaluation and overvaluation of the firm due to asymmetric information distort the firm's investment policy and reduce its value. These distortions disappear when, in $t = 1$, the market learns the true value of x (i.e. $x_1^e = x$ and $I(x_1^e) = I(x)$). To obtain this result, the firm chooses to fully hedge. Indeed, when $h = 1$ the firm sends a perfect signal on x to the market since $\widetilde{\omega} = \widetilde{x}$.

6.2 Non observable hedging policy

The sets of information are as follows:

$$\Gamma_0^F = \{h\}, \Gamma_0^M = \emptyset, \Gamma_1^F = \{x, \varepsilon, h\} \text{ and } \Gamma_1^M = \{\omega\} \tag{12}$$

We call $\Psi(I)$ a measure of the concavity of the investment function $xf(I)$ which we define as follows:

$$\forall I,\ \Psi(I) = -\frac{If''(I)}{f'(I)} \tag{13}$$

The function $\Psi(I)$ is equivalent to the relative risk aversion of utility functions. We call it the relative concavity of the investment function.

We obtain the following proposition:

Proposition 4 *If $\widetilde{\varepsilon}$ is a non observable specific risk and if h is non observable then:*

If $\forall I \geq 0,\ \Psi(I) > \frac{1}{2}$, the full hedging policy $h = 1$ is an equilibrium. This equilibrium Pareto dominates all other equilibrium since we obtain $V_0 = V_0^{SI}$.

If $\forall I \geq 0,\ \Psi(I) = \frac{1}{2}$, then the full hedging policy $h = 1$ is a non strict equilibrium.

If $\forall I \geq 0,\ \Psi(I) < \frac{1}{2}$, the full hedging policy $h = 1$ is not an equilibrium.

Proof. See Appendix 6

In this section, the hedging policy is not observable. Consequently, the expected gain of external financing is not always equal to zero. This gain is equal to zero only at equilibrium when the firm chooses the hedging policy $h = h^e$ that corresponds to the market belief (See Proposition 2). Proposition 4 says that the full hedging policy is not always an equilibrium. In some circumstances, when $h^e = 1$, the firm is incited to deviate from the full hedging policy. Indeed, by deviating from $h = h^e = 1$, the firm can benefit from an attractive "expected gain of external financing". This means that the full hedging policy is not an equilibrium.

For a better understanding of this Proposition, it can be shown (See Appendix 7) that:

$$\frac{\partial I(x_1^e)}{\partial x_1^e} \frac{x_1^e}{I(x_1^e)} = \frac{1}{\Psi(I(x_1^e))} \tag{14}$$

The left term is the elasticity of the investment $I(x_1^e)$ relative to the external financing conditions x_1^e. This elasticity is inversely related to the relative concavity of the investment function. In other words, the less concave the investment function is, the more sensitive the investment is to the external financing conditions.

We are looking for the conditions under which the hedging policy $h = 1$ is an equilibrium. For this, it is sufficient to know whether the firm has an interest in deviating from the strategy $h = 1$ when the market belief is $h^e = 1$.

When $h^e = 1$, the market believes that $\widetilde{\omega}$ is a perfect signal about x. We thus have $x_1^e = \widetilde{\omega}$. Consequently, when the firm decides to deviate $(h \neq 1)$, $x_1^e = \widetilde{\omega}$ is not equal to x and there is still asymmetric information between the firm and the financial market (even if the last one is not aware about this situation). Thus, when the firm deviate $(h \neq 1)$, $x_1^e = \widetilde{\omega}$ is a random variable that fluctuates around x since we have $E_0^F(x_1^e/x) = x$.

Let us consider now the effects of this deviation when $\forall I \geq 0,\ \Psi(I) < \frac{1}{2}$. We know that the investment reacts strongly to variations in x_1^e (See equation (14)). Consequently, for all x, a high $x_1^e\,(x_1^e > x)$ implies a "very high" investment $(I(x_1^e) >> I(x))$ and a low x_1^e $(x_1^e < x)$ implies a "very low" investment $(I(x_1^e) << I(x))$. Therefore, when $x_1^e > x$, the gain in external financing $\left(1 - \frac{x}{x_1^e}\right) I(x_1^e)$ (See equation (9)), due to the overvaluation of the firm, is great because the positive term $\left(1 - \frac{x}{x_1^e}\right)$ is multiplied by a high investment $I(x_1^e)$.

On the other hand, when $x_1^e < x$, the loss of financing $\left(1 - \frac{x}{x_1^e}\right) I\left(x_1^e\right)$, due to an undervaluation of the firm, is small since the negative term $\left(1 - \frac{x}{x_1^e}\right)$ is counterbalanced by an investment $I\left(x_1^e\right)$ which is very low. On average, by deviating, the firm can benefit from a high "expected gain from external financing". However, deviation maintains asymmetric information and introduced some distortions in the firm's investment policy. But, although we are in the case where distortions in the investment policy are great, the NPV of the project is only slightly sensitive to these since the investment function is only slightly concave. Consequently the net present value loss is dominated by the gain in external financing. Therefore the firm, at date 0, tries to benefit from the variations of x_1^e around x by deviating from the full hedging strategy.

In the opposite case, when $\forall I \geq 0$, $\Psi(I) > \frac{1}{2}$, the investment reacts only a little to variations in x_1^e. Overvaluation has thus only a weak advantage over undervaluation and on average the "expected gain from external financing" is low. In this later case, for any x, the loss of NPV due to the distortions in the investment policy dominates the gain in external financing. Indeed, even if the investment distortions are relatively weak, the NPV is very sensitive to them because we are dealing with a "very concave" function of investment. The firm's optimum policy aims to protect itself against the variations in x_1^e around x. Total hedging $h = 1$ achieves this result because in this case $x_1^e = \omega = x$.

Finally, because the hedging policy $h = 1$ restores symmetric information between the firm and the market at date 1, we obtain V_0 is equal to V_0^{SI} which is the maximum equilibrium value of V_0 (See Corollary 1). The full hedging equilibrium is thus a Pareto dominant equilibrium.

7 Public risk with non observable risk exposure

The random fluctuations of exchange rates, interest rates, or prices of raw materials, constitute public risk in the sense that the variations of these variables are public knowledge. On the other hand, the exposure of the firm to such risk θ is not necessarily observable. Calling $\widetilde{\xi}$ the public risk, when θ is not observable, the financial market does not measure the risk $\theta\widetilde{\xi}$ undergone by the firm.

The cash-flow received at date 1 can be rewritten:

$$\widetilde{\omega} = \widetilde{x} + (\theta - h)\,\widetilde{\xi} \tag{15}$$

The following two sections distinguish the case where h is observable from the case where it is not.

7.1 Observable hedging policy

$$\Gamma_0^F = \{\theta, h\}, \Gamma_0^M = \{h\}, \Gamma_1^F = \{\theta, h, x, \xi\} \text{ and } \Gamma_1^M = \{h, \omega, \xi\}$$

We can prove the following Proposition:

Proposition 5 *If $\theta\widetilde{\xi}$ is a public risk with θ non observable and h observable then:*

Any hedging policy independently distributed from θ is a pooling equilibrium.

If $\forall I \geq 0$, $\Psi(I) > \frac{1}{2}$, any hedging policy $h(\theta)$ witch is invertible $\left(h^{-1}\text{ exists}\right)$ is a separating equilibrium. The full hedging policy $h(\theta) = \theta$ is one of these equilibria. These equilibria pareto dominate pooling equilibria since we obtain $V_0 = V_0^{SI}$.

If $\forall I \geq 0$, $\Psi(I) = \frac{1}{2}$, then any invertible hedging policy $h(\theta)$ is a non strict separating equilibrium.

If $\forall I \geq 0$, $\Psi(I) < \frac{1}{2}$, it does not exist any separating equilibrium.

Proof. See Appendix 8

The result obtained is different from that of Proposition 3, where the risk is specific. Actually, here, from date 0, the firm has an informational advantage over the market because the firm knows its risk exposure θ. Since the hedging policy is observable, a signaling game takes place at date zero. Indeed, the firm can use its hedging policy to reveal to the market the true value of θ. It is the case when $h(\theta)$ is invertible (separating equilibrium) and it is not the case when the hedging policy is independently distributed of θ.

When $t = 1$, the term $h\xi$ in the expression of ω (See equation 15) is perfectly well known by the market and can thus be isolated. Consequently, what the market believes, x_1^e, depends only on $x+\theta\xi$ and on the observation of the hedging policy.

First, Proposition 5 says that any hedging policy independently distributed from θ is an equilibrium. Indeed, if the market believes that h brings no information about θ then x_1^e depends only on $x + \theta\xi$. But we know that the firm's value at date 0 can only depend on h through x_1^e. It will therefore be independent of h. Consequently, if the market does not glean any information about θ by observing h, the firm will indeed choose any hedging strategy.

The basis of the rest of the Proposition is very similar to those of Proposition 4. Let us consider any invertible function $h(\theta)$ and show under what conditions $h(\theta)$ is an equilibrium. For this, let us assume that when the market observes the coverage $h(\theta)$, the latter deduces from this that the firm's risk exposure is θ (this is possible only if the function $h(\theta)$ is invertible). Let us also consider a firm whose risk exposure is θ, and under what conditions its optimal hedging policy is really $h(\theta)$. For this, let us call $h\left(\widehat{\theta}\right)$ the hedging strategy chosen by the "firm θ". This strategy is fully revealing when $h\left(\widehat{\theta}\right) = h(\theta)$ (or when $\widehat{\theta} = \theta$ which is equivalent). Now, if the market believes that the firm chooses a revealing strategy, we have $x_1^e = \omega - \left(\widehat{\theta} - h\left(\widehat{\theta}\right)\right)\xi$. We thus have $E_0^F\left(x_1^e/x\right) = x$ and we can therefore apply the same reasoning as for Proposition 4:

When $\forall I \geq 0$, $\Psi(I) > \frac{1}{2}$, we saw that the firm's objective is to restore symmetric information at $t = 1$. Actually, in this case, the loss in NPV associated with investment distortions is greater than the "gain of external financing". By choosing $h\left(\widehat{\theta}\right) = h(\theta)$, the firm signals its risk exposure θ. As risk is public, the market can deduce the value of x by observing ω, ξ and $h(\theta)$ since $x_1^e = \omega - (\theta - h(\theta))\,\xi = x$. In other words, when the value of θ is known by everybody, there is also symmetric information on the value of x at date 1. The ultimate goal of the hedging policy is the same as before: reaching symmetric information on x at date 1 in order to avoid distortions in the investment policy.

On the other hand, when $\forall I \geq 0,\ \Psi(I) < \frac{1}{2}$ the "firm θ" deviates from the revealing strategy $h(\theta)$ to profit from a high "gain of external financing". There is no separating equilibrium.

7.2 Non observable hedging policy

The sets of information are as follows:

$$\Gamma_0^F = \{\theta, h\}, \Gamma_0^M = \emptyset, \Gamma_1^F = \{\theta, h, x, \xi\} \text{ and } \Gamma_1^M = \{\omega, \xi\}$$

We obtain the following Proposition:

Proposition 6 *If $\theta\widetilde{\xi}$ is a public risk with θ observable and if h is non observable then:*

If $\forall I \geq 0,\ \Psi(I) > \frac{1}{2}$, the full hedging policy $h(\theta) = \theta$ is an equilibrium. This equilibrium is Pareto dominant since $V_0 = V_0^{SI}$.

If $\forall I \geq 0,\ \Psi(I) = \frac{1}{2}$, the full hedging policy $h(\theta) = \theta$ is a non strict equilibrium.

If $\forall I \geq 0,\ \Psi(I) < \frac{1}{2}$, the full hedging policy $h(\theta) = \theta$ is not an equilibrium.

Knowing that the total hedging policy is no longer characterized by $h = 1$ but by $h(\theta) = \theta$, the proof for this Proposition is the same as that of Proposition 4.

8 Optimal partial hedging policy

The optimality of the full hedging policy crucially depends on an hypothesis. The quality of the investment project x is assumed to be independent of the hedgeable risk which affects the current cash-flow $\widetilde{\omega}$. Now let us take a french exporting company as an example. A drop in the dollar will reduce the value of the credit stock held by the firm and also the yield resulting from its sales at exportation. The firm's current cash-flow will thus be reduced, in the short term. However, it is possible that future cash-flows and thus the profitability of investments will also be affected by this drop in the dollar.

By using the specific risk's framework to simplify, we call $\widetilde{\varepsilon}_1$ the hedgeable risk that affects the firm's cash-flow at date 1 (effect of the variation in the dollar on firm's current cash-flow) and, to be more general, we introduce $\widetilde{\varepsilon}_2$ the risk that affects the firm's investment cash-flow at date 2 (the equivalent of $\widetilde{\varepsilon}_1$ at date 2). Before date 2, the firm and external investors know only the distribution of $\widetilde{\varepsilon}_2$.

The firm's cash-flow at date 1 is:

$$\widetilde{\omega}_1 = \widetilde{x} + (1-h)\,\widetilde{\varepsilon}_1$$

The cash-flow produced at date 2 by the investment project takes the following form:

$$\widetilde{\omega}_2 = (\widetilde{x} + \alpha\widetilde{\varepsilon}_1 + \widetilde{\varepsilon}_2)\, f(I)$$

The firm's investment quality is no longer $\widetilde{x}$ but it is $\widetilde{z} = \widetilde{x} + \alpha\widetilde{\varepsilon}_1 + \widetilde{\varepsilon}_2$.

The variables $\widetilde{\varepsilon}_1$ and $\widetilde{\varepsilon}_2$ are assumed to be independent[4] with null average. The parameter α represents the long term sensitivity of the firm's cash-flows to past risk (effect of the dollar variation on firm's future cash-flows). We assume $0 \leq \alpha \leq 1$.

$\alpha \geq 0$ means that the risk $\widetilde{\varepsilon}_1$ exerts an influence in the same way on short term and long term flows. $\alpha \leq 1$ means that the effect of the current risk $\widetilde{\varepsilon}_1$ on the long term flows is attenuated relative to its effect on the short term flows. Indeed, in the short term, the depreciation of the currency affects both the value of the credit stock held by the firm and its export turnover. On the other hand, the effects on the long run are only limited to effects on future turnovers.

As in section 6.1, we consider a specific risk with observable hedging policy. This choice is motivated by a simplification consideration. Indeed we can get rid of asymmetric information situation at date 0. The goal of this section is just to show under which conditions partial hedging could be optimal. Of course, it could be interesting to treat all the possible cases as we have done with the full hedging framework but it would be too tedious.

The information sets are the following:

$\Gamma_0^F = \Gamma_0^M = \{\alpha, h\}$, $\Gamma_1^F = \{\alpha, x, \varepsilon_1, h\}$ and $\Gamma_1^M = \{\alpha, \omega_1, h\}$

The market belief on the firm's quality is now:

$$z_1^e = E_1^M\left(\widetilde{x} + \alpha\widetilde{\varepsilon}_1 + \widetilde{\varepsilon}_2\right)$$

The crucial point for the following is that, at date 1, the best prediction[5] of the firm's investment quality is no longer x but $x + \alpha\varepsilon_1$. Indeed, it is the best prediction that the firm itself can make about its quality: $E_1^F\left(\widetilde{x} + \alpha\widetilde{\varepsilon}_1 + \widetilde{\varepsilon}_2\right) = \widetilde{x} + \alpha\widetilde{\varepsilon}_1$. Consequently, there is symmetric information between the firm and the financial market at date 1 when $z_1^e = \widetilde{x} + \alpha\widetilde{\varepsilon}_1$.

We can prove the following Proposition:

Proposition 7 *If the firm's cash-flow at date* 2 *takes the form:* $(x + \alpha\varepsilon_1 + \varepsilon_2)\, f(I)$ *then the optimal hedging policy is the partial hedging policy* $h = 1 - \alpha$.

[4] The hypothesis of the independence of $\varepsilon_1$and ε_2 is a convenience which is not really essential. A process AR(1), for example allows qualitatitively similar conclusions to be obtained.

[5] It is the prediction using all available information about the investment quality $\widetilde{x}+\alpha\widetilde{\varepsilon}_1+\widetilde{\varepsilon}_2$ at date 1.

Proof.

The demonstration is similar to that of Proposition 3 where we replace the function $xf(I)$ by the function $zf(I)$. The "expected gain from external financing" is null since h is observable (See Proof of Proposition 2). Consequently, the value of the firm is determined exclusively by the expected NPV of its investment:

$$V_0 = E_0^F \{zf(I(z_1^e)) - I(z_1^e)\}$$

where $I(z_1^e)$ is determined by the following equation (See equation 6):

$$z_1^e f'(I(z_1^e)) = 1$$

The investment policy that maximizes V_0 is still the investment policy chosen by the firm under symmetric information. It is no longer $I(x)$, but it is now the function $I(x + \alpha\varepsilon_1)$. Indeed, when $h = 1 - \alpha$, we obtain $\omega = x + \alpha\varepsilon_1$ which is the best prediction of $\widetilde{z}$ at date 1. Put differently, when $h = 1 - \alpha$, at the moment of investing (date 1), the market has as much information as the firm as to the quality of the investment project $\widetilde{z} = x + \alpha\varepsilon_1 + \widetilde{\varepsilon}_2$. Consequently, the amount of the investment $I(z_1^e)$ maximizes the investment project's NPV since it takes into account all available information at date 1.

Now, we are able to stipulate the general conclusion of this paper: *the optimal hedging policy is that which ensures the most exact perception of the firm's quality. The objective assigned to corporate risk management is to make it such that the next cash-flow (short term cash-flow) predict, in the best possible way, the future cash-flow (long term cash-flow).*

Lastly, when $\alpha = 1$, the optimal hedging policy is no hedging at all. Thus, our theory doesn't imply that all the firms should systematically hedge.

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Appendix 1

We show here that the scope of this model is not limited to equity financed firms. This model remains valid if the firm's financial structure (e.g. percentage of initial debt and initial equity, etc.) is maintained while financing the investment I.

Indeed, we show successively that under this assumption:

- we still obtain equation (2) that characterizes the part of the firm's cashflows that goes to external investors (new debtholders and new shareholders).

- the firm's objective function and the firm's investment policy are not modified and are still defined by equations (3) and (4).

Assume that the initial firm's financial structure is composed of debt (bonds) and equity. For example, there are n bonds for one stock. Assume also that the firm issue new debt and new equity in the same proportion as before in order to finance the investment I. These new claims are such that they give exactly the same payoffs as the initial one. In other words, we can assimilate new claims to old claims. If q represents the fraction of new shares in the total firm's capital, q also represents the fraction of new debt in the total firm's indebtedness. A fraction q of the firm's dividends goes to the new shareholders and new bondholders receive a fraction q of the firm's debt revenue. Consequently, new claimholders (new shareholders and new debtholders) perceive a fraction q of the firm's future cash-flows. This is consistent with equation (2).

Let D denote the face value of the initial debt. At date 1, the firm's manager maximizes the wealth of the initial shareholders by choosing the investment I:

$$\max_{I} (1-q)\, xf\,(I) - D$$

Using equation (2) and deriving this expression with respect to I, we find back equation (3) that defines the firm's investment policy. Here, we assume that the debt is riskless since the firm's payoff $xf\,(I)$ is enough to reimburse all debtholders. However, we can be more general by introducing a probability of failure. We can assume for example that the firm's cash-flow is equal to $xf\,(I)$ with some probability P and is equal to zero with probability $1-P$. The firm's program becomes:

$$\max_{I} P\left[(1-q)\, xf\,(I) - D\right]$$

which does not change anything in our approach.

Appendix 2

(No strict separating equilibrium)

The functions[6] $I\,(x)$ and $q\,(x)$ characterize a separating equilibrium when the announcement of the investment $I\,(x)$ and of the fraction $q\,(x)$ reveal that the firm's quality is equal to x. That means that a firm of "type x" prefer to choose $I\,(x)$ and $q\,(x)$ rather than $I\,(\widehat{x})$ and $q\,(\widehat{x})$ with $\widehat{x} \neq x$. In other words, the functions $I\,(x)$ and $q\,(x)$ must be such that it is in the firm x's interest to announce its true type x.

Consequently, for these functions to represent a separating equilibrium, they must satisfy the following condition:

$$\forall x, x \in \arg\max_{\widehat{x}} \left(1-q\,(\widehat{x})\right) xf\,(I\,(\widehat{x}))$$

As shown by the latter equation, the true type of the firm only appears as a factor in the firm's objective function. Thus, all firms will choose $\widehat{x}$ which

[6] Functions of x conditional on the realisation of the cash flow ω.

maximizes $(1-q(\widehat{x}))\, f(I(\widehat{x}))$. Consequently, if we want each firm to announce its true quality, the functions $I(x)$ and $q(x)$ must verify:

$$\forall x,\ \ (1-q(x))\, f(I(x)) = C$$

where C is a positive constant.

Each firm x will therefore be indifferent as to whether to announce $I(x)$ and $q(x)$ or announce $I(\widehat{x})$ and $q(\widehat{x})$ with $\widehat{x} \neq x$. Therefore if a separating equilibrium exists, it is not a strict equilibrium.

Appendix 3

(Proof of Proposition 3)

At date zero, the firm chooses its hedging policy in order to maximize its value.

Equation (11) gives us the expression for the firm's value at $t=0$:

$$V_0 = E_0^F \left\{ \left[x \cdot f\left(I\left(x_1^e\right)\right) - I\left(x_1^e\right) \right] \right\}$$

We know that $\forall x$, $xf(I) - I$ is maximum when the firm invests $I(x)$ at date 1 (See equation (6)). In other words, V_0 is maximum when $x_1^e = x$. We obtain this result with $h=1$. Indeed, we have $\omega = x + (1-h)\,\varepsilon = x$ and $x_1^e = E_1^M(x) = E(x/\omega, h=1) = x$. Since $x_1^e = x$, we have symmetric information and $V_0 = V_0^{SI}$.

Appendix 4

(Definition of the function $F\left(x_1^e\right)$)

For a given hedging policy, the value of the firm at $t=0$ is given by the equation (8):

$$V_0 = E_0^F \left\{ \frac{x}{x_1^e} \left[x_1^e \cdot f\left(I\left(x_1^e\right)\right) - I\left(x_1^e\right) \right] \right\}$$

This equation can be also rewritten:

$$V_0 = E_0^F \left\{ x E_0^F \left[\frac{1}{x_1^e} \left[x_1^e \cdot f\left(I\left(x_1^e\right)\right) - I\left(x_1^e\right) \right] / x \right] \right\}$$

or even, noting

$$F\left(x_1^e\right) = f\left(I\left(x_1^e\right)\right) - \frac{I\left(x_1^e\right)}{x_1^e}$$

$$V_0 = E_0^F \left\{ x E_0^F \left[F\left(x_1^e\right) / x \right] \right\}$$

As it will become apparent in Appendix 5, the concavity/convexity of the function $F\left(x_1^e\right)$ will depend upon the degree of concavity of the function $xf(I)$.

Appendix 5

(Concavity of the function $F(x_1^e)$)

The value of the firm in $t=0$ can be written (See Appendix 4):

$$V_0 = E_0^F \left\{ x E_0^F \left[F(x_1^e)/x \right] \right\}$$

with $F(x_1^e)$ as a function of x_1^e.

Let us study the form of function $F(x_1^e)$. For this, let us derive $F(x_1^e)$ with respect to x_1^e :

$$\frac{\partial F(x_1^e)}{\partial (x_1^e)} = f'(I(x_1^e))\, I'(x_1^e) + \frac{I(x_1^e)}{(x_1^e)^2} - \frac{I'(x_1^e)}{x_1^e}$$

But, it is known that (See equation (4)):

$$f'(I(x_1^e)) = \frac{1}{x_1^e} \tag{A5.1}$$

and thus after simplification, we obtain:

$$\frac{\partial F(x_1^e)}{\partial (x_1^e)} = \frac{I(x_1^e)}{(x_1^e)^2}$$

$$\Longrightarrow \frac{\partial^2 F(x_1^e)}{\partial (x_1^e)} = \frac{I'(x_1^e)(x_1^e)^2 - 2x_1^e I(x_1^e)}{(x_1^e)^4} \tag{A5.2}$$

The sign of the expression below is given by the sign of the numerator:

$$I'(x_1^e)\, x_1^e - 2I(x_1^e) \tag{A5.3}$$

To study this sign, let us derive $(A5.1)$ with respect to x_1^e. The result is

$$f'(I(x_1^e)) + x_1^e f''(I(x_1^e))\, I'(x_1^e) = 0 \Leftrightarrow I'(x_1^e)\, x_1^e = -\frac{f'(I(x_1^e))}{f''(I(x_1^e))} \tag{A5.4}$$

The equations $(A5.1)$, $(A5.2)$ and $(A5.4)$ make it possible to write :

$$\begin{aligned} sgn \frac{\partial^2 F(x_1^e)}{\partial x_1^e} &= sgn\left(-\frac{f'(I(x_1^e))}{f''(I(x_1^e))} - 2I(x_1^e) \right) \\ &= sgn\left(\frac{1}{2} + I\frac{f''(I(x_1^e))}{f'(I(x_1^e))} \right) = sgn\left(\frac{1}{2} - \Psi(I(x_1^e)) \right) \end{aligned}$$

where $sgn(\cdot)$ represents the sign of the expression in brackets.

Thus $\frac{\partial^2 F(x_1^e)}{\partial (x_1^e)}$ is of the same sign as $\frac{1}{2} - \Psi(I(x_1^e))$. Consequently,

- If $\forall I \geqslant 0$, $\Psi(I) > \frac{1}{2}$ then $\frac{\partial^2 F(x_1^e)}{\partial x_1^e} < 0$ and $F(x_1^e)$ is concave.

- If $\forall I \geqslant 0,\ \Psi(I) = \dfrac{1}{2}$ then $\dfrac{\partial^2 F(x_1^e(\omega))}{\partial x_1^e(\omega)} = 0$ and $F(x_1^e(\omega))$ is linear.
- If $\forall I \geqslant 0,\ \Psi(I) < \dfrac{1}{2}$ then $\dfrac{\partial^2 F(x_1^e)}{\partial x_1^e} > 0$ and $F(x_1^e)$ is convex.

Appendix 6

(Proof of proposition 4)

When the market believes that the firm fully hedges ($h^e = 1$), we have :

$$x_1^e = \omega$$

Noting h the true hedging policy chosen by the firm, we thus have:

$$x_1^e = \omega = x + (1-h)\varepsilon \qquad \text{(A5.1)}$$

We have shown that the value of the firm at date 0 can be written (See Appendix 4) :

$$V_0 = E_0^F \left\{ x E_0^F \left[F(x_1^e)/x \right] \right\} \qquad \text{(A5.2)}$$

with $F(x_1^e)$ as a function of x_1^e.

It was also shown (See Appendix 4) that if $\forall I \geqslant 0, \Psi(I) > \frac{1}{2}$, $F(x_1^e)$ is concave. Furthermore, we have $= E_0^F \left\{ x E_0^F \left[F(x_1^e)/x \right] \right\} = E_0^F \left\{ x E_0^F \left[F(x+(1-h)\varepsilon)/x \right] \right\}$. Consequently, for each value of x, V_0 is maximum when $h = 1$ since $F(x_1^e)$ is a concave function[7]. Thus, full hedging is an equilibrium this equilibrium is Pareto dominant since $V_0 = V_0^{SI}$.

On the other hand, if $\forall I \geqslant 0, \Psi(I) < \frac{1}{2}$, the function $F(x_1^e)$ is convex (See Appendix 4). Thus the firm will deviate from the strategy $h = 1$.

If $\forall I \geqslant 0, \Psi(I) = \frac{1}{2}$, then $F(x_1^e)$ is linear. The firm's value is independent of h. The equilibrium $h = 1$ is thus not a strict equilibrium since the choice of the hedging strategy is not relevant for the firm when $h^e = 1$.

Appendix 7

(Elasticity of the investment to financing conditions)

At date 1, the firm chooses an investment I that satisfies (See equation (4)):

$$\forall x_1^e, x_1^e f'(I) = 1$$

By differentiating this equation with respect to x_1^e and I, we obtain the following equation:

$$f'(I)\, dx_1^e + x_1^e f''(I)\, dI = 0$$

$$\Longrightarrow \frac{dI}{dx_1^e}\frac{x_1^e}{I} = \frac{-f'(I)}{I f''(I)} = \frac{1}{\Psi(I)}$$

[7] A deviation from $h = 1$ is a mean preserving spread that decreases the value of $E_0^F \{F(x+(1-h)\varepsilon)/x\}$ for each possible value of x.

Appendix 8

(Proof of Proposition 5)

Let us show that any hedging policy h independently distributed from θ is an equilibrium. If external investors believe that h does not give any information on θ then:

$$x_1^e = E\left(x/\omega, h, \xi\right) = E\left(x/x + (\theta - h)\,\xi, h, \xi\right) = E\left(x/x + \theta\xi, \xi\right)$$

It can be seen that x_1^e is independent of the hedging policy set up by the firm. Consequently, the firm's value V_0 is also independent of the hedging policy since it only depends on h through x_1^e. Consequently, the firm chooses any hedging policy h at $t = 0$.

Let us now consider any invertible function $h\,(\theta)$ and show under what conditions $h\,(\theta)$ is an equilibrium. For this, let us assume that when the market observes the coverage $h\,(\theta)$, the latter deduces from this that the firm's risk exposure is θ (this is possible only if the function $h\,(\theta)$ is invertible). Let us also consider a firm whose risk exposure is θ, and under what conditions its optimal hedging policy is really $h\,(\theta)$.

For this, let us assume that the firm θ uses strategy $h\left(\widehat{\theta}\right)$. The market believes that it is dealing with firm $\widehat{\theta}$ and thus we have :

$$x_1^e = E_1^M\left(x\right) = E\left(x/h\left(\widehat{\theta}\right), \omega, \xi\right) = \omega - \left(\widehat{\theta} - h\left(\widehat{\theta}\right)\right)\xi$$

Now,

$$\omega = x + \left(\theta - h\left(\widehat{\theta}\right)\right)\xi$$

Thus,

$$x_1^e = x + \left(\theta - \widehat{\theta}\right)\xi$$

By hypothesis, the random variables $\widetilde{x}$ and $\widetilde{\xi}$ are independent. Thus, any deviation by the firm from the hedging policy $h\,(\theta)$ corresponds to a mean preserving spread**.** Moreover, it is shown in Appendix 4 that the firm's value under $t = 0$ can be written as :

$$V_0 = E_0^F\left\{xE_0^F\left[F\left(x_1^e\right)/x\right]\right\}$$

with $F\left(x_1^e\right)$ as a function of x_1^e.

We thus have:

$$V_0 = E_0^F\left\{xE_0^F\left[F\left(x + \left(\theta - \widehat{\theta}\right)\varepsilon\right)/x\right]\right\}$$

Now it can be shown that (See Appendix 4) if $\forall y \geqslant 0, \Psi\left(y\right) > \frac{1}{2}$ then function $F\left(x_1^e\right)$ is concave. In this case, for each value of x, $E_0^F\left\{\left[F\left(x + \left(\theta - \widehat{\theta}\right)\varepsilon\right)/x\right]\right\}$ is maximum when $\widehat{\theta} = \theta$ as $F\left(x_1^e\right)$ is concave. Thus V_0 is maximum for $\widehat{\theta} = \theta$. The firm's hedging policy is thus $h\left(\widehat{\theta}\right) = h\,(\theta)$. This means that the firm uses its hedging policy in order to announce its true risk exposure. In other words, the firm announces $\widehat{\theta} = \theta$. This equilibrium is Pareto dominant since it establishes a situation of symmetric information between the firm and the market at date 1. This implies that $V_0 = V_0^{SI}$.

Furthermore, it can be shown that (See Appendix 4) if $\forall y \geqslant 0, \Psi(y) < \frac{1}{2}$ then the function $F(x_1^e)$ is convex. In this case $V_0 = E_0^F \left\{ x E_0^F \left[F(x_1^e)/x \right] \right\}$ maximum implies $\widehat{\theta} \neq \theta$. The optimal strategy of the firm $h\left(\widehat{\theta}\right)$ thus differs from $h(\theta)$. There is no separating equilibrium.

If $\forall y \geqslant 0, \Psi(y) = \frac{1}{2}$ then $F(x_1^e)$ is linear. Thus the firm's value does not depend on the hedging policy. The firm is indifferent to strategy $h(\theta)$ and strategy $h\left(\widehat{\theta}\right)$.

Appendix 9

Observable specific risk (or Observable Public Risk Exposure)
and non observable hedging policy.

The risk considered here is noted $\widetilde{\xi}$. It could be either public with θ observable , or specific and observable. On the other hand, the hedging policy is not observable. The sets of information are the following (for specific risk θ must be set $\theta = 1$):

$$\Gamma_0^F \{\theta, h\}, \ \Gamma_0^M = \{\theta\}$$

$$\Gamma_1^F \{\theta, h, x, \xi\} \text{ and } \Gamma_1^M = \{\theta, \omega, \xi\}$$

The following proposition is obtained:

Proposition If $\widetilde{\xi}$ is a public risk and $\Gamma_1^M = \{\theta, \omega, \xi\}$ or if $\widetilde{\xi}$ is a specific risk and $\Gamma_1^M = \{\omega, \xi\}$ then:

- If $\forall I \geqslant 0, \Psi(I) > \frac{1}{2}$, any hedging policy $h(\theta)$ is an equilibrium and $V_0 = V_0^{SI}$.
- If $\forall I \geqslant 0, \Psi(I) = \frac{1}{2}$, any hedging policy $h(\theta)$ is a non strict equilibrium.
- If $\forall I \geqslant 0, \Psi(I) < \frac{1}{2}$, there is no equilibrium.

Let us show that if $\forall I \geqslant 0, \Psi(I) > \frac{1}{2}$ any hedging policy $h(\theta)$ is an equilibrium. For this, let us consider any function $h(\theta)$ and suppose that the market believes that the firm θ chooses the hedging policy $h(\theta)$. Let us consider the true strategy h of "firm θ".

As θ is observable, we have:

$$x_1^e = E(x/\theta, \xi, \omega) = \omega - (\theta - h(\theta))\xi$$

We also have:

$$\omega = x + (\theta - h)\xi$$

Thus, we obtain:

$$x_1^e = x + (h(\theta) - h)\,\xi$$

with

$$E_0^F\left(x_1^e/x\right) = x$$

For each value of x, any deviation from the hedging policy $h(\theta)$ corresponds therefore to a mean preserving spread.

We have shown (See Appendix 4) that

$$V_0 = E_0^F\left\{xE_0^F\left[F\left(x_1^e\right)/x\right]\right\}$$

with $F(x_1^e)$ a function of x_1^e.

Thus,

$$V_0 = E_0^F\left\{xE_0^F\left[F\left(x + (h(\theta) - h)\,\xi\right)/x\right]\right\}$$

Now, we can show that (See Appendix 4) if $\forall I \geqslant 0, \Psi(I) > \frac{1}{2}$, then the function $F(x_1^e)$ is concave. Consequently, the firm's value is maximum when $h = h(\theta)$. The hedging policy $h(\theta)$ is indeed an equilibrium.

On the other hand, if $\forall I \geqslant 0, \Psi(I) < \frac{1}{2}$, the function $F(x_1^e)$ is convex (See Appendix 4). The firm will thus systematically deviate from the strategy $h(\theta)$.

If $\forall I \geqslant 0, \Psi(I) = \frac{1}{2}$, $F(x_1^e)$ is linear. The firm's value does not therefore depend on the hedging policy. The firm is indifferent to $h(\theta)$ and any other hedging policy.