Models for Limits to Arbitrage and Herding in Stock Markets

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§1 Limits to Arbitrage

An investment strategy that is guaranteed to make riskless profit with no cost is an arbitrage strategy. Arbitrage opportunity means the existence of such strategies. The existence of arbitrage opportunity, from an economic point of view, implies that the economy is in a disequilibrium state. Even if irrational traders (*noise traders*) cause this, the rational agents will participate and the price is expected to change until arbitrage is no longer possible. It is not expected to earn profits through arbitrage for a long time in an efficient financial market.

In the traditional financial market setup where rational agents alone are present, prices equal fundamental values [BR01] of stocks. This is the discounted sum of expected future cash flows, where the expectation is taken over the correct probability distribution and the discount rate is compatible with the normative values. Efficient market hypothesis states that the actual price is the fundamental value. In such efficient markets, no investment strategy can earn average returns greater than that are justified by the corresponding risk. This ruling out the possibility of arbitrage has profound implications to the modeling of markets. The principle of no arbitrage helps in analytically determining the prices of derivative securities such as options.

In a market where rational and non-rational agents interact, non-rational behaviors can have an impact on prices. This essentially means there are *limits to arbitrage*. Moreover, the mispricing is sustained for a long time and cannot be arbitraged away i.e. there are *limits to arbitrage*. There are many qualitative reasons for the limits to arbitrage.

Strategies conceived to correct mispricing can themselves be risky, allowing the mispricing to survive. The sources of the risk associated have been summarized in [BR01] [AM02]. These are as follows:

1. *Fundamental risk*: The fundamental value might change over time and the arbitrageurs' models might not coincide with that of the actual data generating process. Consider the actual price is less than that of fundamental. An arbitrageur who buys at the current price is taking a risk that a bad news could cause the value to go down further.

2. *Noise trader risk*: The pessimistic agents who cause the stock to be undervalued could become even more pessimistic lowering the price again. The optimistic agents, on the other hand, overvalue the stock further.

3. *Implementation risk*: This includes the generic transaction costs arbitrageurs pay while implementing their strategies. It also arises because of delay in implementing the strategy.

4. *Observation risk*: When a mispricing occurs, arbitrageurs will not be sure whether it really exists. This uncertainty will limit their position.

   These risks will limit arbitrage and allow deviations from fundamental value to persist. The noise trader risk itself is powerful enough to limit the arbitrage [DL90a] [DL90b].

In the next section, we will explain the basic ideas behind two models for limits to arbitrage.

§2 Marginally Efficient Market and Synchronization Risk Model

Y.C. Zhang proposed the marginally efficient market model [Zha99] where he used conditional entropy to identify the probability of the existence of arbitrage. Shannon entropy, S, is defined as,

$$S = -\sum_{m} p(m) log p(m) \tag{2.1}$$

where $p(m)$ is the probability of the event m. The conditional entropy, $H(n|m)$, is defined as,

$$H(n|m) = -\sum_{m} p(m) \sum_{n} p(n|m) log p(n|m) \tag{2.2}$$

where $p(n|m)$ is the conditional probability that the event n would occur given the event m.

The negative entropy is defined as the difference between the maximal value and the actual value of the conditional entropy H, and is $\Delta H = 1 - H$ for binary case where the events are represented by binary strings. Zhang considered a binary sequence and proved that negative entropy cannot be arbitraged away. He showed that this implies infinite capital is needed to make the price a perfect random walk. The central point of Zhang's work is that inefficiency in the market can be quantified using entropy. His approach is directed towards creating an alternative framework to accommodate inefficiency. Hence, probabilistic arbitrage opportunities do exist. The source of the uncertainty has not been considered. Moreover, the ways to consider the risks discussed in Section 1 have not been taken into account in the marginally efficient market model.

In the synchronization risk model [AM02], Dilip Abreu and Markus Brunnemeier argued that the arbitrage is limited when rational traders face uncertainty when other traders exploit a common arbitrage opportunity. They have proposed that this uncertainty also becomes a risk along with the risks mentioned in the previous section.

There is a single risky asset whose price is p_t. The fundamental value of the risky asset is v_t. The fundamental value before the arrival of information is e^{rt}, where r determines the growth possibilities of the asset. The value changes to $(1 + \beta)e^{rt}$ after the arrival of information. The time of arrival of information is exponentially distributed. Rational arbitrageurs become sequentially aware of the new value and they do not know how early the information is received relative to other arbitrageurs.

Behavioral traders continue to think there are no new information and act as if the fundamental price is e^{rt}. They support the mispricing till the buying (or selling) pressure by rational arbitrageurs lies below a threshold. When this threshold is crossed, the price adjusts and coincides with the fundamental value of the asset.

The main features of this model are: (i) A single arbitrageur cannot correct the mispricing. (ii) Arbitrageurs' opinions about the timing of the price correction are dispersed. (iii) The arbitrageurs incur holding costs in order to exploit an arbitrage opportunity. Sequential awareness of the mispricing is shown to cause delayed arbitrage. Hence, the source of the persistence of arbitrage could be due to the lack of synchronization. This is in addition to the earlier qualitative reasons discussed for the persistence. This model offers one more reason for limits to arbitrage.

In the next section, we would explain the proposed framework for limits to arbitrage and how to incorporate the above features into a single framework.

§3 Proposed System Model

The limitations posed by the market for the arbitrageurs to make use of the arbitrage possibilities are many. Modelling these limits of arbitrage is important to understand the market. Models described in the previous section attempted to quantify these limitations. Dynamical system approach is found to be useful in modelling long term financial accumulation and crisis [And99] and in understanding macroeconomy [And98]. We propose a generic dynamical system model that captures some aspects of the limits of arbitrage. Since our objective is to model those limits through a dynamical system, we focus on the generic aspects rather than the specific details. First, we make an unrealistic assumption that our system is a deterministic one. Even though the rich literature of stochastic dynamical system techniques can be used, discussions in this paper are limited to the deterministic case. This makes our interpretations more clear. But, the proposed approach can be extended to include the stochastic nature of the real market using more sophisticated analysis.

We consider a market with two traders, one is a seller and the other is a buyer.

A transaction takes place when seller's offer price matches with buyer's bid price. If they don't match, seller will decrease the offer and buyer will increase the bid. This might lead to a transaction when the offer is equal to the bid. Sometimes, deadlock would prevail till either new information comes into the market or the traders are willing to change their positions.

Let the offer price, p_s, of the seller be given implicitly by,

$$\dot{p_s} = f_s(a_s, b_s, p_s, u_s, t) \tag{3.3}$$

The bid price, p_b, obeys the equation,

$$\dot{p_b} = f_b(a_b, b_b, p_b, u_b, t) \tag{3.4}$$

a_s, b_s, a_b, and b_b are the parameters associated with the trader. For instance, if any trader wants to change the offer/bid price based only on his earlier price, then he could ignore b_s/b_b. Similarly, he could make a new offer/bid based only on the new arrival of information. In this case, he ignores a_s/a_b.

The u_s/u_b is the information arrival. It is assumed to be a binary sequence. A step input can also be considered as a binary sequence. Piecewise extension is assumed in the continuous domain. p_s must be equal to p_b for the transaction to take place. Let this price at which transaction takes place be p. The difference $p_s - p_b$ and the deviation of the actual price from the fundamental price (r) are the measures of the existence of arbitrage in the sense explained in the previous section. Let y be proportional, with c as the proportional constant, to the first measure ($p_s - p_b$), and the second measure is the misconception about the fundamental value. For the sake of simplicity, a linear first-order system is considered for analysis though both first order and second order systems are considered during the simulation.

$$\dot{p_s} = a_s p_s + b_s u_s \tag{3.5}$$

$$\dot{p_b} = a_b p_b + b_b u_b \tag{3.6}$$

$$y = c(p_s - p_b) \tag{3.7}$$

The structure of the inputs u_s and u_b be,

$$u_s = -k_s y + r_s \tag{3.8}$$

$$u_b = k_b y + r_b \tag{3.9}$$

where r_s and r_b be the corresponding additional information considered by the traders. Any new information that might change the trader's perception of the fundamental value over a long term is represented by these information inputs. In the short term, this refers to the trader's valuation of the market. As y increases, the seller tends to reduce his offer price and buyer would like to increase his bid. The structural difference of u_b is due to this. The analysis will be given based on equilibrium values.

The equations for $\dot{p_s}$ and $\dot{p_b}$ can be written as

$$\dot{p_s} = (a_s - b_s k_s c)p_s + b_s k_s c p_b + b_s r_s \tag{3.10}$$

$$\dot{p_b} = b_b k_b c p_s + (a_b - b_b k_b c)p_b + b_b r_b \tag{3.11}$$

The equilibrium points p_1^* and p_2^* are given by the solutions of

$$(a_s - b_s k_s c)p_s^* + b_s k_s c p_b^* = -b_s r_s \tag{3.12}$$

$$b_b k_b c p_s^* + (a_b - b_b k_b c)p_b^* = -b_b r_b \tag{3.13}$$

Solving the above set of equations,

$$p_s^* = \frac{(b_b k_b c - a_b)b_s r_s + b_s k_s c b_b r_b}{a_s a_b - a_s b_b k_b c - a_b b_s k_s c} \tag{3.14}$$

$$p_b^* = \frac{b_b k_b c b_s r_s + (b_s k_s c - a_s)b_b r_b}{a_s a_b - a_s b_b k_b c - a_b b_s k_s c} \tag{3.15}$$

Suppose the seller gets positive information, which is advantageous to him, he would not like to reduce the price. On the other hand, he would expect a higher p_s^*. Similarly, good news for the buyer means he would like to settle at a lesser price than before, which can be achieved by reducing r_b. It can also be seen that increase in r_b will give an advantage to the seller and the buyer will welcome a decrease in r_s.

By substituting p_s^* and p_b^* in y^*, we get

$$y^* = \frac{c(a_s b_b r_b - a_b b_s r_s)}{a_s a_b - a_s b_b k_b c - a_b b_s k_s c} \tag{3.16}$$

Hence, arbitrage vanishes if

$$\frac{a_s}{b_s r_s} = \frac{a_b}{b_b r_b} \tag{3.17}$$

Fundamental risk is accounted by the change in reference input. If r_s increases with new information, then either r_b or b_b should also increase for *zero arbitrage.* The above expression explains no arbitrage condition in terms of the trader's profile. This is one of the distinct features of the proposed framework. *Noise trader risk* can be incorporated by different feedback gains in various time periods. It can also be noticed that $a_s b_b k_b c + a_b b_s k_s c \neq a_s a_b$. Otherwise, infinite arbitrage would be available. Therefore, the choice of k's should be such that this condition is satisfied.

The synchronization risk can be considered by introducing a delay element. The delay could exist in the following: (i) in deciding a strategy based on new information, (ii) observing the market condition, and (iii) time taken to implement the decisions. In the proposed model, these delays can be taken into account either by a delay element in the feedback loop or by a delay in the forward path.

§4 Simulation results

Simulations have been done using MATLAB tools. The values chosen are as follows: These values are considered arbitrarily to bring out the features of the proposed model and are not from the real market. The positive a's refer to a condition where expectation of a trader is rising with time and the negative ones refer to a situation where the corresponding expectation is bounded. The relative strength of these will decide the value of one with respect to the other and also their impact on the market. Similar interpretation should be considered for the other values also. The relative values are more significant than their absolute values.

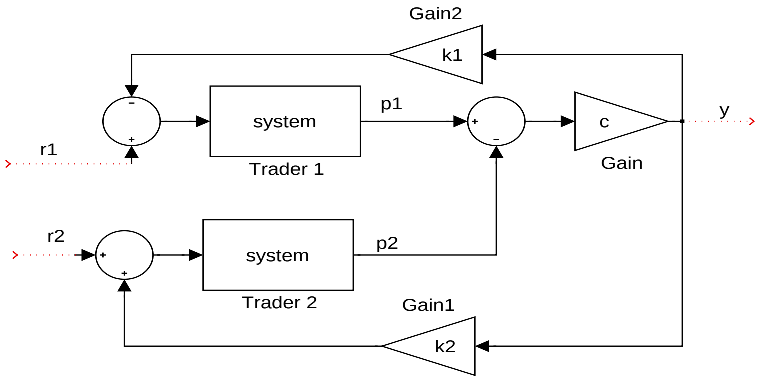

Figure 1: Block diagram of the system with no delay

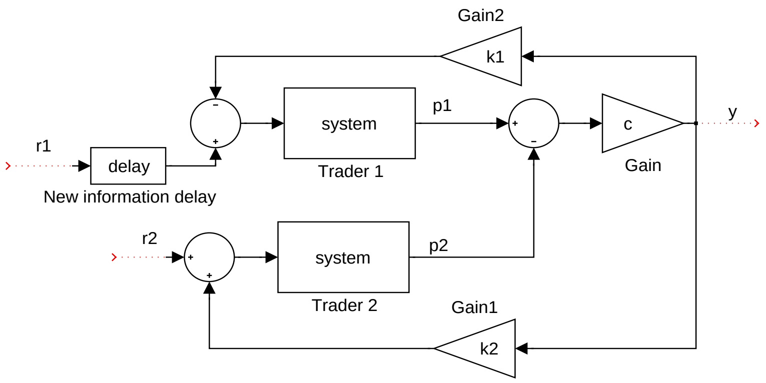

Figure 2: Block diagram of the system with delay in new information

Figure 1 gives the block diagram where delay has not been considered. Figure 5 shows the corresponding y. In Figure 2, the delay in the information input is considered. It can be shown that the response gets slower in the initial stage before settling. Therefore, the information delay makes the market reaction sluggish in a shorter time scale. Figure 3 shows the block diagram where delay in the feedback is taken into account and the corresponding difference in price is shown in Figure 6. The delay in forward path is considered in Figure 4. These delays bring oscillation that makes it longer to settle at equilibrium value. Moreover, oscillation is less pronounced in the case of delay in taking actions (forward path) compared to the delay in knowing (feedback path). The positive a's imply that the response grows in time. It can be made bounded only by external influences outside the system. The control loop is doing exactly that. The financial interpretation is that the trader's decision based

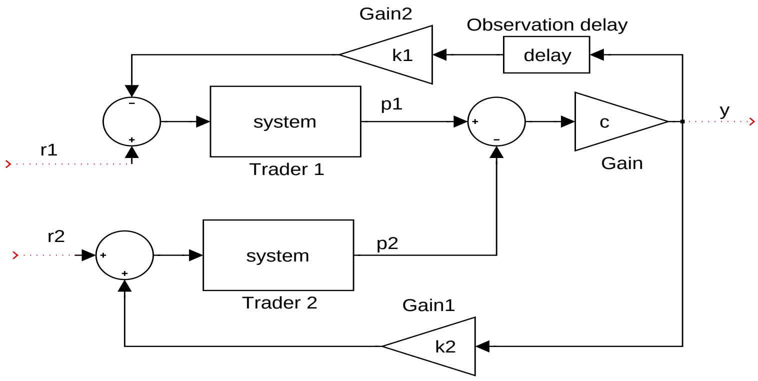

Figure 3: Block diagram of the system with observational delay

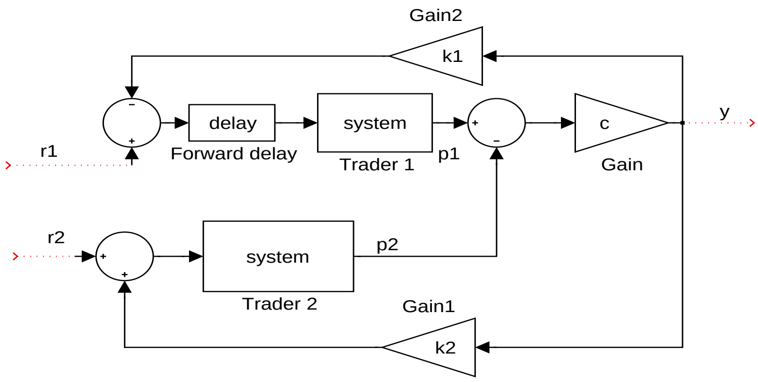

Figure 4: Block diagram for the system with delay in forward path

on observations put a limit to his unbounded expectations. His conception of growth rate could be wrong to start with. But, as he observes more and more, he would correct himself accordingly. The actual value of a refers to his expectation of growth rate. The negative a implies that the response is bounded even in the homogenous case. Further, input will affect only the steady state value. Here, the trader himself has a limited expectation and his observations would change only the exact value. Moreover, the ratio between the a's will measure the dominance each one has in the market behavior. The b's refer to the strength of the decision on his expectation. The higher the value, higher would be the impact of that decision/information on his behavior. Again, the ratios are more significant than the absolute values as this ratio determines the influence of the decisions on his final response. The r's refer to the trader's perception of the fundamental value. If these values are equal, the difference

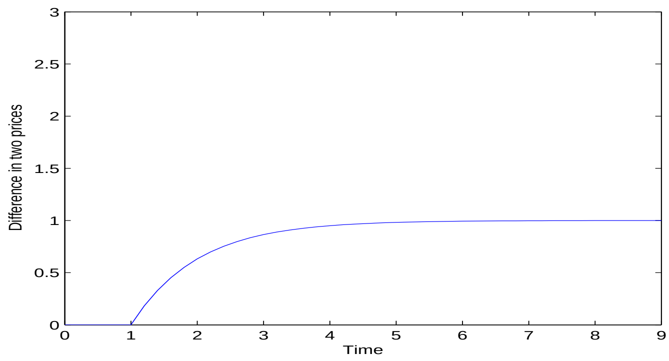

Figure 5: Difference in price for the system with no delay

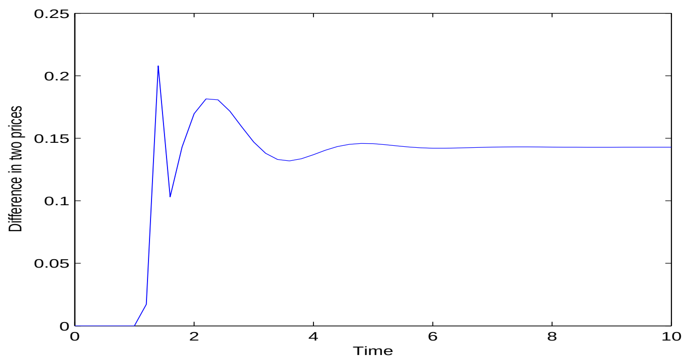

Figure 6: Difference in price for the system with observational delay

in their behavior comes from their strategies employed. The importance of these ratios can be seen in their explicit occurrence in the condition for no-arbitrage. The variation of k's will create different regions in the response, for instance, oscillation might set in. This can be inferred from the fact that they influence the stability and the nature of the response. The exact values of k's have to be obtained by trial and error.

§5 Herding

The characteristic feature of herding in a financial market is the similarity of a group of investors' decisions to buy or sell over a period of time. Institutional herding causes large price movements of individual stocks. An individual is a part of herd if he changes his investment decisions after observing others or inferring what others are doing from publicly available information. Herding tendency may manifest when the investors follow popular opinions and expert's advice. It can also occur when the traders attach more significance to the recent news than they deserve. Disposition effect might also lead to herding. The belief that other traders may know something more about the return on investment, and the conviction that their actions might reveal this information, along with the fear of *losing out* would lead to disposition effect. It is very important to differentiate the actual herding from other spurious herding. Spurious herding occurs when the investors face similar situations wherein the available choices are identical. The decisions might appear to be one of herds but they need not be. The similitude of the decisions by a group of independent traders has severe impact on the market price than that of their diverse decisions. This causes large fluctuations in the market price. The market price fluctuations would be less when they take diverse decisions. Hence, it is very important to study the herding effects to evaluate the risk while participating in the market.

The herding models [Ban93, Ban92, SBW92], *Information based herding and cascades*, are based on the simple idea that agents obtain useful information from observing the actions of other agents, and give more importance to such information ignoring completely their own private information. In such situations, the agents are said to be in an informational cascade. However, when agents are in a cascade, they also know that the cascade is based on little or no information. Therefore, any new arrival of public information or better informed agents or shifts in the underlying value of actions, could result in the disintegration of the cascade. Thus, fragility is a key characteristic of the above information based herding models. The herding models, known as *Information acquisition herding models*, have also been proposed in early nineties. The common theme in these models is that investors decide to follow the same set of stocks or same sources of information. The focus is on short-term horizon of investors, which leads to positive informational spillover. An informed investor wishing to liquidate his position in an asset stands to gain only if other people acting on the same information trade in that asset. The early-informed investors and late-informed have been considered. The early-informed investors trade aggressively in the initial period and reverse their position in the next period to reduce long-term risk, while the late-informed investors cause the price to reflect early-informed investors' information. The early-informed investors make greater profits as the number of late-informed investors increase. In [SJ90], *Principle agent models of herding* have been developed by Scharfstein and Stein. The models are based on the idea that when principals are uncertain of agents' ability to pick the right stocks, it makes sense for agents to mimic the decisions of other agents to preserve the principal's uncertainty about the agents' ability. Similarly, it has been demonstrated that an explicit relative performance clause written for the purpose of adverse selection might lead to herding. A compensation scheme which increases with money manager's performance might also lead to herding. Herding may also be caused by the institutional investors as they share aversion and preference to stocks with certain characteristics like liquidity, risks, and size.

Excessive volatility observed in financial markets is, in some studies, considered as the outcome of a mimic behavior. Scharfstein and Stein [SJ90] presented the evidence of herding in the behavior of fund managers. Welch [SBW92] showed evidence of herding in the forecasts made by financial consultants. Herding behavior is not necessarily *irrational* in the sense that it might be compatible with utility optimization. The important feature of the above models is individuals make their decisions one at a time, taking into account the effect of the decisions of the individuals preceding them. Individuals attempt to estimate from noisy observations and others' decisions sequentially. Non-sequential herding has also been studied in a Bayesian setting by Orlean [Orl95] where any two agents have the same tendency to imitate each other.

In this paper, we have attempted to model herding behavior as a manifestation of reduced order dynamics in a dynamical system framework.

The proposed model does not fall under the three class of herding models considered namely information based herding and cascade models, information acquisition herding models, and principle agent models. The fundamental difference between other models described and the proposed model is those three classes of herding models consider either information or picking up stocks as a key to study herding, whereas the proposed model considers the reduced order dynamics and adaptation in studying herding.

§6 Proposed Systems Approach

Consider a stock market system with n agents. The dynamics of the demand of agents (trader), x_k, at the time instant k is assumed to obey,

$$x_{k+1} = Ax_k + Bu_k \tag{6.18}$$

where $x_k, u_k \in M^{n\times 1}$, A and B are the parameter matrices associated with the traders, and u_k represents the information used by traders at the time instant k. The price formation in the market depends on the aggregate demand. These n agents behave as n_h ($n_h < n$) agents due to herding. This is represented by the transformation,

$$y_k = Cx_k \tag{6.19}$$

where $C \in M^{n_h \times n}$.

The herding phenomenon is translated into a problem of price formation considering the reduced order dynamics rather than the full-order system. Since we are interested in the condition where the possibility of estimating the full-order system information from the reduced order system exists, and the information lost due to herding i.e. reduced order dynamics, we use the well known technique of reduced order observers.

The observer dynamics is of the form [Lew92],

$$z_{k+1} = Fz_k + Gy_k + Hu_k \tag{6.20}$$

with $z_k \in M^{n_h \times n}$.

Let e_k be the estimating error,

$$e_k = z_k - Px_k \tag{6.21}$$

where $P \in M^{n_h \times n}$. The design of the reduced order system dynamics is to select the matrices F, G, and H so that e_k vanishes with time.

It is possible to write the error dynamics as follows if the equality $PA - FP = GC$ is satisfied [Lew92],

$$e_{k+1} = Fe_k \tag{6.22}$$

The error will go to zero with time as long as the matrix F is asymptotically stable. The time taken for the error to go to zero is the risk element associated if the reduced order system is used in the context of financial market. The observer design is dependent on solving the equality $PA - FP = GC$ for F, G, and P such that F is stable and the transformation, W, where W is the matrix obtained by appending C to P, used to obtain the state information from the reduced order system is non-singular. The necessary conditions for the existence of a P such that W is non-singular are: (A, C) is observable and (F, G) is reachable. But there is no guarantee that W is non-singular if these two conditions hold. Therefore, Lewis [Lew92] suggested the following procedure:

1. Choose a $(n - n_h) \times (n - n_h)$ matrix F with desirable eigen values for forcing e_k to go to zero asymptotically. The eigen values of F should be distinct from those of the matrix A with time constants about 5-10 times faster.

2. Choose G so that (F, G) is reachable.

3. Solve for P using $PA - FP = GC$.

4. Check W for full rank. If W is singular, select a different G and go to step-2, or select a different F and go to step-1.

In the above interpretation, herding is interpreted as the effect of reduced order dynamics of a system. The system theory framework offers another interpretation of herding which we will next explain, where a condition for herding can be written in terms of system parameters.

The first trader is represented by the equation,

$$\frac{dx_1}{dt} = -a_1x_1 + b_1u_1 \tag{6.23}$$

The second trader is represented as,

$$\frac{dx_2}{dt} = -a_2x_2 + b_2u_2 \tag{6.24}$$

In the above equations x_1 and x_2 are the trader's estimation of the market price y, u's are the information they use, and a's and b's are the parameters. The market dynamics is represented as,

$$\frac{dy}{dt} = -ay + bu \tag{6.25}$$

The parameters of the market are not known to the traders though they can observe the market price y. They may have to estimate or guess what could be the values of the parameters of the market so that they can reduce the error between their values and the actual value. What schemes they can follow to reduce the error $|y - x_1|$ and $|y - x_2|$? One scheme can be designed using MIT rule which is used in the field of adaptive control theory.

Using the MIT rule [AW95], the market price y can be written in terms of new parameters θ's, a differential operator p.

$$y = \frac{b\theta_1u_1 + b\theta_2u_2}{p + b\theta_3 + a} \tag{6.26}$$

Let,

$$e_1 = y - x_1 \tag{6.27}$$

$$e_2 = y - x_2 \tag{6.28}$$

Traders will modify their input information as follows:

$$\frac{\partial e_1}{\partial \theta_1} = \frac{bu_1}{p + b\theta_3 + a} \tag{6.29}$$

$$\frac{\partial e_2}{\partial \theta_2} = \frac{bu_2}{p + b\theta_3 + a} \tag{6.30}$$

$$\frac{\partial e_1}{\partial \theta_3} = \frac{-by}{p + b\theta_3 + a} \tag{6.31}$$

$$\frac{\partial e_2}{\partial \theta_3} = \frac{-by}{p + b\theta_3 + a} \tag{6.32}$$

These are known as sensitivity derivatives [AW95].

$$\frac{d\theta}{dt} = -c_1 e \frac{\partial e}{\partial \theta} \tag{6.33}$$

The traders cannot use the above dynamics directly as they do not know the knowledge of a and b. But we also observe that when $p + a + b + \theta_a ctual{=}p + a_i$, $i = 1$, and 2, perfect model following occurs. Therefore, the following approximations are used: $p + a + b\theta_3 \approx p + a_1$ (for the first trader), and $p + a + b\theta_3 \approx p + a_2$ (for the second trader).

Substituting the above expressions, we get (i) for the first trader,

$$\frac{d\theta_1}{dt} = -c_1 e_1 \frac{a_1 u_1}{p + a_1} \tag{6.34}$$

$$\frac{d\theta_3}{dt} = -c_1 e_1 \frac{a_1 y}{p + a_1} \tag{6.35}$$

(ii) for the second trader,

$$\frac{d\theta_2}{dt} = -c_2 e_2 \frac{a_2 u_2}{p + a_2} \tag{6.36}$$

$$\frac{d\theta_3}{dt} = -c_2 e_2 \frac{a_2 y}{p + a_2} \tag{6.37}$$

In the above equations, the parameters are also combined in c_1 and c_2. When the traders are trying to reduce the deviation of their model outputs from the market output, it is possible that they might end up matching their outputs with each other rather than with the market. That is, though they are trying to reduce $|y - y_1|$ and $|y - y_2|$, they might actually end up doing the following: $|y_1 - y_2| < |y - y_1|$ and

$|y_1 - y_2| < |y - y_2|$. This is interpreted as herding tendency. The individual traders may not be aware of this as from their point of view they are just trying to match their model output to that of the market. The condition under which the second trader end up matching his model with that of the first trader is,

$$\frac{c_2 a_2 u_2 y}{p + a_2} \approx \frac{c_2 a_2 u_2 y_1}{p + a_2} \tag{6.38}$$

and, when $|y_2| < |y|$,

$$\frac{c_2 a_2 y^2}{p + a_2} \approx \frac{c_2 a_2 y_1^2}{p + a_2} \tag{6.39}$$

If these conditions exist during the evolution of the system, then we consider them as the emergency of the herding tendency. How long this tendency can last is dependent of the sustenance of these conditions. When a trader begins to understand the market, it might happen there could be a large error in his judgments. In that case, he tends to follow others than taking the risk of learning everything on his own provided he knows what are doing. If he does not know, he might as well try to infer that. The second herding condition can be interpreted as something that is connected to this inference. We cannot say more as this inference process is not explicitly taken into account in our model.

§7 Conclusions

A system model to capture the limits to arbitrage is proposed. It is explained that the concepts from the earlier models can be incorporated in the new setup. The impact of the information on the arbitrage can be studied in the proposed model. The explicit condition for no arbitrage is determined using the trader's profile. The ways in which different delays can be implemented in the new framework have been described. The question of utilizing this model to make investment decisions in the real market needs to be addressed. We have also attempted to model herding through an observer interpretation in the dynamical system framework. Herding is interpreted as manifestation of reduced order dynamics of a given system. The risk associated is related to the asymptotic behavior of the error signal. Since an individual cannot guess the asymptotic behavior of the system, he can only choose a risk profile that is acceptable and calculate the risk one is taking by comparing with the asymptotic ones.

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