

Is There a Latent Factor in Stock Returns?

Dr. Hayette Gatfaoui

Associate Professor, Rouen School of Management,
Economics & Finance Department, 1 Rue du Maréchal Juin,
BP 215, 76825 Mont-Saint-Aignan Cedex France;
Phone: 00 33 2 32 82 58 21, Fax: 00 33 2 32 82 58 33;
hayette.gatfaoui@groupe-esc-rouen.fr

March 2006

Abstract

The measurement problems encountered while trying to exhibit the influence of market risk factor on asset returns may be numerous. It seems then difficult to highlight the unique common latent factor underlying stock return evolutions in the market. So far, excess return relationships are mainly and broadly considered. Moreover, basic and common studies require a market factor proxy (i.e., market portfolio benchmark). The chosen proxy usually impacts related results (see Roll [1977]). To bypass such problems, we resort to Kalman filtering methodology to exhibit the common latent factor underlying stock market returns. Of course, when this one exists...

Keywords: CAPM, idiosyncratic risk, Kalman filter, market risk, stock returns, systematic risk.

JEL Codes: C32, D8.

1 Introduction

The stock return puzzle has a long story. Indeed, many authors attempted to explain the global evolution of stock returns. Formerly, Sharpe

(1963,1964), among others, established that stock returns depend on both a market factor as well as an idiosyncratic factor of risk. Such a dependence is usually described by some linear-type relationship. Later, the influence of market variables (e.g., risk free rate, term spread, and yield curve slope) as well as idiosyncratic factors on stock returns is exhibited (see Fama & French [1989,1992], Campbell [1987], Harvey [1989], Breen *et al.* [1989], and Ferson & Harvey [1981,1999]). Specifically, Banz (1981), Berk (1995) and Kothari *et al.* (1995) show the importance of firm size whereas Bhandari (1988) underlines the leverage effect on asset returns. Chan *et al.* (1991) explain stock returns with book-to-market features while Merton (1987) and Amihud & Mendelson (1989) exhibit the informational impact (i.e., news arrival in the market) on stock returns and related liquidity. Recently, Malkiel & Xu (2002,2003) focus on idiosyncratic risk and idiosyncratic volatility in asset returns. In the same line, Campbell *et al.* (2001) exhibit the importance and significance of idiosyncratic risk in asset returns. They find that though idiosyncratic volatility has highly grown over time, stock return global volatility remains driven by market volatility (i.e., global common trend).

However, such linear relationship between stock returns and both market factors as well as idiosyncratic factors suffers from many measurement problems (e.g., heteroskedasticity and autocorrelation; see Fama [1965,1976], Blattberg & Gonedes [1974], and Affleck-Graves & McDonald [1989] among others) leading to a biased explanation of stock return evolution. To solve such problems, some authors resort to specific econometric tools or methods. For example, Shanken (1992) and Jagannathan & Wang (1996) propose a GMM methodology solving the error-in-variables problem. Differently, Ahn & Gadarowski (2000) propose an estimation method, which is robust to conditional heteroskedasticity as well as autocorrelations in asset returns. Recently, Barnes & Hughes (2002) propose a quantile regression methodology (see Buchinsky [1998]), which is robust to error-in-variables bias, omitted variables bias, sensitivity to outliers, and non-normal error distributions. Those authors find results that lead to a rejection of both the unconditional single-factor CAPM and the conditional multi-factor CAPM. More recently, Koutmos & Knif (2002) use conditional time-varying distributions (i.e., GARCH modeling) to assess the influence of systematic risk on stock returns. They consider given market stock indices, and exhibit stationary mean-reverting beta CAPM parameters with a four-day persistence degree. Differently, Gençay *et al.* (2003) use multiscaling wavelet techniques to estimate stock return beta parameters while using the S&P 500 index as

a market portfolio (i.e., systematic risk factor proxy).

Given existing literature, it seems sometimes hard to exhibit the existence of one significant common latent factor in asset returns. Moreover, a market index is always required to proxy the actual market factor of risk. The quality of the chosen market benchmark impacts the accuracy as well as quality or reliability of related measurements (see Roll [1977]). To bypass such problems and open questions, we resort to a robust econometric method to exhibit the latent factor of risk common to any asset in the market. For this purpose, we employ a Kalman filtering methodology, which allows to leave the market factor of risk undetermined (i.e., endogenous to the estimation process). Our paper is then organized as follows. Section 2 introduces the Kalman filter and related EM estimation. Section 3 introduces the data under consideration as well as their statistical properties while section 4 employs Kalman econometric method under our financial framework. Specifically, we consider both US and French data samples. Further investigation is undertaken in section 5 while investigating a common component in both French and US common latent factors. Finally, section 6 draws some concluding remarks and open points for future research.

2 Econometric framework

We expose therein the usefulness of the chosen econometric framework, namely Kalman filter, given our working setting as well as related advantages. Then, we introduce the general econometric estimation process.

2.1 Principle and motivations

The Kalman filter (see Kalman [1960], Harvey [1989a,b], Meinhold & Singpurwalla [1983], Brown & Hwang [1997], Wikle & Cressie [1999], and Cressie & Wikle [2002]) is commonly employed for short term forecasting as well as time series analysis or estimation. This simple econometric method is known to be optimal (i.e., unbiased and minimum error variance algorithm) and robust. The principle is to establish the state, or equivalently, linear dynamic of a given system, and to link such a dynamic to available or observed information about the system at each point of time. Solving such a dependence structure depends on the initial state of the system (i.e., detailed or accurate information about the initial state of the system is required).

Kalman filter is a state-space model describing a system's state as well as its evolution over time. Incidentally, a state-space representation allows for incorporating unobserved variables (i.e., state variables), which are estimated with the observable model (i.e., observed variables or measures). Specifically, Kalman filter is a recursive linear predictor-corrector filter, which minimizes the expected square error between the system's state and corresponding estimate(s) (i.e., quadratic minimization algorithm). For Gaussian random variables, Kalman filter represents the optimal linear predictor and estimator. For non-Gaussian variables, Kalman filter estimator is the best estimator among the linear estimator class. The main interest of this econometric methodology is its ability to forecast a system's state through past, present, and future. In general, observed measures are functions of state variables (i.e., state of the system) insofar as measures are disturbed by a random noise called measurement noise. Hence, Kalman filter attempts to estimate state variables given disturbed observations about the system. Such a forecasting process relies on two sets of equations. The first set of equations is time-updating, and forecasts the system's current state as well as the related error covariance matrix over the next time step. The second set of equations is measure-updating, and corrects the errors committed in the first set of equations (see Chui & Chen [1987]). For this purpose, second order moments of equation noises (i.e., variances of both state and measurement noises) are required.

Finally, Kalman filtering methodology exhibits five advantages (see Lemoine & Pelgrin [2003] among others). First, measure uncertainty is recursively taken into account. Second, ex ante information is taken into account when this one exists. Third, this econometric method can be applied to stationary as well as non-stationary data. Fourth, state and measurement noises can be non-Gaussian. Finally, time-varying estimates are enabled.

In this paper, we use Kalman methodology for filtering purposes. Namely, we look for the best proxy of the current system's state given past and present observations.

2.2 General framework

We introduce here the general modeling framework where state variables are assumed to follow a first order Markovian process. Namely, we consider

following linear measurement and state equations:¹

$$Y_t = Z_t X_t + D_t + \varepsilon_t \quad (1)$$

$$X_t = A_t X_{t-1} + C_t + R_t \eta_t \quad (2)$$

where Y_t is a $N \times 1$ vector of observations (i.e., measure variables); Z_t is a $N \times k$ measurement sensitivity matrix; X_t is a $k \times 1$ vector of state variables;² D_t is a $N \times 1$ vector related to exogenous known variables; ε_t is a $N \times 1$ measurement white noise; A_t is a $k \times k$ state transition matrix; C_t is a $k \times 1$ vector related to exogenous known variables; R_t is a $k \times g$ matrix; η_t is a $g \times 1$ state white noise; and finally t is current time ranging from 1 to T (i.e., multivariate time series with T observations).

We first assume that state and measurement noises follow normal distributions and are independent. Second, initial values of state variables and white noises are independent³ (i.e., causal and invertible state-space model) while initial values of state variables follow a normal law. Namely, we consider:⁴

$$\begin{pmatrix} \eta_t \\ \varepsilon_t \end{pmatrix} \sim \mathcal{N} \left(\begin{bmatrix} 0 \\ 0 \end{bmatrix}, \begin{bmatrix} Q_t & \mathbf{0} \\ \mathbf{0} & H_t \end{bmatrix} \right) \quad (3)$$

$$X_0 \sim \mathcal{N}(m_0, P_0) \quad (4)$$

where m_0 and P_0 are known expectation and covariance matrix parameters of dimensions $k \times 1$ and $k \times k$ respectively. The recursive nature of Kalman filter implies that state and measure variables are functions of the initial system's state, past state errors, past measurement errors, and exogenous variables. Hence, Kalman principle is to estimate state variables at each time t conditional on observed variables (i.e., measure variables) until time t . Specifically, minimizing realized square errors on state variables requires five steps in the estimation process. These five steps are divided into an updating and a forecasting stage as follows:

$$E_{t-1}[X_t] = A_t E_{t-1}[X_{t-1}] + C_t \quad (5)$$

¹For any given observed variable, there exist several possible state-space representations.

² $Z_t X_t$ is considered as a signal at current time t .

³Namely, we assume that $E[\varepsilon_t \eta_t'] = E[\varepsilon_t X_0'] = E[\eta_t X_0'] = 0_{Dim}$ where Dim is $N \times g$, $N \times k$, and $g \times k$ respectively.

⁴At initial time $t_0 = 0$, hidden variables X_0 (i.e., latent common factors, or equivalently, unobserved factors) are Gaussian.

$$Var_{t-1}[X_t] = A_t P_{t-1} A_t' + R_t Q_t R_t' \quad (6)$$

$$E_{t-1}[Y_t] = Z_t E_{t-1}[X_t] + D_t \quad (7)$$

$$v_t = Y_t - E_{t-1}[Y_t] \quad (8)$$

$$F_t = Z_t Var_{t-1}[X_t] Z_t' + H_t \quad (9)$$

$$E_t[X_t] = E_{t-1}[X_t] + K_t v_t \quad (10)$$

$$P_t = (\mathbf{I}_k - K_t Z_t) \times Var_{t-1}[X_t] \quad (11)$$

$$K_t = Var_{t-1}[X_t] Z_t' F_t^{-1} \quad (12)$$

where $E_t[\cdot]$ and $Var_t[\cdot]$ are expectation and covariance operators conditional on available information set at time t ; $P_t = Var_t[X_t]$ is the mean quadratic error on Z_t ; K_t is the Kalman gain matrix; $\mathbf{I}_k$ is the identity matrix of dimension k ; Z_t' is the transposition of matrix Z_t ; F_t is the covariance matrix of v_t ; F_t^{-1} is the inverse matrix of F_t ; and v_t is an innovation process. Notice that $E_{t-1}[X_t]$ and $Var_{t-1}[X_t]$ are the best estimates of X_t and P_t conditional on available information set at time $t - 1$. Analogously, $E_t[X_t]$ is an optimal estimate of X_t given available information and observations at time t . Moreover, relations (11) and (6) are covariance matrix equations, namely Ricatti equations allowing for the computation of Kalman gain series. Relations (10) and (11) deal with state estimate and related covariance matrix updating. Relations (5) and (6) concern forecasting (i.e., time updating). Relation (12) is the gain matrix update; incorporating this matrix in relation (11) increases the estimation accuracy of $E_t[X_t]$ relative to $E_{t-1}[X_t]$. Indeed, the state error covariance matrix Q_t represents a state uncertainty estimate. By the way, $Var_{t-1}[X_t]$ is an ex ante covariance matrix while $Var_t[X_t]$ is an ex post covariance matrix. And, $Var_{t-1}[X_t]$ is the mean quadratic error of forecast estimate $E_{t-1}[X_t]$.

Kalman filter requires to specify starting values for state variables (i.e., initial guess) and to replace unknown matrices with their estimates. Given starting values, unknown matrix parameters are estimated while maximizing Y_t log-likelihood. For this purpose, we assume that Y_t follows a multivariate Gaussian distribution conditional on X_t as well as past values of both X_t and Y_t . Specifically, the log-likelihood under normality assumptions writes:

$$\ell_t = -\frac{N}{2} \ln(2\pi) - \frac{1}{2} \ln|F_t| - \frac{1}{2} v_t' F_t^{-1} v_t \quad (13)$$

where $|F_t|$ is the determinant of matrix F_t . General setting leads to a non stationary (i.e., time-varying) estimation framework whereas we get a stationary case when Z_t , D_t , H_t , A_t , C_t , R_t , and Q_t do not depend on time.

Table 1: Asset denomination

France	USA
ACCOR	AT & T
ALCATEL ALSTOM	DJIA
AXA	DOW JONES
BOUYGUES	FORD MOTOR
L'OREAL	INTL. BUS. MACH. (IBM)
MICHELIN	MERRILL LYNCH
PEUGEOT SA	MICRON TECH.
SBF120	MICROSOFT
TOTAL FINA ELF SA	WALT DISNEY

3 Data and properties

We introduce the data we consider as well as a preliminary statistical analysis.

3.1 Data sets

We consider two different data sets allowing for two different country analyses, namely French and US financial markets. The first set concerns 8 French stock prices and one French stock index price ranging from 01/02/1997 to 07/12/2001, namely 1139 observations per series (see table 1 where DJIA is the Dow Jones Average Industrial index and SBF120 is a diversified French stock index). The second set concerns 8 US stock prices and one US stock index price ranging from 01/02/1997 to 07/12/2001, namely 1142 observations per series. We also consider the global set of 18 asset prices also ranging from 01/02/1997 to 07/12/2001, namely 1111 observations per series after adjusting for non-working day differences. All considered stock prices are daily closing prices.

As we are interested in the common latent component underlying asset return evolutions, we compute asset returns on a continuous basis as follows:

$$R_t = \ln \left(\frac{S_t}{S_{t-1}} \right) \approx \frac{S_t - S_{t-1}}{S_{t-1}} \quad (14)$$

where S_t is the asset price at time t . Hence, we consider 1138 return observations for French assets, 1141 return observations for US assets, and finally

1110 return observations for the merged global data set.

3.2 Statistical profiles

We consider all asset returns on a percentage basis. Our three return data sets exhibit some key statistical features, which are displayed in tables 2, 3 and 17⁵.

Roughly, speaking French and US stock returns are far from being normally distributed. Those returns exhibit both asymmetry and skewness features (i.e., skewed probability distributions). Related probability distributions exhibit fatter tails than Gaussian ones (see related skewness and positive excess kurtosis). Moreover, their positive kurtosis profile is quite very heterogeneous among stock returns. The most volatile French stock returns (i.e., in terms of distance between extreme values, or equivalently, minimum and maximum values) are Alcatel, Axa and Total ones whereas the most volatile US stock returns are Ford Motor and IBM ones. Same conclusions apply to table 17, except that IBM is no more a highly volatile stock return whereas Dow Jones stock return becomes very volatile. Moreover, Kendall's correlation coefficients between asset returns for each financial market exhibit a strong positive link (see tables 4 and 5). Indeed, considering each market separately, the obtained non-parametric correlation coefficients are significant at a 1% bilateral test level. Such a feature underlines the existence of some common trend in stock return evolutions whatever the financial market under consideration.

4 Econometric study

We expose and explain the relevant version of Kalman filtering given our framework as well as related econometric results.

4.1 Model

We only observe stock return data whereas each stock return is driven by both a common latent risk factor as well as an idiosyncratic risk factor. Hence, each stock evolution depends on two unobserved variables (see

⁵This table is exposed in the appendix.

Table 2: French asset return statistics

i	Mean	Std. Dev.	Skewness	Excess kurtosis	Min.	Max.	Median	1st quartile*	3rd quartile*
Accor	2	0.0909	2.4188	-0.0983	-14.7809	10.7692	0.0617	-1.3622	1.4590
Alcatel	3	0.0704	3.6228	-1.9734	-48.4564	14.4352	0.0000	-1.7712	2.0603
Axa	4	0.1537	2.4601	5.3827	-9.9142	45.1151	0.1328	-1.0800	1.2579
Bouygues	5	0.1642	2.9474	0.0745	-17.1909	14.0123	0.0000	-1.4303	1.7247
L'Oréal	6	0.0909	2.4025	0.0530	-10.0285	9.2622	0.0000	-1.4438	1.5263
Michelin	7	0.0004	2.3254	-0.0588	-11.3445	11.8360	0.0000	-1.2826	1.2242
Peugeot	8	0.1106	2.2944	-0.2363	-16.3259	10.4635	0.0000	-1.1462	1.3547
SBF120	1	0.0671	1.3292	-0.2569	-5.3336	5.9459	0.0861	-0.6712	0.9181
Total	9	0.1813	3.1092	10.3303	-13.1709	70.5330	0.1002	-1.3007	1.5753

* Upper bound of the quartile.

Table 3: US asset return statistics

i	Mean	Std. Dev.	Skewness	Excess kurtosis	Min.	Max.	Median	1st quartile*	3rd quartile*
AT & T	6	0.0096	2.8911	0.2564	-23.2620	22.1301	0.0000	-1.6201	1.4453
DJIA	1	0.0209	2.4023	-0.0922	-16.9523	14.2029	0.0000	-1.3585	1.2820
Dow Jones	8	0.0426	1.2074	-0.3804	-7.4549	4.8605	0.0650	-0.6428	0.7991
Ford Motor	9	0.0145	1.8777	-1.4286	-21.4531	8.7601	0.0000	-0.9509	0.9558
IBM	2	0.0142	2.6071	-2.7651	-38.6561	10.6264	0.0000	-1.2807	1.3866
Merrill Lynch	7	0.0749	2.5578	-0.1300	-16.8916	12.3665	0.0936	-1.4402	1.4534
Micron Tech.	4	0.1081	3.1734	0.1772	-12.2978	14.0477	0.0000	-1.8919	2.0379
Microsoft	5	0.1180	4.8077	0.1636	-19.1160	21.7202	0.0000	-3.0511	3.0750
Walt Disney	3	0.1204	2.7225	-0.1585	-16.9577	17.8692	0.0290	-1.4069	1.6963

* Upper bound of the quartile.

Table 4: Kendall's correlation matrix for French asset returns

i	2	3	4	5	6	7	8	1	9
Accor	1.0000	0.2042	0.1961	0.1128	0.1802	0.1790	0.1965	0.3012	0.1731
Alcatel		1.0000	0.2576	0.2684	0.2296	0.1809	0.1776	0.5007	0.1795
Axa			1.0000	0.1464	0.2965	0.2229	0.2136	0.4364	0.1838
Bouygues				1.0000	0.1423	0.0867	0.1426	0.3304	0.1225
L'Oréal					1.0000	0.2127	0.2038	0.4302	0.1779
Michelin						1.0000	0.1971	0.2886	0.1396
Peugeot							1.0000	0.3059	0.1772
SBF120								1.0000	0.3424
Total									1.0000

Table 5: Kendall's correlation matrix for US asset returns

i	6	1	8	9	2	7	4	5	3
AT & T	1.0000	0.2509	0.1602	0.1167	0.1539	0.1942	0.1189	0.1588	0.1528
DJIA		1.0000	0.2702	0.3273	0.3686	0.4232	0.1831	0.3381	0.3056
Dow Jones			1.0000	0.1570	0.1280	0.1974	0.0849	0.1247	0.1448
Ford Motor				1.0000	0.1259	0.1997	0.0785	0.1648	0.1167
IBM					1.0000	0.2167	0.2210	0.2864	0.1496
Merrill Lynch						1.0000	0.1529	0.2743	0.2382
Micron Tech.							1.0000	0.2365	0.0986
Microsoft								1.0000	0.1765
Walt Disney									1.0000

Sharpe [1963,1964] for example). Consequently, we employ Kalman statistical method to describe the dynamics of both latent and idiosyncratic factors insofar as we have incomplete knowledge about the relevant phenomenon underlying those dynamics (i.e., hidden statistical variables). Incidentally, we do not intend to test the CAPM relationship here. We rather attempt to exhibit some common trend inherent to each financial asset's evolution in the market. We address the following question: Do we have a common component in asset return changes? This way, we are able to characterize some market risk structure...

For each stock return i , we set the following dependence structure:

$$R_t^i = \beta^i M_t + e_t^i \quad (15)$$

$$M_t = c_M M_{t-1} + w_t \quad (16)$$

where e_t^i , and w_t are independent Gaussian white noises such that e_t^i incorporates the related idiosyncratic risk factor of asset return R_t^i ; and M_t is the market factor of risk underlying any stock return R_t^i . The only observed variables are asset returns R_t^i where $i \in \{1, \dots, N\}$ with N being 9, 9 and 18 respectively for French, US, and global merged asset data samples. Moreover, $t \in \{1, \dots, T\}$ with T being 1138, 1141 and 1110 respectively for French, US, and global merged data sets. Such a framework can easily be translated into a state-space representation. Namely, the previous system of equations rewrites:

$$\begin{pmatrix} R_t^1 \\ \vdots \\ R_t^N \end{pmatrix} = \begin{pmatrix} \beta^1 \\ \vdots \\ \beta^N \end{pmatrix} \cdot M_t + \begin{pmatrix} e_t^1 \\ \vdots \\ e_t^N \end{pmatrix} \quad (17)$$

$$M_t = c_M M_{t-1} + w_t \quad (18)$$

where relation (17) corresponds to measurement equation (1) and relation (18) corresponds to state equation (2). Hence, we get $Y_t = [R_t^1 \ \dots \ R_t^N]'$, $X_t = M_t$, $D_t = \mathbf{0}$, $\varepsilon_t = [e_t^1 \ \dots \ e_t^N]'$, $k = 1$, $C_t = \mathbf{0}$, $R_t = I_k = 1$, $\eta_t = w_t$, $g = k$, $Z_t = [\beta^1 \ \dots \ \beta^N]'$, and $A_t = c_M$. To sum up, we consider the following linear stat-space model:

$$Y_t = Z_t X_t + \varepsilon_t \quad (19)$$

$$X_t = A_t X_{t-1} + \eta_t \quad (20)$$

We assume that initial conditions m_0 and P_0 about the system are unknown, and elements of Q_t do not depend on P_t . Moreover, we state $Q_t = \sigma_M^2$ such that $P_0 \neq Q_t$, and H_t is a $N \times N$ diagonal covariance matrix as follows:

$$H_t = \begin{pmatrix} \sigma_1^2 & 0 & \cdots & \cdots & 0 \\ 0 & \sigma_2^2 & 0 & 0 & \vdots \\ \vdots & 0 & \ddots & 0 & \vdots \\ \vdots & 0 & 0 & \sigma_{N-1}^2 & 0 \\ 0 & \cdots & 0 & 0 & \sigma_N^2 \end{pmatrix} \quad (21)$$

Therefore, our specification requires to estimate $\beta^1, \dots, \beta^N$ (i.e., measurement equation), M_0 (i.e., the initial state of the system X_0), H_t (i.e., a diagonal⁶ covariance matrix composed of N elements), P_0 , c_M and σ_M (i.e., 1 element of covariance matrix Q_t). Hence, our linear system requires to estimate $2N + 4$ parameters.

4.2 Econometric results

We achieve our state-space model estimation for both French and US assets while employing a Broyden-Fletcher-Goldfarb-Shanno-type optimization method⁷ (i.e., for log-likelihood maximization). The estimates we get are displayed in tables 6 and 7; and we find $P_t^{France} = 4.5128 \times 10^{-12}$ and $P_t^{US} = 0.2113$ whatever time t . Moreover, the accuracy level we set to compute relative gradients is 10^{-6} .

For both markets, the variance of the common latent component⁸ appears to be significant in our state-space formulation. Strikingly, the common latent factor's coefficient c_M is positive and significant on the French market whereas it appears to be negative and insignificant on the US market. By the way, the starting value of the common latent factor is significant only for the US market. However, H_t covariance matrix⁹ as well as beta coefficients are generally significant for the two financial markets under consideration. Recall that beta coefficients represent the impact of the common latent factor

⁶We explicitly assume that stock returns are only correlated through their common latent component.

⁷Non-linear maximization problem.

⁸This one is assumed to remain constant over time.

⁹This covariance matrix is also assumed to be constant over time.

Table 6: Kalman estimates for French asset returns

Parameters	Estimate	Gradient	Std. Dev.	T-Student
β^1	1.5677	0.0003	0.3262	4.8056
β^2	1.3508	0.0007	0.2905	4.6500
β^3	2.7563	0.0020	0.5785	4.7649
β^4	1.4940	0.0006	0.3218	4.6431
β^5	1.7118	-0.0010	0.3549	4.8230
β^6	1.7152	-0.0003	0.3615	4.7449
β^7	1.1700	0.0006	0.2523	4.6374
β^8	1.3099	-0.0014	0.2804	4.6724
β^9	1.4182	-0.0010	0.3070	4.6197
σ_1	0.0000	0.0011	0.0106	0.0003
σ_2	2.1317	0.0096	0.0447	47.7151
σ_3	2.7667	0.0022	0.0578	47.8603
σ_4	2.1136	-0.0036	0.0443	47.7081
σ_5	2.5695	-0.0011	0.0538	47.7305
σ_6	1.9131	-0.0006	0.0401	47.6949
σ_7	2.1026	-0.0099	0.0441	47.7049
σ_8	2.0100	0.0020	0.0421	47.7183
σ_9	2.8723	0.0003	0.0602	47.6959
Q_t	0.8465	0.0000	0.1776	4.7655
P_0	0.6548	0.0048	1.5079	0.4342
M_0	8.9016	0.0000	11.3236	0.7861
c_M	0.0725	0.0001	0.0128	-5.6475

Table 7: Kalman estimates for US asset returns

Parameters	Estimate	Gradient	Std. Dev.	T-Student
β^1	0.7864	0.0030	0.1038	7.5742
β^2	1.0414	-0.0039	0.1395	7.4654
β^3	0.7811	0.0072	0.1095	7.1313
β^4	1.1960	0.0012	0.1822	6.5654
β^5	1.0495	-0.0075	0.1413	7.4269
β^6	0.8219	0.0011	0.1179	6.9742
β^7	1.4912	-0.0085	0.1973	7.5588
β^8	0.5277	0.0108	0.0769	6.8653
β^9	0.7915	0.0058	0.1113	7.1081
σ_1	0.4642	-0.0041	0.0326	14.2237
σ_2	2.0897	0.0028	0.0467	44.7560
σ_3	2.1318	-0.0018	0.0462	46.1350
σ_4	4.5001	-0.0002	0.0966	46.6022
σ_5	2.2828	0.0087	0.0514	44.4024
σ_6	2.6457	0.0004	0.0568	46.5429
σ_7	2.3685	0.0012	0.0560	42.2666
σ_8	1.7222	0.0075	0.0369	46.6931
σ_9	2.3531	-0.0056	0.0506	46.4712
Q_t	1.4182	-0.0062	0.1862	7.6187
P_0	0.9001	0.0000	2.8521	0.3156
M_0	0.9004	0.0054	0.0524	17.1845
c_M	-0.0057	-0.0029	0.0093	-0.6142

Table 8: Statistics for common latent factors in asset returns

	France	USA
Mean	0.0428	0.0513
Std. Dev.	0.8482	0.0397
Skewness	-0.2569	-0.3396
Excess kurtosis	1.2001	3.2187
Min.	-3.4021	-8.1372
Max.	3.7927	5.5226
Median	0.0549	0.0659
1st quartile*	-0.4300	-0.7245
3rd quartile*	0.5862	0.8474

* Upper bound of the quartile.

on asset returns. Considering both financial markets, beta coefficients are all positive, which indicates that asset returns are market driven. Moreover, these coefficients are above unity for all French asset returns as well as IBM, Micron Tech., Microsoft and Merrill Lynch asset returns, those assets magnifying therefore market fluctuations. Moreover, Alcatel stock return amplifies nearly three times market fluctuations. Differently, all the remaining asset returns exhibit beta coefficients below unity, absorbing then market impact. We also get the following statistical profile for the common latent factor M_t inherent to each financial market under consideration (see tables 8 and 9).

Both latent common factors exhibit asymmetric (i.e., leptokurtic) as well as non-normal features. However, we notice structural differences between those two market factors. Indeed, the US market factor return exhibits fatter tails as well as bigger variation bounds (i.e., extreme values) than the French one.

As expected, non-parametric correlation coefficients between common latent factors and related asset returns are positive. Strikingly, we notice a perfect correlation between the French market factor return and SBF120 stock index return. Analogously, the US market factor return is highly correlated with the DJIA return. At a first glance, we can conclude that SBF120 French stock index captures the common latent component inherent to the French financial market in terms of market risk changes (see Gatfaoui [2006]). Differently, in spite of the high previous correlation coefficients, the DJIA US stock index does not capture the whole of market risk changes that are peculiar

Table 9: Correlations of common latent factors with asset returns

i	France		USA	
	Kendall's tau	Spearman's rho	Kendall's tau	Spearman's rho
1	1.0000	1.0000	0.8543	0.9690
2	0.3024	0.4320	0.4218	0.5892
3	0.5017	0.6830	0.3438	0.4878
4	0.4347	0.6046	0.2336	0.3370
5	0.3322	0.4724	0.4087	0.5758
6	0.4293	0.5905	0.2933	0.4212
7	0.2832	0.4100	0.4977	0.6828
8	0.3093	0.4451	0.2962	0.4286
9	0.3450	0.4895	0.3333	0.4730

to the US financial market. Moreover, we attempt to assess the efficiency of the systematic risk factor while explaining stock return evolutions. For this purpose, we realize a set of regressions that are introduced in the appendix (see tables 18 and 19). Results exhibit the general inefficiency of the US systematic risk factor except for AT & T stock return. Namely, the systematic risk factor encompasses the whole information describing the evolution of AT & T stock return. However, such a market factor is generally insufficient to explain the whole evolution of US stock returns, underlining then the significance of the idiosyncratic (i.e., unsystematic) risk component (see Campbell *et al.* [2001]). Analogously, the French market risk factor is inefficient, and fails to explain the whole evolution of French asset returns. Further investigation while comparing French and US financial markets requires to consider all asset returns on the same date scale. Such an investigation is undertaken in the next section.

5 Further investigation

We consider returns on a percentage basis, and on the same date scale (i.e., after adjusting for non-working days¹⁰ in both French and US countries). First, we display all the statistic profiles on the new time scale. Second, we attempt to extract some common component from the latent common factors

¹⁰Some working days in France may correspond to non-working days in USA, and conversely.

Table 10: Kalman estimates for US asset returns

Parameters	Estimate	Gradient	Std. Dev.	T-Student
β^1	0.6019	-0.0001	0.0466	12.9205
β^2	0.7909	-0.0024	0.0634	12.4727
β^3	0.5944	-0.0096	0.0533	11.1424
β^4	0.9268	0.0009	0.0937	9.8957
β^5	0.7791	0.0091	0.0472	16.4934
β^6	0.6180	0.0116	0.0367	16.8373
β^7	1.1459	-0.0070	0.0848	13.5077
β^8	0.4012	0.0010	0.0405	9.9033
β^9	0.6117	-0.0002	0.0592	10.3301
σ_1	0.4601	0.0008	0.0345	13.3241
σ_2	2.1203	-0.0020	0.0478	44.3131
σ_3	2.1634	-0.0008	0.0476	45.4959
σ_4	2.2991	0.0010	0.0994	45.9180
σ_5	4.5650	0.0048	0.0523	43.9870
σ_6	2.6583	-0.0028	0.0580	45.8704
σ_7	2.3959	0.0028	0.0576	41.6293
σ_8	1.7401	-0.0048	0.0377	46.1082
σ_9	2.3793	0.0035	0.0521	45.6791
Q_t	1.8838	-0.0008	0.1344	14.0128
P_0	0.5004	0.0000	16.5527	0.0302
M_0	0.5005	0.0023	19.4252	0.0258
c_M	-0.0033	0.0057	0.0009	-3.6362

inherent to the French and US markets. Finally, we describe briefly the link between this new global common component, and both the French and US market factor ones.

5.1 Statistical profiles

Estimating again our state-space model over the same time scale (and on each market separately) leads to the estimates, which are displayed in tables 11 and 10 with $P_t^{France} = 2.9461 \times 10^{-12}$ and $P_t^{US} = 0.3615$ whatever time t .

Adjusting for non-working dates in both countries changes slightly our pre-

Table 11: Kalman estimates for French asset returns

Parameters	Estimate	Gradient	Std. Dev.	T-Student
β^{10}	1.6604	0.0082	0.4308	3.8542
β^{11}	1.4138	0.0018	0.3768	3.7524
β^{12}	2.9518	-0.0063	0.7780	3.7938
β^{13}	1.5750	-0.0018	0.4169	3.7782
β^{14}	1.8353	0.0043	0.4835	3.7963
β^{15}	1.8058	-0.0066	0.4700	3.8417
β^{16}	1.2540	0.0033	0.3208	3.9090
β^{17}	1.3712	0.0015	0.3587	3.8230
β^{18}	1.4985	0.0038	0.3971	3.7736
σ_{10}	0.0000	-0.0005	0.0198	0.0001
σ_{11}	2.1644	-0.0011	0.0460	47.0964
σ_{12}	2.8350	-0.0028	0.0602	47.0895
σ_{13}	2.1382	0.0029	0.0454	47.1005
σ_{14}	2.6103	0.0011	0.0554	47.1075
σ_{15}	1.9252	0.0002	0.0409	47.0702
σ_{16}	2.1233	0.0006	0.0451	47.0818
σ_{17}	2.0347	0.0005	0.0432	47.1166
σ_{18}	2.9125	0.0006	0.0618	47.1101
Q_t	0.8110	-0.0015	0.2107	3.8495
P_0	0.7106	0.0051	1.6835	0.4221
M_0	8.8777	0.0000	12.2015	0.7276
c_M	0.0687	-0.0003	0.0312	2.2009

vious estimate results.¹¹ We have globally the same behavior as the results introduced in the previous section except for some specific details. First, among high beta coefficients, only Merrill Lynch asset return's beta remains above unity, amplifying then market movements. Second, the initial state of the US market factor M_0 is no more significant while c_M coefficient becomes significant here. Moreover, Q_t variance is higher than previously (i.e., a 46.56% increase).

In the French case, though coefficient estimates changes slightly in level, the same conclusions as in the previous section apply. Namely, all asset returns

¹¹Removing some data is equivalent to remove some information, and such a loss of information impacts slightly our results.

Table 12: Statistics for common latent factors in asset returns (same time scale and percentage basis)

	France	USA
Mean	0.0414	0.0691
Std. Dev.	0.8125	1.7848
Skewness	-0.1951	-0.3436
Excess kurtosis	1.1508	3.0918
Min.	-3.2123	-10.7063
Max.	3.5810	7.2395
Median	0.0494	0.0829
1st quartile*	-0.4206	-0.9741
3rd quartile*	0.5546	1.1309

* Upper bound of the quartile.

amplify significantly market movements. Notice that c_M coefficient of market risk factor remains negative for the US market whereas it remains positive for the French market. Moreover, French and US c_M coefficients are below unity in absolute value. Hence, the French market risk factor evolves in a stable way over our time horizon.

We obtain the following statistical profile for the common latent factor M_t peculiar to each financial market under consideration (see table 12).

The same conclusions as the former section apply here. Briefly, common latent French and US market factors are leptokurtic, the US market factor being more left-asymmetric and having fatter tails than the French one. As a rough guide, we also translate these results into graphs while plotting related histograms as well as related Gaussian distributions (i.e., Normal densities with corresponding moments of French and US market factor returns, see figures 1 and 2). We also display related Q-Q plots in the appendix.

Previous histograms exhibit the higher impact of losses in both French and US financial markets (i.e., higher extreme or more frequent negative returns in absolute value as compared to their positive counterparts). The magnitude of observed losses (i.e., absolute value of negative market factor returns) is higher for the US financial market. The same conclusion holds for the magnitude of observed positive market returns. Consequently, potential losses as

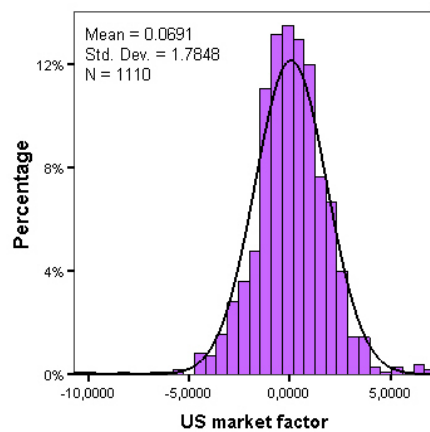


Figure 1: Histogram for US market factor.

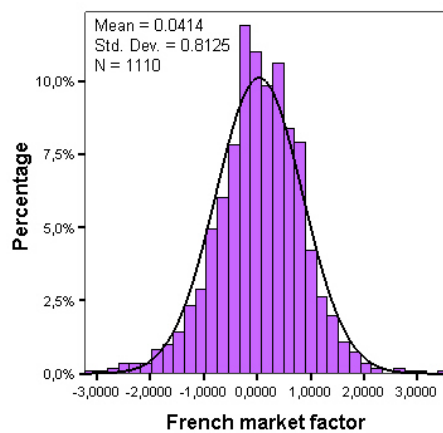


Figure 2: Histogram for French market factor.

Table 13: Correlations of common latent factors with asset returns (same time scale)

France			USA		
i	Kendall's tau	Spearman's rho	i	Kendall's tau	Spearman's rho
1	1.0000	1.0000	10	0.8611	0.9720
2	0.3012	0.4295	11	0.4183	0.5856
3	0.5007	0.6810	12	0.3447	0.4879
4	0.4364	0.6067	13	0.2366	0.3415
5	0.3304	0.4697	14	0.4030	0.5681
6	0.4302	0.5921	15	0.2937	0.4216
7	0.2886	0.4180	16	0.4971	0.6818
8	0.3059	0.4406	17	0.2941	0.4252
9	0.3424	0.4850	18	0.3388	0.4806

well as potential gains are higher on the US market than on the French one. Higher US absolute market returns are balanced with a higher volatility (i.e., higher market risk) on the US financial market as compared to the French financial market's volatility. To get a view about the link between market factors and corresponding asset returns, we consider related non-parametric correlation coefficients (see table 13).

As expected, we get the same results as in the previous section. Namely, correlation coefficients are all positive, and mean that asset returns are market driven whatever the financial market under consideration (see Campbell *et al.* [2001]). For further investigation, we focus on a potential common component in both French and US common latent factors (i.e., French and US market factors).

5.2 Systemic component

We address the question of how to characterize some potential link prevailing between French and US financial markets. Specifically, we look for a relationship between French and US common latent factors. However, we do not focus on the speed at which French and US stock returns may incorporate

market information.¹² We rather focus on the common part of information content, which underlies French and US stock returns at the closing times of their respective financial market places. As a first step, we consider Kendall and Spearman correlation coefficients between US and French market factors, which are respectively $\tau = 0.2834$ and $\rho = 0.4053$. Hence, we exhibit clearly some positive link between these two components. Therefore, at a systemic level of consideration, we attempt to extract a common component in French and US market factors. Such a component may result from business cycle effect, macroeconomic risk or financial integration effect on these two financial markets among others. To this end, we assume the following dependence structure:

$$M_t^{US} = b_1 B_t + e_t^1 \quad (22)$$

$$M_t^{France} = b_2 B_t + e_t^2 \quad (23)$$

$$B_t = c_B B_{t-1} + \eta_t \quad (24)$$

where B_t is the systemic component (i.e., a component common to French and US financial markets); M_t^{France} and M_t^{US} are French and US market factors; (e_t^i) , η_t and c_B as in the previous section. The state-space formulation of such a specification is then:

$$\begin{pmatrix} M_t^{US} \\ M_t^{France} \end{pmatrix} = \begin{pmatrix} b_1 \\ b_2 \end{pmatrix} \cdot B_t + \begin{pmatrix} e_t^1 \\ e_t^2 \end{pmatrix} \quad (25)$$

$$B_t = c_B B_{t-1} + \eta_t \quad (26)$$

with $N = 2$, $k = g = 1$. Recall that we only observe market factors M_t^{France} and M_t^{US} from which we try to infer a common systemic component B_t (i.e., unobserved state variable). The related estimates we get while applying a Kalman methodology are displayed in table 14 with $P_t = 0.0808$.

As expected from correlation coefficients, estimates exhibit a positive link between systemic factor and both French and US market factor returns. Indeed, b_1 and b_2 estimates are positive, significant and above unity. Then, French and US market factors tend to magnify the systemic component's impact. Moreover, coefficient c_B is positive and significant while being below unity (i.e., stable evolution of systemic component over our time horizon). In the same way, variance parameter Q_t is significant. Finally, the statistical profile of the systemic component is summarized in table 15.

¹²Indeed, we do not inquire into simultaneous common information in both US and French stock returns.

Table 14: Kalman estimates for systemic component in asset returns

Parameters	Estimate	Gradient	Std. Dev.	T-Student
b_1	2.3888	0.0157	0.6710	3.5602
b_2	1.2464	0.0155	0.3551	3.5098
σ_1	1.4249	-0.0065	0.0620	22.9762
σ_2	0.5884	0.0111	0.0377	15.6252
Q_t	0.4391	-0.0317	0.1166	3.7654
P_0	1.0150	0.0621	0.0898	0.0001
B_0	3.2020	-0.0173	2.8342	1.1298
c_B	0.2160	0.0301	0.0466	4.6308

Table 15: Statistics for systemic factor in asset returns

	B_t
Mean	0.0207
Std. Dev.	0.3488
Skewness	-0.3636
Excess kurtosis	1.3965
Min.	-1.5214
Max.	1.1667
Median	0.0326
1st quartile*	-0.1761
3rd quartile*	0.2323

* Upper bound of the quartile.

Table 16: Correlations of market and systemic factor returns

Market return	Kendall's tau	Spearman's rho
M_t^{US}	0.5560	0.7405
M_t^{France}	0.7185	0.8902

Analogously to French and US market factors, the systemic factor exhibits non-normal and asymmetric features. Specifically, this component is left-skewed and exhibits fatter tails than the Normal probability distribution. On an absolute value basis, systemic skewness is higher than market skewness both in France and USA. However, systemic excess kurtosis lies between French and US ones. Strikingly, systemic volatility is lower than US and French market volatilities. Finally, we end our study with a quick statistical profile of the obtained systemic factor. Indeed, a brief correlation analysis is displayed in table 16. Then, we plot the related histogram and corresponding Gaussian density (see figure 3). We also display related Q-Q plot in the appendix.

As expected, the correlation coefficients we get exhibit a positive and strong link between systemic factor and both French and US market factors. Hence, French and US financial markets tend to evolve in the same direction (i.e., same structural changes). However, the French market is more sensitive to structural changes than the US one (i.e., higher correlation with the systemic component).

The previous histogram illustrates clearly the statistical profile of systemic factor return. As a rough guide, we also plot the corresponding Gaussian distribution function. Namely, the probability distribution of the systemic factor return is obviously non-normal, fat-tailed as well as left-skewed. Indeed, the magnitude of extreme negative systemic returns is higher than the magnitude of their positive counterparts.

6 Concluding remarks

In this paper, we investigate the existence of a common latent component in asset returns. Our study concerns both the French and US financial markets, and is undertaken in two steps.

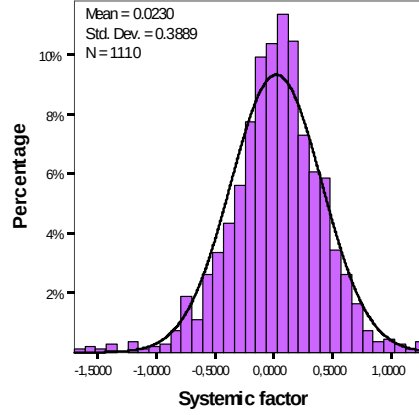


Figure 3: Histogram of systemic risk factor.

First, given that we only observe asset returns, we resort to Kalman filtering methodology to infer some knowledge about the unobservable French and US market factors. This estimation method requires to translate our investigation into a state-space representation. The results we get are powerful in the sense that we find strong evidence of a common latent factor in both financial markets. Moreover, such factors exhibit tail and asymmetric features analogously to their related respective asset returns.

Second, we further investigate a potential link or a potential common component in our two market factors while considering a systemic level. For this purpose, we resort again to Kalman filtering method to infer knowledge about the unobserved systemic component. Such an approach is useful to capture macroeconomic effects and economic as well as financial links between countries or market places. Our results are also significant here and exhibit a strong positive link between the systemic factor and both French and US market factors. However, we chose a linear framework to undertake our study and investigate potential common links between asset returns. Current markets suggest the existence of non-linear link between asset returns. So, some extensions may be undertaken in the lens of non-linearity patterns, and could perhaps lead to even stronger results.

Future research should therefore attempt to apply improved versions of Kalman methodology. Indeed, extensions have been proposed to allow for

relaxing required initial conditions as well as to account for missing data (see Rao [2001]). Moreover, given known non-linear market characteristics (see Gouriéroux & Jasiak [2001] among others), extended Kalman filter (i.e., EKF) and iterated extended Kalman filter (i.e., IEKF) allow for accounting for non-linear system features (see Jaswinsky [1970], Maybeck [1979,1982], Chui & Chen [1987], and Julier & Uhlmann [1998]). To go farther, approximately linear (i.e., slightly non-linear), and non-stationary market evolutions could also be taken into account while employing the generalized flexible least squares (GFLS) approach of Kalaba & Tesfatsion (1990).

7 Appendix

We expose therein computational details as well as complementary explanations and statistics.

7.1 Statistical profiles

For example, table 17 presents the statistical profiles of both French and US stock returns on the same time scale. Results exhibit the fat-tailed as well as asymmetric behavior of related empirical probability distributions.

7.2 Efficiency of French and US market factors

We assess here the efficiency of the obtained systematic risk factors (i.e., market factors of risk) for each financial market under consideration. Namely, we consider the following regression for each stock i , for each financial market, and for time t in $\{1, \dots, T\}$:

$$R_t^i - M_t = \lambda_i M_t + u_t^i \quad (27)$$

where λ_i a constant regression coefficient, and (u_t^i) is a Gaussian noise. Hence, testing for the efficiency of the systematic risk factor consists of testing whether λ_i is significantly zero for each stock return. This is equivalent to test whether the market factor summarizes the whole information that describes asset return evolutions in each considered financial market. Related results are displayed in tables 18 and 19.

Table 17: French and US asset return statistics (same time scale)

i	Mean	Std. Dev.	Skewness	Excess kurtosis	Min.	Max.	Median	1st quartile*	3rd quartile*
Accor	0.0932	2.4491	-0.0908	2.0994	-14.7809	10.7692	0.0614	-1.3637	1.4603
Alcatel	0.0721	3.7140	-1.9344	27.1277	-48.4564	14.4352	0.0000	-1.7712	2.0700
Axa	0.1575	2.4875	5.3428	96.2590	-9.9142	45.1151	0.1107	-1.0933	1.2997
Bouygues	0.1683	3.0020	0.1215	2.6075	-17.1909	14.0123	0.0000	-1.4388	1.7331
L'Oréal	0.0932	2.4195	0.0535	1.0333	-10.0285	9.2622	0.0000	-1.4583	1.5281
Michelin	0.0004	2.3554	-0.0540	2.2005	-11.3445	11.8360	0.0000	-1.2998	1.2604
Peugeot SA	0.1134	2.3175	-0.2112	3.4553	-16.3259	10.4635	0.0000	-1.1562	1.3811
SBF120	0.0688	1.3484	-0.1951	1.1508	-5.3336	5.9459	0.0819	-0.6963	0.9201
Total Fina Elf SA	0.1859	3.1517	10.2020	223.8976	-13.1709	70.5330	0.0575	-1.3258	1.5837
AT & T	0.0099	2.9021	0.3304	9.6706	-23.2620	22.1301	0.0000	-1.6330	1.4661
DJIA	0.0438	1.2229	-0.3677	3.0016	-7.4549	4.8605	0.0608	-0.6492	0.8093
Dow Jones	0.0149	1.8971	-1.4312	17.0840	-21.4531	8.7601	0.0000	-0.9598	0.9864
Ford Motor	0.0146	2.6436	-2.7074	41.9896	-38.6561	10.6264	0.0000	-1.2917	1.3975
IBM	0.0770	2.5903	-0.1285	4.6693	-16.8916	12.3665	0.1026	-1.4513	1.4926
Merrill Lynch	0.1111	3.2231	0.1488	1.3390	-12.2978	14.0477	0.0000	-1.8949	2.0523
Micron Tech.	0.1213	4.8859	0.1404	0.8554	-19.1160	21.7202	0.0000	-3.0678	3.0864
Microsoft	0.1238	2.7249	-0.0395	3.8832	-15.6310	17.8692	0.0423	-1.4497	1.6989
Walt Disney	0.0215	2.4359	-0.0728	4.9720	-16.9523	14.2029	0.0000	-1.3777	1.3666

* Upper bound of the quartile.

Table 18: French asset return regressions

i	R^2	Adjusted R^2	Durbin Watson Stat.	$F(1, 1137)$	λ_i	Student t
Accor	2	0.0191	0.0183	2.1176	0.3508	4.7106
Alcatel	3	0.2250	0.2244	1.7856	1.7563	18.1708
Axa	4	0.0379	0.0370	1.8181	0.4940	6.6901
Bouygues	5	0.0524	0.0516	1.9317	0.7118	7.9296
L'Oréal	6	0.0915	0.0907	2.0659	0.7152	10.7020
Michelin	7	0.0047	0.0038	1.9083	0.1700	2.3141
Peugeot	8	0.0168	0.0160	1.8042	0.3100	4.4142
SBF120	1	1.0000	1.0000	1.9776	0.5677	6.5×10^8
Total	9	0.0151	0.0142	1.9311	0.4182	4.1681

Table 19: US asset return regressions

	i	R^2	Adjusted R^2	Durbin Watson Stat.	$F(1, 1137)$	λ_i	Student t
AT & T	6	0.0017	0.0009	2.0744	1.9984	-0.0816	-1.4136
DJIA	1	0.2759	0.2753	1.9095	434.4350	-0.1212	-20.8431
Dow Jones	8	0.0946	0.0938	2.1528	119.1421	-0.4103	-10.9152
Ford Motor	9	0.0044	0.0036	2.3769	5.0947	-0.1157	-2.2571
IBM	2	0.0116	0.0107	1.9692	13.3583	0.1636	3.6549
Merrill Lynch	7	0.1358	0.1351	2.0458	179.1805	0.6663	13.3858
Micron Tech.	4	0.0101	0.0093	1.9812	11.6760	0.3366	3.4170
Microsoft	5	0.0107	0.0099	1.8885	12.3704	0.1728	3.5172
Walt Disney	3	0.0066	0.0057	2.1442	7.5540	-0.1273	-2.7485

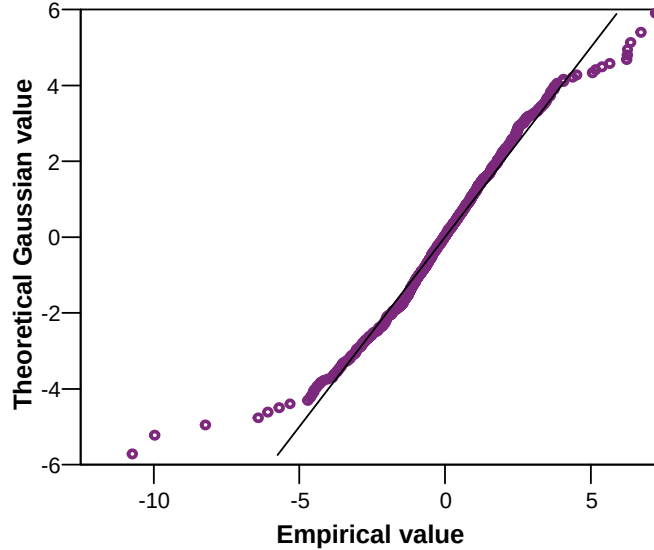


Figure 4: Gaussian Q-Q plot for US market factor.

We also performed the standard regressions of R_t^i on M_t for each financial market under consideration, and found good explanatory powers, and positive and highly significant regression coefficients for both financial markets. These regression coefficients are far below unity for stock returns, and close or equal to unity for stock index returns. Moreover, the explanatory powers indicate that systematic risk factors generally fail to explain the whole evolution of asset returns (except for stock index returns). Indeed, lowest French and US R^2 coefficients are 0.1494 and 0.1390 for Total and Micron Technology stock returns respectively. Conversely, highest French and US R^2 coefficients are 0.4170 and 0.4957 for Alcatel and Merrill Lynch stock returns respectively. As regards market stock indices, related R^2 coefficients are 1 and 0.9524 for SBF120 and DJIA returns respectively. To spare space, we do not report related results, which are available upon request of course.

7.3 Q-Q plots

We display Q-Q plots of US and French market factors as well as systemic factor.

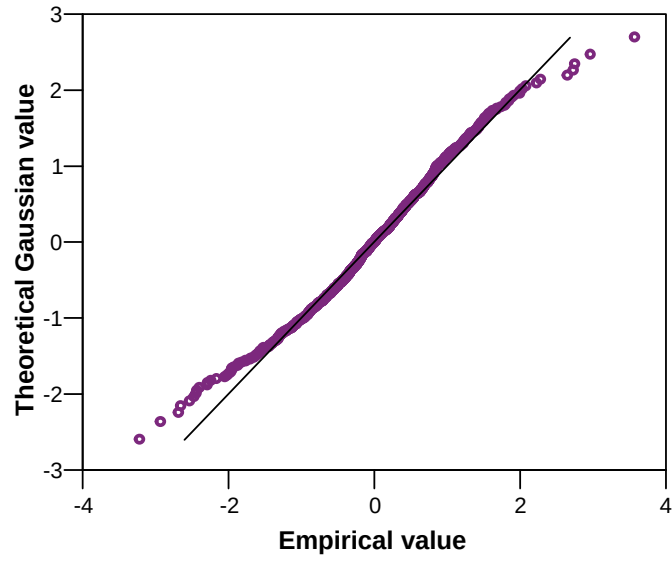


Figure 5: Gaussian Q-Q plot for French market factor.

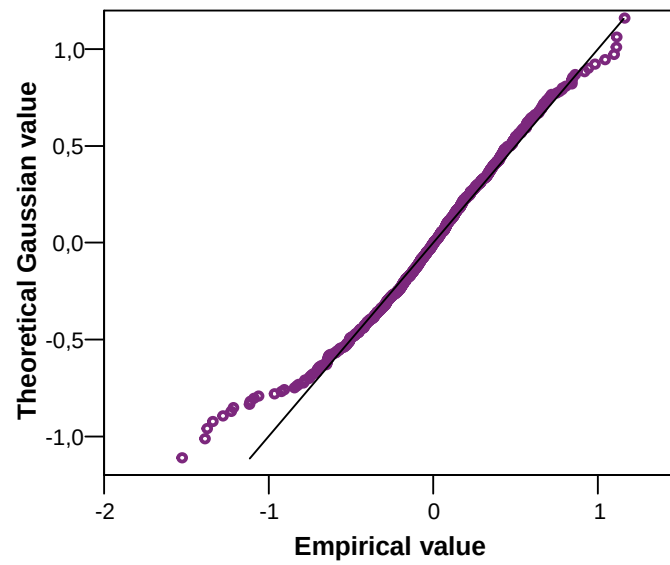


Figure 6: Gaussian Q-Q plot for systemic factor.

Previous Q-Q plots show the gap between empirical probability distributions and the Gaussian density (i.e., distance between reality and theoretical Normal law). Indeed, market and systemic factors are far from being normally distributed given their corresponding extreme values (i.e., fat tails).

References

- Affleck-Graves, J., and B., McDonald. (1989). Non-Normalities and Tests of Asset Pricing Theory. *Journal of Finance*, 44(4): 889-908.
- Ahn, S. C., and C., Gadarowski. (2000). Two-Pass Cross-Sectional Regression of Factor Pricing Models: Minimum Distance Approach. Working Paper, Department of Economics, Arizona State University.
- Amihud, Y., and H. Mendelson. (1989). The Effects of Beta, Bid-Ask Spread, Residual Risk and Size on Stock Returns, *Journal of Finance*, 44(2): 479-486.
- Banz, R. W. (1981). The Relationship Between Return and Market Value of Common Stocks. *Journal of Financial Economics*, 9(1): 3-18.
- Barnes, M. L., and A. W., Hughes. (2002). A Quantile Regression Analysis of the Cross-Section of Stock Market Returns. FRB Boston Series, Working Paper No 02-2.
- Berk, J. B. (1995). A Critique of Size-Related Anomalies. *Review of Financial Studies*, 8(2): 275-286.
- Bhandari, L. C. (1988). Debt/Equity Ratio and Expected Common Stock Returns: Empirical Evidence. *Journal of Finance*, 43(2): 507-528.
- Blattberg, R., and N., Gonedes. (1974). A Comparison of Stable and Student Distributions as Statistical Models for Stock Prices. *Journal of Business*, 47(2): 244-280.
- Breen, W., Glosten, L. R., and R., Jagannathan. (1989). Economic Significance of Predictable Variations in Stock Index Returns. *Journal of Finance*, 44(5): 1177-1189.

Brown, R. G., and P. Y. C., Hwang. (1997). Introduction to Random Signals and Applied Kalman Filtering. Third Edition, John Wiley & Sons, Inc., New York.

Buchinsky, M. (1998). Recent Advances in Quantile Regression Models: A Practical Guideline for Empirical Research. *Journal of Human Resources*, 33(1): 88-126.

Campbell, J. Y. (1987). Stock Returns and the Term Structure. *Journal of Financial Economics*, 18(2): 373-399.

Campbell, J. Y., Lettau, M., Malkiel, B. G., and X., Xu. (2001). Have Individual Stocks Become more Volatile? An Empirical Exploration of Idiosyncratic Risk. *Journal of Finance*, 56(1): 1-43.

Chan, L. K. C., Hamao, Y., and J., Lakonishok. (1991). Fundamentals and Stock Returns in Japan. *Journal of Finance*, 46(5): 1739-1764.

Chui, C. K., and G., Chen. (1987). Kalman Filtering with Real-Time Applications. Springer Series in Information Science, 17, Springer Berlin, Heidelberg.

Cressie, N., and C. K., Wikle. (2002). Space-Time Kalman Filter. In *Encyclopedia of Econometrics*, edited by A. H. El-Shearawi & W. W. Piegorsch, 4: 2045-2049.

Fama, E. F. (1965). The Behavior of Stock Market Prices. *Journal of Business*, 38(1): 34-105.

Fama, E. F. (1976). Foundations of Finance. Basic Books, New York.

Fama, E. F., and K. R., French. (1989). Business Conditions and Expected Returns on Stocks and Bonds. *Journal of Financial Economics*, 25(1): 23-49.

Fama, E. F., and K. R., French. (1992). The Cross-Section of Expected Stock Returns. *Journal of Finance*, 47(2): 427-465.

Fama, E. F., and J. D., MacBeth. (1973). Risk, Return, and Equilibrium: Empirical Tests. *Journal of Political Economy*, 81(3): 607-636.

- Ferson, W. E., and C. R., Harvey. (1991). The Variation of Economic Risk Premiums. *Journal of Political Economy*, 99(2): 385-415.
- Ferson, W. E., and C. R., Harvey. (1999). Conditioning Variables and the Cross-Section of Stock Returns. *Journal of Finance*, 54(4): 1325-1360.
- Gatfaoui, H. (2006). How Does Systematic Risk Impact Stocks? A Study On The French Financial Market. Forthcoming in Greg N. Gregoriou, eds., Chapter 10, *Asset Allocation and International Investments*. Palgrave MacMillan.
- Gençay, R., Selçuk, F., and B., Whitcher. (2003). Systematic Risk and Timescales. *Quantitative Finance* 3(2): 108-116.
- Gourieroux, C., and J., Jasiak. (2001). *Financial Econometrics: Problems, Models, and Methods*. Princeton University Press, USA.
- Harvey, C. R. (1989a). Time-Varying Conditional Covariances in Tests of Asset Pricing Models. *Journal of Financial Economics*, 24(2): 289-317.
- Harvey, C. R. (1989b). *Forecasting Structural Time Series Models and the Kalman Filter*, Cambridge University Press, Cambridge.
- Jagannathan, R., and Z., Wang. (1996). The Conditional CAPM and the Cross-Section of Expected Returns. *Journal of Finance*, 51(1): 3-53.
- Jaswinski, A. M. (1970). *Stochastic Processes and Filtering Theory*. Academic, New York.
- Julier, S. J., and J. K., Uhlmann. (1998). A New Extension of the Kalman Filter to Nonlinear Systems. Working Paper, Department of Engineering Science, University of Oxford.
- Kalaba R., and L., Tesfatsion. (1990). Flexible Least Squares for Approximately Linear Systems. *IEEE Transactions on Systems, Man, and Cybernetics*, 20 (5): 978-989.
- Kalman, R. E. (1960). A New Approach to Linear Filtering and Prediction Problems. *Journal of Basic Engineering, Transactions of the ASME, Series D*, 82(1): 34-45.

Kothari, S. P., Shanken, J., and R. G., Sloan. (1995). Another Look at the Cross-Section of Expected Stock Returns. *Journal of Finance*, 50(1): 185-224.

Koutmos, G., and J., Knif. (2002). Estimating Systematic Risk Using Time-Varying Distributions. *European Financial Management* 8(1): 59-73.

Lemoine, M., and F., Pelgrin. (2003). Introduction aux Modèles Espace-Etat et au Filtre de Kalman. *Revue de l'OFCE*, 86(July): 203-229.

Malkiel, B. G., and Y., Xu. (2002). Idiosyncratic Risk and Security Returns. Working Paper, Department of Economics, Princeton University.

Malkiel, B. G., and Y., Xu. (2003). Investigating the Behavior of Idiosyncratic Volatility. *Journal of Business* 76(4): 613-644.

Maybeck, P. S. (1979). *Stochastic Models, Estimation and Control*. Volume 1, Academic, New York.

Maybeck, P. S. (1982). *Stochastic Models, Estimation and Control*. Volume 2, Academic, New York.

Meinhold, J., and N. D., Singpurwalla. (1983). Understanding the Kalman Filter. *American Statistician*, 37(2): 123-127.

Merton, R. C. (1987). A Simple Model of Capital Market Equilibrium with Incomplete Information. *Journal of Finance*, 42(3): 483-510.

Rao, C. R. (2001). A Note on Kalman Filter. *Proceedings of the National Academy of Science USA*, 98(19): 10557-10559.

Roll, R. (1977). A Critique of the Asset Pricing Theory's Tests. *Journal of Financial Economics*, 4(2): 129-176.

Shanken, J. (1992). On the Estimation of Beta Pricing Models. *Review of Financial Studies*, 5(1): 1-33.

Sharpe, W. F. (1963). A Simplified Model For Portfolio Analysis. *Management Science*, 9(2):499-510.

Sharpe, W. F. (1964). Capital Asset Prices: A Theory of Market Equilibrium Under Conditions of Risk. *Journal of Finance*, 19(3):425-442.

Wikle, C. K., and N., Cressie. (1999). A Dimension-Reduced Approach to Space-Time Kalman Filtering. *Biometrika*, 86(4): 815-829.