Lehman Brothers Seminar

Credit VaR : New Approach

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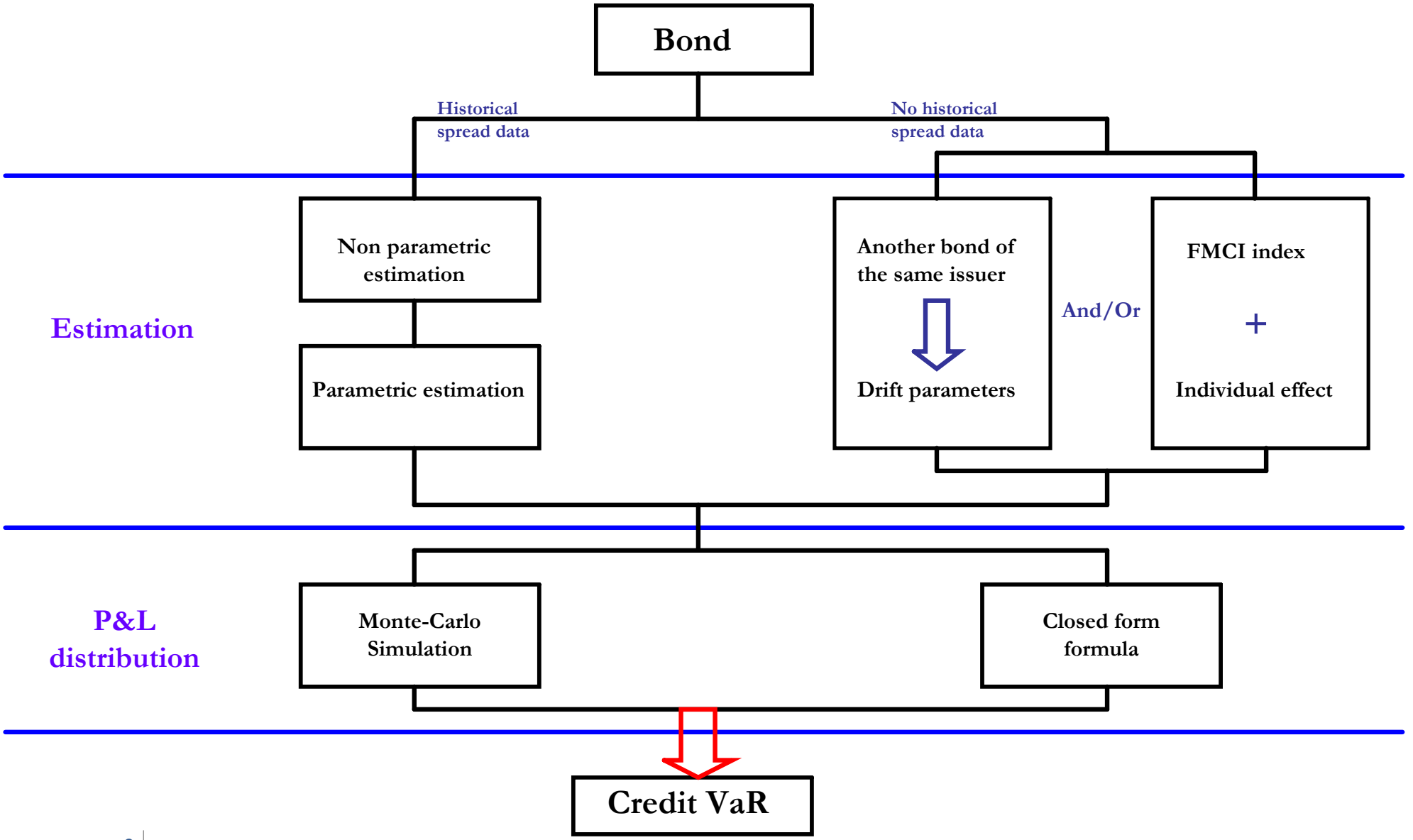
The Story Line



I Methodology & Hypothesis

Methodology & Hypothesis

- The aim of our study is to establish a joint simulation procedure for the spread and rating dynamic
- This procedure will be applied to compute the Monte Carlo Value-at-Risk for one bonds portfolio
- In view of this, we will seek to specify the transition probability :

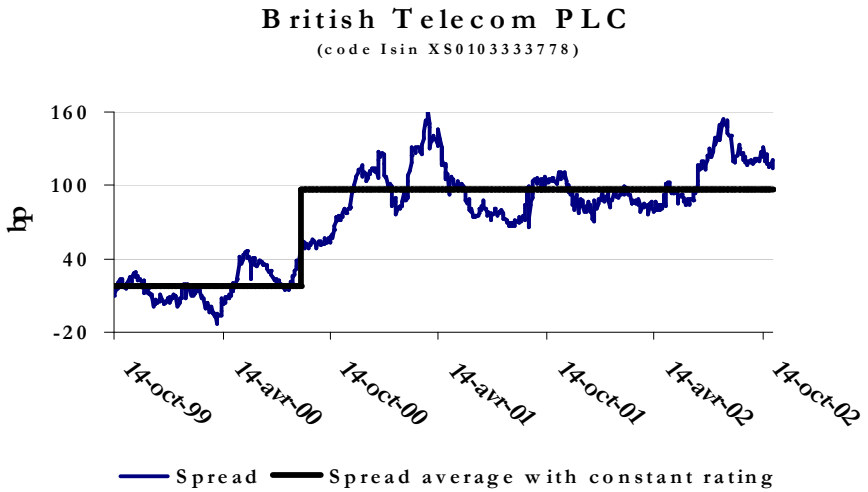
$$\Pi\left((S_t, R_t)\middle|(\underline{S_{t-1}}, \underline{R_{t-1}})\right)$$

- The first step of our study consisted in downloading, from Bloomberg, the internal rate of return for the bonds of a portfolio made up of some CDC-IXIS counterparties, in the US and European markets, chosen from the Telecommunications, Industries and Finance sectors
- In order to ensure good data quality, the following criteria were used for the choice of securities :
  - Security type : Normal (no option embedded)
  - Maturity > 01/01/2004
  - Nominal amount > 250.000.000 USD
  - Price contributors : BGN, which gives a weighted average of the prices given by the banks trading the security
  - Only securities with at least 3 years maturity have been retained
  - The securities must be liquid enough
  - We have eliminated the bonds for which the missing values percentage exceeded 6%
- Ultimately, we have retained 97 securities for our study

Methodology & Hypothesis

Data & descriptive study

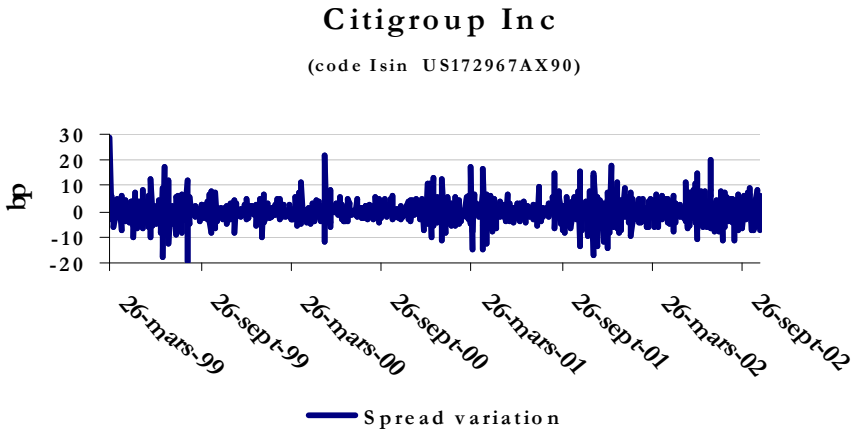
- The spread is computed over the Libor rate
- The second step of the study is a descriptive nature :
  - We have tested the credit spread with constant rating normality hypothesis, which was rejected as expected
  - We have tested the stationarity of the credit spread with constant rating for 45 securities : the hypothesis was accepted by the Dickey-Fuller test for 37 securities, and by the Augmented Dickey-Fuller test for 28 securities
- We have drawn the joint curves of the spread and the rating for the 97 securities, and we noticed the existence of commune statistical regularities. we have drawn the following conclusions :
  - A strong mean-reverting property



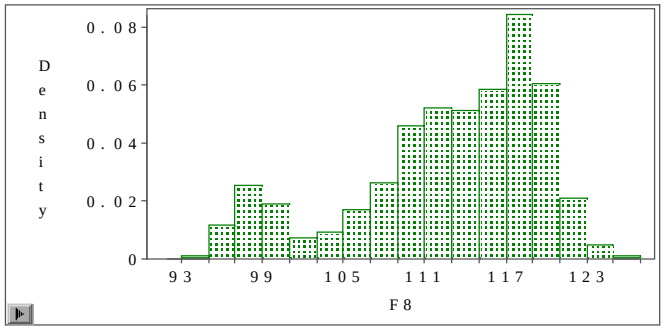
Methodology & Hypothesis

Data & descriptive study

- **The existence of a grouping extreme values**



- **Empirical probability densities functions are not normal, even sometimes with bimodal shape**



Moments

N	468.0000	Sum Wgts	468.0000
Mean	112.7051	Sum	52745.9870
Std Dev	7.0205	Variance	49.2877
Skewness	-0.8552	Kurtosis	-0.0995
USS	5967759.13	CSS	23017.3689
CV	6.2291	Std Mean	0.3245

Methodology & Hypothesis

Rating Spread Influence

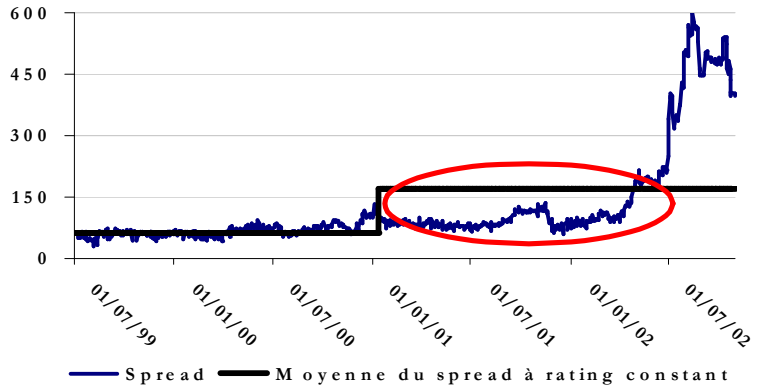
- During the course of the descriptive study, we have noticed that it would be very difficult to model in joint manner the two dynamics. Moreover, only the dynamic of the spread with constant rating seemed to have a stationarity property, which is crucial in the time series theory

- This leads us to search for the dynamic of the spread with constant rating. Using the Bayes Identity, we have :

$$\Pi\left(S_t, R_t \middle| \underline{S_{t-1}}, \underline{R_{t-1}}\right) = \Pi\left(S_t \middle| \underline{S_{t-1}}, \underline{R_{t-1}}, R_t\right) \Pi\left(R_t \middle| \underline{S_{t-1}}, \underline{R_{t-1}}\right)$$

- The relation linking the spread and rating is a reflexive one; the mutual influence between the two take into account the risk-aversion of the economic agents, which involves an appreciation bias. Only the objective free-aversion of risk component of the credit spread can influence the rating, as the rating agency are less averse than investors

- Thumb rule : "The rating react with 6 months delay to the market environment compared to the spread" !



Methodology & Hypothesis

Rating Spread Influence

- Rating influence spread → Shift on the spread level
- Spread influence rating → too complex to model

Take into account the risk aversion

- The hypothesis that the past information about the spread $\underline{S_{t-1}}$ does not influence the present rating is necessary :

$$\Pi\left(R_t \middle| \left(\underline{S_{t-1}}, \underline{R_{t-1}}\right)\right) = \underline{\Pi\left(R_t \middle| \left(\underline{R_{t-1}}\right)\right)} = \Pi\left(\left(R_t\right)\middle|\left(R_{t-1}\right)\right)$$

- The underlined part of the equation indicates that the rating process is a Markov chain, a widely adopted model by the markets participants . The rating agencies publish a transition matrix probability, which allowed us straightforwardly to simulate the rating, using the Hastings-Metropolis algorithm
- In the course of this study, we have sought to specify $\Pi\left(S_t \middle| \left(\underline{S_{t-1}}, \underline{R_{t-1}}, R_t\right)\right)$ We have made the following hypothesis :

$$\Pi\left(S_t \middle| \left(\underline{S_{t-1}}, \underline{R_{t-1}}, R_t\right)\right) = \Pi\left(S_t \middle| \left(S_{t-1}, R_t\right)\right)$$

- This Hypothesis is crucial for simulations, in so far as it make the couple (S_t, R_t) markovian
- We have supposed that the spread with constant rating follows a diffusion process :

$$dS_t = \mu(S_t)dt + \sigma(S_t)dW_t$$

II Nonparametric Estimation

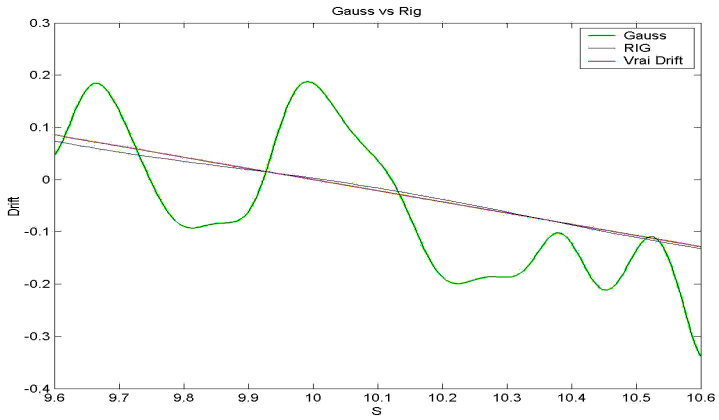
Nonparametric Estimation

- **Which parametric family we will choose for the drift and the volatility ?**
- **Before any parametric estimation of the drift and volatility functions, we want to have an idea about the function's shape**
- **Nonparametric approach is the ideal tool : without making any assumption on one variable's density parametric family, gives a good proxy**
- **Given n observations $S_1, S_2, ..., S_n$ of the spread S, an estimation of the drift and volatility functions at the point x for a given kernel K and a bandwidth h, is given by :**

$$\hat{(\sigma(x))}^2 = \frac{\sum_{i=1}^{n-1}(S_{t+1}-S_t)^2 K(\frac{x-S_t}{h})}{\sum_{t=1}^{n-1} K(\frac{x-S_t}{h})} \qquad \hat{\mu}(x) = \frac{\sum_{t=1}^{n-1}(S_{t+1}-S_t) K(\frac{x-S_t}{h})}{\sum_{t=1}^{n-1} K(\frac{x-S_t}{h})}$$

- **We have chosen a RIG Kernel for the nonparametric approach as the Gaussian is biased and more instable. The following simulation for a series of 10 000 observations depict the assertion :**

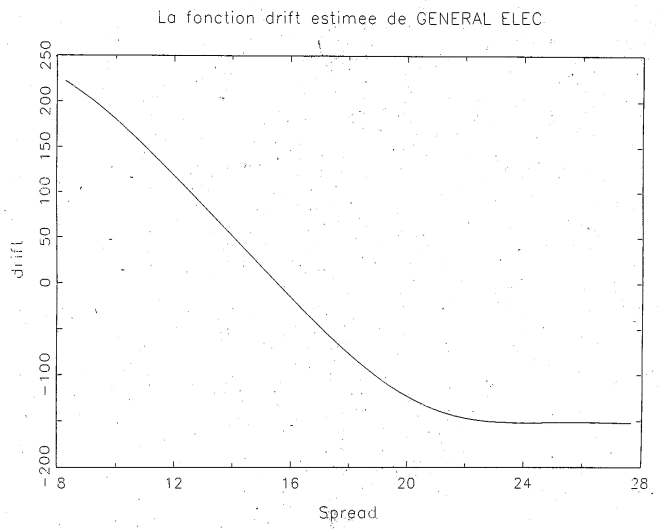
$$K_{RIG(\frac{1}{x-b},\frac{1}{b})}(u) = \frac{1}{\sqrt{2\pi b u}} \exp\left(-\frac{x-b}{2b}\left(\frac{u}{x-b} - 2 + \frac{x-b}{u}\right)\right), x \in \Re_+^*$$



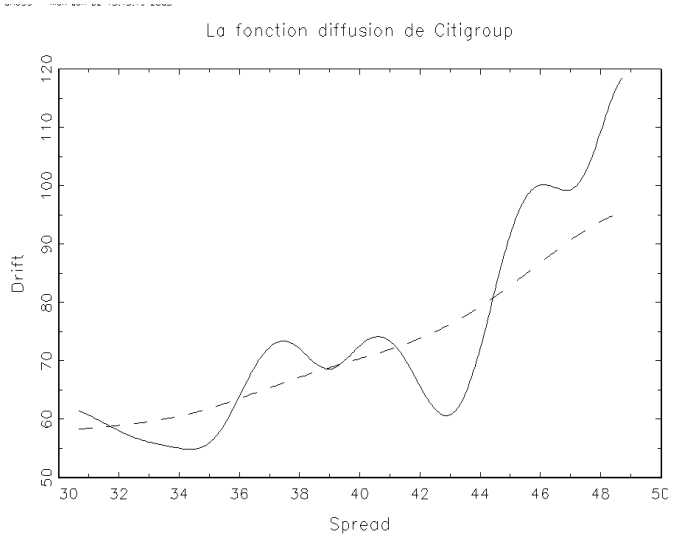
Nonparametric Estimation

- We have drawn the nonparametric estimator for the drift and volatility functions :

Nonparametric estimartor of General Electric Drift



Nonparametric estimartor of Citigroup Volatility



- We remark a linear drift as the mean reverting spread property predict
- We had to take into account the observed negative spreads for the good rated companies
- Ultimately, we have retained this following parametric family :

$$dS_t = a.(b - S_t)dt + (c + d.S_t^2)^{\gamma}\, dW_t$$

Nonparametric Estimation

Ait-Sahalia specification test

- **We want to test the robustness of the family proposed : is the retained family statistically consistent ?**
- **Karlin &Taylor have determined in 1981 the marginal density expression for a stationary diffusion process :**

$$\pi(x,\theta) = \frac{\xi(\theta)}{\sigma^2(x;\theta)} \exp\left(\int_{x_0}^{x} \frac{2\mu(u,\theta)}{\sigma^2(u,\theta)} du\right)$$

  - $\pi(.)$ **: The marginal density of the diffusion process**
  - $\xi(\theta)$ **: A normalization constant**
  - **x0 : Arbitrary**
- **Ait-Sahalia has introduced in 1996 a test of specification for the interest rate models. If we note** $\pi_0(.)$ **the true marginal density of the diffusion process and** $\pi(.,\theta)$ **the marginal density belonging to the parametric family, then the null and alternative Hypothesis of the test are as follows :**

$$H_0 : \exists \theta_0 \in \Theta / \pi(.,\theta_0) = \pi_0(.)$$
$$H_a : \forall \theta \in \Theta, \pi(.,\theta) \neq \pi_0(.)$$

- **Ait-Sahalia defined a distance between the two densities. If this distance is above a certain level, the test is rejected at a certain level of confidence :**

$$\hat{M} = Tb_T \min_{\theta \in \Theta} \frac{1}{T} \sum_{t=1}^{T} (\pi(S_t,\theta) - \hat{\pi}_0(S_t))^2$$

- **As we don't know the true density, Ait Sahalia took the nonparametric estimator as a proxy**
- **The test was applied for 38 bond of spreads series, and our parametric family was accepted for 33 series**

III Estimation

Estimation

- To model the spread conditionally to the rating, we have retained the following parametric family :

$$dS_t = a.(b - S_t)dt + (c + d.S_t^2)^\gamma \, dW_t$$

- We want to estimate the five parameters for a given series of observations. In this goal, we discretise the differential stochastic equation with an Euler scheme :

$$S_{t+\Delta} = (S_t + a(b - S_t).\Delta) + (c + d.S_t^2)^\gamma .\sqrt{\Delta}.\varepsilon_t$$

- Hence, $S_{t+\Delta}$ conditionally to S_t has a normal distribution with $(S_t + a(b - S_t).\Delta)$ as a mean and $(c + d.S_t^2)^\gamma .\sqrt{\Delta}$ as a standard deviation

- This remark allow us to give an expression to the Likelihood for a given series, function of five parameters

$$l(S_n,....,S_0) = l(S_n / S_{n-1}..S_0)...l(S_1 / S_0)l(S_0)$$

- During simulations of the likelihood optimization, we noticed a great sensitivity of the parameters upon the initial value of the optimization algorithm, especially the power parameter Gamma

- In order to have the closest initial point, we used the Ordinary Least Square method applied to the drift and volatility function nonparametric estimator, by depicting a linear relation between the two functions and a powers of the spreads :

$$\hat{\mu}(S_t) = a(b - S_t) + \varepsilon_t \qquad (\hat{\sigma})^{1/\gamma} = c + d.S_t^2 + \eta_t$$

With (ε_t) and (η_t) a random variables with zero mean, and finite second moment

Estimation

Results

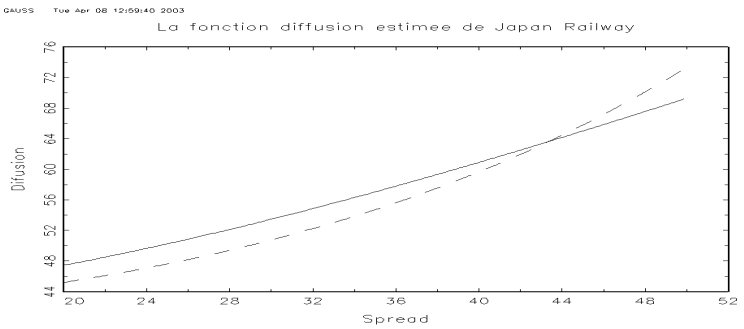
- **Maximum Likelihood estimation for simulation with fixed Gamma gives good results**

	a	b	c	d
True Value	15	30	2000	1
Estimated	15,87	29,38	2060	1,009
Std	3,92	1,57	206	0,221

- **We have ran the maximum likelihood optimization with then without constraint on Gamma, we noticed a freedom degree of the parametric family in term of likelihood :**

Code Isin	a	b	c	d	Gamma constraint	Likelihood Lost
US172967AX90	7,78 -2,39	41,15 4,94	3022 163	1,107 0,125	0,5	0,0318%
XS0097541212	17,52 2,8	25,82 1,75	3188 -135	0,107 0,146	0,5	0,0002%
US00077QAG55	18,37 3,12	48,26 2,08	2989 -279	0,874 0,118	0,5	0,1960%

- **For example, for one "Japan Railway" bond, we estimated two volatility parameters (1763.29 ; 1.2164 ; 0.5) (1.14 ; 9.39 e-6 ; 28.37) under the constraint and non constraint optimisation. These two parameters, apparently very different, provide a very close volatility function :**



V Validation

Validation

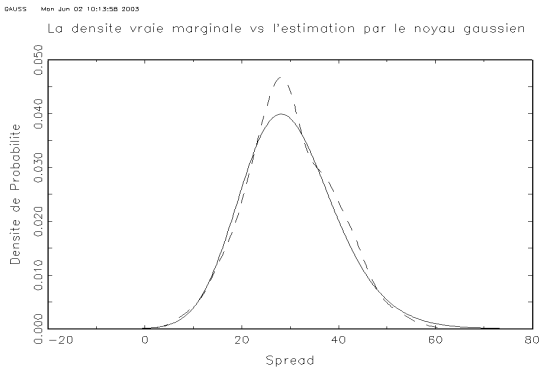
- **We want to validate our choice of the following parametric family modelling the spread dynamic conditionally to a constant rating and the :**

$$dS_t = a.(b - S_t)dt + \sqrt{c + d.S_t^2}\,dW_t$$

- **The Karlin &Taylor formula for the marginal density gives a diffusion process family density proportional to :**

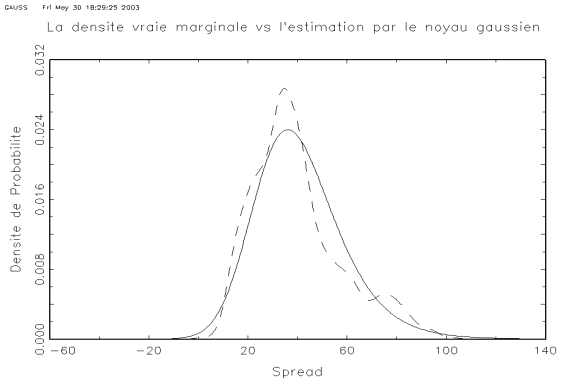
$$\frac{\exp\left(\frac{2ab}{\sqrt{cd}}\,Arc\tan\left(\sqrt{\frac{d}{c}}x\right)\right)}{(c+dx^2)^{1+\frac{a}{d}}}$$

- **Simulations based on 1000 observations length, we compare the true marginal density and the density estimated by the maximum likelihood :**

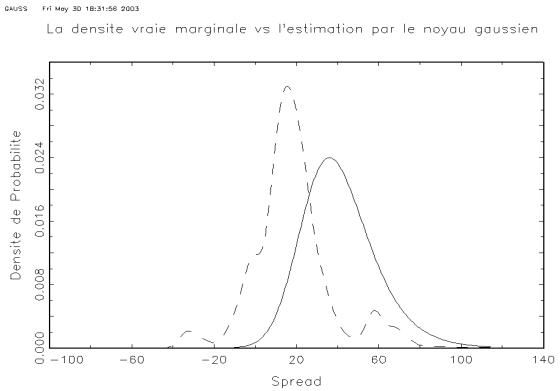


Validation

- **We compared for one bond of Citigroup the true marginal density represented by its nonparametric estimator, and the marginal density inferred by our estimation procedure :**



- **Unfortunately, the parametric family does not reproduce the bimodal shape. Densities for a General Electric Bond :**



Validation

- **The four parameters have an economic interpretation :**
  - **a : the mean reversion speed, belongs to [5, 40]**
  - **b : the long run limit, is equal to the mean spread level**
  - **c : the intrinsic volatility level for one bond at a given rating, belongs to [1000, 4000]**
  - **d : the absolute sensitivity of the volatility square to variation of the spread, belongs to [0, 5]**

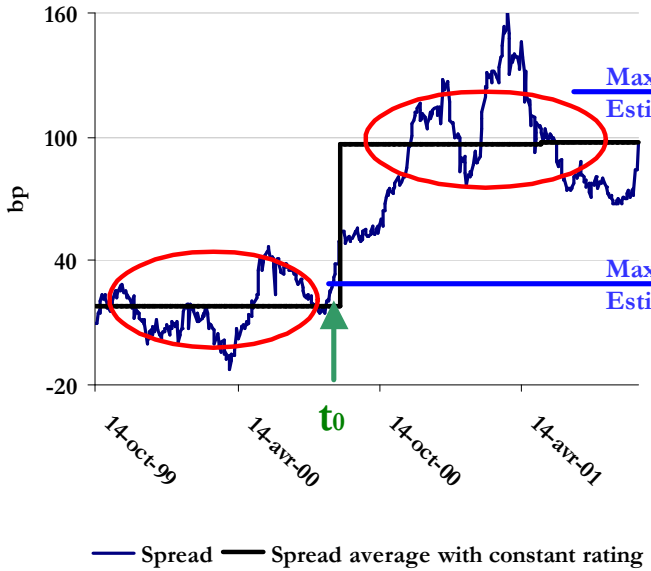
$$d = \frac{1}{2}\left(\frac{1}{S_t}\frac{\partial \sigma_t^2}{\partial S_t}\right)$$

VI Stability

Stability

Problematic

- **Is the four parameters characterising the spread dynamic for a constant rating, except the *b* obviously, are stable during rating changes ?**



Maximum Likelihood Estimation → $l_2 = a_2, c_2, d_2$

Are the two parameters equal ?

Maximum Likelihood Estimation → $l_1 = a_1, c_1, d_1$

$$s_t = s_{t-1} + a_1(b_1 - s_{t-1})\delta + \sqrt{c_1 + d_1 s_{t-1}^2}\sqrt{\delta}\,\varepsilon_t, t \prec t_0$$

$$s_t = s_{t-1} + a_2(b_2 - s_{t-1})\delta + \sqrt{c_2 + d_2 s_{t-1}^2}\sqrt{\delta}\,\varepsilon_t, t \succeq t_0$$

- **To answer this question, we established a Khi test. We made the assumption that the spreads before and after the rating change are independent, which is a reasonable hypothesis, if the lag between the two series is larger than 4. The null and the alternative hypothesis of the tree test are as follows:**

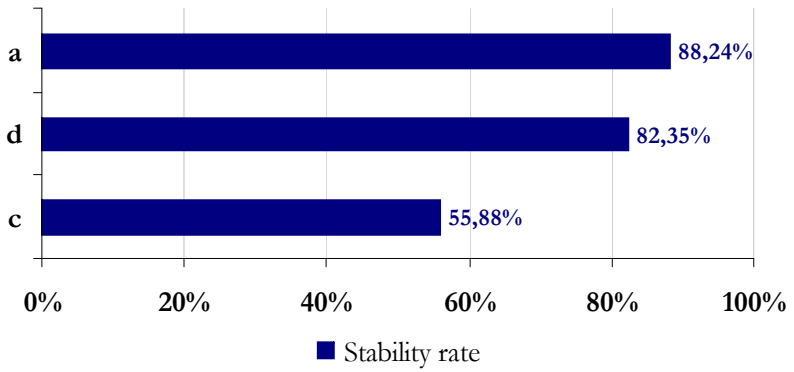
$$H_{01} : a_1 = a_2 \qquad H_{02} : c_1 = c_2 \qquad H_{03} : d_1 = d_2$$
$$H_1 : a_1 \neq a_2 \qquad H_2 : c_1 \neq c_2 \qquad H_1 : d_1 \neq d_2$$

Stability

- As we use a maximum likelihood estimator, then : $\hat{l}_1 \approx N(l_1, \sigma_1^2)$ and $\hat{l}_2 \approx N(l_2, \sigma_2^2)$
- The independence assumption ensure that the estimators are also independent, as it is a measurable function of two set of independent spreads. In consequence, under the null hypothesis :

$$\frac{(\hat{l}_1 - \hat{l}_2)^2}{\hat{\sigma}_1^2 + \hat{\sigma}_2^2} \underset{sous H0}{\approx} \chi^2(1)$$

- Good Test Results :



- The parameter ***c*** is less stable for several reasons :
  - GARCH effect, which will make it function of a lagged volatility
  - Change on the intrinsic level of volatility as the rating change

VII Identification

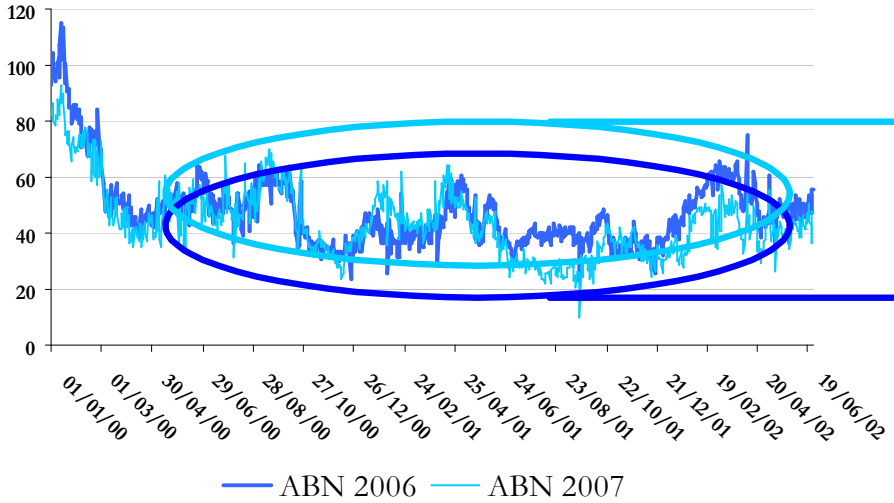
Identification

Problematic

- What we have done at present, allows us to simulate the spread evolution for one bond
- Effectively, we estimate the parameters with enough historical data, then
  - We simulate the rating change
  - We choose the corresponding mean level b
  - We simulate the spread using the differential equation
- Now we will consider situations where we want to simulate one bond which is :
  - Newly issued with a few historical prices
  - Illiquid, hence its historical market prices are not reliable
- In this section, we will attempt to answer the following questions :
  - Can we identify the parameters of two securities of the same issuer ?
  - Do we have the same parameters for two issuers belonging to the same sector, and with the same rating ?
- These properties will allow us to simulate the security with insufficient historical spread using the parameters of other securities of the same risk category, used as a proxy

Identification

Can we identify the parameters of two securities of the same issuer ?



$\theta_1 = (a_1, b_1, c_1, d_1)$

Are the two parameters equal ?

$\theta_2 = (a_2, b_2, c_2, d_2)$

- We can not make the independence assumption for the two estimators, as the two series are obviously correlated
- We set up a multidimensional diffusion model. Given n securities, the new dynamic is :

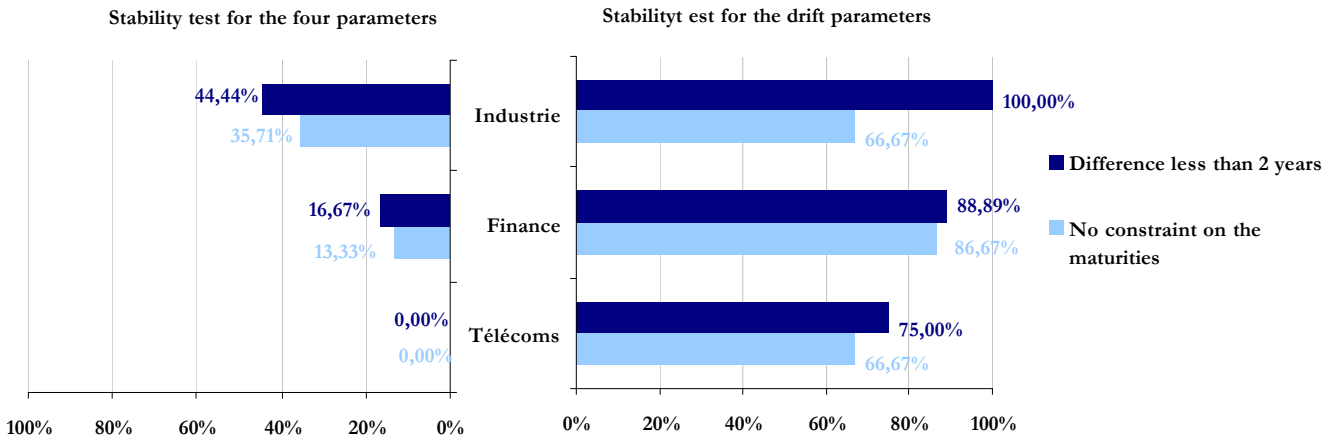
$$\begin{pmatrix} dS_{1,t} \\ \dots \\ dS_{n,t} \end{pmatrix} = \begin{pmatrix} a_1.(b_1 - S_{1,t}) \\ \dots \\ a_n.(b_n - S_{n,t}) \end{pmatrix}.dt + < \begin{pmatrix} \sqrt{c_1 + d_1.S_{1,t}^{\ 2}} \\ \dots \\ \sqrt{c_n + d_n.S_{n,t}^{\ 2}} \end{pmatrix}, \begin{pmatrix} dW_{1,t} \\ \dots \\ dW_{n,t} \end{pmatrix} > \quad \text{with} \quad < \begin{pmatrix} dW_{1,t} \\ \dots \\ dW_{n,t} \end{pmatrix}, \begin{pmatrix} dW_{1,t} \\ \dots \\ dW_{2,t} \end{pmatrix} >= \Omega.dt$$

- This model allow us to use a Maximum Likelihood Test

Identification

Can we identify the parameters of two securities of the same issuer ?

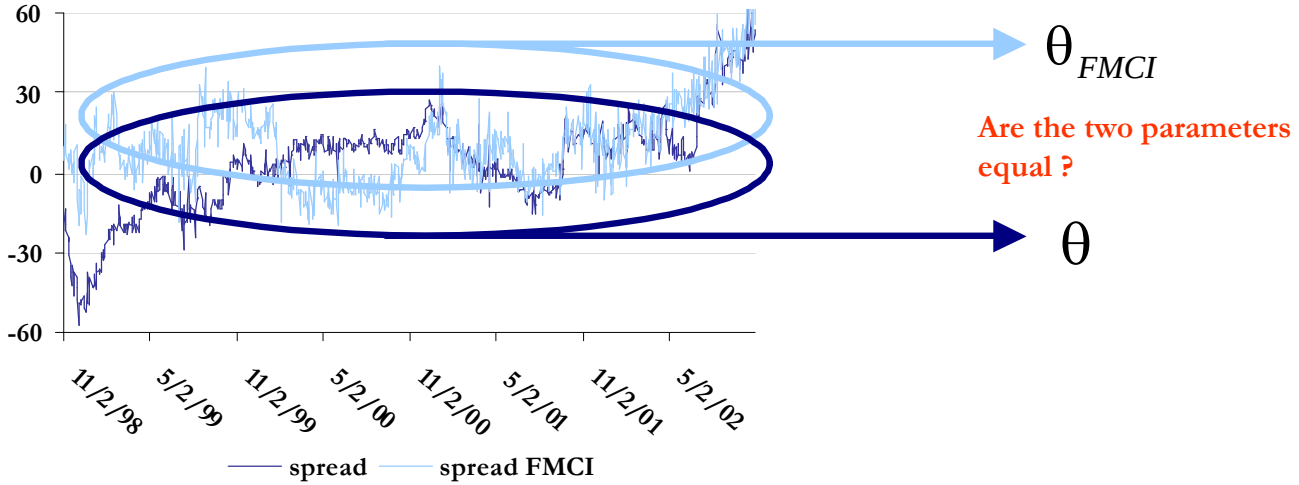
- We performed the test for 34 securities in Finance, Telecom and Industries Sector
- We noticed a great influence of the difference of maturity of the two bonds on the outcome of the test
- Results :



Identification

Fair Market Curve Index

- Fair Market value is a generic yield for a given sector, rating and residual maturity
- The yield at each maturity point represents the composite yield of securities around that maturity
- We will use the FMCI as a Benchmark for the risk category of one security



- The stability property is harder to obtain; only the drift parameters tend to remain stable sometimes, while the volatility parameters are systematically unstable.

Identification

Individual Effect

- We want to model the individual effect of each security compared to its FMCI curve reference
- We propose a multiplicative correction for the FMCI parameters, as in the Car Insurance field

$$S_t = S_{t-1} + a\upsilon_1(b\upsilon_2 - s_{t-1})\delta + \sqrt{c\upsilon_3 + d\upsilon_4 S_{t-1}^2}\sqrt{\delta}\,\varepsilon_t^1$$

$$F_t = F_{t-1} + a(b - F_{t-1})\delta + \sqrt{c + dF_{t-1}^2}\sqrt{\delta}\,\varepsilon_t^2$$

with $\begin{pmatrix}\varepsilon_t^1\\ \varepsilon_t^2\end{pmatrix} \approx N\left(\begin{pmatrix}0\\0\end{pmatrix},\begin{pmatrix}1 & \rho\\ \rho & 1\end{pmatrix}\right)$

- The estimation of one security and its FMCI parameters gives the estimation of the individual effect parameters :

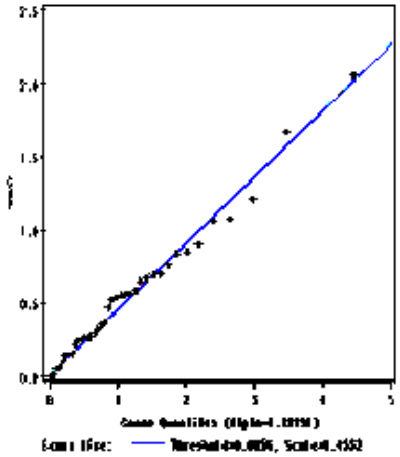
$$\hat{\upsilon}_1 = \frac{a\hat{\upsilon}_1}{\hat{a}}, \hat{\upsilon}_2 = \frac{b\hat{\upsilon}_2}{\hat{b}}, \hat{\upsilon}_3 = \frac{c\hat{\upsilon}_3}{\hat{c}}, \hat{\upsilon}_4 = \frac{d\hat{\upsilon}_4}{\hat{d}}$$

- If we can model the individual effect distribution, we can simulate the spread evolution :
  - We estimate the FMCI parameters
  - We draw four correction parameters $(\upsilon_1, \upsilon_2, \upsilon_3, \upsilon_4)$ from the individual effect distribution
  - We simulate the spread evolution corresponding to this individual effect

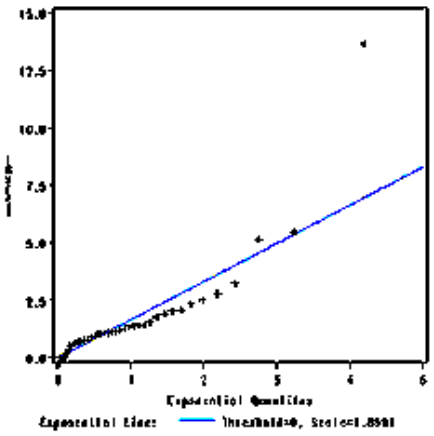
Identification

Individual Effect

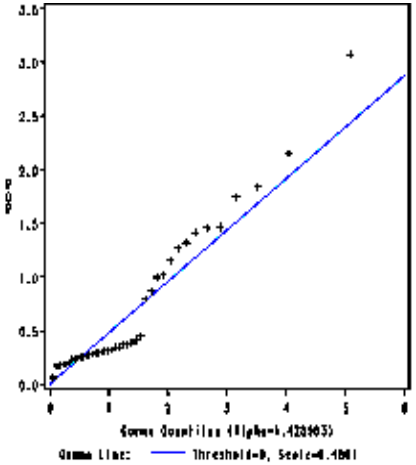
- **We have done the estimation for 41 securities. The Q-Q plot method gives :**



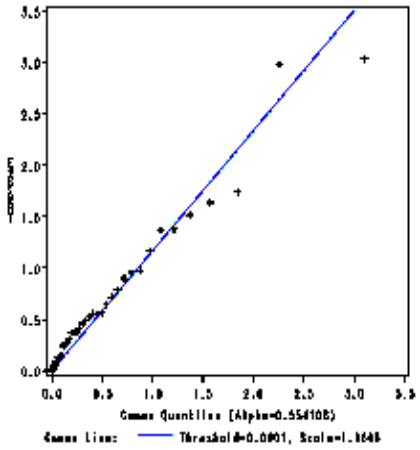
l'adéquation de la loi de v1
avec une loi Gamma



l'adéquation de la loi v2
avec une loi exponentiel



L'adéquation de la loi de v3
avec une loi Gamma



L'adéquation de la loi de v4
avec une loi Gamma

- **We have had a good results except the *C* parameter, matching the densities with Gamma and exponential distributions**

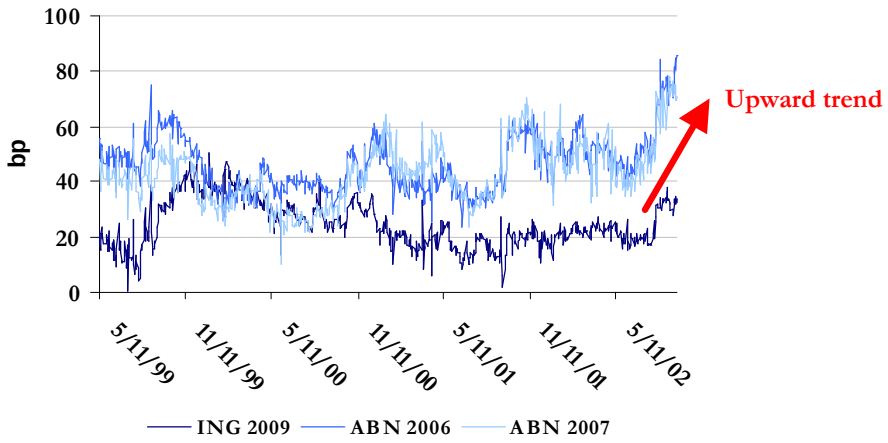
- **In order to model the joint distribution of the four parameters, we had to use the copulas theory results and to study the association measurement**

VIII VaR Measurement

VaR Measurement

Definition

- Value-at-Risk is a market risk management tool that provides a statistical measure of potential losses in the market value of a portfolio resulting from changes in the market over a specified time horizon
- Monte-Carlo VaR generates the market risk factor scenarios by randomly sampling joint distributions of the market factors
- There are several kinds of VaR approaches :
  - Market VaR approach does not take into account the rating changes with a very short horizon, typically a 10-day one
  - Credit VaR approach takes into account the rating changes, and their influence in spreads level, with a typical horizon ranging from one to five years
- We will take an example of a portfolio constituted by three bonds, and we will compare the results obtained by our Markov model, and the Gaussian models which are widely used by the markets participants



VaR Measurement

Portfolio

- **We depict the details of the portfolio, with the estimated parameters :**

Emetteur	Code Isin	a	b	c	d
ING	XS0097541212	18,15	24,71	2718	0,24
ABN Amro	US00077QAG55	25,61	46,61	2193	1,1
ABN Amro	US00077QAA85	26,12	43,15	2201	2,12

$$\begin{pmatrix} dS_{1,t} \\ dS_{2,t} \\ dS_{3,t} \end{pmatrix} = \begin{pmatrix} a_1.(b_1 - S_{1,t}) \\ a_2.(b_2 - S_{2,t}) \\ a_3.(b_3 - S_{3,t}) \end{pmatrix}.dt + < \begin{pmatrix} (c_1 + d_1.S_{1,t}{}^2)^{0.5} \\ (c_2 + d_2.S_{2,t}{}^2)^{0.5} \\ (c_3 + d_3.S_{3,t}{}^2)^{0.5} \end{pmatrix}, \begin{pmatrix} dW_{1,t} \\ dW_{2,t} \\ dW_{3,t} \end{pmatrix} >$$

with $< \begin{pmatrix} dW_{1,t} \\ \dots \\ dW_{n,t} \end{pmatrix}, \begin{pmatrix} dW_{1,t} \\ \dots \\ dW_{2,t} \end{pmatrix} >= \Omega dt$ **and** $\hat{\Omega} = \begin{pmatrix} 1 & 0.405 & 0.2964 \\ 0.405 & 1 & 0.6403 \\ 0.2964 & 0.6403 & 1 \end{pmatrix}$

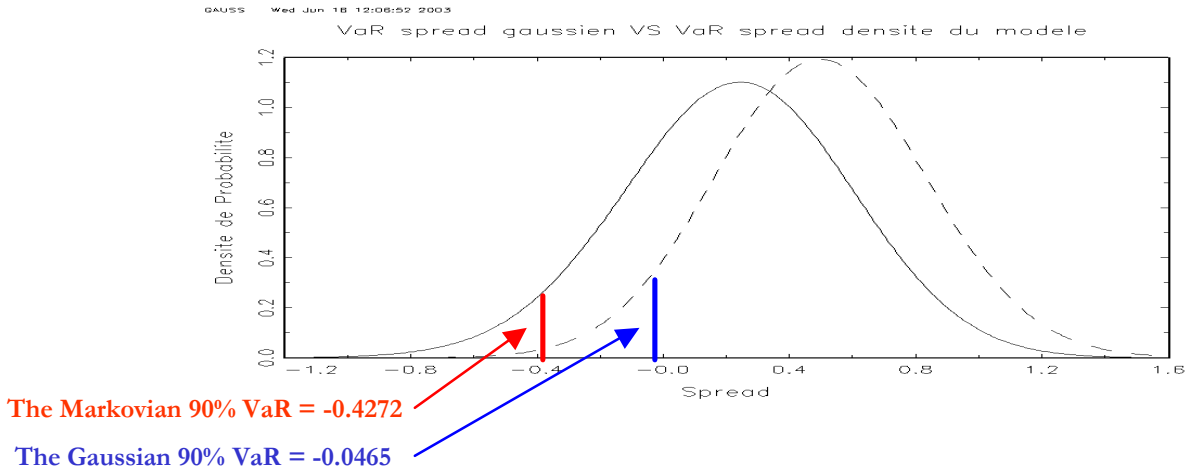
VaR Measurement

Market VaR : Gaussian Spread

- We suppose the spread vector follows a normal density, and each observation is drawn independently from this distribution. We estimate the normal density :

$$\begin{pmatrix} S_{1t} \\ S_{2t} \\ S_{3t} \end{pmatrix} \approx N\left(\begin{pmatrix} 24.55 \\ 46.41 \\ 42.86 \end{pmatrix}, \begin{pmatrix} 71.26 & 14.93 & -3.35 \\ 14.93 & 106.88 & 88.06 \\ -3.35 & 88.06 & 122.36 \end{pmatrix} \right)$$

- We have drawn the P&L profile in 10 days horizon for the gaussian and the markovian model :



- As spreads are in an upward trend, the historical mean of the credit spread is less than the actual spread level

- The gaussian model under-estimate the real VaR as it does not take into account the actual high spread level

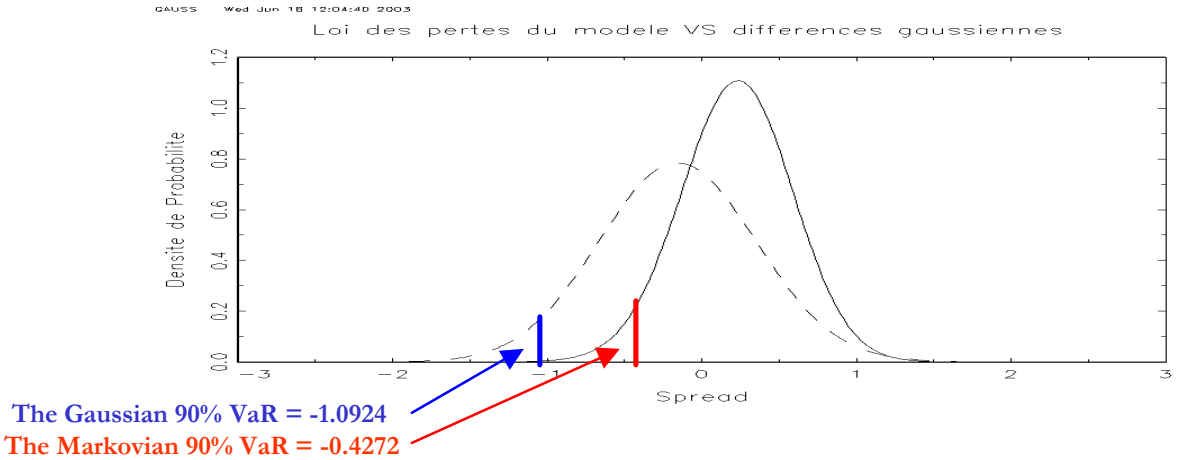
VaR Measurement

Market VaR : Spread variation

- We suppose the spread variation vector follows a normal density, and each observation is drawn independently from this distribution. This hypothesis leads to the spread normality, but with density parameters different from those of gaussian spread. For 10 days horizon, the estimated distribution is :

$$\begin{pmatrix} S_{1,t+10} \\ S_{2,t+10} \\ S_{3,t+10} \end{pmatrix} \approx \begin{pmatrix} 32.69 \\ 85.78 \\ 70.85 \end{pmatrix} + \sqrt{10}.N\left(\begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 13.83 & 11.52 & 10.99 \\ 11.52 & 24.29 & 17.66 \\ 10.99 & 17.66 & 30.10 \end{pmatrix} \right)$$

- We have drawn the P&L profile in 10 days horizon for the variation gaussian and the markovian model :



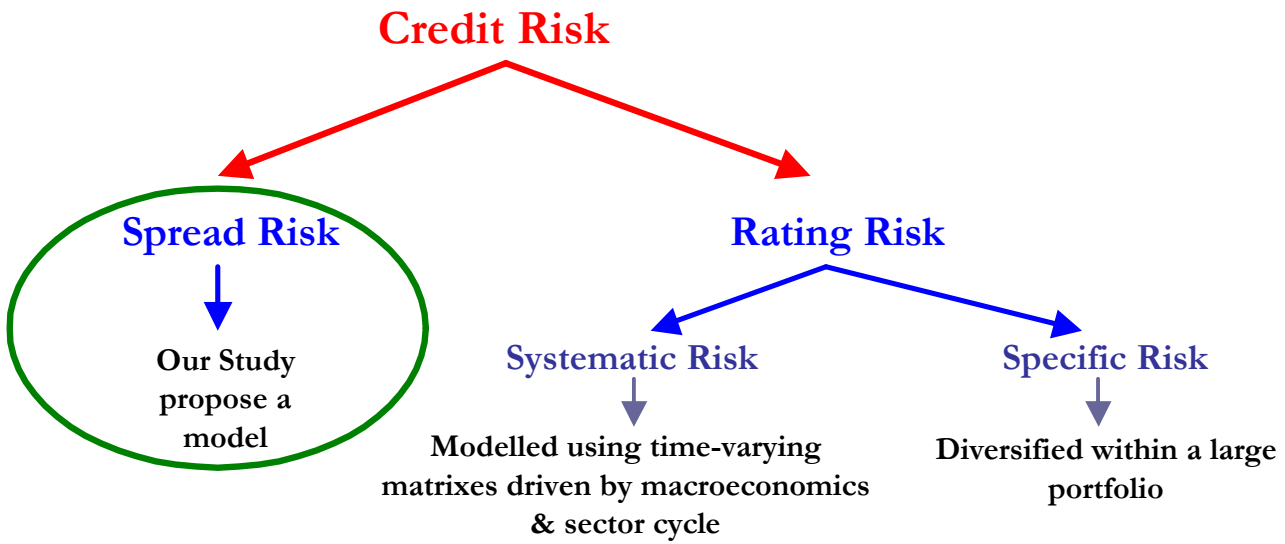
- The gaussian model over-estimates the real VaR as it does not take into account the mean reverting property of the spread process

- In consequence, a high spread compared to the its historical level will probably decrease, unless the rating changes !

VaR Measurement

Credit VaR

- We have establish a procedure to simulate the spread using a given rating transition matrix



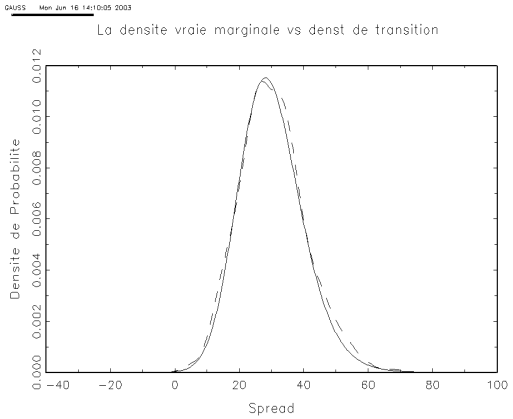
VaR Measurement

Credit VaR

- We know that for a stationary markovian chain, the transition density tend to the marginal density when the horizon of the transition tend to infinity :

$$X_T / X_0 \underset{T \longrightarrow \infty}{\overset{L}{\longrightarrow}} X_0$$

- We have measure the speed of this convergence for our spread diffusion process. We have found that one can make the assumption that the two densities are equal for an horizon greater than 50 business days :



- This approximation is crucial, as in the Monte-Carlo Var procedure, we are trying to estimate the distribution of the bond's spread in a certain horizon, conditionally to its spread level and rating today

- This approximation consist in neglecting the initial value effect and taking the marginal density as a proxy for the transition density

$$\Pi(S_T / S_t, R_t) = \Pi(S_T / R_t)$$

Note ou *Source*

VaR Measurement

Credit VaR

- **Using Bayes formula :** $$\Pi(S_T / R_t) = \sum_{i \in \{rating\}} \Pi(S_T / R_T = i, R_t).\Pi(R_T = i / R_t)$$
- **As the spread depends just on the present rating, we have :** $$\Pi(S_T / R_T = i, R_t) = \Pi(S_T / R_T = i)$$
- **Hence, we obtain the following close formula for the density distribution of the spread in a certain hoorizon conditionally to its present rating :**

$$\Pi(S_T / R_t) = \sum_{i \in \{rating\}} \Pi(S_T / R_T = i).\Pi(R_T = i / R_t)$$

- **This formula tells us that the spread density in the horizon T is the average of all the marginal densities which depend just on the rating, by the spread mean parameter *b*, weighted by the probability of being in a given rating conditionally to the present rating (rating transition matrix)**

Note ou *Source*

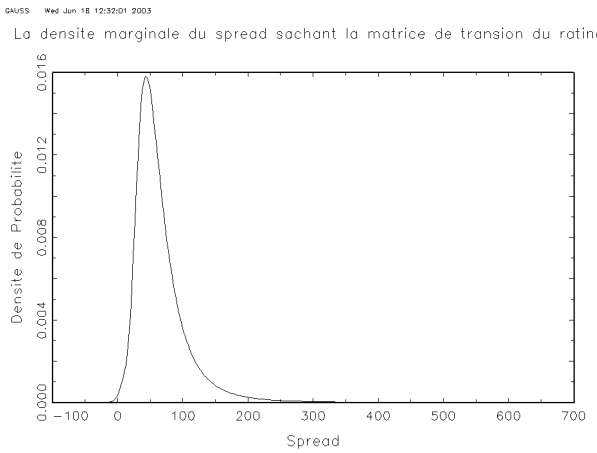
VaR Measurement

Credit VaR: Example

- We took a bond with an initial rating of A and some spread mean for each rating as follow :

Rating	Spread Moyen
AAA	5
AA	20
A	60
BBB	100
BB	150
B	200
CCC	300
D	800

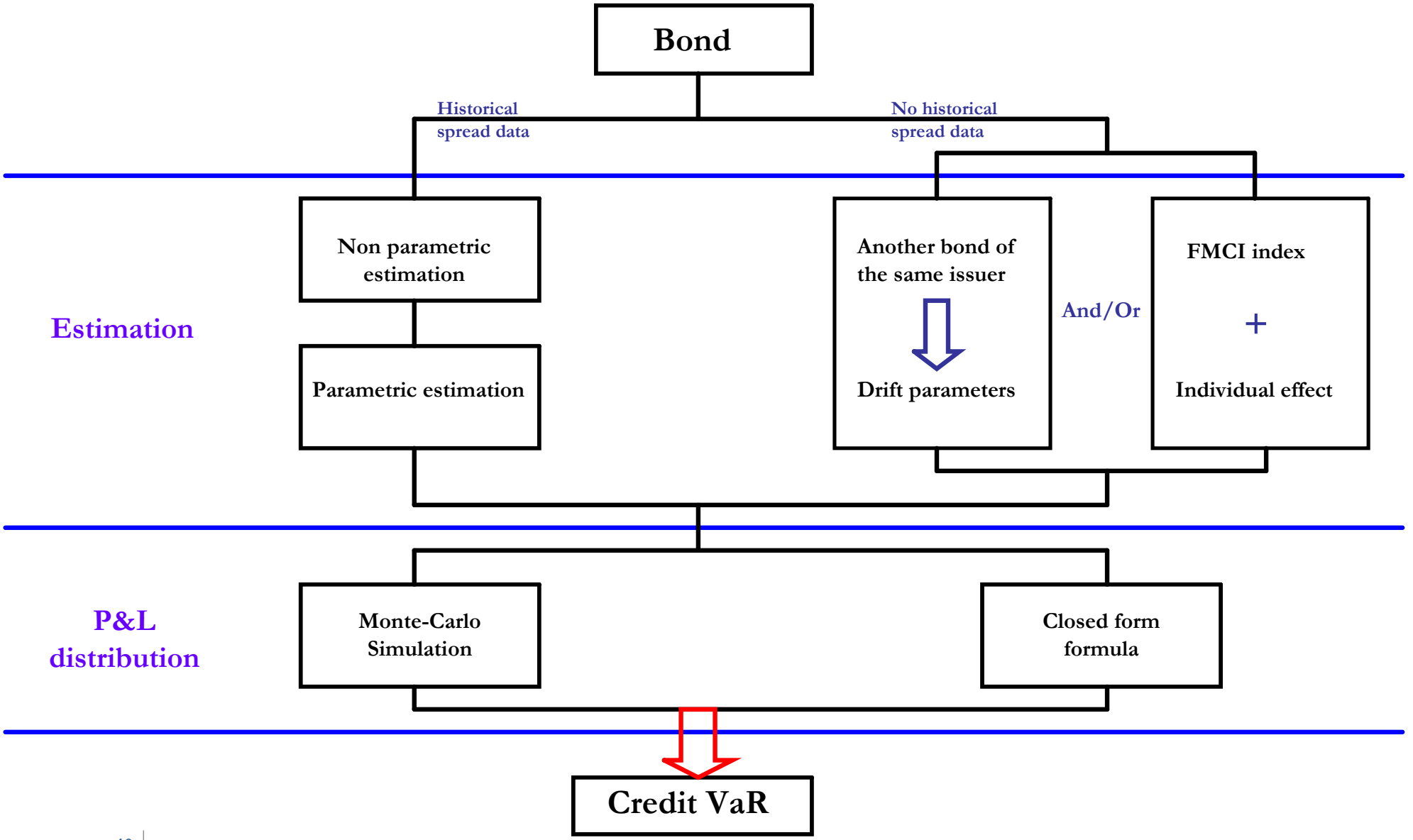
- The formula of the spread density in two years, using the S&P two years transition matrix gives :



- We calculate the 99% quintile at 225 bp, which gives the Var at 99 % confidence level, as the VaR is a monotone function of the spread

IX Conclusion

Conclusion


Bond
Historical spread data
No historical spread data
Estimation
Non parametric estimation
Parametric estimation
Another bond of the same issuer
Drift parameters
And/Or
FMCI index
+
Individual effect
P&L distribution
Monte-Carlo Simulation
Closed form formula
Credit VaR

Conclusion

- We can enrich our diffusion process by a GARCH effect which may overcome some model's drawbacks :
  - The *C* parameter's instability compared to the other parameters
  - The impossibility to reproduce the multi-modal densities

- In order to estimate the bond rating migration matrix, we can use :
  - the risk neutral transition matrix, with a forward looking approach calibrated with market bond prices, but incorporating a risk premium
  - the empirical transition matrix estimated with a set of latent credit cycle variable

- The non-stationarity of the markets is difficult to adjust for. We can enhance the VaR efficiency by using the data update techniques. Fox example, one could update for the volatility by multiplying the historical spreads by a factor $\frac{\hat{V}}{V}$, where V is the historical volatility and $\hat{V}$ is a recent volatility estimate

- The study was conducted with an investment grade bonds. It would be very interesting to see if the good obtained results remain for the speculative grade bonds, with a very stock correlated behaviour