

Valuing Companies by Cash Flow Discounting: Ten Methods and Nine Theories

Pablo Fernández
IESE Business School
Camino del Cerro del Aguila 3
28023 Madrid, Spain

Telephone 34-91-357 08 09
e-mail: fernandezpa@iese.edu

ABSTRACT

This paper is a summarized compendium of all the methods and theories on company valuation using cash flow discounting. The paper shows the ten most commonly used methods for valuing companies by cash flow discounting:

- 1) free cash flow discounted at the WACC;
- 2) equity cash flows discounted at the required return to equity;
- 3) Capital cash flows discounted at the WACC before tax;
- 4) APV (Adjusted Present Value);
- 5) the business's risk-adjusted free cash flows discounted at the required return to assets;
- 6) the business's risk-adjusted equity cash flows discounted at the required return to assets;
- 7) economic profit discounted at the required return to equity;
- 8) EVA discounted at the WACC;
- 9) the risk-free rate-adjusted free cash flows discounted at the risk-free rate; and
- 10) the risk-free rate-adjusted equity cash flows discounted at the required return to assets.

All ten methods always give the same value. This result is logical, since all the methods analyze the same reality under the same hypotheses; they only differ in the cash flows taken as starting point for the valuation.

The disagreements in the various theories on the valuation of the firm arise from the calculation of the value of the tax shields (VTS). The paper shows and analyses 9 different theories on the calculation of the VTS: No-cost-of-leverage, Modigliani and Miller (1963), Myers (1974), Miller (1977), Miles and Ezzell (1980), Harris and Pringle (1985), Damodaran (1994), With-cost-of-leverage and Practitioners method.

The paper contains the most important valuation equations according to these theories, and also shows the changes that take place in the valuation equations when the debt's market value does not match its book value.

JEL Classification: G12, G31, M21

November 19, 2001

Valuing companies by cash flow discounting: Ten methods and nine theories

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Section 1 shows the ten most commonly used methods for valuing companies by cash flow discounting:

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- 3) Capital cash flows discounted at the WACC before tax;
- 4) APV (Adjusted Present Value);
- 5) the business's risk-adjusted free cash flows discounted at the required return to assets;
- 6) the business's risk-adjusted equity cash flows discounted at the required return to assets;
- 7) economic profit discounted at the required return to equity;
- 8) EVA discounted at the WACC;
- 9) the risk-free rate-adjusted free cash flows discounted at the risk-free rate; and
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Section 2 is the application of the ten methods and nine theories to an example. The nine theories are:

- 1) No-cost-of-leverage. Assuming that there are no leverage costs.
- 0) Damodaran (1994). To introduce leverage costs, he assumes that the relationship between the levered and unlevered beta is¹:
$$b_L = b_U + D (1-T) b_U / E$$
- 3) Practitioners method. To introduce higher leverage costs, this method assumes that the relationship between the levered and unlevered beta is:
$$b_L = b_U + D b_U / E$$
- 4) Harris and Pringle (1985) and Ruback (1995). All of their equations arise from the assumption that the leverage-driven value creation or value of tax shields (VTS) is the present value of the tax shields² discounted at the required return to the unlevered equity (K_U). According to them,
$$VTS = PV[D K_d T ; K_U]$$
- 5) Myers (1974) who assumes that the value of tax shields (VTS) is the present value of the tax shields discounted at the required return to debt (K_d). According to Myers
$$VTS = PV[D K_d T ; K_d]$$
- 6) Miles and Ezzell (1980). They state that the correct rate for discounting the tax shield ($D K_d T$) is K_d for the first year, and K_U for the following years.

¹ Instead of the relationship obtained from No-cost-of-leverage: $b_L = b_U + D (1-T) (b_U - b_D) / E$

² The tax shield of a given year is $D K_d T$. D is the value of debt, K_d is the required return to debt, and T is the corporate tax rate. $D K_d$ are the interest paid in a given year. The formulas used in the paper are valid if the debt's interest rate matches the required return to debt (K_d), or, to put it another way, if the debt's market value is identical to its book value. The formulas for the case in which this does not happen are given in Appendix 3.

- 7) Miller (1977) concludes that the leverage-driven value creation or value of the tax shields is zero.
- 8) With-cost-of leverage. This theory assumes that the cost of leverage is the present value of the interest differential that the company pays over the risk-free rate.
- 9) Modigliani and Miller (1963) calculate the value of tax shields by discounting the present value of the tax savings due to interest payments of a risk-free debt ($T D R_F$) at the risk-free rate (R_F).
Modigliani and Miller claim that

$$VTS = PV[R_F; DT R_F]$$

Appendix 1 is a brief overview of the most significant theories on the discounted cash flow valuation

Appendix 2 contains the valuation equations according to these theories.

Appendix 3 shows the changes that take place in the valuation equations when the debt's value does not match its nominal value.

Appendix 4 contains the dictionary of the initials used in the paper.

1. Ten methods for valuing companies by cash flow discounting

There are four basic methods for valuing companies by cash flow discounting:

Method 1. Using the free cash flow and the WACC (weighted average cost of capital).

Equation [1] indicates that the value of the debt (D) plus that of the shareholders' equity (E) is the present value of the expected free cash flows (FCF) that the company will generate, discounted at the weighted average cost of debt and shareholders' equity after tax (WACC):

$$[1] \quad E_0 + D_0 = PV_0 [WACC_t; FCF_t]$$

The definition of WACC or "weighted average cost of capital", is given by [2]:

$$[2] \quad WACC_t = [E_{t-1} Ke_t + D_{t-1} Kd_t (1-T)] / [E_{t-1} + D_{t-1}]$$

Ke is the required return to equity, Kd is the cost of the debt and T is the effective tax rate applied to earnings. $E_{t-1} + D_{t-1}$ are market values³.

Method 2. Using the expected equity cash flow (ECF) and the required return to equity (Ke).

Equation [3] indicates that the value of the equity (E) is the present value of the expected equity cash flows (ECF) discounted at the required return to equity (Ke).

$$[3] \quad E_0 = PV_0 [Ke_t; ECF_t]$$

³ In actual fact, "market values" are the values obtained when the valuation is performed using formula [1]. Consequently, the valuation is an iterative process: the free cash flows are discounted at the WACC to calculate the company's value (D+E) but, in order to obtain the WACC, we need to know the company's value (D+E).

Equation [4] indicates that the value of the debt (D) is the present value of the expected debt cash flows (CFd) discounted at the required return to debt (Kd).

$$[4] \quad D_0 = PV_0 [K_d; CF_d]$$

The expression that relates the FCF with the ECF is⁴:

$$[5] \quad ECF_t = FCF_t + DD_t - I_t (1 - T)$$

DD_t is the increase in debt. I_t is the interest paid by the company.

It is obvious that $CF_d = I_t - DD_t$

The sum of the values given by equations [3] and [4] is identical to the value provided by [1]:⁵

$$E_0 + D_0 = PV_0 [WACC; FCF_t] = PV_0 [K_e; ECF_t] + PV_0 [K_d; CF_d]$$

Method 3. Using the capital cash flow (CCF) and the $WACC_{BT}$ (weighted average cost of capital, before tax).

The capital cash flows⁶ are the cash flows available for all holders of the company's securities, whether these be debt or shares, and are equivalent to the equity cash flow (ECF) plus the cash flow corresponding to the debt holders (CFd).

Equation [6] indicates that the value of the debt today (D) plus that of the shareholders' equity (E), is equal to the capital cash flow (CCF) discounted at the weighted average cost of debt and shareholders' equity before tax ($WACC_{BT}$).

$$[6] \quad E_0 + D_0 = PV[WACC_{BT}; CCF_t]$$

The definition of $WACC_{BT}$ is [7]:

$$[7] \quad WACC_{BT_t} = [E_{t-1} K_e + D_{t-1} K_d] / [E_{t-1} + D_{t-1}]$$

The expression [7] is obtained by making [1] equal to [6]. $WACC_{BT}$ represents the discount rate that ensures that the value of the company obtained using the two expressions is the same⁷:

$$E_0 + D_0 = PV[WACC_{BT}; CCF_t] = PV[WACC; FCF_t]$$

The expression that relates the CCF with the ECF and the FCF is [8]:

$$[8] \quad CCF_t = ECF_t + CF_d = ECF_t - DD_t + I_t = FCF_t + I_t T$$

$$DD_t = D_t - D_{t-1} \quad ; \quad I_t = D_{t-1} K_d$$

⁴ Obviously, the free cash flow is the hypothetical equity cash flow when the company has no debt.

⁵ Indeed, one way of defining the WACC is: the WACC is the rate at which the FCF must be discounted so that equation [2] gives the same result as that given by the sum of [3] and [4].

⁶ Arditti and Levy (1977) suggested that the firm's value could be calculated by discounting the Capital Cash Flows instead of the Free Cash Flow.

⁷ One way of defining the $WACC_{BT}$ is: the $WACC_{BT}$ is the rate at which the CCF must be discounted so that equation [6] gives the same result as that given by the sum of [3] and [4].

Method 4. Adjusted present value (APV)

The adjusted present value (APV) equation [9] indicates that the value of the debt (D) plus that of the shareholders' equity (E) is equal to the value of the unlevered company's shareholders' equity V_u plus the present value of the value of the tax shield (VTS):

$$[9] \quad E_0 + D_0 = V_{u_0} + DVTS_0$$

We can see in Appendix 1 and 2 that there are several theories for calculating the VTS.

If K_u is the required return to equity in the debt-free company (also called the required return to assets), V_u is given by [10]:

$$[10] \quad V_{u_0} = PV_0 [K_u; FCF_t]$$

Consequently,

$$DVTS_0 = E_0 + D_0 - V_{u_0} = PV_0 [WACC_t; FCF_t] - PV_0 [K_u; FCF_t]$$

We can talk of a fifth method (using the business risk-adjusted free cash flow), although this is not actually a new method but is derived from the previous methods:

Method 5. Using the business risk-adjusted free cash flow and K_u (required return to assets).

Equation [11] indicates that the value of the debt (D) plus that of the shareholders' equity (E) is the present value of the expected business risk-adjusted free cash flows ($FCF \backslash K_u$) that will be generated by the company, discounted at the required return to assets (K_u):

$$[11] \quad E_0 + D_0 = PV_0 [K_u; FCF_t \backslash K_u]$$

The definition of the business risk-adjusted free cash flows⁸ ($FCF \backslash K_u$) is [12]:

$$[12] \quad FCF_t \backslash K_u = FCF_t - (E_{t-1} + D_{t-1}) [WACC_t - K_u]$$

Likewise, we can talk of a sixth method (using the business risk-adjusted equity cash flow), although this is not actually a new method but is derived from the previous methods:

Method 6. Using the business risk-adjusted equity cash flow and K_u (required return to assets).

Equation [13] indicates that the value of the equity (E) is the present value of the expected business risk-adjusted equity cash flows ($ECF \backslash K_u$) discounted at the required return to assets (K_u):

$$[13] \quad E_0 = PV_0 [K_u; ECF_t \backslash K_u]$$

The definition of the business risk-adjusted equity cash flows⁹ ($ECF \backslash K_u$) is [14]:

⁸ The expression [12] is obtained by making [11] equal to [1].

⁹ The expression [14] is obtained by making [13] equal to [3].

$$[14] ECF_t \backslash Ku = ECF_t - E_{t-1} [K_e - K_u]$$

Method 7. Using the economic profit and Ke (required return to equity).

Equation [15] indicates that the value of the equity (E) is the equity's book value plus the present value of the expected economic profit (EP) discounted at the required return to equity (Ke).

$$[15] E_0 = E_{vc_0} + PV_0 [K_e; EP_t]$$

The term economic profit (EP) is used to define the accounting net income or profit after tax (PAT) less the equity's book value ($E_{bv_{t-1}}$) multiplied by the required return to equity.

$$[16] EP_t = PAT_t - K_e E_{bv_{t-1}}$$

Method 8. Using the EVA (economic value added) and the WACC (weighted average cost of capital).

Equation [17] indicates that the value of the debt (D) plus that of the shareholders' equity (E) is the book value of the shareholders' equity and the debt ($E_{bv_0} + N_0$) plus the present value of the expected EVA, discounted at the weighted average cost of capital (WACC):

$$[17] E_0 + D_0 = (E_{bv_0} + N_0) + PV_0 [WACC_t; EVA_t]$$

The EVA (economic value added) is the NOPAT (Net Operating Profit After Tax) less the company's book value ($D_{t-1} + E_{vc_{t-1}}$) multiplied by the weighted average cost of capital (WACC). The NOPAT (Net Operating Profit After Taxes) is the profit of the unlevered company (debt-free).

$$[18] EVA_t = NOPAT_t - (D_{t-1} + E_{bv_{t-1}})WACC_t$$

Method 9. Using the risk-free-adjusted free cash flows discounted at the risk-free rate

Equation [19] indicates that the value of the debt (D) plus that of the shareholders' equity (E) is the present value of the expected risk-free-adjusted free cash flows ($FCF \backslash R_f$) that will be generated by the company, discounted at the risk-free rate (R_f):

$$[19] E_0 + D_0 = PV_0 [R_f; FCF_t \backslash R_f]$$

The definition of the risk-free-adjusted free cash flows¹⁰ ($FCF \backslash R_f$) is [20]:

$$[20] FCF_t \backslash R_f = FCF_t - (E_{t-1} + D_{t-1}) [WACC_t - R_f]$$

Likewise, we can talk of a tenth method (using the risk-free-adjusted equity cash flow), although this is not actually a new method but is derived from the previous methods:

Method 10. Using the risk-free-adjusted equity cash flows discounted at the risk-free rate

Equation [21] indicates that the value of the equity (E) is the present value of the expected risk-free-adjusted equity cash flows ($ECF_{t|R_F}$) discounted at the risk-free rate (R_F):

$$[21] \quad E_0 = PV_0 [R_F; ECF_{t|R_F}]$$

The definition of the risk-free-adjusted equity cash flows¹¹ ($ECF_{t|R_F}$) is [22]:

$$[22] \quad ECF_{t|R_F} = ECF_t - E_{t-1} [K_{e_t} - R_{F_t}]$$

We could also talk of a eleventh method; using the business risk-adjusted capital cash flow and K_u (required return to assets), but the business risk-adjusted capital cash flow is identical to the business risk-adjusted free cash flow ($CCF_{t|K_u} = FCF_{t|K_u}$). Therefore, this method would be identical to Method 5.

We could also talk of a twelfth method; using the risk-free-adjusted capital cash flow and R_F (risk-free rate), but the risk-free-adjusted capital cash flow is identical to the risk-free-adjusted free cash flow ($CCF_{t|R_F} = FCF_{t|R_F}$). Therefore, this method would be identical to Method 9.

2. An example. Valuation of the company Toro Inc.

The company Toro Inc. has forecast the balance sheets and income statements for the next few years, as shown in Table 1. After year 3, it is expected that the balance sheet and the income statement will grow at an annual rate of 2%.

Table 1. Balance sheet and income statement forecasts for Toro Inc.

	0	1	2	3	4	5
WCR (working capital requirements)	400	430	515	550	561.00	572.22
Gross fixed assets	1,600	1,800	2,300	2,600	2,913.00	3,232.26
- accumulated depreciation		200	450	720	995.40	1,276.31
Net fixed assets	1,600	1,600	1,850	1,880	1,917.60	1,955.95
TOTAL ASSETS	2,000	2,030	2,365	2,430	2,478.60	2,528
Debt (N)	1,500	1,500	1,500	1,500	1,530.00	1,560.60
Equity (book value)	500	530	865	930	948.60	967.57
TOTAL LIABILITIES	2,000	2,030	2,365	2,430	2,478.60	2,528
Income statement						
Margin		420	680	740	765.00	780
Interest payments		120	120	120	120.00	122
PBT (profit before tax)		300	560	620	645.00	658
Taxes		105	196	217	225.75	230.27
PAT (profit after tax = net income)		195	364	403	419.25	427.64

¹⁰ The expression [20] is obtained by making [19] equal to [1].

¹¹ The expression [22] is obtained by making [21] equal to [3].

Using the forecast balance sheets and income statements in Table 1, we can readily obtain the cash flows given in Table 2. Obviously, the cash flows grow at a rate of 2% after year 4.

Table 2. Cash flow forecasts for Toro Inc

	1	2	3	4	5
PAT (profit after tax)	195	364	403	419.25	427.64
+ depreciation	200	250.00	270.00	275.40	280.91
+ increase of debt	0	0.00	0.00	30.00	30.60
- increase of working capital requirements	-30	-85	-35	-11	-11.22
- investment in fixed assets	-200	-500.00	-300.00	-313.00	-319.26
ECF	165.00	29.00	338.00	400.65	408.66
FCF	243.00	107.00	416.00	448.65	457.62
CFd	120.00	120.00	120.00	90.00	91.80
CCF	285.00	149.00	458.00	490.65	500.46

The unlevered beta (β_u) is 1. The risk-free rate is 6%. The cost of debt is 8%. The corporate tax rate is 35%. The market risk premium is 4%. Consequently, using the CAPM, the required return to assets is 10%.¹² With these parameters, the valuation of this company's equity, using the above equations, is given in Table 3. The required return to equity (K_e) appears in the second line of the table¹³. Equation [3] enables the equity's value to be obtained by discounting the equity cash flows at the required return to equity (K_e)¹⁴. Likewise, equation [4] enables the debt's value to be obtained by discounting the debt cash flows at the required return to debt (K_d)¹⁵. Another way to calculate the equity's value is using equation [1]. The present value of the free cash flows discounted at the WACC (equation [2]) gives us the value of the company, which is the value of the debt plus that of the equity¹⁶. By subtracting the value of the debt from this quantity, we obtain the value of the equity. Another way of calculating the equity's value is using equation [6]. The present value of the capital cash flows discounted at the $WACC_{BT}$ (equation [7]) gives us the value of the company, which is the value of the debt plus that of the equity. By subtracting the value of the debt from this quantity, we obtain the value of the equity. The fourth method for calculating the value of the equity is using the Adjusted Present Value, equation [9]. The company's value is the sum of the value of the unlevered company (equation [10]) plus the present value of the value of the tax shield (VTS)¹⁷.

The business risk-adjusted equity cash flow and free cash flow (ECF_{β_Ku} and FCF_{β_Ku}) are also calculated using equations [14] and [12]. Equation [13] enables the equity's value to be obtained by discounting the business risk-adjusted equity cash flows at the required return to assets (K_u). Another way to calculate the equity's value is using equation [11]. The present value of the business risk-adjusted free cash flows discounted at the required return to assets (K_u) gives us the value of the company, which is the value of the debt plus that of the equity. By subtracting the value of the debt from this quantity, we obtain the value of the equity.

¹² In this example, we use the CAPM: $K_u = R_f + \beta_u P_M = 6\% + 4\% = 10\%$.

¹³ The required return to equity (K_e) has been calculated according to the no-cost-of-leverage theory (see Appendix 1).

¹⁴ The relationship between the value of the equity in two consecutive years is: $E_t = E_{t-1} (1 + K_e) - ECF_t$

¹⁵ The value of the debt matches the nominal value (book value) given in Table 1 because we have considered that the required return to debt matches its cost (8%).

¹⁶ The relationship between the company's value in two consecutive years is:

$$(D+E)_t = (D+E)_{t-1} (1 + WACC_t) - FCF_t$$

¹⁷ As the required return to equity (K_e) has been calculated according to the no-cost-of-leverage theory, we must also calculate the VTS according to the no-cost-of-leverage theory, namely: $VTS = PV(K_u; D \text{ T } K_u)$

The economic profit (EP) is calculated using equation [16]. Equation [15] indicates that the value of the equity (E) is the equity's book value plus the present value of the expected economic profit (EP) discounted at the required return to equity (K_e).

The EVA (economic value added) is calculated using equation [18]. Equation [17] indicates that plus that the equity value (E) is the present value of the expected EVA discounted at the weighted average cost of capital (WACC), plus the book value of the equity and the debt ($Ebv_0 + N_0$) minus the value of the debt (D).

The risk-free-adjusted equity cash flow and free cash flow ($ECF \backslash R_f$ and $FCF \backslash R_f$) are also calculated using equations [22] and [20]. Equation [21] enables the equity's value to be obtained by discounting the risk-free-adjusted equity cash flows at the risk-free rate (R_f). Another way to calculate the equity's value is using equation [19]. The present value of the risk-free-adjusted free cash flows discounted at the required return to assets (R_f) gives us the value of the company, which is the value of the debt plus that of the equity. By subtracting the value of the debt from this quantity, we obtain the value of the equity.

The example of Table 3 shows that the result obtained with all ten valuations is the same. The equity's value today is 3,958.96. As we have already mentioned, these valuations have been performed according to No-cost-of-leverage's theory. The valuations performed using other theories are discussed further on.

Table 3. Valuation of Toro Inc. No cost of leverage

		0	1	2	3	4	5
		K_u	10.00%	10.00%	10.00%	10.00%	10.00%
		K_e	10.49%	10.46%	10.42%	10.41%	10.41%
equation							
[1]	$E + D = PV(WACC; FCF)$	5,458.96	5,709.36	6,120.80	6,264.38	6,389.66	6,517.46
[2]	WACC	9.04%	9.08%	9.14%	9.16%	9.16%	9.16%
	[1] - D = E	3,958.96	4,209.36	4,620.80	4,764.38	4,859.66	4,956.86
[3]	E = PV(K_e; ECF)	3,958.96	4,209.36	4,620.80	4,764.38	4,859.66	4,956.86
[4]	$D = PV(CFd; K_d)$	1,500.00	1,500.00	1,500.00	1,500.00	1,530.00	1,560.60
[6]	$D + E = PV(WACC_{BT}; CCF)$	5,458.96	5,709.36	6,120.80	6,264.38	6,389.66	6,517.46
[7]	$WACC_{BT}$	9.81%	9.82%	9.83%	9.83%	9.83%	9.83%
	[6] - D = E	3,958.96	4,209.36	4,620.80	4,764.38	4,859.66	4,956.86
[10]	$VTS = PV(K_u; D \ T \ K_u)$	623.61	633.47	644.32	656.25	669.38	682.76
	$V_u = PV(K_u; FCF)$	4,835.35	5,075.89	5,476.48	5,608.12	5,720.29	5,834.69
[9]	$VTS + V_u$	5,458.96	5,709.36	6,120.80	6,264.37	6,389.66	6,517.46
	[9] - D = E	3,958.96	4,209.36	4,620.80	4,764.37	4,859.66	4,956.86
[11]	$D + E = PV(K_u; FCF \backslash K_u)$	5,458.96	5,709.36	6,120.80	6,264.37	6,389.66	6,517.46
[12]	$FCF \backslash K_u$		295.50	159.50	468.50	501.15	511.17
	[11] - D = E	3,958.96	4,209.36	4,620.80	4,764.38	4,859.66	4,956.86
[13]	E = PV(K_u; ECF $\backslash K_u$)	3,958.96	4,209.36	4,620.80	4,764.38	4,859.66	4,956.86
[14]	$ECF \backslash K_u$		145.50	9.50	318.50	381.15	388.77
[16]	EP		142.54	308.54	312.85	322.44	328.89
	$PV(K_e; EP)$	3,458.96	3,679.36	3,755.80	3,834.38	3,911.06	3,989.28
[15]	PV(K_e; EP) + $E_{vc} = E$	3,958.96	4,209.36	4,620.80	4,764.38	4,859.66	4,956.86
[18]	EVA		92.23	257.67	264.79	274.62	280.11
	$PV(WACC; EVA)$	3,458.96	3,679.36	3,755.80	3,834.38	3,911.06	3,989.28
[17]	E = PV(WACC; EVA) + $E_{bv} + N - D$	3,958.96	4,209.36	4,620.80	4,764.38	4,859.66	4,956.86
[19]	$D + E = PV(R_f; FCF \backslash R_f)$	5,458.96	5,709.36	6,120.80	6,264.38	6,389.66	6,517.46
[20]	$FCF \backslash R_f$		77.14	-68.87	223.67	250.58	255.59
	[19] - D = E	3,958.96	4,209.36	4,620.80	4,764.38	4,859.66	4,956.86
[21]	E = PV(R_f; ECF $\backslash R_f$)	3,958.96	4,209.36	4,620.80	4,764.38	4,859.66	4,956.86
[22]	$ECF \backslash R_f$		-12.86	-158.87	133.67	190.58	194.39

Tables 4 to 11 contain the most salient results of the valuation performed on the company Toro Inc. according to Damodaran (1994), practitioners method, Harris and Pringle (1985), Myers (1974), Miles and Ezzell (1980), Miller (1977), With-cost-of-leverage theory, and Modigliani and Miller (1963).

Table 4. Valuation of Toro Inc. according to Damodaran (1994)

	0	1	2	3	4	5
$VTS = PV[Ku; DTKu - D(Kd - R_f)(1-T)]$	391.98	398.18	405.00	412.50	420.75	429.16
b	1.261581	1.245340	1.222528	1.215678	1.215678	1.215678
Ke	11.05%	10.98%	10.89%	10.86%	10.86%	10.86%
E	3,727.34	3,974.07	4,381.48	4,520.62	4,611.04	4,703.26
WACC	9.369%	9.397%	9.439%	9.452%	9.452%	9.452%
WACC _{BT}	10.172%	10.164%	10.153%	10.149%	10.149%	10.149%
E+D	5,227.34	5,474.07	5,881.48	6,020.63	6,141.04	6,263.86
EVA		85.63	251.24	257.77	267.57	272.92
EP		139.77	305.80	308.80	318.23	324.59
ECF\Ku		126.00	-10.00	299.00	361.65	368.88
FCF\Ku		276.00	140.00	449.00	481.65	491.28
ECF\R _f		-23.09	-168.96	123.74	180.83	184.44
FCF\R _f		66.91	-78.96	213.74	240.83	245.64

Table 5. Valuation of Toro Inc. according to the practitioners method

	0	1	2	3	4	5
$VTS = PV[Ku; T D Kd - D(Kd - R_f)]$	142.54	144.79	147.27	150.00	153.00	156.06
b	1.431296	1.403152	1.363747	1.352268	1.352268	1.352268
Ke	11.73%	11.61%	11.45%	11.41%	11.41%	11.41%
E	3,477.89	3,720.68	4,123.75	4,258.13	4,343.29	4,430.15
WACC	9.759%	9.770%	9.787%	9.792%	9.792%	9.792%
WACC _{BT}	10.603%	10.575%	10.533%	10.521%	10.521%	10.521%
E+D	4,977.89	5,220.68	5,623.75	5,758.13	5,873.29	5,990.75
EVA		77.82	243.67	249.55	259.31	264.50
EP		136.37	302.45	303.91	313.15	319.41
ECF\Ku		105.00	-31.00	278.00	340.65	347.46
FCF\Ku		255.00	119.00	428.00	460.65	469.86
ECF\R _f		-34.12	-179.83	113.05	170.33	173.73
FCF\R _f		55.88	-89.83	203.05	230.33	234.93

Table 6. Valuation of Toro Inc. according to Harris and Pringle (1985), and Ruback (1995)

	0	1	2	3	4	5
$VTS = PV[Ku; T D Kd]$	498.89	506.78	515.45	525.00	535.50	546.21
b	1.195606	1.183704	1.166966	1.161878	1.161878	1.161878
Ke	10.78%	10.73%	10.67%	10.65%	10.65%	10.65%
E	3,834.24	4,082.67	4,491.93	4,633.12	4,725.79	4,820.30
WACC	9.213%	9.248%	9.299%	9.315%	9.315%	9.315%
WACC _{BT} = Ku	10.000%	10.000%	10.000%	10.000%	10.000%	10.000%
E+D	5,334.24	5,582.67	5,991.93	6,133.12	6,255.79	6,380.90
EVA		88.75	254.27	261.08	270.89	276.31
EP		141.09	307.11	310.72	320.23	326.63
ECF\Ku		135.00	-1.00	308.00	370.65	378.06
FCF\Ku		285.00	149.00	458.00	490.65	500.46
ECF\R _f		-18.37	-164.31	128.32	185.33	189.03
FCF\R _f		71.63	-74.31	218.32	245.33	250.23

Table 7. Valuation of Toro Inc. according to Myers (1974)

	0	1	2	3	4	5
VTs = PV(Kd;D Kd T)	663.92	675.03	687.04	700.00	714.00	728.28
b	1.104529	1.097034	1.087162	1.083193	1.083193	1.083193
Ke	10.42%	10.39%	10.35%	10.33%	10.33%	10.33%
E	3,999.27	4,250.92	4,663.51	4,808.13	4,904.29	5,002.37
WACC	8.995%	9.035%	9.096%	9.112%	9.112%	9.112%
WACC _{BT}	9.759%	9.765%	9.777%	9.778%	9.778%	9.778%
E+D	5,499.27	5,750.92	6,163.51	6,308.12	6,434.29	6,562.97
EVA		93.10	258.59	265.89	275.82	281.34
EP		142.91	308.94	313.48	323.16	329.62
ECF\Ku		148.28	12.50	321.74	384.65	392.34
FCF\Ku		298.28	162.50	471.74	504.65	514.74
ECF\R _F		-11.69	-157.54	135.20	192.33	196.17
FCF\R _F		78.31	-67.54	225.20	252.33	257.37

Table 8. Valuation of Toro Inc. according to Miles and Ezzell

	0	1	2	3	4	5
VTs = PV[Ku; T D Kd] (1+Ku)/(1+Kd)	508.13	516.16	525.00	534.72	545.42	556.33
b	1.190077	1.178530	1.162292	1.157351	1.157351	1.157351
Ke	10.76%	10.71%	10.65%	10.63%	10.63%	10.63%
E	3,843.5	4,092.1	4,501.5	4,642.8	4,735.7	4,830.4
WACC	9.199%	9.235%	9.287%	9.304%	9.304%	9.304%
WACC _{BT}	9.985%	9.986%	9.987%	9.987%	9.987%	9.987%
E+D	5,343.48	5,592.05	6,001.48	6,142.85	6,265.70	6,391.02
EVA		89.01	254.53	261.36	271.17	276.60
EP		141.20	307.22	310.88	320.40	326.80
ECF\Ku		135.78	-0.22	308.78	371.43	378.86
FCF\Ku		285.78	149.78	458.78	491.43	501.26
ECF\R _F		-17.96	-163.90	128.72	185.71	189.43
FCF\R _F		72.04	-73.90	218.72	245.71	250.63

Table 9. Valuation of Toro Inc. according to Miller

	0	1	2	3	4	5
VTs = 0	0	0	0	0	0	0
b	1.539673	1.503371	1.452662	1.438156	1.438156	1.438156
Ke	12.16%	12.01%	11.81%	11.75%	11.75%	11.75%
E = Vu	3,335.35	3,575.89	3,976.48	4,108.13	4,190.29	4,274.09
WACC= Ku	10.000%	10.000%	10.000%	10.000%	10.000%	10.000%
WACC _{BT}	10.869%	10.827%	10.767%	10.749%	10.749%	10.749%
E+D	4,835.35	5,075.89	5,476.48	5,608.13	5,720.29	5,834.69
EVA		73.00	239.00	244.50	254.25	259.34
EP		134.21	300.33	300.84	309.95	316.15
ECF\Ku		93.00	-43.00	266.00	328.65	335.22
FCF\Ku		243.00	107.00	416.00	448.65	457.62
ECF\R _F		-40.41	-186.04	106.94	164.33	167.61
FCF\R _F		49.59	-96.04	196.94	224.33	228.81

Table 10. Valuation of Toro Inc. according to the With-cost-of-leverage theory

	0	1	2	3	4	5
$VTS = PV[Ku; D(KuT + R_F - Kd)]$	267.26	271.49	276.14	281.25	286.88	292.61
b	1.343501	1.321648	1.290998	1.281931	1.281931	1.281931
Ke	11.37%	11.29%	11.16%	11.13%	11.13%	11.13%
E	3,602.61	3,847.38	4,252.61	4,389.38	4,477.16	4,566.71
WACC	9.559%	9.579%	9.609%	9.618%	9.618%	9.618%
$WACC_{BT}$	10.382%	10.365%	10.339%	10.331%	10.331%	10.331%
E+D	5,102.61	5,347.38	5,752.61	5,889.38	6,007.16	6,127.31
EVA		81.82	247.54	253.75	263.53	268.80
EP		138.13	304.18	306.43	315.76	322.08
$ECF \backslash Ku$		115.50	-20.50	288.50	351.15	358.17
$FCF \backslash Ku$		265.50	129.50	438.50	471.15	480.57
$ECF \backslash R_F$		-28.60	-174.40	118.40	175.58	179.09
$FCF \backslash R_F$		61.40	-84.40	208.40	235.58	240.29

Table 11. Valuation of Toro Inc. according to Modigliani and Miller

	0	1	2	3	4	5
$VTS = PV[R_F; D R_F T]$	745.40	758.62	772.64	787.50	803.25	819.31
b	1.065454	1.058571	1.050506	1.045959	1.045959	1.045959
Ke	10.26%	10.23%	10.20%	10.18%	10.18%	10.18%
E	4,080.75	4,334.51	4,749.12	4,895.62	4,993.54	5,093.41
WACC	8.901%	8.940%	9.001%	9.015%	9.015%	9.015%
$WACC_{BT}$	9.654%	9.660%	9.673%	9.672%	9.672%	9.672%
E+D	5,580.75	5,834.51	6,249.12	6,395.62	6,523.54	6,654.01
EVA		94.97	260.52	268.12	278.19	283.75
EP		143.69	309.76	314.75	324.54	331.03
$ECF \backslash Ku$		154.32	18.84	328.41	391.65	399.48
$FCF \backslash Ku$		304.32	168.84	478.41	511.65	521.88
$ECF \backslash R_F$		-8.91	-154.54	138.44	195.83	199.74
$FCF \backslash R_F$		81.09	-64.54	228.44	255.83	260.94

Table 12 is a compendium of the valuations of Toro Inc. performed according to the nine theories. It can be seen that Modigliani and Miller give the maximum equity value (4,080.75) and Miller the minimum (3,335.35). Note that Modigliani and Miller and Myers provide a higher equity value than the No-cost-of-leverage theory. This result is inconsistent as discussed in Fernandez (2002).

Table 12. Valuation of Toro Inc. according to the nine theories

(Value in $t = 0$)	Equity value (E)	Value of tax shield (VTS)	Leverage cost	Ke	
				t=0	t=4
No-cost-of-leverage	3,958.96	623.61	0.00	10.49%	10.41%
Damodaran	3,727.34	391.98	231.63	11.05%	10.86%
Practitioners	3,477.89	142.54	481.07	11.73%	11.41%
Harris and Pringle	3,834.24	498.89	124.72	10.78%	10.65%
Myers	3,999.27	663.92	-40.31	10.42%	10.33%
Miles and Ezzell	3,843.48	508.13	115.48	10.76%	10.63%
Miller	3,335.35	0.00	623.61	12.16%	11.75%
With-cost-of-leverage	3,602.61	267.26	356.35	11.37%	11.13%
Modigliani and Miller	4,080.75	745.40	-121.79	10.26%	10.18%

Table 13 is the valuation of Toro Inc. if the growth after year 3 were 5.6% instead of 2%. Modigliani and Miller and Myers provide a required return to equity (K_e) lower than the required return to unlevered equity ($K_u = 10\%$), which is an inconsistent result because it does not have any economic sense.

Table 13. Valuation of Toro Inc. according to the nine theories if growth after year 3 is 5.6% instead of 2%

(Value in $t = 0$)	Equity value (E)	Value of tax shield (VTS)	Leverage cost	Ke	
				t=0	t=4
No-cost-of-leverage	6,615.67	1,027.01	0.00	10.29%	10.23%
Damodaran	6,234.21	645.55	381.46	10.63%	10.50%
Practitioners	5,823.40	234.75	792.27	11.03%	10.81%
Harris and Pringle	6,410.27	821.61	205.40	10.47%	10.37%
Myers	7,086.10	1,497.44	-470.43	10.00%	9.94%
Miles and Ezzell	6,425.48	836.83	190.19	10.45%	10.36%
Miller	5,588.66	0.00	1,027.01	11.29%	11.01%
With-cost-of-leverage	6,028.81	440.15	586.87	10.82%	10.65%
Modigliani and Miller	12,284.86	6,696.20	-5,669.19	8.15%	8.17%

3. How is the company valued when it reports losses in one or more years?

In such cases, we must calculate the tax rate that the company will pay and this is the rate that must be used to perform all the calculations. It is as if the tax rate were the rate obtained after subtracting the taxes that the company must pay.

Example. The company Campa S.A. reports a loss in year 1. The tax rate is 35%. In year 1, it will not pay any tax as it has suffered losses amounting to 220 million. In year 2, it will pay corporate tax amounting to 35% of that year's profit less the previous year's losses (350 – 220). The resulting tax is 45.5, that is, 13% of the EBT for year 2. Consequently, the effective tax rate is zero in year 1, 13% in year 2, and 35% in the other years.

4. Conclusion

The paper shows the ten most commonly used methods for valuing companies by cash flow discounting always give the same value. This result is logical, since all the methods analyze the same reality under the same hypotheses; they only differ in the cash flows taken as starting point for the valuation. The ten methods analyzed are:

- 1) free cash flow discounted at the WACC;
- 2) equity cash flows discounted at the required return to equity;
- 3) Capital cash flows discounted at the WACC before tax;

- 4) APV (Adjusted Present Value);
- 5) the business's risk-adjusted free cash flows discounted at the required return to assets;
- 6) the business's risk-adjusted equity cash flows discounted at the required return to assets;
- 7) economic profit discounted at the required return to equity;
- 8) EVA discounted at the WACC;
- 9) the risk-free rate-adjusted free cash flows discounted at the risk-free rate; and
- 10) the risk-free rate-adjusted equity cash flows discounted at the required return to assets.

The paper also analyses nine different theories on the calculation of the VTS, which implies nine different theories on the relationship between the levered and the unlevered beta, and nine different theories on the relationship between the required return to equity and the required return to assets. The nine theories analyzed are:

- 1) No-cost-of-leverage,
- 2) Modigliani and Miller (1963),
- 3) Myers (1974),
- 4) Miller (1977),
- 5) Miles and Ezzell (1980),
- 6) Harris and Pringle (1985),
- 7) Damodaran (1994),
- 8) With-cost-of-leverage, and
- 9) Practitioners method.

The disagreements in the various theories on the valuation of the firm arise from the calculation of the value of the tax shields (VTS). Using a simple example we show that Modigliani and Miller (1963) and Myers (1974) provide inconsistent results

The paper contains the most important valuation equations according to these theories (Appendix 2) and also shows the changes that take place in the valuation equations when the debt's market value does not match its book value (Appendix 3).

Appendix 1.

A brief overview of the most significant papers on the discounted cash flow valuation of firms

There is a considerable body of literature on the discounted cash flow valuation of firms. We will now discuss the most salient papers, concentrating particularly on those that proposed different expressions for the present value of the tax savings due to the payment of interest or value of tax shields (VTS). The main problem of most papers is that they consider the value of tax shields (VTS) as the present value of the tax savings due to the payment of interest. Fernandez (2001) argues and proves that the value of tax shields (VTS) is the difference of two present values: the present value of taxes paid by the unlevered firm minus the present value of taxes paid by the levered firm.

Modigliani and Miller (1958) studied the effect of leverage on the firm's value. Their proposition 1 (1958, equation 3) states that, in the absence of taxes, the firm's value is independent of its debt, i.e.,

$$[23] \quad E + D = V_u, \text{ if } T = 0.$$

E is the equity value, D is the debt value, V_u is the value of the unlevered company and T is the tax rate.

In the presence of taxes and for the case of a perpetuity, they calculate the value of tax shields (VTS) by discounting the present value of the tax savings due to interest payments of a risk-free debt ($T D R_F$) at the risk-free rate (R_F). Their first proposition, with taxes, is transformed into Modigliani and Miller (1963, page 436, equation 3):

$$[24] \quad E + D = V_u + PV[R_F; DT R_F] = V_u + D T$$

DT is the value of tax shields (VTS) for a perpetuity. This result is only correct for perpetuities. As **Fernandez (2001)** demonstrates, discounting the tax savings due to interest payments of a risk-free debt at the risk-free rate provides inconsistent results for growing companies. We have seen it in table 13.

Modigliani and Miller' purpose was to illustrate the tax impact of debt on value. They never addressed the issue of the riskiness of the taxes and only treated perpetuities. We have seen that if we relax the no-growth assumption, then new equations are needed.

Myers (1974) introduced the APV (adjusted present value). According to Myers, the value of the levered firm is equal to the value of the firm with no debt (V_u) plus the present value of the tax saving due to the payment of interest (VTS). Myers proposes calculating the VTS by discounting the tax savings ($D T K_d$) at the cost of debt (K_d). The argument is that the risk of the tax saving arising from the use of debt is the same as the risk of the debt. Then, according to Myers (1974):

$$[25] \quad VTS = PV [K_d; D T K_d]$$

Luehrman (1997) recommends to value companies using the Adjusted Present Value and calculates the VTS as Myers. This theory provides inconsistent results for companies other than perpetuities as shown in Fernandez (2001).

Miller (1977) assumes no advantages of debt financing: “I argue that even in a world in which interest payments are fully deductible in computing corporate income taxes, the value of the firm, in equilibrium will still be independent of its capital structure.” According to Miller (1977) the value of the firm is independent of its capital structure, that is,

$$[26] \text{ VTS} = 0.$$

According to **Miles and Ezzell (1980)**, a firm that wishes to keep a constant D/E ratio must be valued in a different manner from the firm that has a preset level of debt. For a firm with a fixed debt target $[D/(D+E)]$ they claim that the correct rate for discounting the tax saving due to debt ($K_d T D_{t-1}$) is K_d for the tax saving during the first year, and K_u for the tax saving during the following years.

The expression of K_e is their equation 22:

$$[27] K_e = K_u + D (K_u - K_d) [1 + K_d (1-T)] / [(1+K_d) E]$$

Although Miles and Ezzell do not mention what the value of tax shields should be, equation [27] relating the required return to equity with the required return for the unlevered company implies that

$$[28] \text{ VTS} = PV[K_u; T D K_d] (1+K_u)/(1+K_d).$$

Lewellen and Emery (1986) also claim that the most logically consistent method is Miles and Ezzell.

Harris and Pringle (1985) propose that the present value of the tax saving due to the payment of interest (VTS) should be calculated by discounting the tax saving due to the debt ($K_d T D$) at the rate K_u . Their argument is that the interest tax shields have the same systematic risk as the firm's underlying cash flows and, therefore, should be discounted at the required return to assets (K_u).

Then, according to Harris and Pringle (1985):

$$[29] \text{ VTS} = PV [K_u; D K_d T]$$

Harris and Pringle (1985, page 242) say “the MM position is considered too extreme by some because it implies that interest tax shields are no more risky than the interest payments themselves. The Miller position is too extreme for some because it implies that debt cannot benefit the firm at all. Thus, if the truth about the value of tax shields lies somewhere between the MM and Miller positions, a supporter of either Harris and Pringle or Miles and Ezzell can take comfort in the fact that both produce a result for unlevered returns between those of MM and Miller. A virtue of either Harris and Pringle compared to Miles and Ezzell is its simplicity and straightforward intuitive explanation.”

Ruback (1995) reaches equations that are identical to those of Harris-Pringle (1985). **Kaplan and Ruback (1995)** also calculate the VTS “discounting interest tax shields at the discount rate for an all-equity firm”. **Tham and Vélez-Pareja (2001)**, following an arbitrage argument, also claim that the appropriate discount rate for the tax shield is K_u , the required return to unlevered equity. **Fernandez (2002)** shows that Harris and Pringle (1985) provides inconsistent results.

Damodaran (1994, page 31) argues that if all the business risk is borne by the equity, then the equation relating the levered beta (b_L) with the asset beta (b_u) is:

$$[30] b_L = b_u + (D/E) b_u (1 - T).$$

It is important to note that equation [30] is exactly equation [22] assuming that $b_d = 0$. One interpretation of this assumption is that “all of the firm's risk is borne by the stockholders (i.e., the

beta of the debt is zero)”¹⁸. But we think that it is difficult to justify that the debt has no risk (unless the cost of debt is the risk-free rate) and that the return on the debt is uncorrelated with the return on assets of the firm. We rather interpret equation [30] as an attempt to introduce some leverage cost in the valuation: for a given risk of the assets (b_u), by using equation [30] we obtain a higher b_L (and consequently a higher K_e and a lower equity value) than with equation [22]. Equation [30] appears in many finance books and is used by some consultants and investment banks.

Although Damodaran does not mention what the value of tax shields should be, his equation [30] relating the levered beta with the asset beta implies that the value of tax shields is:

$$[31] \text{ VTS} = \text{PV}[K_u; D \text{ T } K_u - D (K_d - R_f) (1-T)]$$

Another way of calculating the levered beta with respect to the asset beta is the following:

$$[32] b_L = b_u (1 + D/E).$$

We will call this method the **Practitioners’ method**, because consultants and investment banks often use it¹⁹. It is obvious that according to this equation, given the same value for b_u , a higher b_L (and a higher K_e and a lower equity value) is obtained than according to [22] and [30].

One should notice that equation [32] is equal to equation [30] eliminating the $(1-T)$ term. We interpret equation [32] as an attempt to introduce still higher leverage cost in the valuation: for a given risk of the assets (b_u), by using equation [32] we obtain a higher b_L (and consequently a higher K_e and a lower equity value) than with equation [30].

Equation [30] relating the levered beta with the asset beta implies that the value of tax shields is:

$$[33] \text{ VTS} = \text{PV}[K_u; D \text{ T } K_d - D(K_d - R_f)]$$

By comparing [33] to [31] it can be seen that [33] provides a VTS that is $\text{PV}[K_u; D \text{ T } (K_u - R_f)]$ lower than [31]. We interpret this difference as additional leverage cost (on top of the leverage cost of Damodaran) introduced in the valuation.

Inselbag and Kaufold (1997) argue that if the firm targets the dollar values of debt outstanding, the VTS is given by the Myers (1974) equation. However, if the firm targets a constant debt/value ratio, the VTS is given by the Miles and Ezzell (1980) equation.

Copeland, Koller y Murrin (2000) treat the Adjusted Present Value in their Appendix A. They only mention perpetuities and only propose two ways of calculating the VTS: Harris y Pringle (1985) and Myers (1974). They conclude “*we leave it to the reader’s judgment to decide which approach best fits his or her situation*”. They also claim that “the finance literature does not provide a clear answer about which discount rate for the tax benefit of interest is theoretically correct.” It is quite interesting to note that Copeland et al. (2000, page 483) only suggest Inselbag and Kaufold (1997) as additional reading on Adjusted Present Value.

We will consider two additional theories to calculate the value of the tax shields. We label these two theories No-Costs-Of-Leverage, and With-Costs-Of-Leverage.

¹⁸ See page 31 of Damodaran (1994)

We label the first theory the **No-Costs-Of-Leverage** equation because as may be seen in Fernandez (2001), is the only equation that provides consistent results when there are not leverage costs. According to this theory, the VTS is the present value of DTKu (not the interest tax shield) discounted at the unlevered cost of equity (Ku).

[34] $PV[Ku; D T Ku]$

Equation [34] is the result of considering that the value of tax shields (VTS) is the difference of two present values: the present value of taxes paid by the unlevered firm minus the present value of taxes paid by the levered firm. It can be seen in **Fernandez (2002)**.

Comparing [31] to [34], it can be seen that [31] provides a VTS that is $PV[Ku; D (Kd - R_f) (1-T)]$ lower than [34]. We interpret this difference as leverage cost introduced in the valuation by Damodaran.

Comparing [33] to [34] it can be seen that [33] provides a VTS that is $PV[Ku; D T (Ku - Kd) + D(Kd - R_f)]$ lower than [34]. We interpret this difference as leverage cost introduced in the valuation by the practitioners' method.

With-Costs-Of-Leverage. This theory provides another way of quantifying the VTS:

[35] $VTS = PV[Ku; D Ku T - D (Kd - R_f)]$

One way of interpreting equation [35] is that the leverage costs (with respect to [34]) are proportional to the amount of debt and to the difference of the required return on debt minus the risk-free rate.²⁰

By comparing [35] to [34], it can be seen that [40] provides a VTS that is $PV[Ku; D (Kd - R_f)]$ lower than [34]. We interpret this difference as leverage cost introduced in the valuation.

¹⁹ One of the many places where it appears is Ruback (1995), p. 5.

²⁰ This formula can be completed with another parameter j that takes into account that the cost of leverage is not strictly proportional to debt. j should be lower for small leverage and higher for high leverage. Introducing the parameter, the value of tax shields is $VTS = PV [Ku; D T Ku - j D (Kd - R_f)]$.

The following table is the synthesis of the 9 theories mentioned about the value of tax shields applied to level perpetuities.

Perpetuities. Value of tax shields (VTS) according to the 9 theories.

	Theories	Equation	VTS
1	No-Costs-Of-Leverage equation	[34]	DT
2	Damodaran	[31]	$DT - [D(K_d - R_f)(1-T)]/K_u$
3	Practitioners	[33]	$D[R_f - K_d(1-T)]/K_u$
4	Harris-Pringle	[29]	$T D K_d / K_u$
5	Myers	[25]	DT
6	Miles-Ezzell	[28]	$TDK_d(1+K_u)/[(1+K_d)K_u]$
7	Miller (1977)	[26]	0
8	With-Costs-Of-Leverage	[35]	$D(K_u T + R_f - K_d)/K_u$
9	Modigliani&Miller	[24]	DT

Appendix 2

Valuation equations according to the main theories

Market value of the debt = Nominal value

	No-cost-of-leverage	Damodaran (1994)	Practitioners
Ke	$Ke = Ku + \frac{D(1-T)}{E} (Ku - Kd)$	$Ke = Ku + \frac{D(1-T)}{E} (Ku - R_F)$	$Ke = Ku + \frac{D}{E} (Ku - R_F)$
β_L	$\beta_L = \beta_u + \frac{D(1-T)}{E} (\beta_u - \beta_d)$	$\beta_L = \beta_u + \frac{D(1-T)}{E} \beta_u$	$\beta_L = \beta_u + \frac{D}{E} \beta_u$
WACC	$Ku \left(1 - \frac{DT}{E+D} \right)$	$Ku \left(1 - \frac{DT}{E+D} \right) + D \frac{(Kd - R_F)(1-T)}{(E+D)}$	$Ku - D \frac{R_F - Kd(1-T)}{(E+D)}$
WACC_{BT}	$Ku - \frac{DT(Ku - Kd)}{(E+D)}$	$Ku - D \frac{T(Ku - R_F) - (Kd - R_F)}{(E+D)}$	$Ku + \frac{D(Kd - R_F)}{(E+D)}$
VTs	$PV[Ku; DTKu]$	$PV[Ku; DTKu - D(Kd - R_F)(1-T)]$	$PV[Ku; T D Kd - D(Kd - R_F)]$
$ECF_t \backslash Ku$	$ECF_t - D_{t-1} (Ku_t - Kd_t) (1-T)$	$ECF_t - D_{t-1} (Ku_t - R_F) (1-T)$	$ECF_t - D_{t-1} (Ku_t - R_F)$
$FCF_t \backslash Ku$	$FCF_t + D_{t-1} Ku_t T$	$FCF_t + D_{t-1} Ku_t T - D_{t-1} (Kd - R_F) (1-T)$	$FCF_t + D_{t-1} [R_{F,t} - Kd_t (1-T)]$
$ECF_t \backslash R_F$	$ECF_t - D_{t-1} (Ku_t - Kd_t) (1-T) - E_{t-1} (Ku_t - R_{F,t})$	$ECF_t - D_{t-1} (Ku_t - R_F) (1-T) - E_{t-1} (Ku_t - R_{F,t})$	$ECF_t - (E_{t-1} + D_{t-1}) (Ku_t - R_{F,t})$
$FCF_t \backslash R_F$	$FCF_t + D_{t-1} Ku_t T - (E_{t-1} + D_{t-1}) (Ku_t - R_{F,t})$	$FCF_t + D_{t-1} Ku_t T - D_{t-1} (Kd - R_F) (1-T) - (E_{t-1} + D_{t-1}) (Ku_t - R_{F,t})$	$FCF_t + D_{t-1} [R_{F,t} - Kd_t (1-T)] - (E_{t-1} + D_{t-1}) (Ku_t - R_{F,t})$

	Harris-Pringle (1985) Ruback (1995)	Myers (1974)	Miles-Ezzell (1980)
Ke	$Ke = Ku + \frac{D}{E} (Ku - Kd)$	$Ke = Ku + \frac{Vu - E}{E} (Ku - Kd)$	$Ke = Ku + \frac{D}{E} (Ku - Kd) \left[1 - \frac{T Kd}{1 + Kd} \right]$
β_L	$\beta_L = \beta_u + \frac{D}{E} (\beta_u - \beta_d)$	$\beta_L = \beta_u + \frac{Vu - E}{E} (\beta_u - \beta_d)$	$\beta_L = \beta_u + \frac{D}{E} (\beta_u - \beta_d) \left[1 - \frac{T Kd}{1 + Kd} \right]$
WACC	$Ku - D \frac{Kd T}{(E+D)}$	$Ku - \frac{D VTS (Ku - Kd) + D Kd T}{(E+D)}$	$Ku - D \frac{Kd T}{(E+D)} \frac{1 + Ku}{1 + Kd}$
WACC_{BT}	Ku	$Ku - \frac{D VTS (Ku - Kd)}{(E+D)}$	$Ku - \frac{D T Kd}{(E+D)} \frac{(Ku - Kd)}{(1 + Kd)}$
VTs	$PV[Ku; T D Kd]$	$PV[Kd; T D Kd]$	$PV[Ku; T D Kd] \frac{(1 + Ku)}{(1 + Kd)}$
$ECF_t \backslash Ku$	$ECF_t - D_{t-1} (Ku_t - Kd_t)$	$ECF_t - (Vu - E) (Ku_t - Kd_t)$	$ECF_t - D (Ku_t - Kd_t) \frac{1 + Kd (1-T)}{(1 + Kd)}$
$FCF_t \backslash Ku$	$FCF_t + T D_{t-1} Kd_t$	$FCF_t + T D Kd + VTS (Ku - Kd)$	$FCF_t + T D Kd (1 + Ku) / (1 + Kd)$
$ECF_t \backslash R_F$	$ECF_t - D_{t-1} (Ku_t - Kd_t) - E_{t-1} (Ku_t - R_{F,t})$	$ECF_t - (Vu - E) (Ku_t - Kd_t) - E_{t-1} (Ku_t - R_{F,t})$	$ECF_t - D (Ku_t - Kd_t) \frac{1 + Kd (1-T)}{(1 + Kd)} - E_{t-1} (Ku_t - R_{F,t})$
$FCF_t \backslash R_F$	$FCF_t + T D_{t-1} Kd_t - (E_{t-1} + D_{t-1}) (Ku_t - R_{F,t})$	$FCF_t + T D Kd + VTS (Ku - Kd) - (E_{t-1} + D_{t-1}) (Ku_t - R_{F,t})$	$FCF_t + T D Kd (1 + Ku) / (1 + Kd) - (E_{t-1} + D_{t-1}) (Ku_t - R_{F,t})$

	Miller	With-cost-of-leverage	Modigliani-Miller
Ke	$Ke = Ku + \frac{D}{E} [Ku - Kd(1-T)]$	$Ke = Ku + \frac{D}{E} [Ku(1-T) + KdT - R_F]$	$Ke = Ku + \frac{D}{E} [Ku - Kd(1-T) - (Ku - g) \frac{VTS}{D}]^*$
β_L	$\beta_L = \beta_u + \frac{D}{E} (\beta_u - \beta_d) + \frac{D T Kd}{E P_M}$	$\beta_L = \beta_u + \frac{D}{E} [\beta_u(1-T) + \beta_d T]$	$\beta_L = \beta_u + \frac{D}{E} [\beta_u - \beta_d + \frac{T Kd}{P_M} - \frac{VTS (Ku - g)}{D P_M}]^*$
WACC	Ku	$Ku - \frac{D (KuT + R_F - Kd)}{E + D}$	$\frac{D Ku - (Ku - g) VTS}{(E + D)}^*$
WACC_{BT}	$Ku + D \frac{Kd T}{(E + D)}$	$Ku - \frac{D [(Ku - Kd)T + R_F - Kd]}{E + D}$	$\frac{D Ku - (Ku - g) VTS + D T Kd}{(E + D)}^*$
VTs	0	$PV[Ku; D (KuT + R_F - Kd)]$	$PV[R_F; T D R_F]$
$ECF_t \backslash Ku$	$ECF_t - D_{t-1} [Ku_t - Kd_t (1-T)]$	$ECF_t - D_{t-1} [Ku_t (1-T) + KdT - R_{F,t}]$	$ECF_t - D_{t-1} [Ku_t - Kd_t (1-T) - (Ku - g) VTS / D]^*$
$FCF_t \backslash Ku$	FCF_t	$FCF_t + D_{t-1} [Ku_t T - Kd_t + R_{F,t}]$	$FCF_t + E_{t-1} Ku + (Ku - g) VTS^*$
$ECF_t \backslash R_F$	$ECF_t - D_{t-1} [Ku_t - Kd_t (1-T)] - E_{t-1} (Ku_t - R_{F,t})$	$ECF_t - D_{t-1} [Ku_t (1-T) + KdT - R_{F,t}] - E_{t-1} (Ku_t - R_{F,t})$	$ECF_t - D_{t-1} [Ku_t - Kd_t (1-T) - (Ku - g) VTS / D] - E_{t-1} (Ku_t - R_{F,t})^*$
$FCF_t \backslash R_F$	$FCF_t - (E_{t-1} + D_{t-1}) (Ku_t - R_{F,t})$	$FCF_t + D_{t-1} [Ku_t T - Kd_t + R_{F,t}] - (E_{t-1} + D_{t-1}) (Ku_t - R_{F,t})$	$FCF_t + E_{t-1} Ku + (Ku - g) VTS - (E_{t-1} + D_{t-1}) (Ku_t - R_{F,t})^*$

* Valid only for growing perpetuities

Appendix 2 (cont.)

Valuation equations according to the main theories
Market value of the debt = Nominal value

Equations common to all methods:

WACC and WACC_{BT}:

$$WACC_t = \frac{E_{t-1} K_{e_t} + D_{t-1} K_{d_t} (1-T)}{(E_{t-1} + D_{t-1})} \quad WACC_{BT_t} = \frac{E_{t-1} K_{e_t} + D_{t-1} K_{d_t}}{(E_{t-1} + D_{t-1})}$$

$$WACC_{BT_t} - WACC_t = \frac{D_{t-1} K_{d_t} T}{(E_{t-1} + D_{t-1})}$$

Relationships between cash flows:

$$ECF_t = FCF_t + (D_t - D_{t-1}) - D_{t-1} K_{d_t} (1-T) \quad CCF_t = FCF_t + D_{t-1} K_{d_t} T$$

$$CCF_t = ECF_t - (D_t - D_{t-1}) + D_{t-1} K_{d_t}$$

Cash flows\\K_u:

$$ECF\\K_u = ECF_t - E_{t-1} (K_{e_t} - K_u)$$

$$FCF\\K_u = FCF_t - (E_{t-1} + D_{t-1})(WACC_t - K_u) = CCF\\K_u = CCF_t - (E_{t-1} + D_{t-1})(WACC_{BT_t} - K_u)$$

Cash flows\\R_f:

$$ECF\\R_f = ECF_t - E_{t-1} (K_{e_t} - R_f)$$

$$FCF\\R_f = FCF_t - (E_{t-1} + D_{t-1})(WACC_t - R_f) = CCF\\R_f = CCF_t - (E_{t-1} + D_{t-1})(WACC_{BT_t} - R_f)$$

$$ECF\\R_f = ECF\\K_u - E_{t-1} (K_u - R_f) \quad FCF\\R_f = FCF\\K_u - (E_{t-1} + D_{t-1})(K_u - R_f)$$

$$FCF\\K_u - ECF\\K_u = D_{t-1} K_{u_t} - (D_t - D_{t-1}) \quad FCF\\R_f - ECF\\R_f = D_{t-1} R_{f_t} - (D_t - D_{t-1})$$

Appendix 3

Valuation equations according to the main theories when the debt's market value (D) does not match its nominal or book value (N)

This appendix contains the expressions of the basic methods for valuing companies by cash flow discounting, when the debt's market value (D) does not match its nominal value (N). If the debt's market value (D) does not match its nominal value (N), it is because the required return to debt (Kd) is different from the debt's cost (r).

The interest paid in a period t is: $I_t = N_{t-1} r_t$. The increase in debt in a period t is: $\Delta N_t = N_t - N_{t-1}$. Consequently, the debt cash flow in a period t is: $CF_d = I_t - \Delta N_t = N_{t-1} r_t - (N_t - N_{t-1})$.

$$\text{Consequently, the debt's value at } t=0 \text{ is: } D_0 = \sum_{t=1}^{\infty} \frac{N_{t-1} r_t - (N_t - N_{t-1})}{\prod_{i=1}^t (1 + K_{d_i})}$$

It is easy to show that the relationship between the debt's market value (D) and its nominal value (N) is:

$$D_t - D_{t-1} = N_t - N_{t-1} + D_{t-1} K_{d_t} - N_{t-1} r_t$$

Consequently: $\Delta D_t = \Delta N_t + D_{t-1} K_{d_t} - N_{t-1} r_t$

The fact that the debt's market value (D) does not match its nominal value (N) affects several equations given in section 1 in this paper. The equations [1], [3], [4], [6], [7], [9] and [10] continue to be valid, but the other equations change.

The expression of the WACC in this case is:

$$[2^*] \text{ WACC} = \frac{E K_e + D K_d - N r T}{E + D}$$

The expression relating the ECF with the FCF is:

$$[5^*] \text{ ECF}_t = \text{FCF}_t + (N_t - N_{t-1}) - N_{t-1} r_t (1 - T)$$

The expression relating the CCF with the ECF and the FCF is:

$$[8^*] \text{ CCF}_t = \text{ECF}_t + \text{CF}_{d_t} = \text{ECF}_t - (N_t - N_{t-1}) + N_{t-1} r_t = \text{FCF}_t + N_{t-1} r_t T$$

	No-cost-of-leverage	Damodaran (1994)	Practitioners
WACC	$K_u - \frac{N r T + D T (K_u - K_d)}{(E + D)}$	$K_u - \frac{N r T + D [T (K_u - R_F) - (K_d - R_F)]}{(E + D)}$	$K_u - \frac{N r T - D (K_d - R_F)}{(E + D)}$
VTS	$PV[K_u; D T K_u + T (N r - D K_d)]$	$PV[K_u; T N r + D T (K_u - R_F) - D (K_d - R_F)]$	$PV[K_u; T N r - D (K_d - R_F)]$
FCF_t / K_u	$\text{FCF}_t + D_{t-1} K_u T + T (N_{t-1} r_t - D_{t-1} K_d)$	$\text{FCF}_t + D_{t-1} K_u T + T (N_{t-1} r_t - D_{t-1} K_d) - D_{t-1} (K_d - R_F) (1 - T)$	$\text{FCF}_t + T (N_{t-1} r_t - D_{t-1} K_d) + D_{t-1} (R_F - K_d) (1 - T)$

	Harris-Pringle (1985) Ruback (1995)	Myers (1974)	Miles-Ezzell (1980)
WACC	$K_u - \frac{N r T}{(E + D)}$	$K_u - \frac{D VTS (K_u - K_d) + N r T}{(E + D)}$	$K_u - N \frac{r T}{(E + D)} \frac{1 + K_u}{1 + K_d}$
VTS	$PV[K_u; T N r]$	$PV[K_d; T N r]$	$PV[K_u; N_{t-1} r_t T] (1 + K_u) / (1 + K_d)$
FCF_t / K_u	$\text{FCF}_t + T N_{t-1} r_t$	$\text{FCF}_t + T N r + VTS (K_u - K_d)$	$\text{FCF}_t + T N r (1 + K_u) / (1 + K_d)$

Equations common to all the methods:

WACC and WACC_{BT}:

$$\text{WACC}_t = \frac{E_{t-1} K_{e_t} + D_{t-1} K_{d_t} - N_{t-1} r_t T}{(E_{t-1} + D_{t-1})} \quad \text{WACC}_{BTt} = \frac{E_{t-1} K_{e_t} + D_{t-1} K_{d_t}}{(E_{t-1} + D_{t-1})}$$

$$\text{WACC}_{BTt} - \text{WACC}_t = \frac{N_{t-1} r_t T}{(E_{t-1} + D_{t-1})}$$

Relationships between the cash flows:

$$\text{ECF}_t = \text{FCF}_t + (N_t - N_{t-1}) - N_{t-1} r_t (1 - T) \quad \text{CCF}_t = \text{FCF}_t + N_{t-1} r_t T \quad \text{CCF}_t = \text{ECF}_t - (N_t - N_{t-1}) + N_{t-1} r_t$$

APPENDIX 4

Dictionary

b_d = Beta of debt

b_L = Beta of levered equity

b_u = Beta of unlevered equity = beta of assets

D = Value of debt

E = Value of equity

E_{bv} = Book value of equity

ECF = Equity cash flow

EP = Economic Profit

EVA = Economic value added

FCF = Free cash flow

g = Growth rate of the constant growth case

I = interest paid

K_u = Cost of unlevered equity (required return to unlevered equity)

K_e = Cost of levered equity (required return to levered equity)

K_d = Required return to debt = cost of debt

N = Book value of the debt

NOPAT = Net Operating Profit After Tax = profit after tax of the unlevered company

PAT = Profit after tax

PBT = Profit before tax

P_M = Market premium = $E (R_M - R_F)$

PV = Present value

r = Cost of debt

R_F = Risk-free rate

T = Corporate tax rate

VTs = Value of the tax shield

V_u = Value of shares in the unlevered company

WACC = weighted average cost of capital

$WACC_{BT}$ = weighted average cost of capital before taxes

WCR = Working capital requirements = net current assets

REFERENCES

- Arditti, F. D. and H. Levy (1977), “The Weighted Average Cost of Capital as a Cutoff Rate: A Critical Examination of the Classical Textbook Weighted Average”, *Financial Management* (Fall), pp. 24-34.
- Copeland, T. E., T. Koller and J. Murrin (2000). *Valuation: Measuring and Managing the Value of Companies*. Third edition. New York: Wiley.
- Damodaran, A (1994), *Damodaran on Valuation*, John Wiley and Sons, New York.
- Fernandez, Pablo (2002, forthcoming). *Valuation Methods and Shareholder Value Creation*, Academic Press.
- Fernandez, Pablo (2001), “The value of tax shields is NOT equal to the present value of tax shields” SSRN Working Paper.
- Harris, R.S. and J.J. Pringle (1985), “Risk-Adjusted Discount Rates Extensions form the Average-Risk Case“, *Journal of Financial Research* (Fall), pp. 237-244.
- Inselbag, I. and H. Kaufold (1997), “Two DCF Approaches for Valuing Companies under Alternative Financing Strategies (and How to Choose Between Them)”, *Journal of Applied Corporate Finance* (Spring), pg. 114-122.
- Kaplan, Steven and Richard Ruback (1995), “The Valuation of Cash Flow Forecast: An Empirical Analysis”, *Journal of Finance*, Vol 50 No 4, September.
- Lewellen, W.G. and D.R. Emery (1986), “Corporate Debt Management and the Value of the Firm”, *Journal of Financial Quantitative Analysis* (December), pg. 415-426.
- Luehrman, Timothy A (1997), “What’s Worth: A General Manager’s Guide to Valuation”, and “Using APV: A Better Tool for Valuing Operations”, *Harvard Business Review*, (May-June), pg. 132-154.
- Miles, J.A. and J.R. Ezzell (1980) “The Weighted Average Cost of Capital”, Perfect Capital Markets and Project Life: A Clarification,” *Journal of Financial and Quantitative Analysis* (September), pp. 719-730.
- Miles, J.A. and J.R. Ezzell, (1985) “Reequationing Tax Shield Valuation: A Note,” *Journal of Finance* Vol XL, 5 (December), pp. 1485-1492.
- Miller, Merton H. (1977), “Debt and Taxes”, *Journal of Finance* (May), pg. 261-276.
- Modigliani, Franco, and Merton Miller (1958), “The Cost of Capital, Corporation Finance and the Theory of Investment”, *American Economic Review* 48, 261-297.
- Modigliani, Franco, and Merton Miller (1963), “Corporate Income Taxes and the Cost of Capital: A Correction”, *American Economic Review* (June), pp. 433-443.
- Myers, S.C. (1974), “Interactions of Corporate Financing and Investment Decisions - Implications for Capital Budgeting”, *Journal of Finance* (March), pp. 1-25
- Ruback, Richard S. (1995), “A Note on Capital Cash Flow Valuation”, Harvard Business School, 9-295-069.
- Tham, Joseph, and Ignacio Vélez-Pareja (2001), “The correct discount rate for the tax shield: the N-period case”, SSRN Working Paper