Does the consciousness of the disposition effect increase the equity premium?

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The disposition effect is a well established phenomenon in the empirical and experimental financial literature. It leads to sell winners too early and to hold losers too long. In this paper, we show that this phenomenon has two non negligible consequences. First, it decreases the welfare of individuals and, second, it increases the risk premium required by investors. In fact, when agents know they can suffer from this bias at a future date, they are also conscious that the probability distribution of their final wealth will be different from the final probability distribution of their initial portfolio. It may lead these investors to require a greater risk premium to invest in stocks.

Classification JEL: G11, G14
Key words: disposition effect; equity premium puzzle; loss aversion; behavioral finance

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1. Introduction

The disposition effect is the tendency of investors to sell winning stocks too early and to hold losing stocks too long. It was first analyzed by Shefrin and Statman (1985) and confirmed on individual data by Odean (1998), among others. Experimental evidence of the disposition effect has also been obtained in the first place by Weber and Camerer (1998).

Selling winning stocks too early can refer to self-control problems, to aversion to regret or to a belief in mean reversion of prices. It also suggests a possible time-inconsistency in successive decisions. It is as if investors were changing their horizon of investment, depending on the evolution of stock prices and such investors are usually called *disposition investors*. The empirical evidence shows that disposition investors dynamically revise their portfolios in a sub-optimal way. Thus, even if an investor decides to

[1] We want to thank Marie-Hélène Broihanne and Maxime Merli for their helpful comments.

buy stocks at date 0, she may be conscious that the final probability distribution of her wealth will not be the final distribution of the value of the portfolio set up at date 0. This leads to ask the question we address in this paper. Does the consciousness of the disposition effect generate an increase in the equity premium required by an investor? And if it is the case, is the risk premium variation sizeable?

To the best of our knowledge, this question has never been directly addressed in the literature. The influence of the disposition effect on the equity premium has been analyzed through the role it plays in changing the demand function of disposition investors. In particular, Grinblatt and Han (2005) present a model in which current prices are jointly determined by the fundamental value of the stock and by the reference price of the investor (the initial buying price in the standard case). Working on the database of Odean (1998), Goetzmann and Massa (2003)[2] test the model of Grinblatt and Han and show that the existence of disposition investors has a non negligible effect on prices and returns.

In this paper, we consider a simple two-period model of an exchange economy where two assets are traded: a risky asset, called the stock, and a risk-free asset, called the bond. The price of the risky asset is assumed to be driven by the usual Cox-Ross-Rubinstein model (1979). Agents' preferences are described by a simplified version of prospect theory[3]. Investors are characterized by a piecewise linear valuation function so that the expectation of their value function can be expressed as the value of a portfolio of options.

We introduce the disposition effect by assuming a strictly positive probability of selling the stock at date 1 when an up-state has occurred. To simplify the formulation of the model, we assume that the probability of switching from stocks to bonds after a down-state is equal to 0. We could obtain similar results by defining a probability of switching in each state as long as the probability in the up-state is greater than the corresponding one in the down-state.

Our main results are the following:

[2] The paper of Grinblatt and Han was already circulating as a NBER working paper in 2002.
[3] Kahneman and Tversky (1979) and Tversky and Kahneman (1992).

1. Disposition investors reach a lower level of welfare and the loss of welfare increases with the intensity of the disposition effect.
2. When investors find optimal to invest in stocks for one period, they also find optimal to invest in stocks for two periods, even if they are conscious of being disposition investors.
3. In the case evoked in the preceding point, investors require a higher risk premium. However, the increase doesn't exceed 0.5 % for a reasonable intensity of the disposition effect.

The paper is organized as follows. In section 2, we analyse the related literature on the disposition effect. We recall the essential empirical and experimental results showing the presence of this effect among individual and professional investors.

In section 3, we first analyze the one-period problem. We define what we call a *critical investor*, that is an agent characterized by a (critical) loss aversion parameter which keeps her indifferent between stocks and bonds. We then study, in a two-period model, the relationship between this critical loss aversion parameter and the horizon of investment and show that it is decreasing. This result is an argument in favour of time-diversification, even if the reference wealth level is chosen as the initial wealth capitalized at the risk-free rate, as in Barberis *et al.*(2001). The conclusion is obviously reinforced if a lower reference wealth level is chosen.

In section 4, we introduce the disposition effect in the two-period model. We assume that there is a positive probability of switching from stocks to bonds at date 1, after a stock price increase in the first-period. We show that the risk premium required by investors increases with the intensity of the disposition effect and we illustrate our result with numerical simulations referring to the historical levels of the parameters, especially those which concern the equity premium and the volatility of stock returns.

Section 5 concludes the papers. All the proofs are reported in the appendix.

2. Related literature

2.1 What drives the disposition effect?

Though our essential purpose is not to analyze the theoretical explanations of the disposition effect, it is worth to stress some of the arguments evoked in the literature to explain this phenomenon. The question in the title of this subsection appears in the title of two papers by Zuchel (2001) and Barberis and Xiong (2006). In fact, following Shefrin and Statman (1985), a number of authors (for example Odean (1998) and Weber and Camerer (1998)) have justified the disposition effect by prospect theory (PT) preferences, especially the S-shape valuation function assumed in PT. The argument is, roughly speaking, the following: after a gain (loss), agents are in the concave (convex) part of the valuation function so they become risk averse (lovers).

When agents are risk-averse over gains and risk lovers in the domain of losses, they prefer to realize paper gains and keep paper losses. However, the above arguments are quite informal; Barberis and Xiong (2006) (see also Hens and Vlcek (2005)) have analyzed the theoretical foundations of this explanation. They get controversial results. They show that the disposition effect is observed for some values of the essential variables (the expected stock return and the horizon), but also find the opposite effect for other reasonable values of these variables. The intuition of their result, also found in Hens and Vlcek, may be simply summarized in a two-period framework. Investors initially buying stocks for two periods and prone to sell their stocks after a gain at date 1, would not have bought the stock in the first place at date 0. These authors exhibit, in a number of situations, a contradiction between the first purchase at date 0 and the decision to sell at date 1. In a continuous-time model, Kyle *et al.*(2006) also show that, depending on the risk-return characteristics of the stocks, the disposition effect or the opposite can arise at the theoretical level. Roughly speaking, investors exhibit a propensity to hold on to losers for stocks with relatively low Sharpe ratios. They may liquidate early their winning stocks when the current profits are rising or when they drop to the break-even point.

The results of these theoretical works show that there is no consensus about the explanation of the disposition effect. However, as we will see in the next section, this is a very robust phenomenon on the empirical and experimental point of view.

2.2 The empirical evidence

The label *disposition effect* was first given by Shefrin and Statman (1985) to translate the tendency of investors to sell their winners too early and hold on to their losers too long. The early studies analyzed the abnormal trading volumes on stocks that had risen in price over previous periods. Lakoniskok and Smidt (1986) found much more volume for winners on NYSE and Amex stocks and Ferris *et al.* (1988) showed on 30 U.S stocks that current volume is negatively correlated with volume on the preceding days, when the price was higher than the current price. The reverse correlation appears for stocks whose prices were lower on the preceding days. As these studies were concerning aggregate volume, they were not revealing the individual decision process of buyers and sellers.

Concerning the behaviour of individual investors, the reference study is Odean (1998) who analyzed 10 000 individual accounts at a large discount broker between 1987 and 1993. The two main results of Odean may be summarized as follows:

a. The proportion of realized gains is significantly larger than the proportion of realized losses, except in December (essentially for tax reasons).
b. The winning stocks (which were sold) perform better than the losing stocks (which were kept).

An elementary interpretation of the first result is that a paper loss is perceived as less painful than a realized loss or, symmetrically, a realized gain generates more utility than a paper gain. The other usual interpretations are that investors erroneously believe that stock prices revert to the mean; they then realize gains and retain losing investments to wait for positive returns, expected to compensate preceding paper losses. Selling the winners may also be justified to rebalance portfolios or to avoid high transactions costs on low-price stocks. Odean found that the disposition effect was persistent even when controlling for these two arguments. Lehenkari and Pertunnen (2004) analyzed individual trades on the Finnish markets and also found that capital losses reduce the selling propensity of investors; however, they didn't find the opposite effect for capital gains.

Concerning professional traders, Garvey and Murphy (2004) analyzed the behaviour of a proprietary stock-trading team (15 traders) specialized on NASDAQ stocks. This team generated $1.4 million in intraday trading profits for a 3-month period (8 march-13 june 2000), in a downward-trending market. The members of such teams obviously work with a very short horizon (a few minutes) and their trades are not motivated by diversification needs or capital constraints. The activity of each trader in the team focuses on one or two stocks. Garvey and Murphy show that the mean duration of a losing roundtrip is 268 seconds and the duration of a winning roundtrip is only 166 seconds. When a day is divided into three periods of about two hours each, the difference between the durations of loosing and winning roundtrips is significant in each period, even if mean durations are much shorter in the morning. The authors also distinguish three categories of roundtrips, depending on the size of the trade. They also found a significant difference between the mean durations in each category. Finally, Garvey and Murphy show that closing profitable positions early and holding losing positions longer is not optimal since it reduces the global profitability of the team. Coval and Shumway (2005), Frino and *al.* (2005) and Locke and Mann (2003) obtain the same kind of results on different futures markets. In the same vein, Jordan and Diltz (2004) show that a large majority of day traders hold losing trades longer than profitable trades. Shapira and Venezia (2001) also observe the disposition effect for professional and individual investors on the Israeli market. Shu *et al.* (2005) show that tawainese investors are much more reluctant to realize their losses than U.S investors. However, they observe the reverse effect on low-return, high-price stocks. They interpret their findings by saying that Taiwanese individual investors exhibit a stronger belief in mean reversion than U.S investors. Finally, Genesove and Mayer (2001) illustrate the differences in the behaviour of buyers and sellers on the housing market, leading to conclude to the existence of a disposition effect.

2.3 The experimental evidence

Weber and Camerer (1998) conducted an experiment with six stocks characterized by different (but constant) expected returns. The price changes were exogenous so that players could infer (after a number of periods) with reasonable accuracy the stocks with an upward (downward) trend or no trend at all. This design was chosen to mitigate the disposition effect or, symmetrically, to reinforce the result if this effect was observed. On the aggregate, 51 % of sales were concerning winning stocks and only 39 %, losing stocks.

Two other interesting points have to be mentioned in this study. Participants were informed about the independence of successive price changes. They then could not rationally believe in mean reversion of stock prices. As they were learning about the trend after each price change, one could have expected that players were eager to keep winning stocks and to sell losing stocks, a behaviour contradicting the disposition effect.

The second experiment presented by Weber and Camerer (1998) was also performed. In this one, stocks were automatically sold at the end of each of the 14 periods of the game. Participants had the possibility to buy back the stocks immediately without bearing transaction costs. The authors observed that the disposition effect was largely reduced when this automatic selling procedure was used. In other words, players were not so eager to buy back the losing stocks. This observation leads to evoke the question of self-control problems. When agents have to decide themselves, they irrationally believe that stocks will bounce back after a price decrease, even if they were convinced at preceding dates that successive price changes are independent random variables. Roughly speaking, they may be subject to the gambler's fallacy. After losing periods, they start to believe that "luck" is due in the next draws.

3 Loss aversion in a discrete-time model

3.1 Preliminaries

We first consider a one-period model with two dates, denoted as 0 and 1. Loss averse investors are characterized by valuation functions defined as follows:

$$v(x) = (x - x^*)_+ - \lambda (x^* - x)_+$$

where x is the final wealth, $\lambda > 1$ is the loss aversion coefficient and x^* is the reference wealth level.

Assume now that two assets are traded on the market at date 0: first, there is a stock, the random terminal payment of which being denoted as *S*, taking N terminal values $(x_i, i=1,...,n)$ with probabilities $(p_i, i=1,...,n)$. Second, there is a risk-free asset, the bond, generating a known rate of return, denoted as $r-1$. The initial price of the bond is normalized to 1 and the terminal values of the risky asset are assumed to be ranked in the increasing order. This assumption is not crucial since the terminal values of any portfolio containing stocks and bonds will be ranked in the same order.

An investor endowed with an initial wealth W_0 chooses to invest in stocks if her expected valuation function is greater to the one obtained by investing in bonds. We assume that the date-1 reference level is rW_0, the terminal wealth level for a 100 % investment in the bond. A consequence of this assumption is that the valuation function is equal to 0 when W_0 is fully invested in bonds.

It also allows us to assume, without loss of generality, that the initial wealth is equal to 0. In this framework, an investor who wants to buy stocks, borrows at date 0 at the risk-free rate $r-1$. The net cash-flow obtained at the terminal date is then equal to $W_1 - rW_0$ which can be simply written as $N(S_1 - rS_0)$ where S_t, $t = 0,1$, stands for the date-*t* price of the stock and *N* is the number of stocks bought at date $t = 0$. Without loss of generality, we assume $N = 0$ in the following.

As decisions are taken at date 0, the investor bases her evaluation on the probability distribution of her final wealth. Purchasing the stock is then desirable only if the following inequality is satisfied:

$$E\left[v(S_1)\right] = \sum_{i=1}^{n} p_i \left(x_i - rS_0\right)_+ - \lambda \sum_{i=1}^{n} p_i \left(rS_0 - x_i\right)_+ \geq 0 \tag{1}$$

When $E\left[v(S_1)\right] = 0$, the investor is indifferent between stocks and bonds. Thanks to the ranking of the terminal values of the stock, equation (1) can be rewritten as follows:

$$E\left[v(S_1)\right] = \sum_{i=i_0}^{n} p_i \left(x_i - rS_0\right) - \lambda \sum_{i=1}^{i_0-1} p_i \left(rS_0 - x_i\right) \geq 0$$

where the index i_0 is defined by $i_0 = \inf\left\{i \in \{1,2,...n\} / x_i \geq rS_0\right\}$.

The first question we address in the following is to calculate the loss aversion parameter which keeps the investor indifferent between stocks and bonds.

3.2 The critical loss aversion index in the two-state, one-period model

We now assume that $n = 2$. The first state is called the down-state and the second the up-state. We simply get the following preliminary result.

Proposition 1

1) The loss aversion parameter λ^*, keeping the investor indifferent between the two assets, is defined by:

$$\lambda^* = \frac{p_2}{p_1}\frac{u-r}{r-d} \tag{2}$$

where u and d are defined by $x_2 = uS_0$ and $x_1 = dS_0$.

2) $\lambda^* > 1 \Leftrightarrow E\left(\frac{S_1}{S_0}\right) > r$

u and *d* define the gross returns on the risky asset in the two states. These notations are the ones used by Cox-Ross-Rubinstein (1979) in their seminal paper on the binomial option pricing model.

An agent characterized by the parameter λ^* will be called a *critical investor*.

The formulation of λ^* in equation (2) has a natural interpretation. Assume for a moment that the probabilities of the two states are equal to 0.5 and consider a distortion of parameters *u* and *d* such that the expected return on the stock remains constant but the variance increases. For example, if *u* becomes $u + a$ and *d* becomes $d – a$, the parameter λ^* decreases when *a* is positive. It simply means that some investors preferring stocks in the first economy would choose bonds in the other, more risky, economy.

The second point in proposition 1 is also intuitive. The existence of a positive risk premium is equivalent to a loss averse critical investor ($\lambda^* > 1$). In fact, if the critical investor was risk-neutral (or loss-neutral), there would be no reason to get a risk premium on the stock.

Finally, the critical value obtained in proposition 1 has a nice interpretation in terms of probabilities. It can be seen as the ratio of two probability ratios by writing:

$$\lambda^* = \frac{p_2 / p_1}{(r-d)/(u-r)}$$

The numerator is the ratio of the real probabilities of the two states and the denominator is the ratio of the corresponding risk-neutral probabilities.

Finally, λ^* is a decreasing function of the risk-free rate. In fact, we have :

$$\frac{\partial \lambda^*}{\partial r} = \frac{p_2}{p_1} \frac{d-u}{(r-d)^2} < 0$$

This result is also quite intuitive; all other things being equal, a lower proportion of investors choose the risky asset when the risk premium is lower or, equivalently, when the risk-free asset is more attractive.

Suppose now that our critical investor wants to hedge her stock portfolio by buying a put option with an exercise price equal to $K = rS_0$. Remember that this exercise price is the one which prevents the investor's wealth to become negative at date 1. The following corollary shows that the reservation price of this put option is equal to the arbitrage-free price obtained in the usual binomial model.

Corollary 2

The reservation price of a put option with a strike price $K = rS_0$ which prevents the critical investor from getting a negative final wealth is equal to:

$$\phi_0 = \frac{1}{r}(r-d)S_0\left(\frac{u-r}{u-d}\right) \tag{4}$$

This is in fact not surprising since our critical investor is indifferent between the stock and the bond. It means that the critical loss aversion index establishes the link between real and risk-neutral probabilities, as mentioned before. Indeed, the formulation of the put price does not depend on the real probability of a down-state. It is so because, for the critical investor under consideration, the risk of the stock is exactly compensated by its expected return.

3.3 The two-period model

To study the time-diversification effect in the discrete-time model, we consider a two-period binomial model with constant parameters u and d. The following proposition shows that the critical investor of the preceding section would get a strictly positive expected valuation function by investing in stocks.

Proposition 3

If $\lambda = \lambda^* > 1$, *we get* $E\left[v(S_2)\right] > 0$.

This result can be interpreted in several ways. First, if we assume that investors are characterized by the same loss aversion coefficient, those with a short horizon sell their stocks to investors with longer horizons. However, one cannot be satisfied by this idea since it leads to a quite surprising result. In equilibrium, the only investors possessing stocks are the ones with the longest horizon.

In the preceding two-period model, investors do not decide anything at date 1. However, the market is open and they could change their mind by this date, switching from bonds (stocks) to stocks (bonds). In the following section we introduce the disposition effect by allowing our critical investor to sell her stocks at date 1.

4 The disposition effect and the equity premium

4.1 Introduction of the disposition effect

When investors are prone to the disposition effect, it may have some consequences on the equity premium they require on stocks. In fact, assume that an agent with a given horizon T decides at date 0 to invest in stocks. After a gain (a stock price increase), she may be prone to switch to bonds, even if she decided at date 0 with a two-period horizon in mind. This tendency to sell winners too early depends on several factors like willpower or self-control. However, an investor knowing that she has self-control problems[4] also knows that the distribution of returns she will face at date T is not the one corresponding to a 100 % stock portfolio.

In the following, we introduce this effect in the two-period model of section 3. More precisely, we consider that the investor will switch to bonds with a probability θ at date 1 when the stock price is uS_0.

Figure 1 around here

Figure 1 depicts the two-period tree in this framework. At dates 0 and 1, the lower cell in each group contains the vector of probabilities of reaching the successors, starting by the probability of an up-state at the next date. Compared to the model in section 3, the essential modification is the introduction of a value urS_0 at date t = 2, which is possibly reached with a probability θ when starting in the up-state at t = 1.

This change in the two-period tree takes into account a switch from stocks to bonds at date 1, due to the disposition effect.

Let us assume S_0 = 1 to simplify the notations. The value of the portfolio of the investor can now take four values at date 2, respectively u^2, ur, ud and d^2 with probabilities $p^2(1-\theta)$, $p\theta$, $p(1-p)(2-\theta)$ and $(1-p)^2$.

Remember that in the binomial model, we need to distinguish the two cases $ud > r^2$ and $ud < r^2$. We will first consider the case $ud > r^2$ which is the most plausible when the volatility of stock returns is not too high and the expected return not too low[5].

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[4] Obviously, the problems we refer to here are not pathological. Everybody has experienced such problems. You take a decision today, quit smoking tomorrow for example, and you continue to smoke the day after, and so on.

[5] In fact, let $u = r + \pi + \sigma$ and $d = r + \pi - \sigma$ where π stands for the equity premium and assume here that p = 0.5. σ is then the standard deviation of returns. We get:

$$ud - r^2 = (r + \pi + \sigma)(r + \pi - \sigma) - r^2 = 2\pi r + \pi^2 - \sigma^2$$

If π is comparable to the historical equity premium (around 6 %) and if we assume that r = 1, corresponding to a risk-free rate equal to 0, the volatility of the stock return must be higher than 40 % for the right hand-side of the above equation to be negative. Obviously, this remark is reinforced if the risk-free rate is positive.

4.2 The disposition effect and the agent's welfare

We now change our notations and denote W_2 as the final wealth of the investor at date 2. This change of notations is justified because an investor starting with stocks may end with bonds at date 2 by switching at date 1 in the up-state. The date-2 stock price is then not her final wealth in this case. We also use $v(W_2, \lambda, \theta)$ to denote the value function because it now depends on the probability of a switch at date 1.

Proposition 4

If the equity premium is positive, we have:

$$\frac{\partial E\left[v(W_2,\lambda^*,\theta)\right]}{\partial\theta} < 0 \;\; and \;\; E\left[v(W_2,\lambda^*,1)\right] > 0$$

This result concerning the derivative is intuitive when $ud - r^2 > 0$. In this situation, the second period will be equivalent to play a lottery which generates only gains. As our value function is piecewise linear, it is as if the investor was risk-neutral on this part of the curve. Consequently, the investment in stocks would generate value because the risk premium is positive and the investor would not require any risk premium here. It explains why it is in fact not optimal to realize the gain at date 1. When $ud - r^2 < 0$, two conflicting effects are at work. The first one is the equity premium, which is an incentive to hold stocks in the second period. The second one is the fact that a price decrease in period 2 generates a loss. It may then be an incentive to switch to bonds. Even if a down-state occurs after switching, the final wealth corresponds to a gain $ur - r^2$. Proposition 4 means that the first effect is always greater for the one-period critical investor. At a first glance, it seems surprising; however the result doesn't concern any investor but only the critical investor (and investors with a lower loss aversion coefficient).

The second inequality in proposition 4 shows that the expected valuation function of the one-period critical investor remains positive when she is prone to the disposition effect. In other words, the equity premium puzzle cannot be explained only by the disposition effect.

However, the first part of this proposition implies that the equity premium required by investors is larger when they take into account the disposition effect. The following subsection addresses the question of the measurement of the variation in the equity premium, coming from the existence of this effect.

4.3 The variation of the equity premium due to the disposition effect

The purpose of this section is to compare the risk premia required by two investors characterized by the same loss aversion coefficient. More precisely, we will take as a benchmark an investor who is not prone to the disposition effect and determine the risk premium required by a second investor prone to this effect with a probability θ.

The benchmark investor is then the **two-period** critical investor characterized by a loss aversion parameter $\lambda = \lambda' > 1$. To allow a direct comparison of risk premia, we consider the simple case $p = 0.5$, $u = r + \pi + \sigma$ and $d = r + \pi - \sigma$. The parameter π is then the risk premium and σ is the standard deviation of stock returns.

Proposition 5

The critical two-period loss aversion parameter λ' is defined by:

$$\lambda' = \frac{4\pi^2 - (\sigma - \pi)^2 + 2r(3\pi + \sigma)}{(\sigma - \pi)(2r + \pi - \sigma)} \text{ if } ud - r^2 > 0 \qquad (5)$$

$$\lambda' = \frac{4\sigma(r + \pi)}{(\sigma + \pi)^2 - 4\pi^2 + 2r(\sigma - 3\pi)} \text{ if } ud - r^2 < 0 \qquad (6)$$

In footnote 2 we recalled that the relationship $ud = r^2$ induces the following link between the volatility and the risk premium:

$$\sigma^2 = \pi(2r + \pi) \qquad (7)$$

In this particular case, the value of λ' is given by:

$$\lambda' = \frac{(\sigma + \pi)(2r + \sigma + \pi)}{(\sigma - \pi)(2r + \pi - \sigma)} \qquad (8)$$

To estimate the critical loss aversion parameter, consider the following figures reported in Barberis *et al.* (2001) concerning Treasury Bills and NYSE stock data on the period 1926-1995:

$$r = 1.0058; \pi = 5.45\%; \sigma = 20.02\%$$

These values correspond to the first part of proposition 5, that is $ud - r^2 > 0$. We obtain $\lambda^{'} = 2.65$, a value very close to the estimation by Benartzi and Thaler (1995) for a piecewise linear utility function (2.77), but slightly greater than the one estimated by Tversky and Kahneman (1992) which was 2.25 (but obtained with an S-shape valuation function).

4.4 Sensitivity analysis

We now analyze the supplementary premium required by an investor characterized by $\lambda^{'} = 2.65$, if she is prone to the disposition effect with intensity θ. In other words, we study the relationship between θ and π,the other parameters being given. According to the historical levels of the risk-free rate and to the volatility of stock returns, we only consider the case $ud - r^2 > 0$.

The expected valuation function may now be written as:

$$\begin{aligned}
E\left[v(W_2,\lambda',\theta)\right] &= 0.25((1-\theta)\left[(r+\pi+\sigma)^2 - r^2\right] + 2\theta r(\pi+\sigma) \\
&\quad +(2-\theta)\left[(r+\pi+\sigma)(r+\pi-\sigma) - r^2\right] + \lambda'\left[(r+\pi-\sigma)^2 - r^2)\right]) \\
&= 0.25((1-\theta)\left[(\pi+\sigma)^2 + 2r(\pi+\sigma)\right] \\
&\quad +2\theta r(\pi+\sigma) + (2-\theta)(\pi^2 - \sigma^2 + 2r\pi) \\
&\quad -\lambda'\left[(\pi-\sigma)^2 + 2r(\pi-\sigma)\right]) \\
&= 0.25((1-\theta)(\pi+\sigma)^2 + 2r(\pi+\sigma) \\
&\quad +(2-\theta)(\pi^2 - \sigma^2 + 2r\pi) + \lambda'\left[(\pi-\sigma)^2 + 2r(\pi-\sigma)\right])
\end{aligned}$$

(9)

The numerical results are summarized on figure 2 which represents the equity premium required by the critical investor as a function of θ. Using the abovementioned parameters, we get a risk premium varying from 5.45 %, when $\theta = 0$, to 7 % when $\theta = 1$, that is a premium variation of 1.5 %. However, the intensity of the disposition effect cannot be expected to be very strong, corresponding to values of θ around 0.2. It leads to an increase of about 25 basis points for the risk premium. Consequently, even if the role of the disposition effect is not negligible, the disposition effect cannot justify a significant part of the equity premium. The following proposition uses the implicit function theorem to calculate the derivative of the equity premium with respect to the intensity of the disposition effect.

Proposition 6

On the level curve $E\left[v(W_2,\lambda',\theta)\right]=0$ *the derivative of the equity premium with respect to* θ *is given by:*

$$\frac{d\pi}{d\theta}=\frac{\pi(\pi+\sigma+r)}{r+(1-\theta)(\pi+\sigma)+(2-\theta)(r+\pi)+\lambda'\left(r+\pi-\sigma\right)}$$

The risk premium required by an investor is then an increasing function of θ, as illustrated on figure 2. More over, the derivative in proposition 6 also increases in θ. It justifies the convex curve obtained on figure 2. Referring to empirical and experimental works quoted in section 2, a value of θ, around 0.2 or 0.3 seems reasonable. In this case, the variation of the equity premium is around 25 basis points. Though it is not an important increase, it is a non negligible part of the equity premium and deserves attention.

4. Concluding remarks

The model presented in this paper is simple and parsimonious because of the assumption of a piecewise linear valuation function. Only two parameters are important, the loss aversion coefficient and the probability of switching from stocks to bonds.

The disposition effect is one of the more robust investor drawbacks documented in the literature. We have analyzed the impact of the intensity of this effect on the equity premium. Our essential results show that a one-period critical investor still invest in stocks for two periods even if she is prone to the disposition effect. However, the two-period critical investor requires a higher risk premium to continue to invest in stocks when she is a disposition investor. Moreover, the risk premium is an increasing and convex function of the intensity of the disposition effect.

Further research is needed in several directions. First, considering a multi-period model would be interesting to analyze the persistence of the effect. However, it leads to some difficulties and necessitates some arbitrary assumptions about the investor strategy. Especially, in a *n*- period model, what is the decision process of an investor who switches at an intermediate date. Does he switch only once, or can she come back to stocks a few periods later?

A second direction could be to consider a continuous-time model. The advantage of a piecewise linear valuation function is that it can be written as the terminal payoff of a portfolio of call and put options. Switching from bonds to stocks before the horizon of investment is equivalent to an early exercise of the two types of options. Though the rules of optimal early exercise for a put option are well known, the corresponding rules for a portfolio of calls and puts are not so obvious.

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Appendix

Proof of proposition 1

1) u and d define the gross returns on the risky asset in the two states. These notations are the ones used by Cox-Ross-Rubinstein (1979) in their seminal paper on the binomial model.

It is well known that the absence of arbitrage opportunities implies $d < r < u$. As mentioned before, indifference between the stock and the risk-free asset means:

$$E\left[v(S_1/r)\right]=\frac{1}{r}\left(p_2\left(uS_0-rS_0\right)-\lambda p_1\left(rS_0-dS_0\right)\right)=0 \tag{A1}$$

Equation (2) follows immediately.

2) The expected gross return on the stock is equal to:

$$E\left[S_1/S_0\right]=p_2u+p_1d$$

We also have:

$$\lambda^*>1\Leftrightarrow p_2(u-r)>p_1(r-d)\Leftrightarrow p_2u+p_1d>r$$

The result in point (2) is then obvious. ■

Proof of corollary 2

As the initial wealth is assumed to be 0, the agent borrows ϕ_0+S_0 at time 0, the put being devoted to protect the risky portfolio against returns lower than *r*; writing the expected valuation function gives the following relationship :

$$p_2\left(uS_0-(S_0+\phi_0)r\right)-\lambda^*(1-p_2)\left[(S_0+\phi_0)r-dS_0-(rS_0-dS_0)\right]=0$$

The first term corresponds to an up-state in which the put option is not exercised. The second term corresponds to a down-state and includes three values: the terminal value of the stock, the terminal payment of the put option and, finally, the reimbursement of the amount borrowed at date 0.

After elementary simplifications, we get:

$$p_2\left(uS_0-(S_0+\phi_0)r\right)-\lambda^*(1-p_2)\phi_0r=0$$

The result announced in the corollary is immediately obtained by replacing λ^* by its value.■

Proof of proposition 3

To simplify the notations, we denote $p_2 = p$ and consider an initial price of the stock equal to 1. In fact, the sign of $E\left[v(S_2)\right]$ is independent of S_0. Using this simplifying assumption, the stock price takes three values at date 2, respectively u^2, ud and d^2 with probabilities p^2, $2p(1\text{-}p)$ and $(1-p)^2$.

Two cases must be considered, depending on the position of ud with respect to r^2. If $ud > r^2$ an up-down sequence generates a gain. On the contrary, if $ud < r^2$ the same sequence generates a loss.

1) $ud > r^2$

$$E\left[v(S_2)\right] = p^2(u^2 - r^2) + 2p(1-p)(ud - r^2) - \lambda^*(1-p)^2(r^2 - d^2)$$

Replacing λ^* by its value and dividing the result by p leads to:

$$\frac{E\left[v(S_2)\right]}{p} = p(u^2 - r^2) + 2(1-p)(ud - r^2) - (1-p)(r+d)(u-r)$$

A few lines of elementary calculations reduce this expression to:

$$\frac{E\left[v(S_2)\right]}{p} = (u+r)\big(p(u-r) + (1-p)(d-r)\big) \tag{A2}$$

The second term in the RHS of (A2) is the excess return on the stock, relative to the risk-free rate. In proposition 1, we proved that it is positive as soon as the critical investor is loss averse (corresponding to $\lambda^* > 1$).

2) $ud < r^2$

$$\begin{aligned}
E\left[v(S_2)\right] &= p^2(u^2 - r^2) - \lambda^*\big(2p(1-p)(r^2 - ud) + (1-p)^2(r^2 - d^2)\big) \\
&= p^2(u^2 - r^2) - p\frac{u-r}{r-d}\big(2p(r^2 - ud) + (1-p)(r^2 - d^2)\big) \\
&= p\frac{u-r}{r-d}\Big[p(u+r)(r-d) - \big(2p(r^2 - ud) + (1-p)(r^2 - d^2)\big)\Big]
\end{aligned}$$

A few more transformations give:

$$E\left[v(S_2)\right] = p\frac{u-r}{r-d}(r+d)\left[p(u-d) - (r-d)\right]$$

We now use the point (2) of proposition 1, saying that the excess return on the stock is positive. It implies that:

$$p(u-d) - (r-d) > p(u-d) - (pu + (1-p)d - d) = 0$$

We then get $E\left[v(S_2)\right] > 0$.

Another way to get this result is to remark that a positive risk premium is equivalent to an inequality between the risk-neutral probability of an up-state and the corresponding real probability, the former being lower than the latter. This inequality is in fact:

$$p > \frac{r-d}{u-d} \Leftrightarrow p(u-d)-(r-d) > 0$$

■

Proof of proposition 4

1) We first analyze the case $ud > r^2$.

The expected valuation function of the one-period critical investor is worth:

$$\begin{aligned} E\left[v(W_2,\lambda^*,\theta)\right] = &\left[p^2(1-\theta)(u^2-r^2)+p\theta(ur-r^2)+(2-\theta)p(1-p)(ud-r^2)\right] \\ &-\lambda^*(1-p)^2(r^2-d^2) \end{aligned}$$

where $\lambda^* = \dfrac{p}{1-p}\dfrac{u-r}{r-d}$. We then obtain :

$$\begin{aligned} E\left[v(W_2,\lambda^*,\theta)\right] &= p\left[p(1-\theta)(u^2-r^2)+\theta(ur-r^2)+(2-\theta)(1-p)(ud-r^2)\right]-\lambda^*(1-p)^2(r^2-d^2) \\ &= p\left[p(1-\theta)(u^2-r^2)+\theta(ur-r^2)+(2-\theta)(1-p)(ud-r^2)\right]-\frac{u-r}{r-d}p(1-p)(r^2-d^2) \\ &= p\left[p(1-\theta)(u^2-r^2)+\theta(ur-r^2)+(2-\theta)(1-p)(ud-r^2)-(1-p)(u-r)(r+d)\right] \\ &= p\left[p(1-\theta)u^2+\theta ur+(2-\theta)(1-p)ud-(1-p)(ur+ud-rd)-r^2\right] \\ &= p\left[p(1-\theta)u^2+\theta ur+(1-\theta)(1-p)ud-r(1-p)(u-d)-r^2\right] \end{aligned}$$

We now remark that:

$$\begin{aligned} \frac{\partial E\left[v(W_2,\lambda^*,\theta)\right]}{\partial\theta} &= p\left[-pu^2+ur-(1-p)ud\right] \\ &= -pu\left[(1-p)d+pu-r\right] < 0 \end{aligned}$$

The derivative of the valuation function with respect to θ is negative as soon as the risk premium (which is the term between brackets) on the stock is positive.

We now calculate the valuation function at θ = 1.

$$
\begin{aligned}
E\left[v(W_2,\lambda^*,1)\right] &= p\left[ur - r(1-p)(u-d) - r^2\right] \\
&= p\left[r(u-r) - r(1-p)(u-d)\right] \\
&= pr(u-d)\left[\frac{u-r}{u-d} - (1-p)\right]
\end{aligned}
$$

The term between brackets is the difference between the risk-neutral probability of a down-state and the corresponding actual probability. It is well-known that when the risk premium on the risky asset is positive, the risk-neutral probability of the up-state is lower than the corresponding real probability. It follows that the reverse inequality is satisfied by the probability of reaching the down-state. It follows that

$$E\left[v(W_2,\lambda^*,1)\right] > 0$$

2) When $ud < r^2$ the expected value function is written as:

$$
\begin{aligned}
E\left[v(W_2,\lambda^*,\theta)\right] = &\left[p^2(1-\theta)(u^2-r^2) + \theta(ur-r^2)\right] \\
&-\lambda^*\left[(2-\theta)p(1-p)(r^2-ud) + (1-p)^2(r^2-d^2)\right]
\end{aligned}
$$

because a up-down sequence now generates a loss.

Replacing λ^* by its value, and simplifying by $(1-p)$ leads to:

$$
\begin{aligned}
E\left[v(W_2,\lambda^*,\theta)\right] = &\left[p^2(1-\theta)(u^2-r^2) + p\theta(ur-r^2)\right] \\
&-p\frac{u-r}{r-d}\left[(2-\theta)p(r^2-ud) + (1-p)(r^2-d^2)\right] \\
= &\ p\left[p(1-\theta)(u^2-r^2) + \theta(ur-r^2) - \frac{u-r}{r-d}\left[(2-\theta)p(r^2-ud) + (1-p)(r^2-d^2)\right]\right]
\end{aligned}
$$

Taking the derivative with respect to θ gives:

$$\begin{aligned}
\frac{\partial E\left[v(W_2,\lambda^*,\theta)\right]}{\partial\theta} &= p\left[-p(u^2-r^2)+(ur-r^2)+\frac{u-r}{r-d}p(r^2-ud)\right] \\
&= p\left[-r^2(1-p-\frac{u-r}{r-d}p)-p(u^2)+(ur)-\frac{u-r}{r-d}p(ud)\right] \\
&= p\left[(-r^2+ur)(1-p\frac{u-d}{r-d})\right] \\
&= rp\left[(u-r)(1-p\frac{u-d}{r-d})\right] \\
&= rp\left[\frac{u-r}{r-d}(r-d-p(u-d))\right] = rp\left[\frac{u-r}{r-d}(r-pu-(1-p)d)\right]
\end{aligned}$$

Just as in point (1), the positive risk premium allows us to conclude that $\frac{\partial E\left[v(W_2,\lambda^*,\theta)\right]}{\partial\theta}<0$.

We now calculate the valuation function at θ = 1.

$$\begin{aligned}
E\left[v(W_2,\lambda^*,1)\right] &= p\left[(ur-r^2)-\frac{u-r}{r-d}\left[p(r^2-ud)+(1-p)(r^2-d^2)\right]\right] \\
&= p(u-r)\left[r-\frac{1}{r-d}\left[p(r^2-ud)+(1-p)(r^2-d^2)\right]\right] \\
&= p\frac{u-r}{r-d}\left[r^2-rd-p(r^2-ud)-(1-p)(r^2-d^2)\right] \\
&= p\frac{u-r}{r-d}\left[-rd+pud+(1-p)d^2\right] \\
&= pd\frac{u-r}{r-d}\left[pu+(1-p)d-r\right]>0
\end{aligned}$$

Proof of proposition 5

$\lambda^{'}$ depends on whether $ud - r^2$ is positive or negative. Consider first the case $ud - r^2 > 0$. We get for the first investor:

$$E\left[v(W_2,\lambda',0)\right]=\frac{1}{4}\left[(u^2-r^2)+2(ud-r^2)+\lambda'(d^2-r^2)\right]=0$$

We deduce:

$$\lambda'=\frac{(u^2-r^2)+2(ud-r^2)}{(r^2-d^2)}=\frac{\left[(r+\pi+\sigma)^2-r^2+2((r+\pi+\sigma)(r+\pi-\sigma)-r^2)\right]}{(r^2-(r+\pi-\sigma)^2)}$$

After elementary simplifications, we obtain the desired result:

$$\lambda' = \frac{4\pi^2 - (\sigma - \pi)^2 + 2r(3\pi + \sigma)}{(\sigma - \pi)(2r + \pi - \sigma)}$$

Consider now the case $ud - r^2 < 0$ (even if it doesn't correspond to the historical value of the parameters). The valuation function is worth:

$$E\left[v(W_2)\right] = \frac{1}{4}\left[(u^2 - r^2) + \lambda'\left(2ud + d^2 - 3r^2\right)\right]$$

We deduce the critical loss aversion parameter:

$$\lambda' = \frac{u^2 - r^2}{3r^2 - 2ud - d^2}$$

Replacing *u*, *r* and *d* by their values we get, after a few lines of calculations:

$$\lambda' = \frac{4\sigma(r + \pi)}{(\sigma + \pi)^2 - 4\pi^2 + 2r(\sigma - 3\pi)}$$

Proof of proposition 6

$$\begin{aligned}
E\left[v\right] = 0.25(&(1-\theta)((r+\pi+\sigma)^2 - r^2) + 2\theta((r+\pi+\sigma)r - r^2) \\
&+(2-\theta)((r+\pi+\sigma)(r+\pi-\sigma) - r^2) \\
&+\lambda'\left((r+\pi-\sigma)(r+\pi-\sigma) - r^2\right)) \\
= 0.25(&(1-\theta)((\pi+\sigma)^2 + 2r(\pi+\sigma)) + 2\theta r(\pi+\sigma) \\
&+(2-\theta)(\pi^2 - \sigma^2 + 2r\pi) \\
&+\lambda'\left((\pi-\sigma)^2 + 2r(\pi-\sigma)\right))
\end{aligned}$$

We then get:

$$\begin{aligned}
\frac{\partial E\left[v\right]}{\partial \theta} &= -0.25(((\pi+\sigma)^2 + 2r(\pi+\sigma)) + 2r(\pi+\sigma) \\
&\quad -(\pi^2 - \sigma^2 + 2r\pi)) \\
&= -0.25((\pi+\sigma)^2 + (\pi^2 - \sigma^2 + 2r\pi)) \\
&= -0.5\pi(\pi+\sigma+r)
\end{aligned}$$

$$\begin{aligned}\frac{\partial E[v]}{\partial \pi} &= 0.25(2(1-\theta)(r+\pi+\sigma)+2r\theta+2(2-\theta)(r+\pi) \\ &\quad +2\lambda'(r+\pi-\sigma)) \\ &= 0.5((1-\theta)(r+\pi+\sigma)+r\theta+(2-\theta)(r+\pi) \\ &\quad +\lambda'(r+\pi-\sigma))\end{aligned}$$

Finally, the implicit function theorem leads to:

$$\frac{d\pi}{d\theta} = \frac{\pi(\pi+\sigma+r)}{(1-\theta)(r+\pi+\sigma)+r\theta+(2-\theta)(r+\pi)+\lambda'(r+\pi-\sigma)}$$

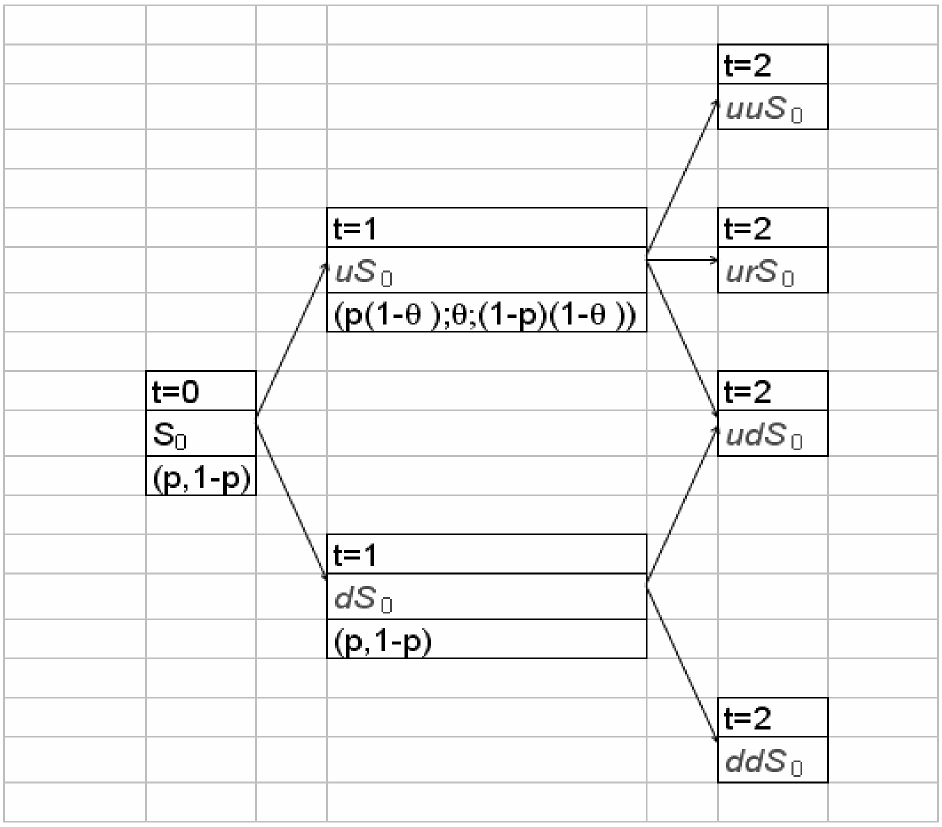

Figure 1: The two-period model when the disposition effect is introduced

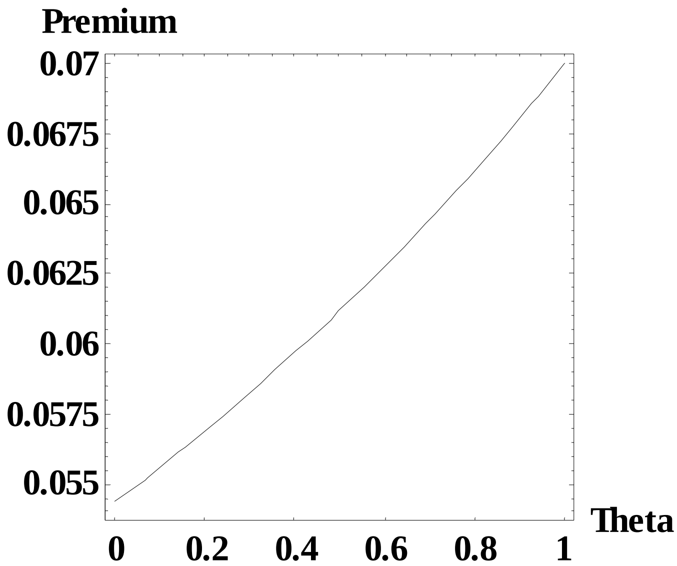

Figure 2: The risk premium as a function of θ