Chapter 8 – Options and their Valuation

What is a financial option?

An option is a contract which gives its holder the right, but not the obligation, to buy (or sell) an asset at some predetermined price within a specified period of time. Options are a type of derivative.

NOTE: It does not obligate its owner to take any action. It merely gives the owner the right to buy or sell an asset.

Calls and Puts:

Options fall into two basic groups, *calls* and *puts*. Each represents a *class* of options.

Call options: Call options, or more simply *calls*, give their owner (holder) the right but not the obligation to <u>buy</u> a specific quantity of some asset from the option writer for a set period of time at a fixed price. The asset is called the *underlying asset*, the set period of time is called the *time to expiration* or *time to expiry*, and the fixed price is called the *strike price* or *exercise price*.

Put options: Put options, or more simply *puts,* give their owner (holder) the right but not the obligation to <u>sell</u> a specific quantity of some asset to the option writer for a set period of time at a fixed price. The asset is called the *underlying asset*, the set period of time is called the *time to expiration* or *time to expiry*, and the fixed price is called the *strike price* or *exercise price*.

Option Terminology

- Option Price (Premium): The market price of the option contract.
- Expiration date: The date the option matures.
- Exercise value (or Intrinsic value): The value of a call option if it were exercised today = MAX[Current stock price - Strike price, 0].

  Note: The exercise value is zero if the stock price is less than the strike price.

- American Option: An option which can be exercised any time before it expires.
- European Option: An option which can only be exercised on its expiration date.

- Covered option: A call option written against stock held in an investor's portfolio.
- Naked (uncovered) option: An option sold without the stock to back it up.
- In-the-money call: A call whose exercise price is less than the current price of the underlying stock.

- Out-of-the-money call: A call option whose exercise price exceeds the current stock price.
- LEAPS: Long-term Equity AnticiPation Securities that are similar to conventional options except that they are long-term options with maturities of up to 2 1/2 years.

Uses of Options

- Options are typically granted to executives and other employees as a means of compensation. They are a form of compensation which is directly tied to maximizing the shareholder's wealth (increasing the stock price), therefore providing an incentive for executives and employees to work harder.
- Companies like the fact that granting options requires no immediate cash expenditure, though it might dilute shareholder wealth if the options are later exercised.

Long versus Short:

From the buyer's perspective an option is a *right but not an obligation*.

From the seller's perspective an option is a *contingent liability*.

The buyer (*holder*) of an option is often described as *long the option*.

The seller (*writer/grantor*) of an option is often described as *short the option.*

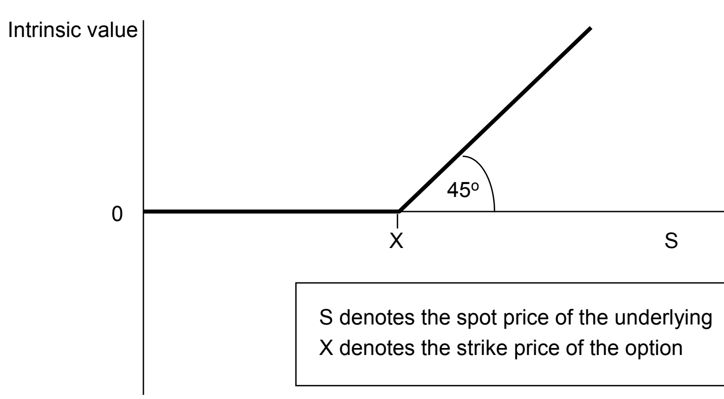
- Types of underlying assets:
  - *common stocks*
  - *stock indexes*
  - bonds
  - interest rates
  - exchange rates
  - commodities

- What exchanges trade standard equity options in the U.S.?
  - Chicago Board Options Exchange (CBOE)
  - International Securities Exchange (ISE)
  - American Stock Exchange (AMEX)
  - Philadelphia Stock Exchange (PHLX, “Philly”)
  - Pacific Coast Stock Exchange (PCX, “P-Co”)

Create a table which shows (S) stock price, (X) strike price, and intrinsic value of a Call

Price of Stock (S)	Strike Price (X)	Intrinsic Value of Option MAX[S-X, 0]
$10.00	$25.00	$0.00
15.00	25.00	0.00
20.00	25.00	0.00
25.00	25.00	0.00
30.00	25.00	5.00
35.00	25.00	10.00
40.00	25.00	15.00
45.00	25.00	20.00
50.00	25.00	25.00

Intrinsic value of a call option



S denotes the spot price of the underlying
X denotes the strike price of the option

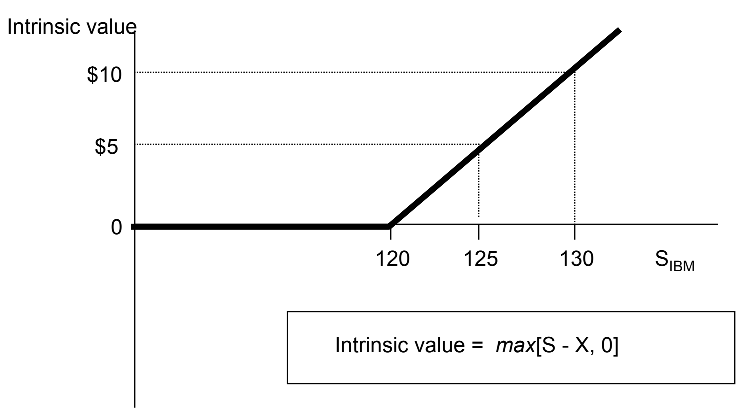
Example: Suppose that today is July 10 and the price of IBM stock is $125 a share. You would like to buy an American call option on IBM having a strike price of $120 and September expiration. This would be described as a *September 120 IBM call*.

This option would give you the right to buy 100 shares of IBM stock from the option writer for a price of $120 a share anytime between today and the expiration of the option on September 18 (third Friday of September).

Series:

All options of the same class with the same strike price and the same expiration date make up an *option series*.

Intrinsic value of a September 120 IBM call

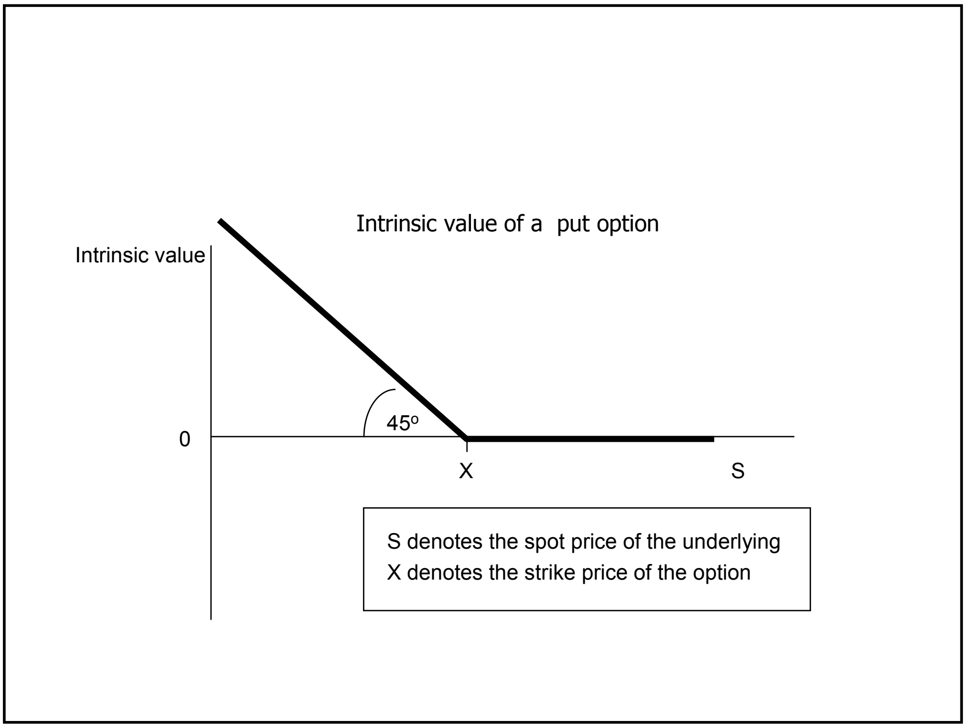


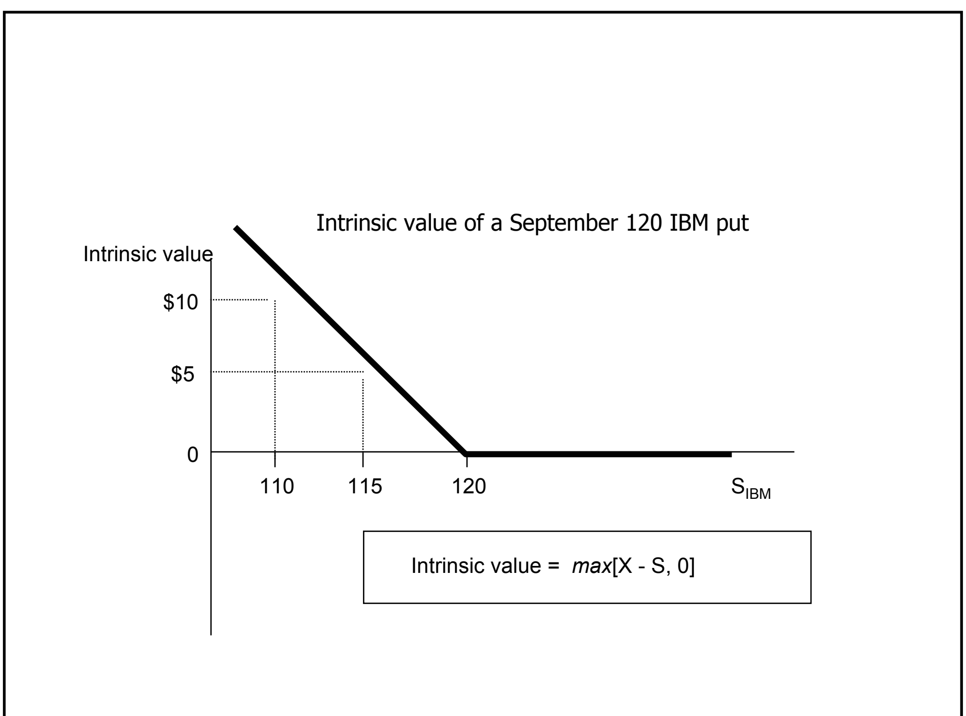
Put Options

- A put option represents the right to sell the underlying asset at the strike price.
- The exercise value of a put option is MAX[X-S, 0]. If the stock price is greater than the strike price, you would not exercise the put option.

Create a table which shows (S) stock price, (X) strike price, and intrinsic value of a Put

Price of Stock (S)	Strike Price (X)	Intrinsic Value of Option MAX[X-S, 0]
$10.00	$25.00	$15.00
15.00	25.00	10.00
20.00	25.00	5.00
25.00	25.00	0.00
30.00	25.00	0.00
35.00	25.00	0.00
40.00	25.00	0.00
45.00	25.00	0.00
50.00	25.00	0.00


Intrinsic value of a put option
Intrinsic value
45°
0
X
S
S denotes the spot price of the underlying
X denotes the strike price of the option


Intrinsic value of a September 120 IBM put
Intrinsic value
$10
$5
0
110
115
120
S IBM
Intrinsic value = max[X - S, 0]

Moneyness:

	Call	Put
if S > X	in-the money (ITM)	out-of-the money (OTM)
if S = X	at the money (ATM)	at-the-money (ATM)
if S < X	out-of-the-money (OTM)	in-the-money (ITM)

Other moneyness terms:

near the money
deep in the money
deep out of the money

Why does an option have value?

Options have two distinct types of value.

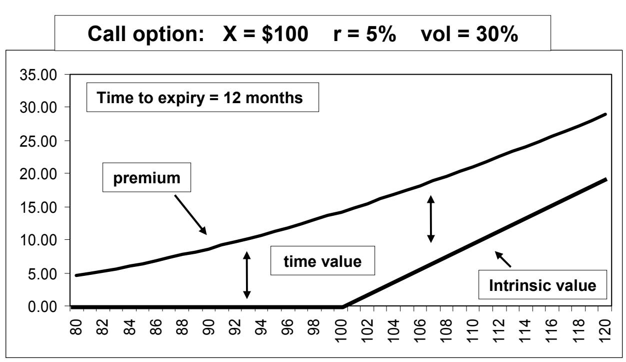
Intrinsic value: The value that would be captured if the option is exercised immediately.

Time value: The value that represents the potential for the option to acquire more intrinsic value before it expires.

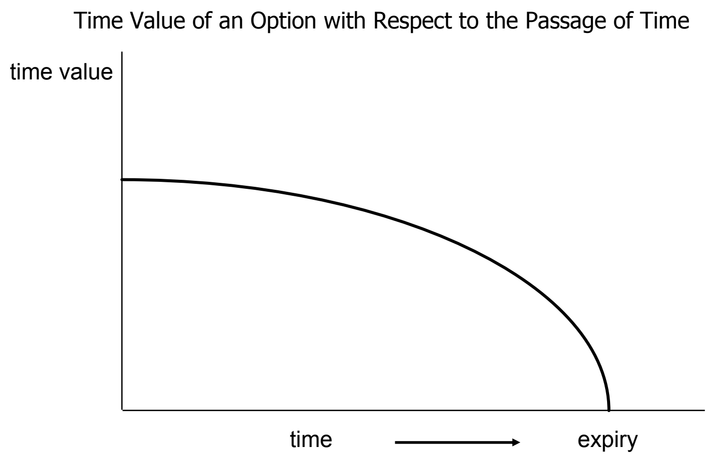
Premium = intrinsic value + time value

- The time value of an option represents the potential to acquire more intrinsic value.

- Time value is a function of (Value Drivers):
  - the *volatility* of the price of the underlying asset (measured on an annual basis)
  - the amount of *time* remaining until the option expires (measured in years)
  - the *moneyness* of the option
  - the risk-free *rate of interest*
  - *dividends* that the underlying will pay



The time value is the difference between the premium and the intrinsic value!

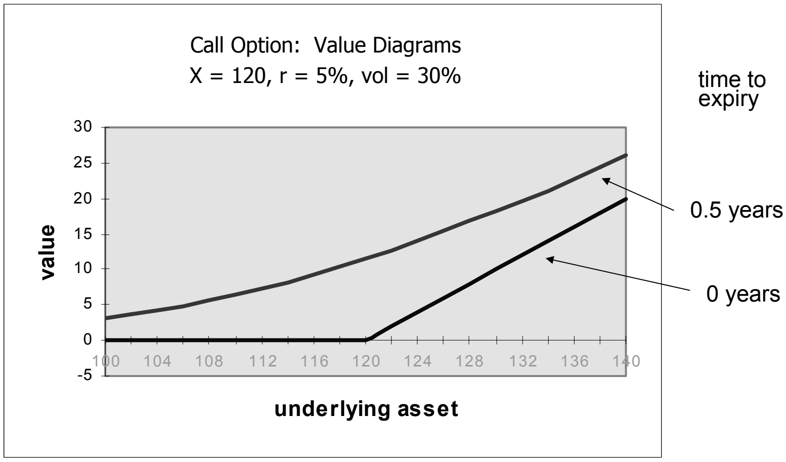


Value diagrams:

A value diagram is a visual depiction of the value of an option with respect to the underlying asset.

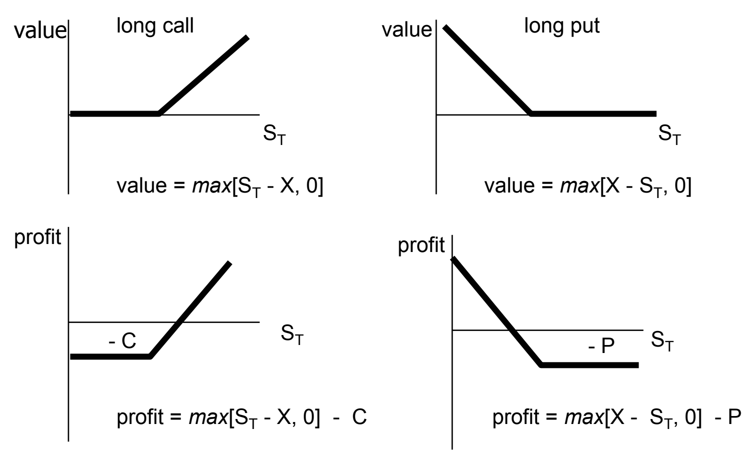
There are an infinite number of value diagrams associated with an option. The passage of time, changes in the risk-free rate of interest, and changes in volatility will all change an option's value diagram. This is so because all of these impact the time value component.

At the end of the option's life, the time value component will be zero, and so only the intrinsic value remains. It is for this reason that option valuation diagrams are most often drawn on the assumption that the option is at expiration.

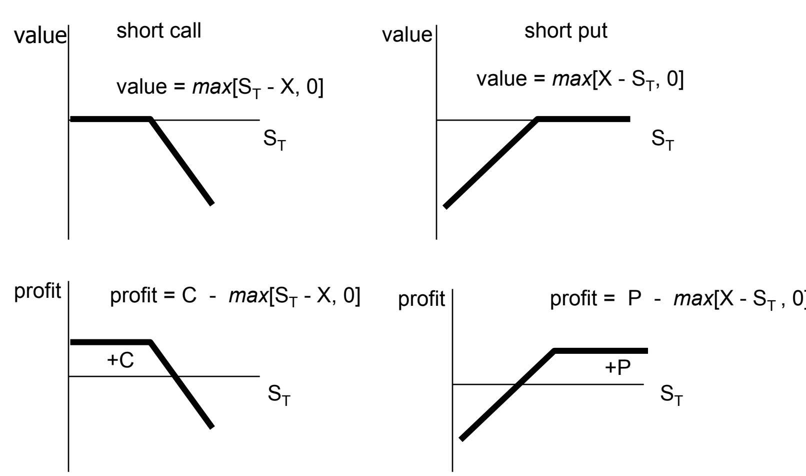


Profit diagrams (payoff profiles): An option's value diagram does not depict the profit or loss from buying or selling (writing) an option. This is so because it only depicts the value of the option at the end of the option's life. To get a true picture of the profit or loss, we need to take into consideration what was originally paid or received for the option when it was purchased or sold, respectively.

When we do this, the resultant graph is called a *profit diagram*. Sometimes it is called a payoff profile. However, the term "payoff profile" has to be used carefully because it is sometimes used to mean "value" and it sometimes used to mean "profit."



long call
value
S_T
value = $max[S_T - X, 0]$
profit
- C
S_T
profit = $max[S_T - X, 0]$ - C
long put
value
S_T
value = $max[X - S_T, 0]$
profit
- P
S_T
profit = $max[X - S_T, 0]$ - P



short call
value
value = $max[S_T - X, 0]$
S_T
profit
profit = C - $max[S_T - X, 0]$
+C
S_T
short put
value
value = $max[X - S_T, 0]$
S_T
profit
profit = P - $max[X - S_T , 0]$
+P
S_T

Valuing Options: Black-Scholes Option Pricing Model

Analytical Methods:

This approach was first successfully employed by Fischer Black and Myron Scholes, with assistance from Robert Merton, in 1969. It resulted in the publication of the now famous Black/Scholes model (1973) and the subsequent, more general, Merton model. Together, these models are now known as the Black/Scholes/Merton model.

In this approach, the model developer begins with a clearly defined set of assumptions. From these assumptions, the modeler derives directly a complete (closed form) solution that takes the form of a formula. The formula requires specific inputs and produces an unambiguous solution (option value).

What are the assumptions of the Black-Scholes Option Pricing Model?

- The stock underlying the call option provides no dividends during the call option's life.
- There are no transactions costs for the sale/purchase of either the stock or the option.
- R_{RF} is known and constant during the option's life.

(More...)

- Security buyers may borrow any fraction of the purchase price at the short-term risk-free rate.
- No penalty for short selling and sellers receive immediately full cash proceeds at today's price.
- Call option can be exercised only on its expiration date.
- Security trading takes place in continuous time, and stock prices move randomly in continuous time.

Black/Scholes Model Notation

σ: The volatility of the price of the underlying asset, measured as the standard deviation of the percentage price changes continuously compounded (called continuous return).

t: The time to option expiry measured in years or fractions of a year.

r: The risk-free rate of interest continuously compounded.

P: The current spot price of the underlying asset.

X: The strike price of the option.

N(d): The area under a cumulative standard normal distribution from $-\infty$ to the value d.

exp(z): The value *z* raised to the power "e" (on some calculators, this is e^x).

What are the three equations that make up the Black-Scholes OPM?

$C = P[N(d_1)] - Xe^{-r_{RF}t}[N(d_2)].$

$$d_1 = \frac{\ln(P/X) + [r_{RF} + (\sigma^2/2)]t}{\sigma\sqrt{t}}.$$

$d_2 = d_1 - \sigma\sqrt{t}.$

What is the value of the following call option according to the OPM? Assume: P = \$27; X = \$25; r_{RF} = 6%; t = 0.5 years: σ^2 = 0.11

$C = \$27[N(d_1)] - \$25e^{-(0.06)(0.5)}[N(d_2)].$

$$\mathbf{d_1} = \frac{\ln(\$27/\$25) + [(0.06 + 0.11/2)](0.5)}{(0.3317)(0.7071)}$$

$= 0.5736.$

$d_2 = d_1 - (0.3317)(0.7071) = d_1 - 0.2345$

$= 0.5736 - 0.2345 = 0.3391.$

$N(d_1) = N(0.5736) = 0.5000 + 0.2168$
$= 0.7168.$

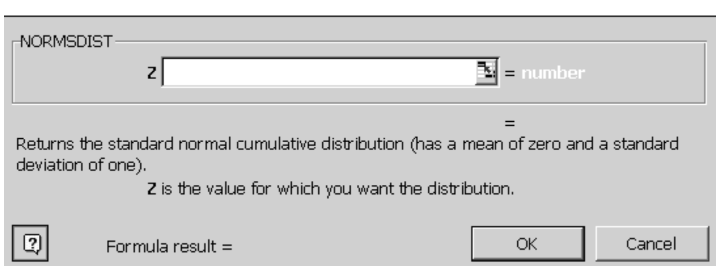
$N(d_2) = N(0.3391) = 0.5000 + 0.1327$
$= 0.6327.$

Note: Values obtained from Excel using NORMSDIST function.

$C = \$27(0.7168) - \$25e^{-0.03}(0.6327)$
$= \$19.3536 - \$25(0.97045)(0.6327)$
$= \$4.0036.$

Using Excel to find N(d)

- N(d): The area under a cumulative standard normal distribution from $-\infty$ to the value d.



Put Options Black-Scholes Equation

Put Option's Value = $C - P + Xe^{-r_{RF}t}$

There are many different types of options and new types are being introduced all the time. Some of these lend themselves to analytical solutions but others do not.

The derivation of a closed form solution is exceedingly difficult and not always possible.

Valuing Options: Binomial Approach

Numeric Methods:

Numeric methods are a group of techniques that arrive at option valuation via a sequence of finite steps that get closer and closer to the true value (based on the assumptions employed).

The most widely used of the numeric methods are lattice models, the most common of which is the *binomial option pricing model*. We will build this model. The binomial option pricing model was developed by John Cox, Stephen Ross, and Mark Rubinstein (Cox/Ross/Rubinstein or CRR) and published in 1979.

Binomial Option Pricing Model

Advantages:

1. Easy to understand without a knowledge of advanced mathematics.
2. Assumptions can easily be changed to accommodate different types of options.

Disadvantages:

1. The principal disadvantage of the binomial model is that it is computationally intensive, often requiring many millions of calculations to get a sufficiently good approximation. Until recently, computers simply lacked the necessary speed to produce usable results in real time.
2. The "Greeks" have to be derived numerically, rather than analytically.

Binomial Option Pricing Model

If we make the same assumptions made in the derivation of an analytical solution, the binomial model should give approximately the same result. However, as we will see, when using numeric methods, there is a trade-off between accuracy and computational time.

We will illustrate the methodology making the same assumptions that Black and Scholes made.

Binomial Model Notation

σ: The volatility of the price of the underlying asset, measured as the standard deviation of the percentage price changes continuously compounded (called continuous return).

τ: The time to option expiry measured in years or fractions of a year.

T: The number of periods into which the life of the option will be divided.

t: The current period.

r: The risk-free rate of interest continuously compounded.

S: The current spot price of the underlying asset.

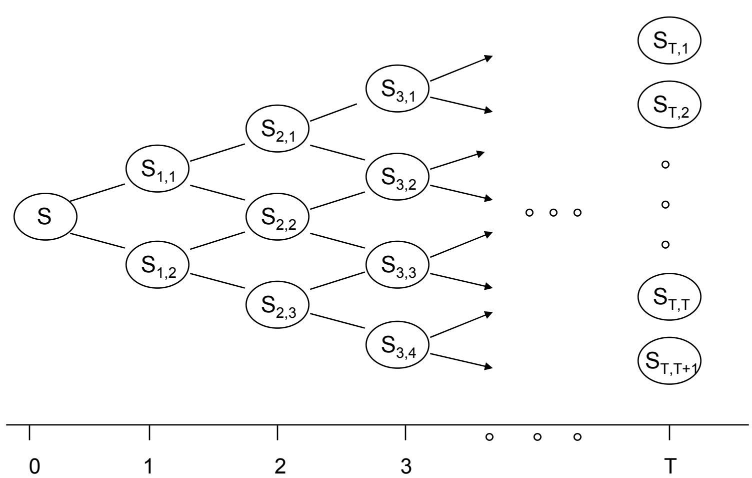
X: The strike price of the option.

exp(z): The value *z* raised to the power "e" (on some calculators, this is e^x).

We begin by asking how the price of the underlying asset would evolve over a period of time if we divide that period of time into some number of discrete intervals.

We will divide time into *T* discrete intervals and we will denote the successive intervals as 1, 2, 3, ..., T.   The current time is 0.

In a binomial framework, the price next period can *rise to one and only one new higher price* or *fall to one and only one new lower price*.

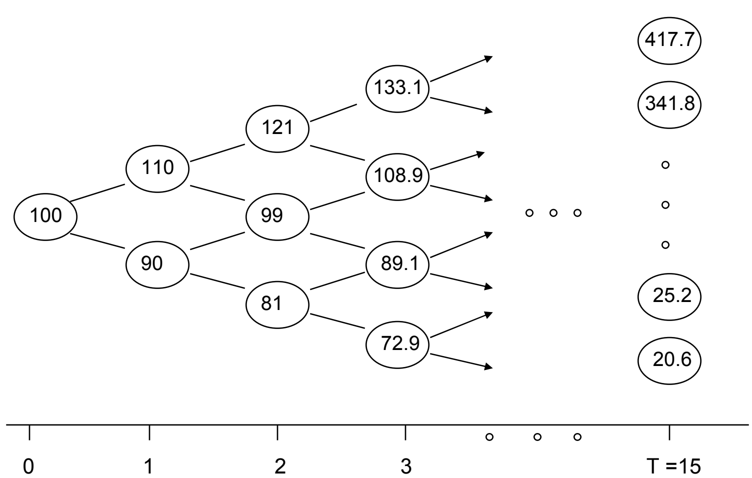


Next, we need to consider how the price one period is related to the price in the previous period.

One possible assumption would be that the price each period rises/falls by a fixed dollar amount. But this can lead to negative values.

Alternatively, we can assume that the price each period rises/falls by a fixed percentage amount. Suppose that we assume that the price rises/falls by 10% each period (measured as an effective annual rate).

This result is better because the stock price cannot go below zero. It is also more intuitively appealing, based on the assumption of constant volatility where that volatility is measured on a percentage change basis.



Because of distributional properties, however, in option pricing analytics, percentage price changes are measured on a continuous basis.

10% as a periodic rate is equivalent to 9.531% on a continuous basis.

$$S_{1,1} = \quad exp(+.09531) \times S$$
$$S_{1,2} = \quad exp(-.09531) \times S$$

The next question is what should the percentage change from period-to-period be? We assumed 10% (9.531% continuous) for convenience.

It turns out that the price change should be the volatility (that is, after all, what volatility measures).

However, the time intervals we employ are not necessarily one year long and volatility is routinely measured on an annual basis. Thus, we need to measure volatility on a periodic basis.

We exploit the following relationship:

$$\sigma_{periodic} = \sqrt{\sigma} \times \tau/T$$

$$\sigma_{periodic} = \sqrt{\sigma} \quad \times \quad \tau/T$$

Example:

Suppose that annual volatility is 30%. The total period of time is three months (τ = .25). Time will be divided into one-month periods (therefore T = 3). What is the periodic volatility?

Solution: $\sigma_{periodic} = \sqrt{.3} \quad \times \quad .25/3$

$= 8.66\%$

Therefore: $S_{1,1} = exp(+8.66\%) \times S$

and $S_{1,2} = exp(-8.66\%) \times S$

In the CRR binomial option pricing model the value:

$exp(\sigma \times \sqrt{\tau/T}\ )$ is called the Up Multiplier and denoted *U*

and, $exp(-\sigma \times \sqrt{\tau/T}\ )$ is called the Down Multiplier and denoted *D*

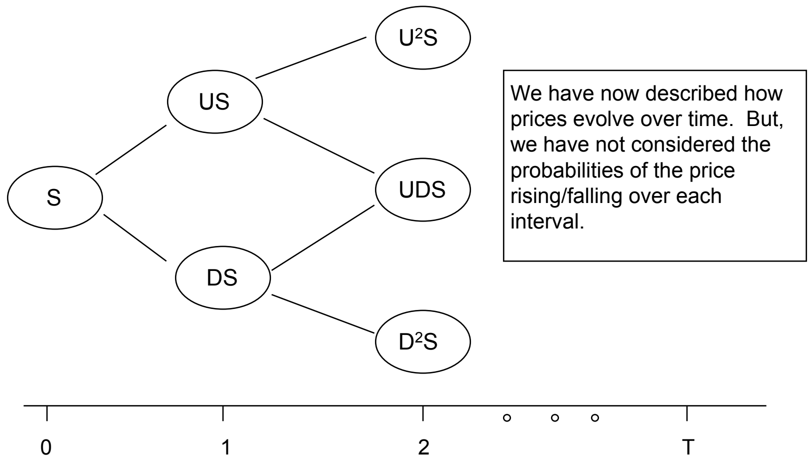
Thus

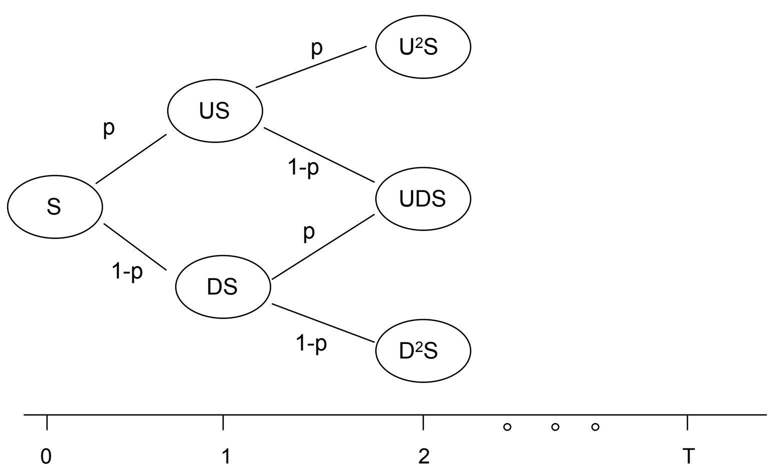
$$S_{1,1} = U \times S$$
$$S_{1,2} = D \times S$$

and

$$S_{2,1} = U^2 \times S$$
$$S_{2,2} = UD \times S$$
$$S_{2,3} = D^2 \times S$$

and so forth.


U²S
US
UDS
S
DS
D²S
0
1
2
T
We have now described how prices evolve over time. But, we have not considered the probabilities of the price rising/falling over each interval.


p
U²S
p
US
1-p
UDS
S
p
1-p
DS
1-p
D²S
0
1
2
T

Probabilities:

The key to deriving the probabilities is *risk-neutral pricing*.

Essentially, the mean return on the security should be the risk-free rate. This is a consequence of arbitrage. Therefore:

$$p \times US_t \quad + \quad (1 - p) \times DS_t \quad = \quad exp(\, r \times (\tau/T))\, S_t$$

implying that

$$p \times U \quad + \quad (1 - p) \times D \quad = \quad exp(\, r \times (\tau/T))$$

From this, we can solve for p and for (1-p).

$$p = \frac{exp(r \times (\tau/T)) - D}{U - D}$$

Suppose that you want to know the value of an ATM call option on a stock that is currently trading at $100. The option expires in 3 months (.25 years).

The stock's vol is 30 and the continuous risk free rate of interest is 5%. We will divide the life of the option into 3 subperiods of 1 month each (i.e., T = 3).

Suppose that you want to know the value of an ATM call option on a stock that is currently trading at $100. The option expires in 3 months (.25 years).

The stock's vol is 30 and the continuous risk-free rate of interest is 5%. We will divide the life of the option into 3 subperiods of 1 month each (i.e., T = 3).

$$U = exp(+.3 \times \sqrt{(.25/3)}) = 1.09046$$

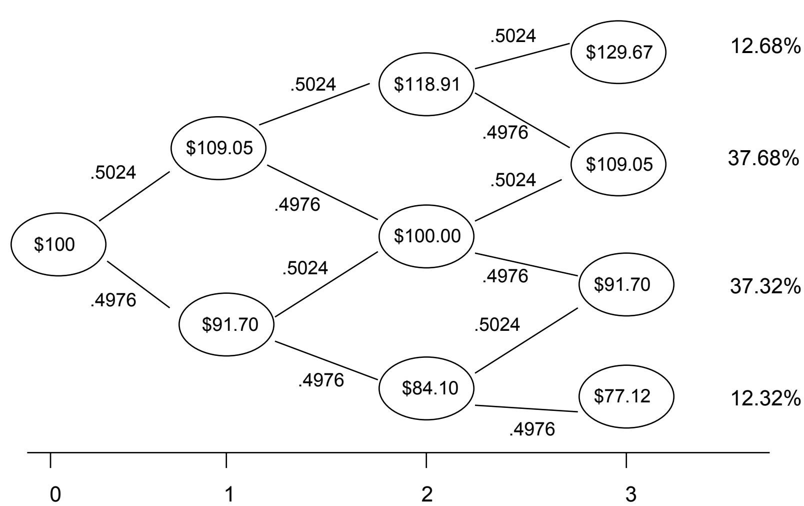
$$D = exp(-.3 \times \sqrt{(.25/3)}) = 0.91704$$

$$p \times 1.09046 + (1-p) \times 0.91704 = exp(.05 \times (.25/3))$$

solving:

$$p = 0.5024392 \quad \text{and} \quad (1-p) = 0.4975608$$

Step 1. Determine all possible future values for the stock and their associated probabilities.



Step 2. Get all possible terminal values for the option and their associated probabilities.

Recall that the option is ATM, therefore we know it is struck at $100 (X = $100). What is the expected terminal value of this call option?

-----Terminal Value-----		
Stock (S_T)	Call Option (C_T)	Probability
$129.67	$29.67	12.68%
109.05	9.05	37.68%
91.70	0.00	37.32%
77.12	0.00	12.32%

Step 3. Get the "expected" terminal value of the option.

-----Terminal Value-----			
Stock (S_T)	Call Option (C_T)	Probability	Probability × C_T
$129.67	$29.67	12.68%	$3.762
109.05	9.05	37.68%	3.410
91.70	0.00	37.32%	0.000
77.12	0.00	12.32%	0.000
			$**7.172**

Step 4. Get the present value of the option. This is the fair premium.

-----Terminal Value-----			
Stock (S_T)	Call Option (C_T)	Probability	Probability $\times$ C_T
$129.67	$29.67	12.68%	$3.762
109.05	9.05	37.68%	3.410
91.70	0.00	37.32%	0.000
77.12	0.00	12.32%	0.000
			$**7.172**

Finally, we want to calculate the current value of the call. We do this by simply discounting the expected terminal value of the call by the risk-free rate.

$$\text{Current value of the call} = \frac{\$7.172}{exp(.05 \times .25)} = \mathbf{\$7.083}$$

Payoffs are Equal,Values Must Be Equal. Put-Call Parity Relationship

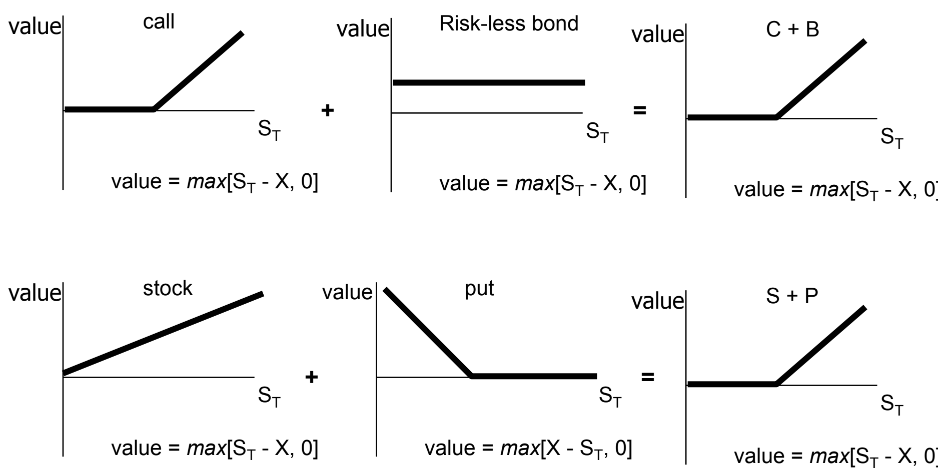
Put + Stock = Call + PV of Exercise Price

$$Put + P = V + Xe^{-r_{RF}t}$$

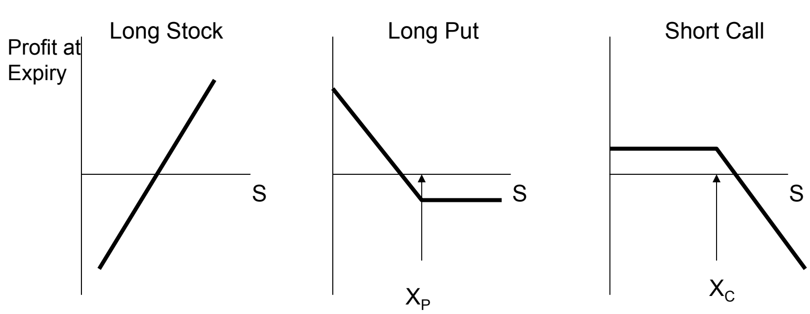
$$Put = V - P + Xe^{-r_{RF}t}$$

Alternatively you can think of this as a risk-less bond.

Put-Call Parity Relationship

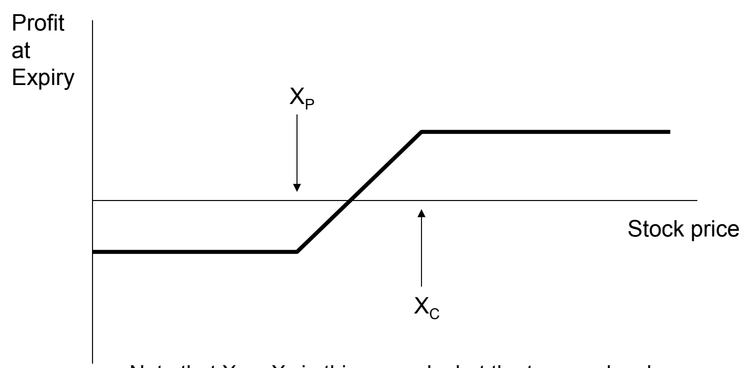


Uses of Options: Hedging

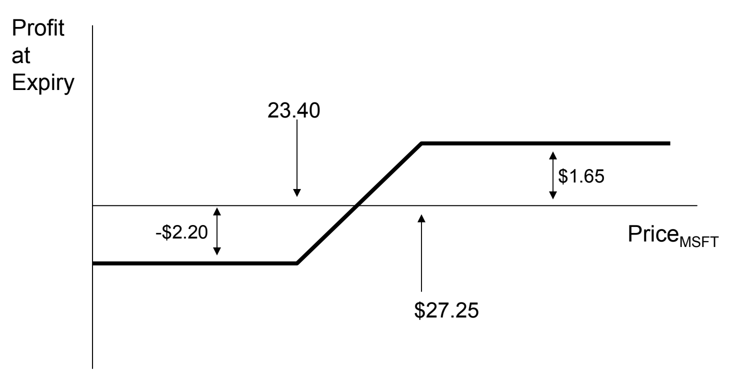


What happens when these three positions are combined?

Collared Shares



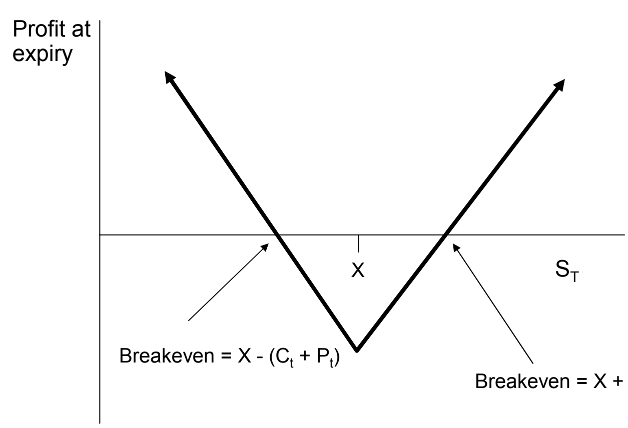
Note that $X_C > X_P$ in this example, but the two can be close together, or even identical.



Uses of Options: Speculation

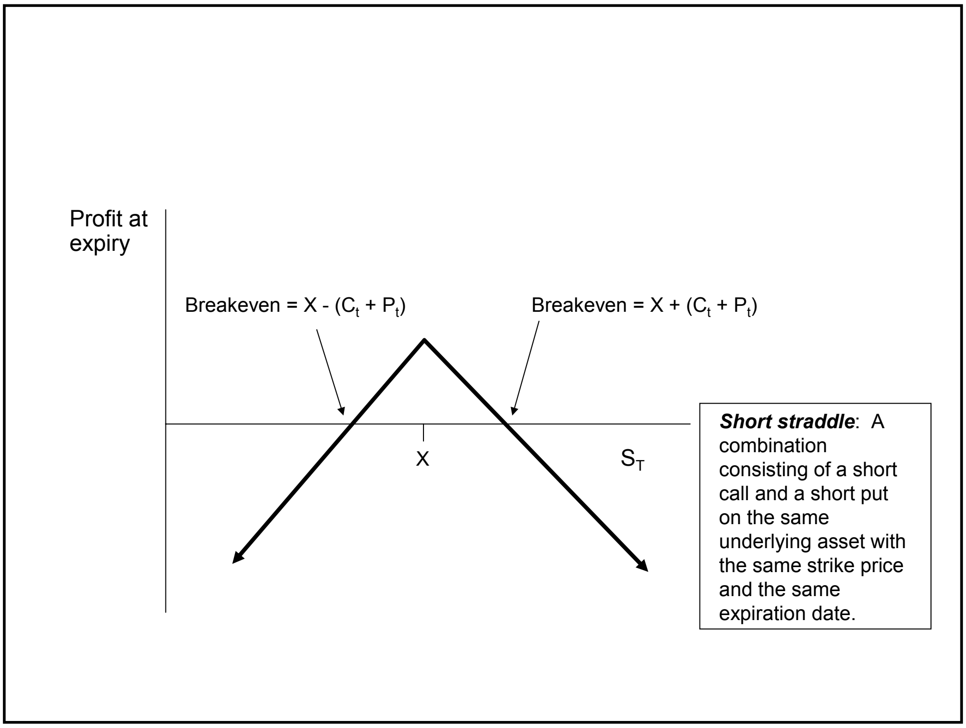
There are all sorts of combinations. Here are a few:

- Straddles
- Strips
- Straps
- Strangles

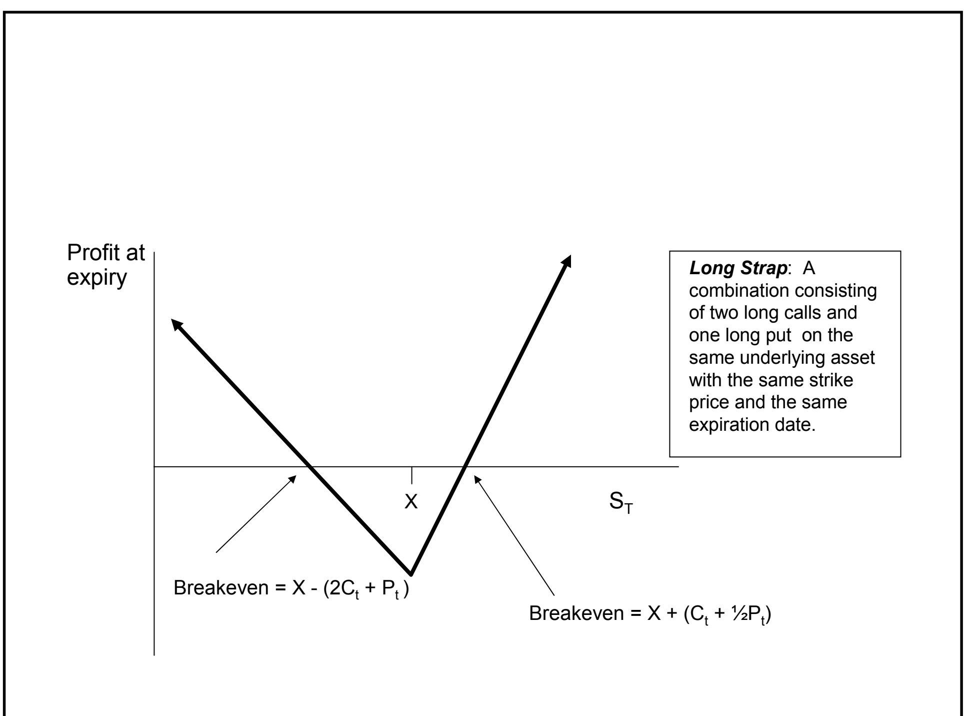


Long straddle: A combination consisting of a long call and a long put on the same underlying asset with the same strike price and the same expiration date.

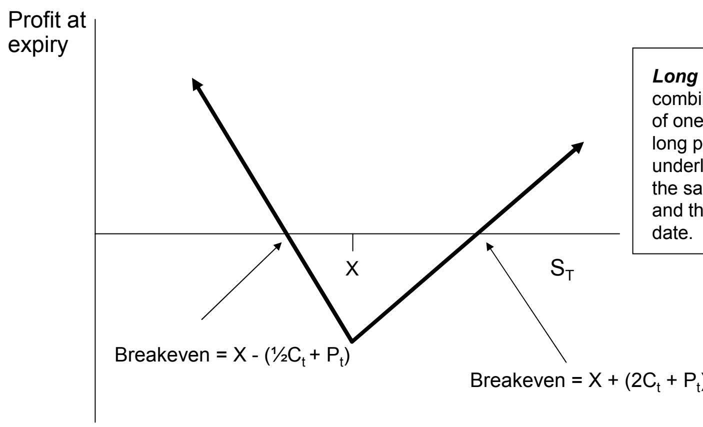
What would a short straddle look like?



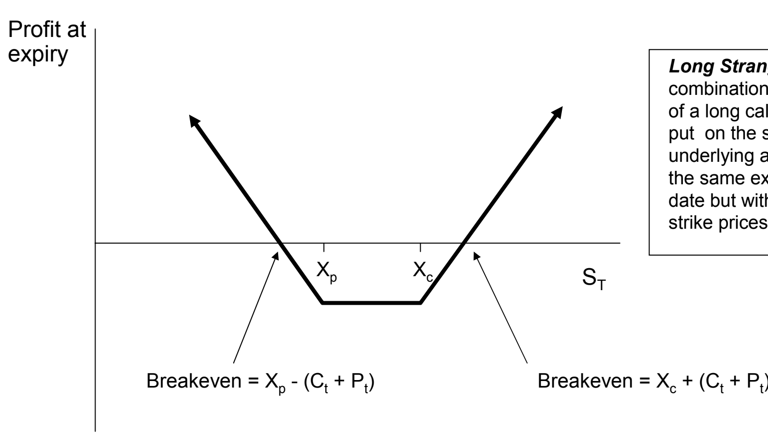
Profit at expiry
Breakeven = X - (C_t + P_t)
Breakeven = X + (C_t + P_t)
X
S_T
Short straddle: A combination consisting of a short call and a short put on the same underlying asset with the same strike price and the same expiration date.



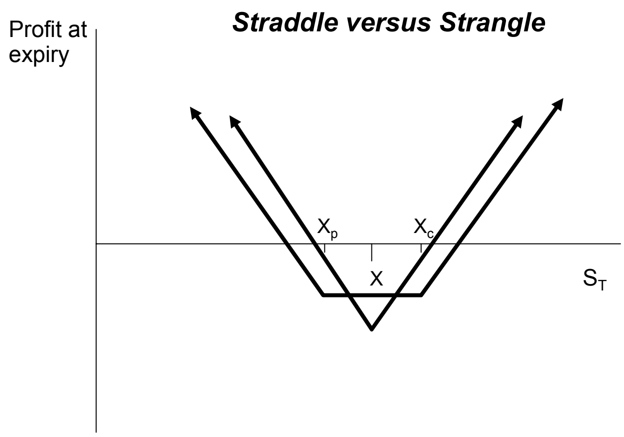
Profit at expiry
Long Strap: A combination consisting of two long calls and one long put on the same underlying asset with the same strike price and the same expiration date.
X
S_T
Breakeven = X - (2C_t + P_t)
Breakeven = X + (C_t + ½P_t)



Long Strip: A combination consisting of one long call and two long puts on the same underlying asset with the same strike price and the same expiration date.



Long Strangle: A combination consisting of a long call and a long put on the same underlying asset with the same expiration date but with different strike prices.


Straddle versus Strangle
Profit at expiry
Xp
Xc
X
ST