Derivatives: Basics

A derivative is a financial instrument whose value depends on values of other financial instruments or on some measure of the state of the economy or of nature. The instrument or measure on whose value the derivative depends is called the "underlying".

A derivative is an agreement with two sides.

One of the most important questions in finance is how to price a derivative.

Forward Contracts

One of the simplest kinds of derivatives is a *forward contract*, which is an agreement to buy or to sell an asset at a specified time at a specified price.

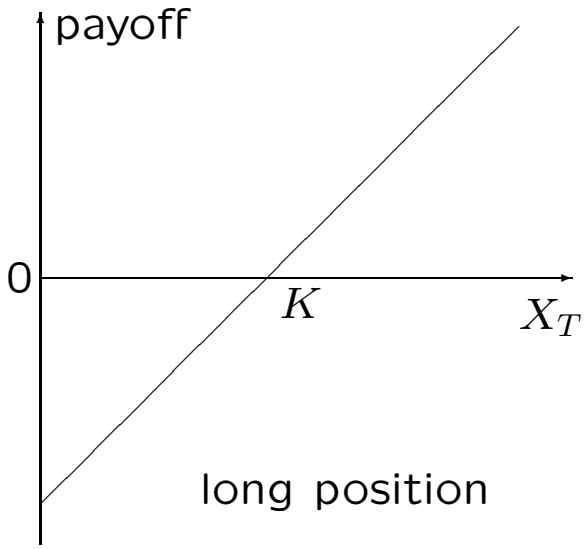
The agreement to buy is a *long position* and the agreement to sell is a *short position*. The agreed upon price is the *delivery price*. The consumation of the agreement is an *execution*.

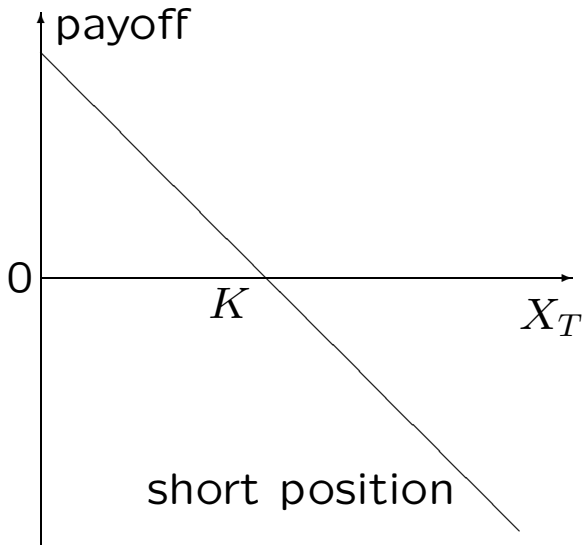
Forward contracts are relatively simple to price, and their analysis is important for developing pricing methods for other derivatives.

Analysis of Forward Contracts

Using standard notation, let K be the delivery price or "strike price", at time T, and let X_t be the value of the underlying.

The result at settlement is shown below.





The important question is what is the price of the derivative as a function of X_T and of time to settlement.

For a forward contract it is easy. The answer is

$$F_0 = X_0 e^{rT} - K,$$

where r is the (annual) riskfree rate of growth, and T is the time (in years).

Derivatives: Basics

There is a variety of modifications to the basic forward contract that involve

- nature of the underlying
  - asset
    - investment
    - consumable
    - income producing
  - nonasset (e.g., index, price of electricity, weather)
- negociability of the instrument (market, possibility of short positions, intermediate party, etc.)

Derivatives: Basics

Other modifications to the basic forward contract involve

- nature of the agreement (right, that is, "contingent claim", or obligation)
- flexibility of time of execution (at a specified time or up to a specified time)
- method of settlement (cash or delivery)
- dependence of the derivative value on the path of the value of the underlying

Types of Derivatives

These variations on the basic forward contract are all interesting. Only a few of them are actually available.

Some variations are much easier to analyze than others. The simple ones are interesting for classroom analyses and they may provide useful approximations for derivatives that are actually available.

For individual investors, there is a relatively small set of derivatives available (realistically).

For all of them there is a ready market (always) through a third party, and short sales are possible.

Most of the readily traded derivatives are options, or contingent claims. That kind of derivative is a right; not an obligation.

Therefore, a long position is a right and a short position (in the derivative) is an obligation. The right "expires" at the settlement date.

Derivatives That Have Markets

The common types of derivatives are

- Stock options
- Index options
- Commodity futures
- Rate futures

Uses

Stock options are used by individual investors and by investment companies for leverage, hedging, and income.

Index options are used by individual investors and by investment companies for hedging and speculative income.

Commodity futures are used by individual investors for speculative income, by investment companies for income, and by producers and traders for hedging.

Rate futures are used by individual investors for speculative income, by investment companies for income and for hedgins, and by traders for hedging.

Types of Common Derivatives

The variations depend on the nature of the underlying.

- Stock (investment asset, possibly income-producing). The buy side is a "call" and the sell side is a "put".
  - Settlement (exercise) is by delivery.
  - Exercise can be any time prior to expiration date ("American style").
- Index (investment asset, not income-producing). The buy side is a "call" and the sell side is a "put".
  - Settlement is by cash.
  - Exercise can only be at expiration date.
- Commodity (consumable asset, not income-producing).
  - Settlement is by delivery.
  - Exercise can only be at expiration date.

U.S. Stock Options

A market for stock options in the U.S. is one of the national security exchanges: Amex, CBOE, NYSE, Pacific Exchange, and Philadelphia Exchange.

All (almost all) options are initiated with and through the Options Clearing Corporation, owned by the exchanges and headquartered on LaSalle St.

Options (and also futures) are regulated by the Commodity Futures Trading Commission (the analogue of the SEC). The SEC also regulates options through its regulatory oversight of the exchanges.

Analysis of Stock Options

Another important difference between stock options and forward contracts is that stock options are *rights*, not obligations. The payoff therefore cannot be negative.

Because the payoff cannot be negative, there must be a cost to obtain a stock option.

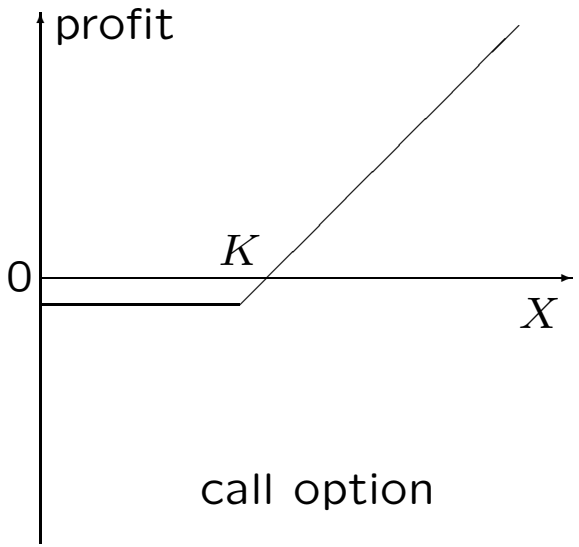
The profit is the difference between the payoff and the price paid.

Another difference in stock options and forward contracts is that (real-world) stock options can be exercised at any time (during trading hours) prior to expiration. We will, however, often consider a modification, the "European option", which can only be exercised at a specified time. (There are some European options that are actually traded, but they are generally for large amounts, and they are rarely traded by individuals.)

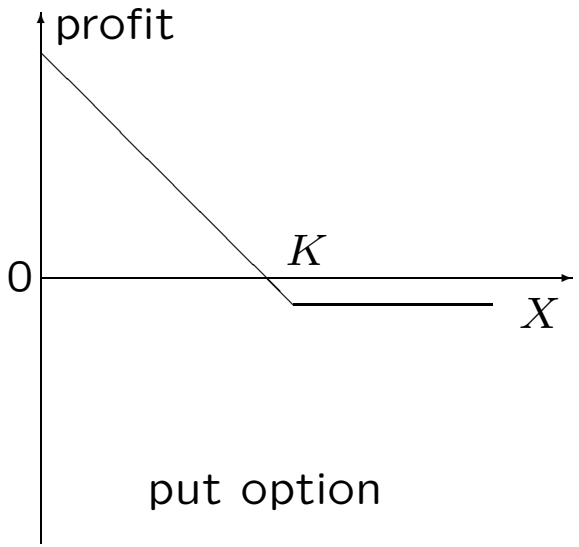
Analysis of Stock Options

The two sides of a forward contract result in the two types of stock options.

The results *at expiration* are



call option



put option

For short positions, just flip the graphs.

Market Models for Derivative Pricing

A simple model of the market assumes two assets:
a *riskless asset* with price at time t of β_t,
and a *risky asset* with price at time t of X_t.

The price of a derivative can be determined based on trading strategies involving these two assets.

The price of the riskless asset follows the deterministic ordinary differential equation

$$\mathrm{d}\beta_t = r\beta_t \mathrm{d}t,$$

where r is the instantaneous riskfree interest rate.

The price of the risky asset follows the stochastic differential equation

$$\mathrm{d}X_t = \mu X_t \mathrm{d}t + \sigma X_t \mathrm{d}W_t.$$