18

Transport Problems

18.1 INTRODUCTION

Transport problems typically deal with the action of shifting goods between a series of locations using a series of limited resources. In the wider business context the same methodology is used to look at the movement of transactions through a system, particularly where there are a limited number of servers or a limited amount of processing time available. The following is an illustration of how to solve this type of problem.

18.2 TRANSPORT PROBLEM

A company has undertaken an exercise and decided to outsource all of their transaction processing. The outsourced supplier has four locations available where processing can be undertaken and this provides a useful business continuity advantage to the company.

They currently have three different systems that produce information, each of which can be processed at any location, for example, System A can be processed at any one of the four possible locations. System A actually requires five hours of processing to enable its output to be produced. Each location has a set amount of time available for this piece of work and location A can only take on three hours of additional processing at this stage.

The challenge is to work out what would be the optimal processing strategy given the number of options that are available. The processing quantities available at each location and the differing system requirements are shown in Table 18.1.

Table 18.1 Processing supply and demand

System	Required	Location	Capacity
A	5	1	3
B	5	2	4
C	7	3	5
		4	5
Total	**17**		**17**

As we can see, the hours required for processing, at 17, is also the capacity, so in this case supply and demand balance. Were this not the case a slack variable would need to be introduced in the form of an additional system or location to absorb/generate the required excess. To solve the problem you will need to undertake a matrix-based approach, but this will need to consider other relevant factors, for example the cost of processing. These are the costs (time and money) that occur in moving a single unit from the production system to the processing location and are displayed in Table 18.2.

The objective for the outsourcing supplier is to undertake the processing for the three systems at the minimum cost.

Table 18.2 Processing costs (£)

	Location			
System	1	2	3	4
A	5	8	6	5
B	7	6	7	4
C	5	4	5	6

The procedure adopted is similar to the simplex method that is discussed in Chapter 17. In this case the process works by starting from the top left-hand corner of a supply/demand matrix. You first attempt to satisfy the processing demand by looking at the first location. The first location only has three hours available and this is less than the five hours required to process the output from System A. There will then be two hours of processing still required on the output from System A, which will now be processed at the second location. This process continues until all of the systems processing requirements have been accounted for. This provides the initial values for the analysis as shown in Table 18.3.

Table 18.3 Initial values: Cycle 1

	Location				
System	1	2	3	4	**Available**
A	3	2	0	0	**5**
B	0	2	3	0	**5**
C	0	0	2	5	**7**
Total	**3**	**4**	**5**	**5**	**17**

As you can see the way this has worked is to consider each of the locations in order. The resulting total processing cost can be calculated by using the information from the matrix given in Table 18.2. The total processing cost is therefore calculated as follows:

$$(3 \times £5) + (2 \times £8) + (2 \times £6) + (3 \times £7) + (2 \times £5) + (5 \times £6) = £104$$

However, this solution has only been reached by taking the four sites in an order that may not in fact be optimal. There is no reason to believe that it will be cost effective for the excess from processing System A to actually be processed by Location 2. It may be that Location 3 or 4 would be more appropriate, or that Location 1 should actually be processing something else altogether. It is therefore necessary to search for a lower cost solution.

To achieve this we need to introduce additional ghost costs. These imaginary costs are split into two parts, a component for each system and another for each location. Since only relative magnitudes are of interest, the first ghost cost is assigned the value zero. Only the unit costs of the cells used in the previous cost calculation (Table 18.2) are of interest. If the unit cost from system i to location j is C_{ij}, then ghost costs S_i and L_j need to be chosen such that $C_{ij} = S_i + L_j$. The resulting values are shown in Table 18.4.

The order in which the values are assigned is S_A(the ghost cost for System A), then the first location (L_1), followed by the second, L_2. The second ghost cost, for System B is then introduced, S_B, then successively L_3, S_C and L_4. We are assigning the value zero to S_A, since

Table 18.4 Allocation of the ghost costs: Cycle 1

	Location				
System	1	2	3	4	$\mathbf{S}_i$
A	5	8			**0**
B		6	7		**−2**
C			5	6	**−4**
L_j	**5**	**8**	**9**	**10**	

$C_{A1} = 5$ and $C_{A2} = 8$. This gives $L_1 = 5$ and $L_2 = 8$. Since $C_{B2} = 6$ and $L_2 = 8$ this makes $S_B = -2$ and so on.

For the remaining cells, ghost costs $\tilde{C}_{ij} = S_i + L_j$ can be calculated. These values should then be compared to the true costs. Any differences, $C_{ij} - \tilde{C}_{ij}$, should then also be calculated. If any cell were showing a negative value then this would indicate that you have not yet reached the optimal solution and that further improvement can be achieved.

This difference gives an indication of the savings expected on employing the different processing allocation to the four sites. In practice you will normally adopt a presentational format using special notation designed to encapsulate all of this information. To achieve this you need to construct a new cell in the form shown in Figure 18.1 for each existing cell to show the relationship between the ghost and true costs.

Going back to the example shown in Table 18.4, for the cells that have already had a value allocated to them, the difference will be zero. This leads to the grid that is shown in Figure 18.2.

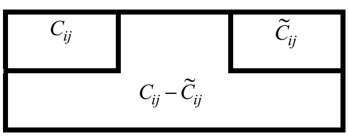

Figure 18.1 The relation between the ghost and true costs.

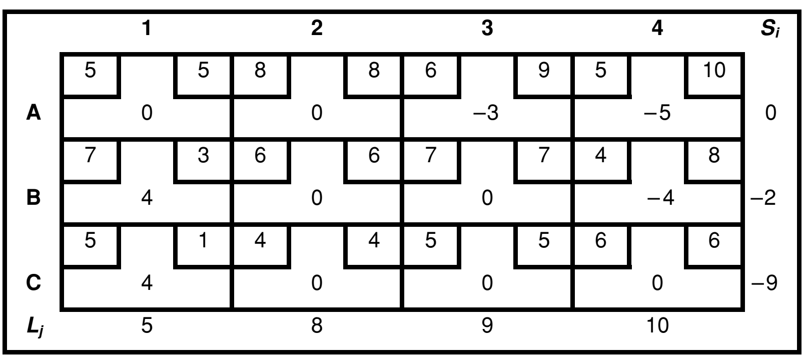

Figure 18.2 Ghost costs: cycle 1.

Table 18.5 Selection of initial values: Cycle 2

	Location				
System	1	2	3	4	**Available**
A	3	$2-x$	0	x	**5**
B	0	$2+x$	$3-x$	0	**5**
C	0	0	$2+x$	$5-x$	**7**
Total	**3**	**4**	**5**	**5**	

Since the number of non-zero cells will correspond to the number of system and location ghost costs that are required, the number of non-zero entries must be less than the number of ghost costs. Therefore, selecting too many cells in Table 18.3 will lead to an insoluble problem. The key is to initially use as few cells as possible.

Since there are still three negative values appearing in Figure 18.2, this is an indication that the optimal solution has not yet been reached. Since the cell A4 gives the largest negative value, it would make sense to see if we can increase its usage. Assume that the utilisation is increased by a value x, then this will create a capacity excess. One possible solution is shown in Table 18.5.

Since we need all of the values to be non-negative, the best that can be done is take x to be 2, since otherwise A2 would become negative. The expected savings are shown in Table 18.6, with the total savings being referred to as the improvement index.

There must be an even number of entries in the path and in this case the net saving is $5x$, which given that $x = 2$ is the highest possible value of x, means that the savings are 10. The new initial values are shown in Table 18.7.

At this stage the cost is 94, as expected, and an improvement has been achieved. The steps are now repeated to see if a further improvement can be achieved. For the ghost costs the values are shown in Table 18.8.

Table 18.6 Expected savings

Path	A4	C4	C3	B3	B2	A2	Total
Cost	5	6	5	7	6	8	
Change	x	$-x$	x	$-x$	x	$-x$	$-5x$

Table 18.7 Initial values: Cycle 2

	Location				
System	1	2	3	4	**Available**
A	3	0	0	2	**5**
B	0	4	1	0	**5**
C	0	0	4	3	**7**
Total	**3**	**4**	**5**	**5**	

Table 18.8 Allocation of the ghost costs: Cycle 2

	Location				
System	1	2	3	4	S_i
A	5			5	**0**
B		6	7		**3**
C			5	6	**1**
L_j	**5**	**3**	**4**	**5**	

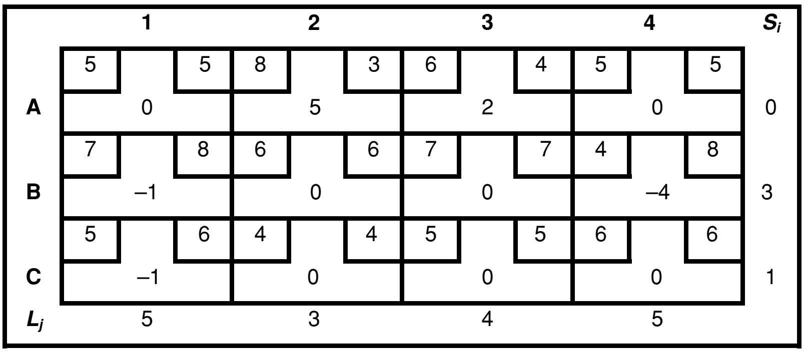

Figure 18.3 Ghost costs: cycle 2.

The order in which the values are assigned is S_A, L_1, L_4, S_C, L_3, S_B and finally L_2. The resulting difference grid is shown in Figure 18.3.

The most negative cell is now B_4. On assigning a further utilisation x to this system, the output available will give the results shown in Table 18.9, and the improvement index is shown in Table 18.10.

Table 18.9 Selection of initial values: Cycle 3

	Location				
System	1	2	3	4	**Available**
A	3	0	0	2	**5**
B	0	4	$1-x$	x	**5**
C	0	0	$4+x$	$3-x$	**7**
Total	**3**	**4**	**5**	**5**	

Table 18.10 Expected savings

Path	B4	B3	C3	C4	Total
Cost	4	7	5	6	
Change	x	$-x$	x	$-x$	$-4x$

Table 18.11 Initial values: Cycle 3

	Location				
System	1	2	3	4	**Available**
A	3	0	0	2	**5**
B	0	4	0	1	**5**
C	0	0	5	2	**7**
Total	**3**	**4**	**5**	**5**	

Table 18.12 Allocation of the ghost costs: Cycle 3

	Location				
System	1	2	3	4	S_i
A	5			5	**0**
B		6		4	**–1**
C			5	6	**1**
L_j	**5**	**7**	**4**	**5**	

The maximum allowed value for x is now only 1 since otherwise cell B3 would go negative. This therefore gives an expected improvement of –4. The initial values are now as shown in Table 18.11. The resulting cost is 90 with ghost costs shown in Table 18.12.

The order in which the values are assigned is S_A, L_1, L_4, S_C, S_B, L_3 and L_2. The resulting difference grid is shown in Figure 18.4.

You then continue to use the algorithm in a similar fashion until no further improvement is possible. After five cycles, since there are no negative cells, an optimal solution will have been obtained. This final solution is shown in Table 18.13. At this stage the cost has reduced to 78.

Therefore the optimal solution is for system A to send 3 hours of work to Location 1 and 2 hours to Location 3. System B should send 5 hours of work to Location 4 and System C should send 4 hours to Location 2 and 3 hours of work to Location 3.

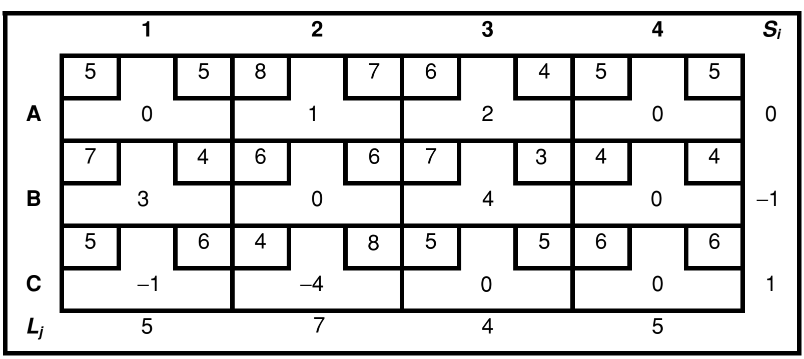

Figure 18.4 Ghost costs: cycle 3.

Table 18.13 Final values

	Location				
System	1	2	3	4	**Available**
A	3	0	2	0	**5**
B	0	0	0	5	**5**
C	0	4	3	0	**7**
Required	**3**	**4**	**5**	**5**	

In this instance the solution is unique.

While the adoption of the north-west corner as our starting point may be the simplest starting strategy, it is also rather naïve. A superior starting point is to select the minimum cost routes, in this case the starting guess actually corresponded to the final solution which was confirmed in one additional cycle.

When applying the approach to actual business problems there is a risk that a solution that is too simplistic will be adopted and that a number of important factors may be overlooked. There may be additional external constraints that need to be considered, including the training of staff required to undertake the work, or savings that may result from sending all the work from a single system to a single location.

This type of analysis can be easily applied to machine time and scheduling problems, including computer applications. The procedure could also be employed in reverse to decide the optimal number of locations or systems that meet a specific need. So the main concern is always to get your problem to fit into this type of analysis and then to make sure that all the constraints have been properly considered.