CHAPTER 12

VALUATION: PRINCIPLES AND PRACTICE

In this chapter, we look at how to value a firm and its equity, given what we now know about investment, financing and dividend decisions. We will consider two approaches to valuation. The first and most fundamental approach to valuing a firm is *discounted cash flow valuation*, which extends the present value principles that we developed to analyze projects to value a firm. The value of any firm is determined by four factors – its capacity to generate cash flows from assets in place, the expected growth rate of these cash flows, the length of time it will take for the firm to reach stable growth, and the cost of capital. Consequently, to increase the value of a firm, we have to change one or more of these variables.

The second way of valuing a firm or its equity is to based the value on how the market is valuing similar or comparable firms; this approach is called *relative valuation*. This approach can yield values that are different from a discounted cashflow valuation and we will look at some of the reasons why these differences may occur.

In a departure from the previous chapters, we will take the perspective of investors in financial markets in estimating value. Investors assess the value of a firm's stock in order to decide whether to buy the stock or, if they already own it, whether to continue to it.

Discounted Cash Flow Valuation

In discounted cash flow valuation, we estimate the value of any asset by discounting the expected cash flows on that asset at a rate that reflects their riskiness. In a sense, we measure the intrinsic value of an asset. The value of any asset is a function of the cash flows generated by that asset, the life of the asset, the expected growth in the cash flows and the riskiness associated with them. In other words, it is the present value of the expected cash flows on that asset.

$$\text{Value of Asset} = \sum_{t=1}^{t=N} \frac{E(\text{Cash Flow}_t)}{(1+r)^t}$$

where the asset has a life of N years and r is the discount rate that reflects both the riskiness of the cash flows and financing mix used to acquire the asset. If we view a firm

as a portfolio of assets, this equation can be extended to value a firm, using cash flows to the firm over its life and a discount rate that reflects the collective risk of the firm's assets.

This process is complicated by the fact that while some of the assets of a firm have already been created, and thus are assets-in-place, a significant component of firm value reflects expectations about future investments. Thus, we not only need to measure the cash flows from current investments, but we also must estimate the expected value from future investments. In the sections that follow, we will introduce the discounted cash flow model in steps. We begin by discussing two different ways of approaching valuation – equity and firm valuation – and then move on to consider how best to estimate the inputs into valuation models, and then consider how to go from the value of a firm to the value of equity per share.

Equity Valuation versus Firm Valuation

There are two paths to discounted cash flow valuation -- the first is to value just the equity stake in the business; the second is to value the entire firm, including equity and any other claimholders in the firm (bondholders, preferred stockholders, etc.). While both approaches discount expected cash flows, the relevant cash flows and discount rates are different for each.

> ***Value of Equity:*** This is the value of the equity stake in a business; in the context of a publicly traded firm, it is the value of the common stock in the firm.

The value of equity is obtained by discounting expected cash flows to equity — i.e., the residual cash flows after meeting all expenses, tax obligations, and interest and principal payments — at the cost of equity — i.e., the rate of return required by equity investors in the firm.

$$\text{Value of Equity} = \sum_{t=1}^{t=n} \frac{\text{CF to Equity}_t}{(1+k_e)^t}$$

where

CF to Equity_t = Expected Cash flow to Equity in period t

k_e = Cost of Equity

The dividend discount model is a specialized case of equity valuation, where the value of a stock is the present value of expected future dividends.

The value of the firm is obtained by discounting expected cash flows to the firm, i.e., residual cash flows after meeting all operating expenses and taxes, but prior to debt payments — at the weighted average cost of capital — i.e., the cost of the different components of financing used by the firm, weighted by their market value proportions.

> ***Value of Firm:*** The value of the firm is the value of all investors who have claims on the firm; thus, it includes lenders and debt-holders, who have fixed claims and equity investors, who have residual claims.

$$\text{Value of Firm} = \sum_{t=1}^{t=n} \frac{\text{CF to Firm}_t}{(1+\text{WACC})^t}$$

where

CF to Firm_t = Expected Cash flow to Firm in period t

WACC = Weighted Average Cost of Capital

While the two approaches use different definitions of cash flow and discount rates, they will yield consistent estimates of value as long as the same set of assumptions is applied for both. It is important to avoid mismatching cash flows and discount rates, since discounting cash flows to equity at the weighted average cost of capital will lead to an upwardly biased estimate of the value of equity, while discounting cash flows to the firm at the cost of equity will yield a downward biased estimate of the value of the firm.

12.1. ☞**: Firm Valuation and Leverage**

It is often argued that equity valuation requires more assumptions than firm valuation, because cash flows to equity require explicit assumptions about changes in leverage whereas cash flows to the firm are pre-debt cash flows and do not require assumptions about leverage. Is this true?

a. Yes

b. No

Explain.

Choosing the Right Valuation Model

All discounted cash flow models ultimately boil down to estimating four inputs - current cashflows, an expected growth rate in these cashflows, a point in time when the firm will be growing a rate it can sustain forever and a discount rate to use in discounting these cashflows. In this section, we will examine the choices available in terms of each of these inputs.

In terms of cashflows, there are three choices - dividends or free cashflows to equity for equity valuation models, and free cashflows to the firm for firm valuation models. Discounting dividends usually provides the most conservative estimate of value for the equity in any firm, since most firms pay less in dividends than they can afford to. In the dividend policy section, we noted that the free cash flow to equity, i.e., the cash flow left over after meeting all investment needs and making debt payments, is the amount that a firm can pay in dividends. The value of equity, based upon the free cash flow to equity, will therefore yield a more realistic estimate of value for equity, especially in the context of a takeover. Even if a firm is not the target of a takeover, it can be argued that the value of equity has to reflect the possibility of a takeover, and hence the expected free cash flows to equity. The choice between free cash flows to equity and free cash flows to the firm is really a choice between equity and firm valuation. Done consistently, both approaches should yield the same values for the equity in a business. As a practical concern, however, cash flows to equity are after net debt issues or payments and become much more difficult to estimate when financial leverage is changing over time, whereas cashflows to the firm are pre-debt cash flows and are unaffected by changes in the leverage. Ease of use dictates that firm valuation will be more straightforward under this scenario.

While we can estimate any of these cashflows from the most recent financial statements, the challenge in valuation is in estimating these cashflows in future years. In most valuations, this takes the form of an expected growth rate in earnings that is then used to forecast earnings and cash flows in future periods. The growth rates estimated should be consistent with our definition of cashflows. When forecasting cashflows to equity, we will generally forecast growth in net income or earnings per share that are

measures of equity earnings. When forecasting cashflows to the firm, the growth rate that matters is the growth rate in operating earnings. [1]

The choice of discount rates will be dictated by the choice in cash flows. If the cash flow that is being discounted is dividends or free cash flows to equity, the appropriate discount rate is the cost of equity. If the cash flow being discounted is the cash flow to the firm, the discount rate has to be the cost of capital.

The final choice that all discounted cash flow models have to make relates to expected growth patterns. Since firms have infinite lives, the way in which we apply closure is to estimate a terminal value at a point in time and dispense with estimating cash flows beyond that point. To do this in the context of discounted cash flow valuation, we have to assume that the growth rate in cash flows beyond this point in time are constant forever, an assumption that we refer to as "stable" growth. If we do this, the present value of these cash flows can be estimated as the present value of a growing perpetuity. There are three questions that every valuation then has to answer:

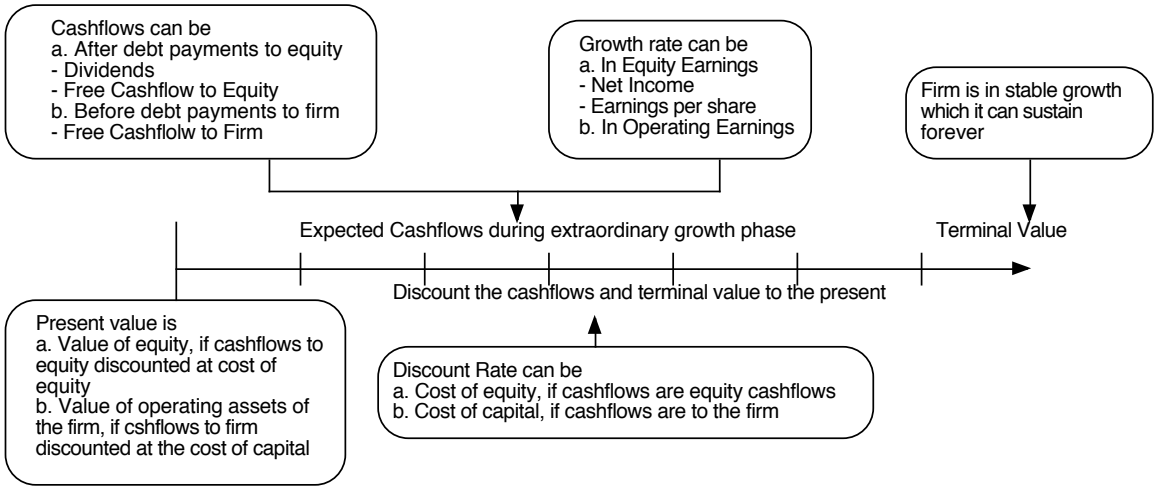
1. How long in the future will a company be able to grow at a rate higher than the stable growth rate?
2. How high will the growth rate be during the high growth period and what pattern will it follow?
3. What will happen to the firm's fundamentals (risk, cash flow patterns etc.) as the expected growth rate changes?

At the risk of being simplistic, we can broadly classify growth patterns into three categories - firms which are in stable growth already, firms which expect to maintain a 'constant" high growth rate for a period and then drop abruptly to stable growth and firms which will have high growth for a specified period and then grow through a transition phase to reach stable growth at a point in time in the future. As a practical point, it is important that as the growth rate changes, the firm's risk and cash flow characteristics change as well. In general, as expected growth declines towards stable growth, firms should see their risk approach the "average" and reinvestment needs decline. These

[1] We should generally become much more conservative in our growth estimates as we move up the income statements. Generally, growth in earnings per share will be lower than the growth in net income, and growth in net income will be lower than the growth in operating income.

choices are summarized in Figure 12.1. We will now examine each of these valuation models in more detail in the next section.

Figure 12.1: The Ingredients in a Valuation



 model.xls: This spreadsheet allows you to pick the right discounted cash flow valuation model for your needs, given the characteristics of the business that you are valuing.

In Practice: What Is A Stable Growth Rate?

Determining when your firm will be in stable growth is difficult to do without first defining what we mean by a stable growth rate. There are two insights to keep in mind when estimating a "stable" growth rate. First, since the growth rate in the firm's cashflows is expected to last forever, the firm's other measures of performance (including revenues, earnings and reinvestment) can be expected to grow at the same rate. Consider the long-term consequences of a firm whose earnings grow 6% a year forever, while its dividends grow at 8%. Over time, the dividends will exceed earnings. Similarly, if a firm's earnings grow at a faster rate than its dividends in the long term, the payout ratio will converge towards zero, which is also not a steady state. The second issue relates to what growth rate is reasonable as a 'stable' growth rate. Again, the assumption that this growth rate will last forever establishes rigorous constraints on "reasonableness". A firm cannot in the long term grow at a rate significantly greater than the growth rate in the economy in which it operates. Thus, a firm that grows at 8% forever in an economy

growing at 4% will eventually become larger than the economy. In practical terms, the stable growth rate cannot be larger than the nominal (real) growth rate in the economy in which the firm operates, if the valuation is done in nominal (real) terms.

Can a stable growth rate be much lower than the growth rate in the economy? There are no logical or mathematical limits on the downside. Firms that have stable growth rates much lower than the growth rate in the economy will become smaller in proportion to the economy over time. Since there is no economic basis for arguing that this cannot happen, there is no reason to prevent analysts from using a stable growth rate much lower than the nominal growth rate in the economy. In fact, the stable growth rate can be a negative number. Using a negative stable growth rate will ensure that your firm peaks in your last year of high growth and becomes smaller each year after that.

There is one rule of thumb that works well in setting a cap on the stable growth rate. The stable growth rate should generally not exceed the riskfree rate used in a valuation. Why should the two be related? The riskfree rate can be decomposed into an expected inflation rate and an expected real interest rate. If we assume that the real growth rate of an economy will be equal to the real interest rate in the long term, the riskfree rate becomes a proxy for the nominal growth rate in the economy.

12.2 ☞**: Cyclical firms and Constant Growth Rates**

Models that are built on the assumption of an expected constant growth rate over time cannot be used for cyclical firms, whose earnings growth is likely to be very volatile over time - high during economic booms, and very low or negative during recessions.

a. True

b. False

Explain.

Estimation in DCF Models

While all discounted cashflow models require the same four ingredients – cashflows, a discount rate, a period of high growth and a growth rate during the period, there are different estimation challenges we face with each model. In this section, we will

begin by estimating these inputs to the simplest of the three models, the dividend discount model, and then extend the discussion to cashflow to equity and firm valuation models.

I. Dividend Discount Models

When an investor buys stock, he generally expects to get two types of cash flows - dividends during the holding period and an expected price at the end of the holding period. Since this expected price is itself determined by future dividends, the value of a stock is the present value of just expected dividends. The dividend discount model is therefore the most direct and the most conservative way of valuing a stock since it counts only those cashflows that are actually paid out.

Setting up the Model

In it's most general form, the value of a stock in the dividend discount model is the present value of the expected dividends on the stock in perpetuity.

$$\text{Value per share of stock} = \sum_{t=1}^{t=\infty} \frac{\text{Expected Dividends in period t}}{(1+\text{Cost of Equity})^t}$$

Since we cannot estimate dividends in perpetuity, we generally allow for a period where dividends can grow at extraordinary rates but we allow for closure in the model by assuming that the growth rate will decline to a stable rate that can be sustained forever at some point in time in the future. By assuming stable growth at some point in time in the future, we can stop estimating dividends and estimate what we think the stock will be worth at the end of the extraordinary growth period.

> ***Terminal Value:*** This is the expected price of a stock (or equity) at the end of a specified holding period.

$$\text{Value}_0 = \sum_{t=1}^{t=n} \frac{\text{E(Dividends)}_t}{(1+r)^t} + \frac{\text{Terminal Value}_n}{(1+r)^n} \quad \text{where Terminal Value}_n = \frac{\text{E(Dividends)}_{n+1}}{(r_n - g_n)}$$

where r is the cost of equity and g_n is the expected growth rate in dividends in perpetuity after year n.[2] Note that it is possible for a firm to already be in stable growth, in which case this model collapses into itls simplest form:

Value of a stock in stable growth = Expected Dividends next year/ (Cost of equity – g_n)

This model is called the Gordon Growth model and is a special case of the dividend discount model. It can be used only for firms that are already in stable growth.[3]

Estimating Model Inputs

Breaking down the general version of the dividend discount model, there are four basic components. The first is the length of the high growth period, during which the firm can sustain extraordinary growth. The second is the expected dividends each year during the high growth period. The third is the cost of equity that stockholders will demand for holding the stock, based upon their assessments of risk. The final input is the expected price at the end of the high growth period – the terminal value. In this section, we will consider the challenges associated with estimating each of these components.

a. Length of High-Growth Period

The question of how long a firm will be able to sustain high growth is perhaps the most difficult of all to answer in a valuation, but two points are worth keeping in mind. One is that it is not a question of whether but when; all firms will ultimately be stable growth firms, because high growth makes firms larger, and the firm's size will eventually become a barrier to further high growth. The second is that high growth in valuation, at least high growth that creates value, comes from firms earning high returns on their marginal investments. Using the terminology that we have used before in investment analysis, it comes from firms having a return on equity (capital) that is well in excess of the cost of equity (capital). Thus, when we assume that a

> ***High Growth Period:*** This is a period during which a company's earnings or cash flows are expected to grow at a rate much higher than the overall growth rate of the economy.

[2] The cost of equity can be different for the high growth and stable growth periods. Hence, r_n is the cost of equity for the stable growth period.

[3] When the Gordon Growth model is used to value high growth companies, it is entirely possible that g>r and the model will yield a negative value. The problem is not with the model but in its msapplication to a high growth firm.

firm will experience high growth for the next 5 or 10 years, we are also implicitly assuming that it will earn excess returns (over and above the cost of equity or capital) during that period. In a competitive market, these excess returns will eventually draw in new competitors, and the excess returns will disappear.

We should look at three factors when considering how long a firm will be able to maintain high growth.

1. *Size of the firm*: Smaller firms are much more likely to earn excess returns and maintain these excess returns than otherwise similar larger firms. This is so because they have more room to grow and a larger potential market. When looking at the size of the firm, we should look not only at its current market share, but also at the potential growth in the total market for its products or services. Thus, Microsoft may have a large market share of the computer software market, but it may be able to grow in spite of it because the entire software market is growing. On the other hand, Boeing dominates the market for commercial airliners, but we do not expect the overall market for airliners to increase substantially. Boeing, therefore, is far more constrained in terms of future growth.
2. *Existing growth rate and excess returns:* Although the returns we would like to estimate are the marginal returns on new investments, there is a high correlation between the returns on current investments and these marginal returns. Thus, a firm earning excess returns of 20% on its current investments is far more likely to have large positive excess returns and a long growth period than a firm currently earning excess returns of 2%. There are cases where this rule will not work, such as in new industries going through major restructuring.
3. *Magnitude and Sustainability of Competitive Advantages*: This is perhaps the most critical determinant of the length of the high growth period. If there are significant barriers to entry and sustainable competitive advantages, firms can maintain high growth for longer periods. If, on the other hand, there are no or minor barriers to entry, or if the firm's existing competitive advantages are fading, we should be far more conservative about allowing for long growth periods. The quality of existing

management also influences growth. Some top managers[4] have the capacity to make the strategic choices that increase competitive advantages and create new ones.

Again, the sensitivity of value to changes in the length of the high growth period can always be estimated. While some analysts use growth periods greater than 10 years, the combination of high growth rates and long growth periods creates a potent mix in terms of increasing the size of the firm, in many cases well beyond the realm of what is reasonable. Looking back, there are very few firms that have been able to grow at high rates for more than 10 years.

Illustration 12.1: Length of High Growth Period

To assess how long high growth will last at Disney, Aracruz and Deutsche Bank, we assessed their standings on each of the above characteristics in Table 12.1:

Table 12.1: Assessment of Length of High Growth Period

	Disney	*Aracruz*	*Deutsche Bank*
Firm Size/Market Size	Firm is one of the largest players in the entertainment and theme park businesses but the businesses are redefining themselves and expanding.	Firm has a small market share of the paper/pulp business, but the business is mature.	Firm has a significant market share of a mature business.
Current Excess Returns	Firm is earning less than its cost of capital, and has done so for last few years	Returns on capital are largely a function of paper/pulp prices but, on average, have been less than the cost of capital.	Firm has a return on equity that has lagged its cost of equity in recent years.
Competitive Advantages	Has some of the most recognized brand names in the world. Knows more about operating theme parks than any other firm in the world. Has skilled animation studio staff.	Cost advantages because of access to Brazilian rainforests. Has invested in newer, updated plants and has skilled workforce.	Has an edge in the commercial banking business in Germany but this advantage is dissipating in the EU.
Length of High Growth period	10 years, entirely because of its strong competitive advantages (which have been wasted over the last few years) but the excess returns are likely to be	5 years, largely due to access to cheap raw material.	5 years, mostly to allow firms to recover to pre-downturn levels.

[4] Jack Welch at GE and Robert Goizueta at Coca Cola are good examples of CEOs who made a profound difference in the growth of their firms.

	small.		

What about Bookscape? The single biggest competitive advantage possessed by the firm is its long-term lease, at favorable terms, in a superb location in New York City. It is unlikely that the firm will be able to replicate this advantage elsewhere. Adding to this the fact that this is a private firm leads us to conclude that there will be no high growth period.

12.3. ☞**: Length of High Growth Period and Barriers to Entry**

Assume that you are analyzing two firms, both of which are enjoying high growth. The first firm is Earthlink Network, an internet service provider, which operates in an environment with few barriers to entry and extraordinary competition. The second firm is Biogen, a biotechnology firm which is enjoying growth from two drugs to which it owns patents for the next decade. Assuming that both firms are well managed, which of the two firms would you expect to have a longer high growth period?

a. Earthlink Network
b. Biogen
c. Both are well managed and should have the same high growth period

b. Expected Dividends during High Growth Period

The first step in estimating expected dividends during the high growth period is to estimate the expected earnings for each year. This can be done in one of two ways – you can apply an expected growth rate to current earnings or you can begin by estimating future revenues first and then estimate net profit margins in each year. The first approach is easier but the second approach provides for more flexibility since margins can change over time. The resulting expected earnings are paired with estimated dividend payout ratios in each period, which may change over the high growth period. This may seem like an awkward procedure, since expected dividends could well be estimated using the current dividends and applying a dividend growth rate, but it is used for two reasons. First, most analyst projections for growth are stated in

Historical Growth Rate (in Earnings): This is the growth rate over the past few periods in earnings - it can be calculated either by averaging the year-specific growth rates (arithmetic average) or by estimating at the compounded growth rate over the whole period.

terms of revenues and earnings rather than dividends. Second, separating earnings forecasts from dividend payout provides more flexibility in terms of changing dividend payout ratios as earnings growth rates change. In particular, it allows us to raise dividend payout ratios as earnings growth rates decline.

The growth rate in earnings can be estimated using one of three approaches. The first is to look at the past and measuring the historical growth rate in earnings over previous years. In measuring earnings growth, we will have to consider both how far back to go in time and whether to use arithmetic average or geometric average growth rates. In general, geometric growth rates yield more meaningful values than arithmetic average growth rates. The second is to look at estimates made by others following the same stock. In fact, growth estimates made by equity research analysts following a stock are public information and are easily accessible.[5] The third is to consider fundamentals and to estimate a growth rate based upon a firm's investment policy. In particular, the growth in earnings per share of a firm can be written as the product of two variables – the percentage of the earnings per share that is retained in the firm to generate future growth (retention ratio) and a the return earned on equity in these new investments:

Expected Growth Rate = Retention Ratio * Return on Equity

Thus, a firm with a return on equity of 20% and a retention ratio of 70% should have earnings growth of 14% a year. Reverting back to our discussion of dividend policy in chapter 10, note that the retention ratio and the payout ratio are two sides of the same coin:

Retention Ratio = 1 – Payout Ratio

Since the retention ratio cannot exceed 100%, the expected growth in earnings per share in the long term for a firm cannot exceed its return on equity.

Assuming that we can obtain all three estimates of the growth rate in earnings for a firm, which one should we use in valuing a company? Historical growth should be weighted the least, because earnings are volatile and past growth has generally not been

[5] I/B/E/S, First Call and Zacks all track equity research analyst forecasts continuously and the consensus estimate across all analysts is publicly available.

highly correlated with future growth.[6] Analyst estimates are useful signposts of what the investment community thinks about a company and could include information that is not be in the financial statements. In particular, it could reflect changes in both the company's management and strategic plans. However, trusting analysts, no matter how well informed they may be, to come up with the most important input in a valuation is not prudent. Ultimately, the fundamental growth equation offers the most promise because it relates growth back to what the firm does and also constrains us to pay for growth (by requiring firms to reinvest) as we estimate value.

12.4. ☞**: Differences in Growth Rates**

The growth rates from historical earnings, analyst projections and fundamentals can often be very different. These differences can be best explained by:

a. As firms become larger, the differences between growth rates will increase.
b. Analysts are biased towards making optimistic estimates of growth
c. The inputs used to estimate fundamental growth reflect what happened last year rather than what we expect will happen in the future.
d. All of the above

Illustration 12.2: Growth in Earnings per share: Deutsche Bank

In 2003, Deutsche Bank reported net income of $1,365 million on a book value of equity of $29,991 million at the end of 2002. The resulting return on equity for the firm is 4.55%:

Return on Equity = Net Income$_{2003}$/ Book Value of Equity$_{2002}$ = 1365/29,991 = 4.55%

This is lower than the cost of equity for the firm, which is 8.76%, and the average return on equity for European banks, which is 11.26%. In the four quarters ended in March 2004, Deutsche Bank paid out dividends per share of 1.50 Euros on earnings per share of 4.33 Euros. The resulting retention ratio is 65.36%.

Retention Ratio = 1 – Dividends per share/ Earnings per share = 1 – 1.50/4.33 = 65.36%

[6] One of the most famous studies of growth was titled "Higgledy Piggledy Growth" (Little, I.M.D., 1962, *Higgledy Piggledy Growth*, Institute of Statistics, Oxford.) precisely because earnings growth was so difficult to predict based upon history.

If Deutsche maintains its existing return on equity and retention ratio for the long term, its expected growth rate will be anemic.

Expected Growth Rate $_{\text{Existing Fundamentals}}$ = Retention Ratio * ROE = .6536*.0455 = 2.97%

For the next five years, we will assume that the return on equity will improve to the industry average of 11.26% while the retention ratio will stay unchanged at 65.36%. The expected growth in earnings per share is 7.36%.

Expected Growth Rate $_{\text{Modified Fundamentals}}$ = .6536 * .1126 = .0736

c. Cost of Equity

The dividends and terminal price should be discounted back at a rate that reflects the risk in the investment to stockholders to arrive at the current value. In chapter 4, we argued that the only risk that diversified investors see in a stock is market risk and that this risk can be measured with a beta (in the capital asset pricing model) or multiple betas (in the arbitrage pricing or multi-factor models). The same reasoning applies here. In fact, the costs of equity that we estimated for Disney, Deutsche Bank and Aracruz in chapter 4 will be the costs of equity that will be used if we were valuing stock in these companies using a dividend discount model. The only point that relates specifically to valuation is that high-growth firms tend to have higher betas than do low-growth firms. Building on this premise, it is important that, as we change growth rates over time, we also adjust risk accordingly. Thus, when a firm goes from high growth to low growth, its beta should be moved towards one to reflect the lower growth.

d. Terminal Value

The last component of the model is the value that you attach to the equity at the end of a period of high growth. This value is estimated from expected dividends in the first time period following the high-growth period, the cost of equity in the stable phase, and the expected stable growth rate in dividends as follows:

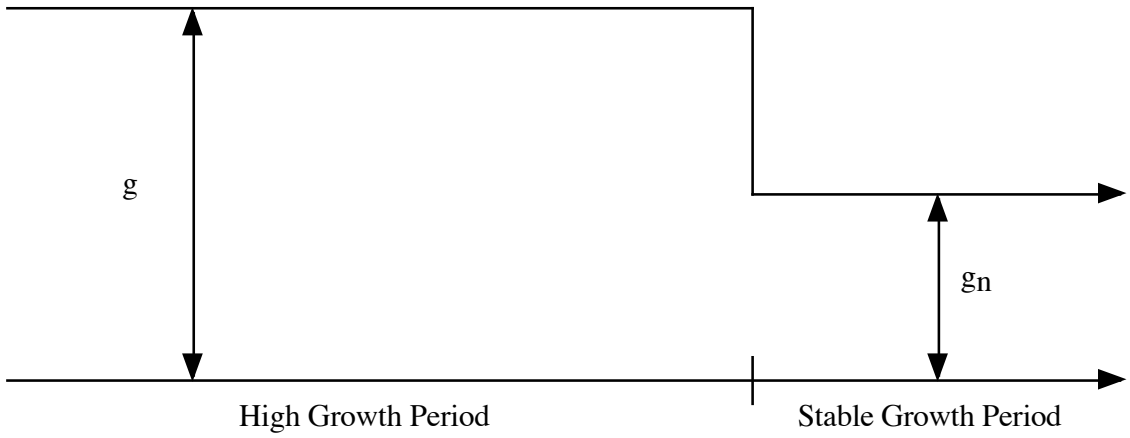
$$\text{Value of Equity in year n} = \frac{\text{Expected Dividends}_{n+1}}{r_n - g_n}$$

where r_n is the cost of equity in the stable growth period and g_n is the expected growth rate in dividends beyond year n (forever).

Before you estimate terminal value, you need to map out a path for the earnings growth during the high growth phase to move towards the stable growth rate. The

simplest assumption to make is that your earnings growth rate is constant for the high growth period, after which the growth rate drops to the stable level, as shown in Figure 12.2.

Figure 12.2: Two-Stage Growth Model



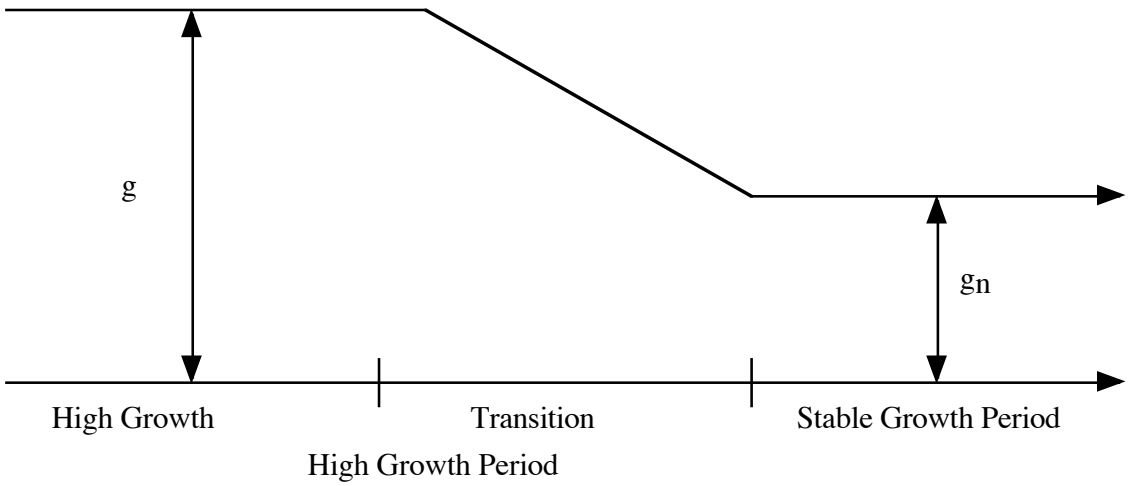
This is a two-stage model, and its limitation is obvious. It assumes that the growth rate is high during the initial period and is transformed overnight to a lower, stable rate at the end of the period. While these sudden transformations in growth can happen, it is much more realistic to assume that the shift from high growth to stable growth happens gradually over time. The assumption that the growth rate drops precipitously from its level in the initial phase to a stable rate also implies that this model is more appropriate for firms with modest growth rates in the initial phase. For instance, it is more reasonable to assume that a firm growing at 12% in the high growth period will see its growth rate drop to 4%, than it is for a firm growing at 40% in the high-growth period. If we assume that the growth rate and payout ratio are fixed for the high growth period, the present value of the dividends during the high growth period can be estimated as follows:[7]

$$\text{PV of High-growth dividends}_0 = \frac{\text{Dividends}_0 * (1+g) * \left(1 - \frac{(1+g)^n}{(1+r)^n}\right)}{r-g}$$

[7] Unlike the stable growth model equation, this one can be used even if the expected growth rate exceeds the discount rate. While this makes the denominator negative, it will also result in a negative numerator, and the net effect will be positive. The only condition when it will not work if g=r, but the PV of dividends in that case will just be the product of the number of years of growth and dividends today since the growth and the discounting effects each year will cancel out.

A more general formulation would allow for growth during the high growth period, followed by a gradual reduction to stable growth over a transition period, as illustrated in figure 12.3:

Figure 12.3: High Growth followed by transition



This model allows for growth rates and payout ratios to change gradually during the transition period.

Whatever path you devise to get your firm to stable growth, it is not just the growth rate that should change in stable growth. The other characteristics of the firm should also change to reflect the stable growth rates.

- The cost of equity should be more reflective of that of a mature firm. If it is being estimated using a beta, that beta should be closer to one in stable growth even though it can take on very high or very low values in high growth.
- The dividend payout ratio, which is usually low or zero for high growth firms, should increase as the firm becomes a stable growth firm. In fact, drawing on the fundamental growth equation from the last section, we can estimate the payout ratio in stable growth:

  Dividend Payout Ratio = 1 – Retention Ratio = 1 – Expected growth rate/ ROE

  If we expect the stable growth rate to be 4% and the return on equity in stable growth to be 12%, the payout ratio in stable growth will be 66.67% (1- 4/12).
- The return on equity in stable growth, if used to estimate the payout ratio, should be also reflective of a stable growth firm. The most conservative estimate to make in stable

growth is that the return on equity will be equal to the cost of equity, thus denying the firm the possibility of excess returns in perpetuity. If this is too rigid a framework, you can assume that the return on equity will converge on an industry average in the stable growth phase.

If there is a transition period for growth, as in figure 12.3, the betas and payout ratios should adjust in the transition period, as the growth rate changes.

12.5. ☞**: Terminal Value and Present Value**

The bulk of the present value in most discounted cash flow valuations comes from the terminal value. Therefore, it is reasonable to conclude that the assumptions about growth during the high growth period do not affect value as much as assumptions about the terminal value.

a. True

b. False

Explain.

Closing Thoughts on the Dividend Discount Model

Many analysts view the dividend discount model as outmoded but it is a useful starting point in valuing all companies and may be the only choice in valuing companies where estimating cashflows is not feasible. As we noted in chapter 11, estimating free cashflows for financial service companies is often difficult to do both because the line between operating and capital expenses is a fuzzy one and because working capital, defined broadly, could include just about all of the balance sheet. While we can arrive at approximations of cashflows by making assumptions about capital expenditures, we are often left in the uncomfortable position of assuming that dividends represent free cashflows to equity for these firms. Even for firms where we can estimate free cash flows to equity with reasonable precision, the dividend discount model allows us to estimate a "floor value" in most cases since firms tend to pay out less in dividends than they have available in free cashflows to equity.

It is often argued that the dividend discount model cannot be used to value high growth companies that pay little in dividends. That is true only if we use the inflexible version of the model where future dividends are estimated by growing current dividends.

In our more flexible version, where both payout ratios and earnings growth can change over time, the dividend discount model can be extended to cover all types of firms.

There is one final point worth making in this section. We can estimate the value of equity on a per share basis by using dividends per share or we can obtain the aggregate value of equity using total dividends paid. The two approaches will yield the same results, if there are no management options, warrants or convertible bonds outstanding. If there are equity options, issued by the firm, that are outstanding, it is safest to value the equity on an aggregate basis. We will consider how best to deal with equity options in arriving at a value per share later in this chapter.

12.6. ☞**: Payout Ratios and Expected Growth**

The dividend discount model cannot be used to value stock in a company with high growth, which does not pay dividends.

a. True

b. False

Explain.

 This file on the web contains, by sector, the industry averages for returns on capital, retention ratios, debt equity ratios and interest rates.

Illustration 12.3: Valuing equity using the Dividend Discount Model: Deutsche Bank

In illustration 12.2, we estimated the annual growth rate for the next 5 years at Deutsche Bank to be 7.36%, based upon an estimated ROE of 11.26% and a retention ratio of 65.36%. In 2003, the earnings per share at Deutsche Bank were 4.33 Euros, and the dividend per share was 1.50 Euros. Our earlier analysis of the risk at Deutsche Bank provided us with an estimate of beta of 0.98, which used in conjunction with the Euro riskfree rate of 4.05% and a risk premium of 4.82%, yielded a cost of equity of 8.76% (see illustration 4.11).

Based upon these inputs, we estimate the expected earnings per share and dividends per share for the next 5 years, and the present value of these dividends in table 12.2:

Table 12.2: Present Value of Expected Dividends for High Growth Period

Year	EPS	Payout Ratio	DPS	PV at 8.76%
1	€4.65	34.64%	€1.61	€1.48
2	€4.99	34.64%	€1.73	€1.46
3	€5.36	34.64%	€1.86	€1.44
4	€5.75	34.64%	€1.99	€1.42
5	€6.18	34.64%	€2.14	€1.41
Present value of expected dividends =				€7.22

Note that we could have arrived at the same present value, using the short cut described earlier (since the payout ratio and the cost of equity remain unchanged for the high growth period):

$$\text{PV of High-growth dividends}_0 = \frac{1.50 * (1.0736) * \left(1 - \frac{(1.0736)^5}{(1.0876)^5}\right)}{.0876 - .0736} = 7.22$$

At the end of year 5, we will assume that Deutsche Bank's earnings growth will drop to 4% and stay at that level in perpetuity. In keeping with the assumption of stable growth, we will also assume that

- The beta will rise marginally to 1, resulting in a slightly higher cost of equity of 8.87%.

  Cost of Equity = Riskfree Rate + Beta * Risk Premium = 4.05%+ 4.82% = 8.87%

- The return on equity will drop to the cost of equity of 8.87%, thus preventing excess returns from being earned in perpetuity.
- The payout ratio will adjust to reflect the stable period growth rate and return on equity.

  Stable Period Payout Ratio = 1 – g/ ROE = 1- .04/.0887 = .5490 or 54.9%

  The expected dividends in year 6 is calculated using this payout ratio:

  Expected Dividends in year 6 = Expected EPS_6 * Stable period payout ratio

  =€6.18 (1.04) * .549 = €3.5263

The value per share at the end of the fifth year can be estimated using these inputs:

Terminal Value per share = Expected Dividends in year 6/ (Cost of equity – g)

= €3.5263/(.0887 - .04) = €72.41

The present value of the terminal value is computed using the high growth period cost of equity:

Present value of terminal value = Terminal Value/ $(1+r)^n$ = $72.41/1.0876^5$ = 47.59

The total value per share is the sum of this value and the present value of the expected dividends in the high growth period:

Value per share = PV of expected dividends in high growth + PV of terminal value

= €7.22 + €47.59 = €54.80

The market price of Deutsche Bank at the time of this valuation was 66 Euros per share. Based upon out assumptions, Deutsche Bank looks over valued.

II. Free Cashflow to Equity Models

In Chapter 11, while developing a framework for analyzing dividend policy we estimated the free cash flow to equity as the cash flow that the firm can afford to pay out as dividends, and contrasted it with the actual dividends. We noted that many firms do not pay out their FCFE as dividends; thus, the dividend discount model may not capture their true capacity to generate cash flows for stockholders. A more appropriate model is the free cash flow to equity model.

Setting up the Model

The FCFE is the residual cash flow left over after meeting interest and principal payments and providing for capital expenditures to maintain existing assets and to create new assets for future growth. The free cash flow to equity is measured as follows:

FCFE = Net Income + Depreciation - Capital Expenditures - Δ Working Capital - Principal Repayments + New Debt Issues

where Δ Working Capital is the change in non-cash working capital.

In the special case where the capital expenditures and the working capital are expected to be financed at the target debt ratio δ, and principal repayments are made from new debt issues, the FCFE is measured as follows:

FCFE = Net Income + (1-δ) (Capital Expenditures - Depreciation) + (1-δ) Δ Working Capital

There is one more way in which we can present the free cash flow to equity. If we define the portion of the net income that equity investors reinvest back into the firm as the equity reinvestment rate, we can state the FCFE as a function of this rate.

Equity Reinvestment Rate

$$= \frac{\text{(Capital Expenditures - Depreciation + } \Delta \text{ Working Capital) (1- } \delta)}{\text{Net Income}}$$

FCFE = Net Income (1 – Equity Reinvestment Rate)

Once we estimate the FCFE, the general version of the FCFE model resembles the dividend discount model, with FCFE replacing dividends in the equation:

Value of the Stock = PV of FCFE during high growth + PV of terminal price

$$\text{Value}_0 = \sum_{t=1}^{t=n} \frac{E(FCFE)_t}{(1+r)^t} + \frac{\text{Terminal Value}_n}{(1+r)^n} \text{ where Terminal Value}_n = \frac{E(FCFE)_{n+1}}{(r_n - g_n)}$$

where the expected free cashflows to equity are estimated each year for the high growth period, r is the cost of equity and g_n is the stable growth rate.

There is one key difference between the two models, though. While the dividends can never be less than zero, the free cashflows to equity can be negative. This can occur even if earnings are positive, if the firm has substantial working capital and capital expenditure needs. In fact, the expected free cashflows to equity for many small, high growth firms will be negative at least in the early years, when reinvestment needs are high, but will become positive as the growth rates and reinvestment needs decrease.

In Practice: Estimating Capital Expenditure and Working Capital Needs

Two components go into estimating reinvestments. The first is net capital expenditures, which is the difference between capital expenditures and depreciation. While these numbers can easily be obtained for the current year for any firm in the United States[8], they should be used with the following caveats:

1. Firms seldom have smooth capital expenditure streams. Firms can go through periods when capital expenditures are very high, followed by periods of relatively light capital expenditures. Consequently, when estimating the capital expenditures to use

for forecasting future cash flows, we should look at capital expenditures over time and normalize them by taking an average or we should look at industry norms.

2. If we define capital expenditures are expenditures designed to generate benefits over many years, research and development expenses are really capital expenditures. Consequently, R&D expenses need to be treated as capital expenditures, and the research asset that is created as a consequence needs to be amortized, with the amortization showing up as part of depreciation.[9]
3. Finally, in estimating capital expenditures, we should not distinguish between internal investments (which are usually categorized as capital expenditures in cash flow statements) and external investments (which are acquisitions). The capital expenditures of a firm, therefore, need to include acquisitions. Since firms seldom make acquisitions every year, and each acquisition has a different price tag, the point about normalizing capital expenditures applies even more strongly to this item.

The second component of reinvestment is the cash that needs to be set aside for working capital needs. As in the chapters on investment analysis, we define working capital needs as non-cash working capital, and the cash flow effect is the period-to-period change in this number. Again, while we can estimate this change for any year using financial statements, it has to be used with caution. Changes in non-cash working capital are volatile, with big increases in some years followed by big decreases in the following years. To ensure that the projections are not the result of an unusual base year, we tie the changes in working capital to expected changes in revenues or costs of goods sold at the firm over time. For instance, we use the non-cash working capital as a percent of revenues, in conjunction with expected revenue changes each period, to estimate projected changes in non-cash working capital. As a final point, non-cash working capital can be negative, which can translate into positive cash flows from working capital as

[8] It is actually surprisingly difficult to obtain the capital expenditure numbers even for large, publicly traded firms in some markets outside the United States. Accounting standards, in these markets, often allow firms to lump investments together and report them in the aggregate.

[9] Capitalizing R&D is a three-step process. First, you need to specify, on average, how long it takes for research to pay off (amortizable life). Second, you have to collect R&D expenses from the past for an equivalent period. Third, the past R&D expenses have to be written off (straight line) over the amortizable life.

revenue increases. It is prudent, when this occurs, to set non-cash working capital needs to zero[10].

Estimating Model Inputs

Just as in the dividend discount model, there are four basic inputs needed for this model to be usable. First, the *length of the high growth period* is defined. Second, the *free cash flow to equity* each period during the growth period is specified; this means that net capital expenditures, working capital needs and the debt financing mix are all estimated for the high growth period. Third, the *rate of return* stockholders will demand for holding the stock is estimated. Finally, the *terminal value of equity at the end of the high growth period* is calculated, based upon the estimates of stable growth, the free cash flows to equity and required return after the high growth ends. Of the four inputs, the length of the high growth period and the rate of return required by stockholders are the same for the dividend discount and FCFE valuation models. On the other two, the differences in the other two inputs are minor but still worth emphasizing.

a. Estimating FCFE during High Growth Period

As in the dividend discount model, we start with the earnings per share and estimate expected growth in earnings. Thus, the entire discussion about earnings growth in the dividend discount model applies here as well. The only difference is in the estimation of fundamental growth. When estimating fundamental growth in the dividend discount model, we used the retention ratio and the return on equity to estimate the expected growth in earnings. When estimating fundamental growth in the FCFE valuation model, it is more consistent to use the equity reinvestment rate that we defined in the last section and the return on equity to estimate expected growth:

Expected Growth in Net Income = Equity Reinvestment Rate * Return on Equity

Unlike the retention ratio, which cannot exceed 100% or be less than 0%, the equity reinvestment rate can be negative (if capital expenditures drop below depreciation) or greater than 100%. If the equity reinvestment rate is negative and is expected to remain so for the foreseeable future, the expected growth in earnings will be negative. If the

[10] While it is entirely possible that firms can generate positive cash flows from working capital decreasing

equity reinvestment rate is greater than 100%, the net income can grow at a rate higher than the return on equity though the firm will have to issue new stock to fund the reinvestment.[11]

Once the earnings are estimated, the net capital expenditures, working capital needs, and debt financing needs have to be specified in order to arrive at the FCFE. Just as the dividend payout ratio was adjusted to reflect changes in expected growth, the net capital expenditure and working capital needs should change as the growth rate changes. In particular, high growth companies will have relatively higher net capital expenditures and working capital needs. In other words, the equity reinvestment rate will generally be high in high growth and decline as the growth rate declines. A similar point can be made about leverage. High growth, high risk firms generally do not use much leverage to finance investment needs; as the growth tapers off, however, the firm will be much more willing to use debt, suggesting that debt ratios will increase as growth rates drop.

There is one final point worth making about equity valuations. Since the net income includes both income from operations and income from cash and marketable securities, we have two choices when it comes to equity valuations. The first and easier, albeit less precise one, is to discount the total free cashflow to equity (including the income from cash holdings) at a cost of equity that is adjusted to reflect the cash holdings.[12] The present value of equity will then incorporate the cash holdings of the company. The second and more precise way is to discount the net income, without including the interest income from cash, at a cost of equity that reflects only the operations of the firm and then to add the cash and marketable securities on to this present value at the end.

 This file on the web contains, by sector, the industry averages for net capital expenditures and working capital as a percent of revenues.

for short periods, it is dangerous to assume that this can occur forever.

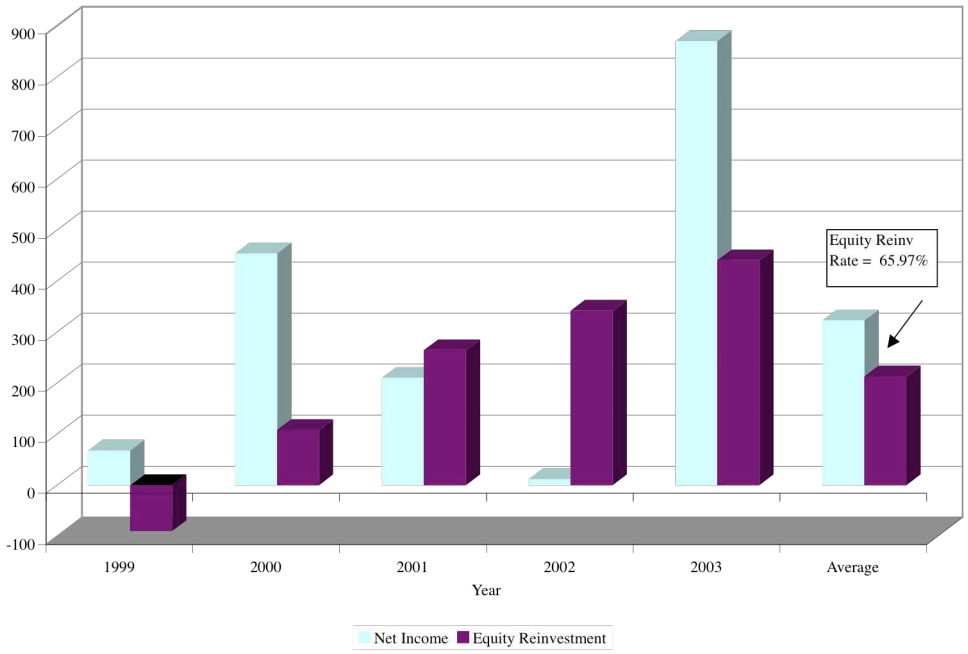
[11] If the equity reinvestment rate exceeds 100%, the net income of the firm is insuffuicient to cover the equity reinvestment needs of the firm. Fresh equity will have to be issued to fund the difference. This will increase the number of shares outstanding.

[12] The beta for equity will be based upon an unlevered beta, adjusted for the cash holdings of the company. In other words, if the company is 20% cash and 80% operations, the unlevered beta will be estimated attaching a 20% weight to cash and a beta of zero for cash.

Illustration 12.4: Estimating Growth Rate in Net Income

Like many manufacturing firms, Aracruz has volatile reinvestment outlays and the cashflows from debt swing wildly from year to year. In figure 12.4, we report on net income and equity reinvestment (capital expenditures – depreciation + change in non-cash working capital – net cashflow from debt) each year from 1999 to 2003:

Figure 12.4: Equity Reinvestment and Net Income at Aracruz- 1999 to 2003



Rather than base the equity reinvestment rate on the most recent year's numbers, we will use the average values for each of the variables over the entire period to compute a "normalized" equity reinvestment rate:

Normalized Equity Reinvestment Rate = Average Equity Reinvestment$_{99\text{-}03}$/ Average Net Income$_{99\text{-}03}$ = 213.17/323.12 = 65.97%

To estimate the return on equity, we look at only the portion of the net income that comes from operations (ignoring the income from cash and marketable securities) and divide by the book value of equity net of cash and marketable securities. This non-cash ROE is a cleaner measure of the returns on equity in operating assets.

Non-cash ROE = (Net Income – After-tax Interest income on cash)$_{2003}$/ (BV of Equity – Cash)$_{2002}$

Aracruz had net income of $148.09 million in 2003, interest income before taxes of $43.04 million and faced a tax rate of 34%. The book value of equity at the end of 2002 was $1760.58 million, of which cash represented $273.93 million.

Non-cash $ROE_{Aracruz}$ = (148.09 – 43.04(1-.34))/ (1760.58-273.93) = .0805 or 8.05%

The expected growth in net income can be computed as the product of the non-cash ROE and the equity reinvestment rate.

Expected Growth in Net Income = Equity Reinvestment Rate * Non-cash ROE

= 65.97% * 8.05% = 5.31%

Based upon fundamentals, we would expect Aracruz's net income to grow 5.31% a year.

In Practice: Paths to a Higher ROE

The expected growth rate in earnings per share and net income are dependent upon the return on equity that a firm makes on its new investments. The higher the return one equity, the higher the expected growth rate in earnings. But how do firms generate higher returns on equity? Algebraically, the return on equity can be decomposed into a return on capital and a leverage effect:

Return on Equity = Return on capital + D/E (Return on capital – Cost of debt (1-tax rate))

where,

Return on capital = EBIT (1-tax rate)/ (Book value of debt + Book value of equity)

D/E = Book value of debt/ Book value of equity

The second term in the equation reflects the influence of debt. To the extent that a firm can earn a return on capital that exceeds the after-tax cost of debt, its return on equity will increase as it uses more debt. A firm with a return on capital of 12%, a debt to equity ratio of 0.5 and an after-tax cost of debt of 4% will have a return on equity of 16%. Lest firms view this as a free lunch, we should hasten to point out that using more debt will also increase the firm's beta and cost of equity and the value of equity may very well decrease with higher borrowing, even though the return on equity and expected growth rate may be higher.

b. Estimating Terminal Value

As with the dividend discount model, the terminal value in the FCFE model is determined by the stable growth rate and the cost of equity. The difference between this

model and the dividend discount model lies primarily in the cash flow used to calculate the terminal price: the latter uses expected dividends in the period after the high growth period, whereas the former uses the free cash flow to equity in that period:

$$\text{Terminal value of equity}_n = \frac{FCFE_{n+1}}{r - g_n}$$

In estimating that cash flow, the net capital expenditures and working capital needs should be consistent with the definition of stability. The simplest way to ensure this is to estimate an equity reinvestment rate from the stable period return on equity:

Equity Reinvestment rate in stable growth = Stable growth rate/ Stable period ROE

This is exactly the same equation we used to compute the retention ratio in stable growth in the dividend discount model.

Many analysts assume that stable growth firms have capital expenditures that offset depreciation and no working capital requirements. This will yield a equity reinvestment rate of zero which is consistent only with a stable growth rate of zero. Using a stable growth rate of 3 or 4% while allowing for no reinvestment essentially allows your firm to grow without paying for the growth and will yield too high a value for the firm.

Reconciling FCFE and Dividend Discount Model Valuations

The FCFE discounted cash flow model can be viewed as an alternative to the dividend discount model. Since the two approaches sometimes provide different estimates of value, however, it is worth a comparison.

There are two conditions under which the value obtained from using the FCFE in discounted cash flow valuation will be the same as the value obtained from using the dividend discount model. The first is obvious: when the dividends are equal to the FCFE, the value will be the same. The second is more subtle: when the FCFE is greater than dividends, but the excess cash (FCFE - Dividends) is invested in projects with a net

present value of zero, the values will also be similar. For instance, investing in financial assets that are fairly priced should yield a net present value of zero.[13]

More often, the two models will provide different estimates of value. First, when the FCFE is greater than the dividend and the excess cash either earns below-market returns or is invested in negative net present value projects, the value from the FCFE model will be greater than the value from the dividend discount model. This is not uncommon. There are numerous case studies of firms that having accumulated large cash balances by paying out low dividends relative to FCFE, have chosen to use this cash to finance unwise takeovers (the price paid is greater than the value received). Second, the payment of smaller dividends than the firm can afford lowers debt-equity ratios; accordingly, the firm may become underleveraged, reducing its value.

In those cases where dividends are greater than FCFE, the firm will have to issue new shares or borrow money to pay these dividends leading to at least three negative consequences for value are possible. One is the flotation cost on these security issues, which can be substantial for equity issues. Second, if the firm borrows the money to pay the dividends, the firm may become overleveraged (relative to the optimal), leading to a loss in value. Finally, paying too much in dividends can lead to capital rationing constraints, whereby good projects are rejected, resulting in a loss of wealth.

When the two models yield different values, two questions remain: (1) What does the difference between the two models tell us? (2) Which of the two models is the appropriate one to use in evaluating the market price? In most cases, the value from the FCFE model will exceed the value from the dividend discount model. The difference between the value obtained from the FCFE model and the value obtained from the dividend discount model can be considered one component of the value of controlling a firm – that is, it measures the value of controlling dividend policy. In a hostile takeover, the bidder can expect to control the firm and change the dividend policy (to reflect FCFE), thus capturing the higher FCFE value. In the more infrequent case —the value from the dividend discount model exceeds the value from the FCFE — the difference has

[13] Mechanically, this will work out only if you keep track of the cash build up in the dividend discount model and add it to the terminal value. If you do not do this, you will under value your firm with the dividend discount model.

less economic meaning but can be considered a warning on the sustainability of expected dividends.

As for which of the two values is more appropriate for evaluating the market price, the answer lies in the openness of the market for corporate control. If there is a significant probability that a firm can be taken over or its management changed, the market price will reflect that likelihood; in that case, the value from the FCFE model would be a more appropriate benchmark. As changes in corporate control become more difficult, either because of a firm's size and/or legal or market restrictions on takeovers, the value from the dividend discount model will provide a more appropriate benchmark for comparison.

12.7. ☞**: FCFE and DDM Value**

Most firms can be valued using FCFE and DDM valuation models. Which of the following statements would you most agree with on the relationship between these two values?

a. The FCFE value will always be higher than the DDM value
b. The FCFE value will usually be higher than the DDM value
c. The DDM value will usually be higher than the FCFE value
d. The DDM value will generally be equal to the FCFE value

Illustration 12.5: FCFE Valuation: Aracruz

To value Aracruz, using the FCFE model, we will use the expected growth in net income that we estimated in illustration 12.4 and value the equity in operating assets first and then add on the value of cash and other non-operating assets. We will also value the company in U.S. dollars, rather than Brazilian real, because the firm generates so much of its cashflows in dollars. Summarizing the basic information that we will be using:

- The net income for the firm in 2003 was $148.09 million but $28.41 million of this income represented income from financial assets.[14] The net income from non-operating assets is $119.68 million.

[14] The pre-tax income from financial assets was $43.04 million. We used a 34% tax rate to arrive at

- In 2003, capital expenditures amounted to $ 228.82 million, depreciation was $191.51 million and non-cash working capital increased by $10.89 million. The net cashflow from debt was $531.20 million, resulting in a large negative equity reinvestment in that year.

  Equity Reinvestment $Rate_{2003}$ = (228.82 – 191.51 + 10.89 -531.20)/ 119.68 = -403.58%

  We will use the average equity reinvestment rate of 65.97%, based upon the average values from 199-2003, that we computed in illustration 12.4 as the equity reinvestment rate for the next 5 years. In conjunction, with the non-cash return on equity of 8.05% that we computed in that illustration, we estimate an expected growth rate of 5.31% a year for the next 5 years.
- In illustration 4.7, we estimated a beta for equity of 0.7576 for the paper business that Aracruz.[15] With a nominal U.S. dollar riskfree rate of 4% and an equity risk premium of 12.49% for Brazil (also estimated in chapter 4), we arrive at a dollar cost of equity of 13.46%

  Cost of equity = 4% + 0.7576 (12.49%) = 13.46%

  After year 5, we will assume that the beta will remain at 0.7576 and that the equity risk premium will decline to 8.66%.[16] The resulting cost of equity is 10.56%.

  Cost of equity in stable growth = 4% + 0.7576 (8.66%) = 10.56%
- After year 5, we will assume that the growth in net income will drop to the inflation rate (in U.S. dollar terms) of 2% and that the return on equity will rise to 10.56% (which is also the cost of equity). The equity reinvestment rate in stable growth can then be estimated as follows:

  Equity Reinvestment $Rate_{\text{Stable Growth}}$ = Expected Growth Rate/ Return on Equity

  = 2%/10.56% = 18.94%

To value the equity in Aracruz, we begin by estimating the free cashflows to equity from operations in table 12.3:

[15] We used the equity beta of just the operating asssets in this valuation. If we had chosen to include the cash from financial holdings as part of net income, we would have used Aracruz's consolidated equity beta of 0.7040.

[16] We halved the country risk premium from 7.67% to 3.84%. We are assuming that as Brazil grows, it will become a less risky country to invest in.

Table 12.3: Expected FCFE at Aracruz – Years 1-5

	1	*2*	*3*	*4*	*5*
Net Income (non-cash)	$126.04	$132.74	$139.79	$147.21	$155.03
Equity Reinvestment Rate	65.97%	65.97%	65.97%	65.97%	65.97%
FCFE	$42.89	$45.17	$47.57	$50.09	$52.75
Present Value at 13.46%	$37.80	$35.09	$32.56	$30.23	$28.05

FCFE = Net Income (1- Reinvestment Rate)

To estimate the terminal value of equity, we first estimate the free cashflow to equity in year 6:

FCFE in year 6 = Net Income in year 6 (1- Equity Reinvestment Rate$_{\text{Stable Growth}}$)

= 155.03 (1.02) (1- .1894) = $128.18 million

The terminal value is then computed using the stable period cost of equity of 10.56%:

Terminal value of equity = 128.18/(.1056-.02) = $1497.98 million

The current value of equity is the sum of the present values of the expected cashflows in table 12.3, the present value of the terminal value of equity and the value of cash and non-operating assets today:

Present Value of FCFEs in high growth phase =	$163.73
+ Present Value of Terminal Equity Value = 1497.98/1.1346^5 =	$796.55
Value of equity in operating assets =	$960.28
+ Value of Cash and Marketable Securities =	$352.28
Value of equity in firm =	$1,312.56

Dividing by the 859.59 million shares outstanding yields a value per share of $1.53. Converted into Brazilian Real at the exchange rate of 3.15 BR/$ prevailing at the time of this valuation, we get a value per share of 4.81 BR per share, well below the market price of 7.50 BR/share.

In Practice: Reconciling your value with the market price

When you value a company and arrive at a number very different from the market price, as we have with both Aracruz and Deutsche Bank, there are three possible explanations. The first is that we are mistaken in our assumptions and that our valuations are wrong while the market is right. Without resorting to the dogma of efficient markets,

this is a reasonable place to start since this is the most likely scenario. The second is that the market is wrong and that we are right, in which case we have to decide whether we have enough confidence in our valuations to act on them. If we find a company to be under valued, this would require buying and holding the stock. If the stock is over valued, we would have to sell short. The problem, though, is that there is no guarantee that markets, even if they are wrong, will correct their mistakes in the near future. In other words, a stock that is over valued can become even more over valued and a stock that is under valued may stay undervalued for years, wreaking havoc on our portfolio. This also makes selling short a much riskier strategy since we generally can do so only for a few months.

One way to measure market expectations is to solve for a growth rate that will yield the market price. In the Aracruz valuation, for instance, we would need an expected growth rate of 19.50% in earnings over the next 5 years to justify the current market price. This is called an implied growth rate and can be compared to the estimate of growth we used in the valuation of 5.31%.

III. Free Cashflow to the Firm Models

The dividend discount and FCFE models are models for valuing the equity in a firm directly. The alternative is to value the entire business and then to use this value to arrive at a value for the equity. That is precisely what we try and do in firm valuation models where we focus on the operating assets of the firm and the cashflows that they generate.

Setting up the Model

The cash flow to the firm can be measured in two ways. One is to add up the cash flows to all of the different claim holders in the firm. Thus, the cash flows to equity investors (which take the form of dividends or stock buybacks) are added to the cash flows to debt holders (interest and net debt payments) to arrive at the cash flow to the firm. The other approach to estimating cash flow to the firm, which should yield equivalent results, is to estimate the cash flows to the firm prior to debt payments, but after reinvestment needs have been met:

Earnings before interest and taxes (1 - tax rate)

– (Capital Expenditures - Depreciation)

– Change in Non-cash Working Capital

= Free Cash Flow to the Firm

The difference between capital expenditures and depreciation (net capital expenditures) and the increase in non-cash working capital represent the reinvestment made by the firm to generate future growth. Another way of presenting the same equation is to add the net capital expenditures and the change in working capital, and state that value as a percentage of the after-tax operating income. This ratio of reinvestment to after-tax operating income is called the **reinvestment rate**, and the free cash flow to the firm can be written as:

$$\text{Reinvestment Rate} = \frac{(\text{Capital Expenditures} - \text{Depreciation} + \Delta \text{ Working Capital}))}{\text{EBIT}(1 - \text{tax rate})}$$

Free Cash Flow to the Firm = EBIT (1-t) (1 – Reinvestment Rate)

Note that the reinvestment rate can exceed 100%[17], if the firm has substantial reinvestment needs. If that occurs, the free cash flow to a firm will be negative even though after-tax operating income is positive. The cash flow to the firm is often termed an unlevered cash flow, because it is unaffected by debt payments or the tax benefits[18] flowing from these payments.

As with the dividends and the FCFE, the value of the operating assets of a firm can be written as the present value of the expected cashflows during the high growth period and a terminal value at the end of the period:

$$\text{Value}_0 = \sum_{t=1}^{t=n} \frac{E(FCFF)_t}{(1+r)^t} + \frac{\text{Terminal Value}_n}{(1+r)^n} \quad \text{where Terminal Value}_n = \frac{E(FCFF)_{n+1}}{(r_n - g_n)}$$

where r is the cost of capital and g_n is the expected growth rate in perpetuity.

[17] In practical terms, this firm will need external financing, either from debt or equity or both, to cover the excess reinvestment.

[18] If you are wondering where the tax benefits from interest payments, which are real cash benefits, show up, it is in the discount rate, when we compute the after-tax cost of debt. If we add this tax benefit as a cash flow to the free cash flow to the firm, we will double count the tax benefit.

Estimating Model Inputs

As with the dividend discount and the FCFE discount models, there are four basic components that go into the value of the operating assets of the firm – a period of high growth, the free cashflows to the firm during that period, the cost of capital to use as a discount rate and the terminal value for the operating assets of the firm. We have additional steps we need to take to get to the value of equity per share. In particular, we have to incorporate the value of non-operating assets, subtract out debt and then consider the effect of options outstanding on the equity of the firm.

a. Estimating FCFF during High Growth Period

We base our estimate of a firm's value on expected future cash flows, not current cash flows. It is the forecasts of earnings, net capital expenditures and working capital that will yield these expected cash flows. One of the most significant inputs into any valuation is the expected growth rate in operating income. As with the growth rates we estimated for dividends and net income, the variables that determine expected growth are simple. The expected growth in operating income is a product of a firm's reinvestment rate, i.e., the proportion of the after-tax operating income that is invested in net capital expenditures and non-cash working capital, and the quality of these reinvestments, measured as the after-tax return on the capital invested.

$$\text{Expected Growth}_{EBIT} = \text{Reinvestment Rate} * \text{Return on Capital}$$

where,

$$\text{Reinvestment Rate} = \frac{\text{Capital Expenditure} - \text{Depreciation} + \Delta \text{ Non-cash WC}}{\text{EBIT}(1 - \text{tax rate})}$$

$$\text{Return on Capital} = \text{EBIT}(1\text{-t}) / (\text{Book value of Debt} + \text{Book value of equity})$$

Both measures should be forward looking, and the return on capital should represent the expected return on capital on future investments. In the rest of this section, we will consider how best to estimate the reinvestment rate and the return on capital.

The reinvestment rate is often measured using a firm's past history on reinvestment. Although this is a good place to start, it is not necessarily the best estimate of the future reinvestment rate. A firm's reinvestment rate can ebb and flow, especially in firms that invest in relatively few large projects or acquisitions. For these firms, looking at an average reinvestment rate over time may be a better measure of the future. In addition, as

firms grow and mature, their reinvestment needs (and rates) tend to decrease. For firms that have expanded significantly over the last few years, the historical reinvestment rate is likely to be higher than the expected future reinvestment rate. For these firms, industry averages for reinvestment rates may provide a better indication of the future than using numbers from the past. Finally, it is important that we continue treating R&D expenses and operating lease expenses consistently. The R&D expenses, in particular, need to be categorized as part of capital expenditures for purposes of measuring the reinvestment rate.

The return on capital is often based upon the firm's return on capital on existing investments, where the book value of capital is assumed to measure the capital invested in these investments. Implicitly, we assume that the current accounting return on capital is a good measure of the true returns earned on existing investments, and that this return is a good proxy for returns that will be made on future investments. This assumption, of course, is open to question if the book value of capital is not a good measure of the capital invested in existing projects and/or if the operating income is mis-measured or volatile. Given these concerns, we should consider not only a firm's current return on capital, but also any trends in this return as well as the industry average return on capital. If the current return on capital for a firm is significantly higher than the industry average, the forecasted return on capital should be set lower than the current return to reflect the erosion that is likely to occur as competition responds.

Finally, any firm that earns a return on capital greater than its cost of capital is earning an excess return. These excess returns are the result of a firm's competitive advantages or barriers to entry into the industry. High excess returns locked in for very long periods imply that this firm has a permanent competitive advantage.

In Practice: After-tax Operating Income

The income statement for a firm provides a measure of the operating income of the firm in the form of the earnings before interest and taxes (EBIT) and a tax rate in the form of an effective tax rate. Since the operating income we would like to estimate is before capital and financing expenses, we have to make at least three adjustments to the accounting operating income:

- The first adjustment is for financing expenses that accountants treat as operating expenses. The most significant example is operating leases. Since these lease payments constitute firm commitments into the future, they are tax deductible, and the failure to make lease payments can result in bankruptcy, we treat these expenses as financial expenses. The adjustment, which we describe in detail in chapter 4, results in an increase in both the operating income and the debt outstanding at the firm.
- The second adjustment is to correct for the incidence of one-time or irregular income and expenses. Any expense (or income) that is truly a one-time expense (or income) should be removed from the operating income and should not be used in forecasting future operating income. While this would seem to indicate that all extraordinary charges should be expunged from operating income, there are some extraordinary charges that seem to occur at regular intervals – say once every four or five years. Such expenses should be viewed as "irregular" rather than extraordinary expenses and should be built into forecasts. The easiest way to do this is to annualize the expense. Put simply, this would mean taking one-fifth of any expense that occurs once every five years, and computing the income based on this apportioned expense.

As for the tax rate, the effective tax rates reported by most firms are much lower than the marginal tax rates. As with the operating income, we should look at the reasons for the difference and see if these firms can maintain their lower tax rates. If they cannot, it is prudent to shift to marginal tax rates in computing future after-tax operating income.

Illustration 12.6: Estimating Growth Rate in Operating Income - Disney

We begin by estimating the reinvestment rate and return on capital for Disney in 2003, using the numbers from the latest financial statements. We did convert operating leases into debt and adjusted the operating income and capital expenditure accordingly.[19]

$$\text{Reinvestment Rate}_{2003} = (\text{Cap Ex} - \text{Depreciation} + \text{Chg in non-cash WC})/ \text{EBIT}\ (1-t)$$

[19]The book value of debt is augmented by the $1,753 million in present value of operating lease commitments. The unadjusted operating income for Disney was $2,713 million. The operating lease adjustment adds the inputed interest expense on the PV of operating leases to the operating income (5.25% of $1753 million= $92 million), the current year's operating lease expense to capital expenditures ($556 million) and the depreciation on the leased asset to depreciation ($195 million).

$= (1735 – 1253 + 454)/(2805(1-.373)) = 53.18\%$

Return on capital$_{2003}$ $= EBIT\ (1-t)_{2003}/\ (BV\ of\ Debt_{2002} + BV\ of\ Equity_{2002})$

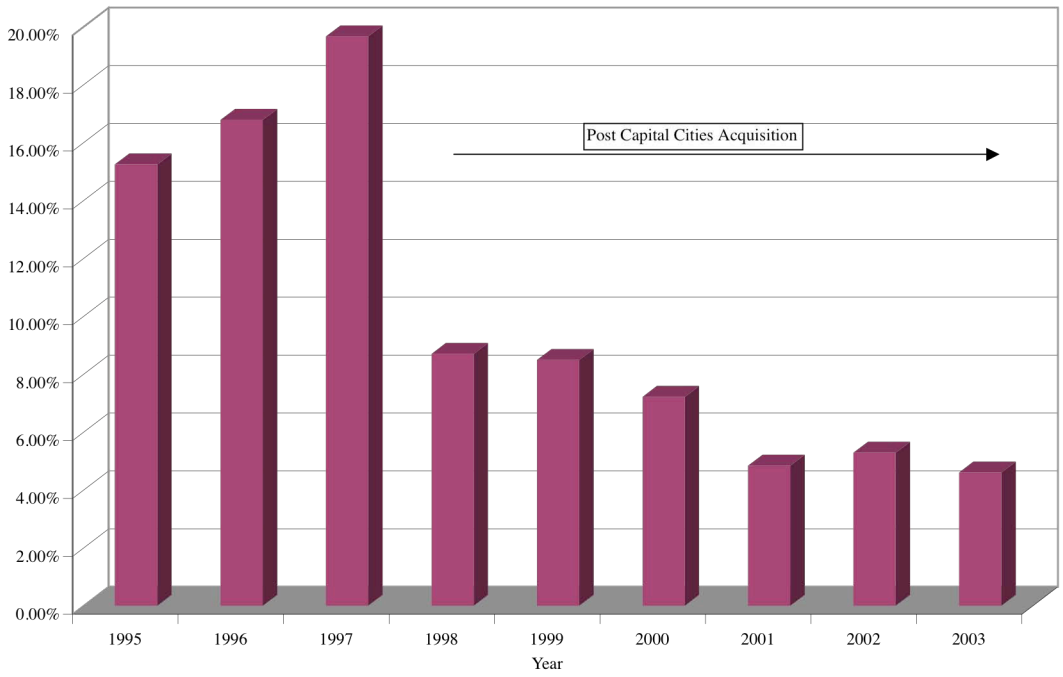
$= 2805\ (1-.373)/\ (15{,}883+23{,}879) = 4.42\%$

If Disney maintains its 2003 reinvestment rate and return on capital for the next few years, its growth rate will be only 2.35%.

Expected Growth Rate from existing fundamentals = 53.18% * 4.42% = 2.35%

To make our estimates for the future, we look at Disney's returns on capital and reinvestment rates from 1995 to 2003 in Figure 12.5:

Figure 12.5: Return on Capital - Disney



Note the dramatic drop in the return on capital after the Capital Cities acquisition from more than 19% in 1997 to less than 10% in 1998.[20] While this may sound optimistic, we will assume that Disney will be able to earn a return on capital of 12% on its new investments and that the reinvestment rate will be 60% for the immediate future.[21] The expected growth in operating income over this period will be 7.20%.

[20] Part of this drop can be explained by accounting factors; the acquisition of Capital Cities increased the book value of equity and debt substantially.

[21] We are, however, leaving the return on capital on existing investments at 4.55% since improving those returns will be much more difficult to do.

Expected Growth Rate in operating income = Return on capital * Reinvestment Rate

= 12% * .60 = 7.20%

 *fundgrEB.xls*: There is a dataset on the web that summarizes reinvestment rates and return on capital by industry group in the United States for the most recent quarter.

b. Estimating Cost of Capital

Unlike equity valuation models, where the cost of equity is used to discount cashflows to equity, the cost of capital is used to discount cashflows to the firm. The cost of capital is a composite cost of financing that includes the costs of both debt and equity and their relative weights in the financing structure:

$$\text{Cost of Capital} = k_{equity} (\text{Equity}/(\text{Debt}+\text{Equity}) + k_{debt} (\text{Debt}/(\text{Debt} + \text{Equity})$$

where the cost of equity represents the rate of return required by equity investors in the firm, and the cost of debt measures the current cost of borrowing, adjusted for the tax benefits of borrowing. The weights on debt and equity have to be market value weights. We discussed the cost of capital estimation extensively earlier in this book, in the context of both investment analysis and capital structure. We will consider each of the inputs in the model, in the context of valuing a firm.

The cost of equity, as we have defined it through this book, is a function of the non-diversifiable risk in an investment, which, in turn, is measured by a beta (in the single factor model) or betas (in the multiple factor models). We argued that the beta(s) are better measured by looking at the average beta(s) of other firms in the business, i.e, bottom-up estimates, and reflecting a firm's current business mix and leverage. This argument is augmented, when we value companies, by the fact that a firm's expected business mix and financial leverage can change over time, and its beta will change with both. As the beta changes, the cost of equity will also change, from year to year.

Just as the cost of equity can change over time as a firm's exposure to market risk changes, so too can the cost of debt, as its exposure to default risk changes. The default risk of a firm can be expected to change for two reasons. One is that the firm's size will change as we project earnings further into the future; the volatility in these earnings is also likely to change over time. The second reason is that changes occur in financial

leverage. If we expect a firm's financial leverage to change over time, it will affect its capacity to service debt and hence its cost of borrowing. The after-tax cost of debt can also change as a consequence of expected changes in the tax rate over time.

As a firm changes its leverage, the weights attached to equity and debt in the cost of capital computation will change. Should a firm's leverage be changed over the forecast period? The answer to this depends upon two factors. The first is whether the firm is initially under or over levered. If it is at its appropriate leverage, there is a far smaller need to change leverage in the future. The second is the views of the firm's management and the degree to which they are responsive to the firm's stockholders. Thus, if the management of a firm is firmly entrenched and steadfast in its opposition to debt, an under levered firm will stay under levered over time. In an environment where stockholder have more power, there will eventually be pressure on this firm to increase its leverage toward its optimal level.

Illustration 12.7: Cost of capital - Disney

Recapping the inputs we used to estimate the cost of capital in Disney, we will make the following assumptions:

- The beta for the first 5 years will be the bottom-up beta of 1.2456 that we estimated in illustration 4.5. In conjunction with a riskfree rate of 4% and market risk premium of 4.82%, this yields a cost of equity of 10%.

Cost of Equity = Riskfree Rate + Beta * Risk Premium = 4% + 1.2456(4.82%) = 10%

- The cost of debt for Disney for the first 5 years, based upon its rating of BBB+, is 5.25%. Using Disney's tax rate of 37.30% gives us an after-tax cost of debt of 3.29%:

After-tax cost of debt = 5.25% (1-.373) = 3.29%

- The current market debt ratio of 21% debt, estimated in illustration 4.15 will be used as the debt ratio for the first 5 years of the valuation. Keep in mind that this debt ratio is computed using the market value of debt (inclusive of operating leases) of $14,668 million and a market value of equity of $55,101 million.

The cost of capital for Disney, at least for the first 5 years of the valuation, is 8.59%.

Cost of capital = Cost of Equity (E/(D+E))) + After-tax cost of debt (D/(D+E))

= 10% (.79) + 3.29% (.21) = 8.59%

12.8. ☞**: Firm Valuation and Leverage**

A standard critique of the use of cost of capital in firm valuation is that it assumes that leverage stays stable over time (through the weights in the cost of capital). Is this true?

a. Yes

b. No

 *wacc.xls*: This dataset summarizes the costs of capital for firms in the United States, categorized by industry group.

c. Estimating Terminal Value

The approach most consistent with a discounted cash flow model assumes that cash flows, beyond the terminal year, will grow at a constant rate forever, in which case the terminal value can be estimated as follows:

Terminal value$_n$ = Free Cashflow to Firm$_{n+1}$ / (Cost of Capital$_{n+1}$ - g_n)

where the cost of capital and the growth rate in the model are sustainable forever. We can use the relationship between growth and reinvestment rates that we noted earlier to estimate the reinvestment rate in stable growth:

Reinvestment Rate in stable growth = Stable growth rate / ROC_n

where the ROC_n is the return on capital that the firm can sustain in stable growth. This reinvestment rate can then be used to generate the free cash flow to the firm in the first year of stable growth:

$$\text{Terminal Value} = \frac{EBIT_{n+1}(1-t)\left(1-\frac{g_n}{ROC_n}\right)}{(\text{Cost of Capital}_n - g_n)}$$

In the special case where ROC is equal to the cost of capital, this estimate simplifies to become the following:

$$\text{Terminal Value}_{ROC=WACC} = \frac{EBIT_{n+1}(1-t)}{\text{Cost of Capital}_n}$$

Thus, in every discounted cash flow valuation, there are two critical assumptions we need to make on stable growth. The first relates to when the firm that we are valuing will become a stable growth firm, if it is not one already. The second relates to what the characteristics of the firm will be in stable growth, in terms of return on capital and cost of capital. We examined the first question earlier in this chapter when we looked at the dividend discount model. Let us consider the second question now.

As firms move from high growth to stable growth, we need to give them the characteristics of stable growth firms. A firm in stable growth will be different from that same firm in high growth on a number of dimensions. For instance,

- As we noted with equity valuation models, high growth firms tend to be *more exposed to market risk* (and have higher betas) than stable growth firms. Thus, although it might be reasonable to assume a beta of 1.8 in high growth, it is important that the beta be lowered, if not to one, at least toward one in stable growth[22].
- High growth firms tend to have *high returns on capital and earn excess returns*. In stable growth, it becomes much more difficult to sustain excess returns. There are some who believe that the only assumption sustainable in stable growth is a zero excess return assumption; the return on capital is set equal to the cost of capital. While we agree, in principle, with this view, it is difficult in practice to assume that all investments, including those in existing assets, will suddenly lose the capacity to earn excess returns. Since it is possible for entire industries to earn excess returns over long periods, we believe that assuming a firm's return on capital will move towards its industry average yields more reasonable estimates of value.
- Finally, high growth firms tend to *use less debt* than stable growth firms. As firms mature, their debt capacity increases. The question whether the debt ratio for a firm should be moved towards its optimal cannot be answered without looking at the incumbent managers' power, relative to their stockholders, and their views about debt. If managers are willing to change their debt ratios, and stockholders retain some power, it is reasonable to assume that the debt ratio will move to the optimal level in stable growth; if not, it is safer to leave the debt ratio at existing levels.

12.9. ☞**: Net Capital Expenditures, FCFE and Stable Growth**

Assume that you are valuing a high-growth firm with high risk (beta) and large reinvestment needs (high reinvestment rate). You assume the firm will be in stable growth after 5 years, but you leave the risk and reinvestment rate at high growth levels. Will you under value or over value this firm?

a. Under value the firm
b. Over value the firm

Illustration 12.8: Stable Growth Inputs and Transition Period - Disney

We will assume that Disney will be in stable growth after year 10. In its stable growth phase, we will assume the following:

- The beta for the stock will drop to one, reflecting Disney's status as a mature company. This will lower the cost of equity for the firm to 8.82%.

  Cost of Equity = Riskfree Rate + Beta * Risk Premium = 4% + 4.82% = 8.82%

- The debt ratio for Disney will rise to 30%. This is the optimal we computed for Disney in chapter 8 and we are assuming that investor pressure will be the impetus for this change. Since we assume that the cost of debt remains unchanged at 5.25%, this will result in a cost of capital of 7.16%

  Cost of capital = 8.82% (.70) + 5.25% (1-.373) (.30) = 7.16%

- The return on capital for Disney will drop from its high growth period level of 12% to a stable growth return of 10%. This is still higher than the cost of capital of 7.16% but the competitive advantages that Disney has are unlikely to dissipate completely by the end of the 10th year.

- The expected growth rate in stable growth will be 4%. In conjunction with the return on capital of 10%, this yields a stable period reinvestment rate of 40%:

  Reinvestment Rate = Growth Rate / Return on Capital = 4% /10% = 40%

The values of all of these inputs adjust gradually during the transition period, from years 6 to 10, from high growth levels to stable growth values.

[22] As a rule of thumb, betas above 1.2 or below 0.8 are inconsistent with stable growth firms. Two-thirds of all US firms have betas that fall within this range.

c. From Operating Asset Value to Firm Value

The operating income is the income from operating assets, and the cost of capital measures the cost of financing these assets. When the operating cash flows are discounted to the present, we value the operating assets of the firm. Firms, however, often have significant amounts of cash and marketable securities on their books. The value of these assets should be added to the value of the operating assets to arrive at firm value.

Cash and marketable securities can easily be incorporated into firm value, whereas other non-operating assets are more difficult to value. Consider, for instance, minority holdings in other firms and subsidiaries, where income statements are not consolidated[23]. If we consider only the reported income[24] from these holdings, we will miss a significant portion of the value of the holdings. The most accurate way to incorporate these holdings into firm value is to value each subsidiary or firm in which there are holdings and assign a proportional share of this value to the firm. If a firm owns more than 50% of a subsidiary, accounting standards in the U.S. require that the firm fully consolidate the income and assets of the subsidiary into its own. The portion of the equity that does not belong to the firm is shown as minority interest on the balance sheet and should be subtracted out to get to the value of the equity in the firm.[25]

There is one final asset to consider. Firms with defined pension liabilities sometimes accumulate pension fund assets in excess of these liabilities. While the excess does belong to the owners of the firm, they face a tax liability if they claim it. The conservative rule would be to assume that the social and tax costs of reclaiming the excess pension funds are so large that few firms would ever even attempt to do it.

Illustration 12.9: Value of Non-Operating Assets at Disney

At the end of 2003, Disney reported holding $1,583 million in cash and marketable securities. In addition, Disney reported a book value of $1.849 million for

[23] When income statements are consolidated, the entire operating income of the subsidiary is shown in the income statement of the parent firm. Firms do not have to prepare consolidated financial statements if they hold minority stakes in firms and take a passive role in their management.

[24] When firms hold minority, passive interests in other firms, they report only the portion of the dividends they receive from these investments.

[25] Optimally, we would like to subtract out the market value of the minority interests rather than the book value which is reported in the balance sheet.

minority investments in other companies, primarily in non-US Disney theme parks.[26] In the absence of detailed financial statements for these investments, we will assume that the book value is roughly equal to the market value. Note that we consider the rest of the assets on Disney's balance sheet including the $6.2 billion it shows in capitalized television and film costs and $19.7 billion it shows in goodwill and intangibles to be operating assets that we have already captured in the cashflows.[27]

Finally, Disney consolidates its holdings in a few subsidiaries where it owns less than 100%. The portion of the equity in these subsidiaries that does not belong to Disney is shown on the balance sheet as a liability (minority interests) of $428 million. As with its holdings in other companies, we will assume that this is also the estimated market value and subtract it from firm value to arrive at the value of equity in Disney.

 *cash.xls*: There is a dataset on the web that summarizes the value of cash and marketable securities by industry group in the United States for the most recent quarter.

d. From Firm Value to Equity Value

The general rule that you should use is that the debt you subtract from the value of the firm should be at least equal to the debt that you use to compute the cost of capital. Thus, if you decide to convert operating leases to debt to compute the cost of capital, you should subtract out the debt value of operating leases from the value of operating assets to estimate the value of equity. If the firm you are valuing has preferred stock, you would use the market value of the stock (if it is traded) or estimate a market value[28] (if it is not) and deduct it from firm value to get to the value of common equity.

There may be other claims on the firm that do not show up in debt for purposes of computing cost of capital but that you should subtract out from firm value.

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[26] Disney owns 39% of Euro Disney and 43% of the proposed Hong Kong Disney park. It also owns 37.5% of the A&E network and 39.6% of E! Television.

[27] Adding these on to the present value of the cashflows would represent double counting.

[28] Estimating market value for preferred stock is relatively simple. Preferred stock generally is perpetual and the estimated market value of the preferred stock is therefore:

$$\text{Value of preferred stock} = \frac{\text{Preferred Dividend}}{\text{Cost of preferred stock}}$$

The cost of preferred stock should be higher than the pre-tax cost of debt, since debt has a prior claim on the cash flows and assets of the firm.

- *Expected liabilities on lawsuits*: You could be analyzing a firm that is the defendant in a lawsuit, where it potentially could have to pay tens of millions of dollars in damages. You should estimate the probability that this will occur and use this probability to estimate the expected liability. Thus, if there is a 10% chance that you could lose a case that you are defending and the expected damage award is $1 billion, you would reduce the value of the firm by $100 million (probability * expected damages). If the expected liability is not expected to occur until several years from now, you would compute the present value of the payment.
- *Unfunded Pension and Health Care Obligations*: If a firm has significantly under funded a pension or a health plan, it will need to set aside cash in future years to meet these obligations. While it would not be considered debt for cost of capital purposes, it should be subtracted from firm value to arrive at equity value.
- *Deferred Tax Liability*: The deferred tax liability that shows up on the financial statements of many firms reflects the fact that firms often use strategies that reduce their taxes in the current year while increasing their taxes in the future years. Of the three items listed here, this one is the least clearly defined, since it is not clear when or even whether the obligation will come due. Ignoring it, though, may be foolhardy, since the firm could find itself making these tax payments in the future. The most sensible way of dealing with this item is to consider it an obligation, but one that will come due only when the firm's growth rate moderates. Thus, if you expect your firm to be in stable growth in 10 years, you would discount the deferred tax liability back ten years and deduct this amount from the firm value to get to equity value.

e. From Equity Value to Equity Value per share

Once the value of the firm, inclusive of non-operating assets, has been estimated, we generally subtract the value of the outstanding debt to arrive at the value of equity and then divide the value of equity by the number of shares outstanding to estimate the value per share. This approach works only when common stock is the only equity outstanding. When there are warrants and employee options outstanding, the estimated value of these options has to be subtracted from the value of the equity, before we divide by the number of shares outstanding. The same procedure applies when the firm has convertible bonds outstanding, since these conversion options represent claims on equity, as well.

While the approach described above will provide the precise value, there are two short cuts available. One is to divide the value of equity by the fully diluted[29] number of shares outstanding rather than by the actual number. This approach will underestimate the value of the equity, because it fails to consider the cash proceeds from option exercise. The other shortcut, which is called the *treasury stock approach*, adds the expected proceeds from the exercise of the options (exercise price multiplied by the number of options outstanding) to the numerator before dividing by the number of shares outstanding. While this approach will yield a more reasonable estimate than the first one, it does not include the time value of the options outstanding. Thus, it tends to overstate the value of the common stock.


warrants.xls: This spreadsheet allows you to value the options outstanding in a firm, allowing for the dilution effect.

Illustration 12.10: Value of Equity Options

Disney has granted considerable numbers of options to its managers. At the end of 2003, there were 219 million options outstanding, with a weighted average exercise price of $26.44 and weighted average life of 6 years. Using the current stock price of $26.91, an estimated standard deviation[30] of 40, a dividend yield of 1.21%. a riskfree rate of 4% and an option pricing model, we estimate the value of these equity options to $2.129 billion.[31] The value we have estimated for the options above are probably too high, since we assume that all the options are exercisable. In fact, a significant proportion of these options (about 50%) are not vested[32] yet, and this fact will reduce their estimated value. We will also assume that these options, when exercised, will generate a tax benefit to the firm equal to 37.3% of their value:

After-tax value of equity options = 2129 (1-.373) = $1334.67 million

[29] We assume that all options will be exercised, and compute the number of shares that will be outstanding in that event.

[30] We used the historical standard deviation in Boeing's stock price to estimate this number.

[31] The option pricing model used is the Black-Scholes model. It is described in more detail in the appendix.

[32] When options are not vested, they cannot be exercised. Firms, when providing options to their employees, firms often require that they continue as employees for a set period before they can exercise these options.

Reconciling Equity and Firm Valuations

This model, unlike the dividend discount model or the FCFE model, values the firm rather than equity. The value of equity, however, can be extracted from the value of the firm by subtracting out the market value of outstanding debt. Since this model can be viewed as an alternative way of valuing equity, two questions arise - Why value the firm rather than equity? Will the values for equity obtained from the firm valuation approach be consistent with the values obtained from the equity valuation approaches described in the previous chapter?

The advantage of using the firm valuation approach is that cashflows relating to debt do not have to be considered explicitly, since the FCFF is a pre-debt cashflow, while they have to be taken into account in estimating FCFE. In cases where the leverage is expected to change significantly over time, this is a significant saving, since estimating new debt issues and debt repayments when leverage is changing can become increasingly messy the further into the future you go. The firm valuation approach does, however, require information about debt ratios and interest rates to estimate the weighted average cost of capital.

The value for equity obtained from the firm valuation and equity valuation approaches will be the same if you make consistent assumptions about financial leverage. Getting them to converge in practice is much more difficult. Let us begin with the simplest case – a no-growth, perpetual firm. Assume that the firm has $166.67 million in earnings before interest and taxes and a tax rate of 40%. Assume that the firm has equity with a market value of $600 million, with a cost of equity of 13.87%, and debt of $400 million, with a pre-tax cost of debt of 7%. The firm's cost of capital can be estimated:

$$\text{Cost of capital} = (13.87\%)\left(\frac{600}{1000}\right) + (7\%)(1 - 0.4)\left(\frac{400}{1000}\right) = 10\%$$

$$\text{Value of the firm} = \frac{\text{EBIT}(1 - t)}{\text{Cost of capital}} = \frac{166.67(1 - 0.4)}{0.10} = \$1{,}000$$

Note that the firm has no reinvestment and no growth. We can value equity in this firm by subtracting out the value of debt.

Value of equity = Value of firm – Value of debt = $ 1,000 - $400 = $ 600 million

Now let us value the equity directly by estimating the net income:

Net Income = (EBIT – Pre-tax cost of debt * Debt) (1-t) = (166.67 - 0.07*400) (1-0.4) = 83.202 million

The value of equity can be obtained by discounting this net income at the cost of equity:

$$\text{Value of equity} = \frac{\text{Net Income}}{\text{Cost of equity}} = \frac{83.202}{0.1387} = \$\,600 \text{ million}$$

Even this simple example works because of the following assumptions that we made implicitly or explicitly during the valuation.

1. The values for debt and equity used to compute the cost of capital were equal to the values that we obtained in the valuation. Notwithstanding the circularity in reasoning – you need the cost of capital to obtain the values in the first place – it indicates that a cost of capital based upon market value weights will not yield the same value for equity as an equity valuation model, if the firm is not fairly priced in the first place.
2. There are no extraordinary or non-operating items that affect net income but not operating income. Thus, to get from operating to net income, all we do is subtract out interest expenses and taxes.
3. The interest expenses are equal to the pre-tax cost of debt multiplied by the market value of debt. If a firm has old debt on its books, with interest expenses that are different from this value, the two approaches will diverge.

If there is expected growth, the potential for inconsistency multiplies. You have to ensure that you borrow enough money to fund new investments to keep your debt ratio at a level consistent with what you are assuming when you compute the cost of capital.


fcffvsfcfe.xls: This spreadsheet allows you to compare the equity values obtained using FCFF and FCFE models.

Illustration 12.11: FCFF Valuation: Disney

To value Disney, we will consider all of the numbers that we have estimated already in this section. Recapping those estimates:

- The operating income in 2003, before taxes and adjusted for operating leases, is $2,805 million. While this represents a significant come back from the doldrums of 2002, it is still lower than the operating income in the 1990s and results in an after-tax return on capital of only 4.42% (assuming a tax rate of 37.30%).

- For years 1 through 5, we will assume that Disney will be able to raise its return on capital on new investments to 12% and that the reinvestment rate will be 60%. (See illustration 12.6). This will result in an expected growth rate of 7.20% a year.
- For years 1 through 5, we will assume that Disney will maintain its existing debt ratio of 21% and its current cost of capital of 8.59% (see illustration 12.7).
- The assumptions for stable growth (after year 10) and for the transition period are listed in illustration 12.8.

In table 12.4, we estimate the after-tax operating income, reinvestment and free cashflow to the firm each year for the next 10 years:

Table 12.4: Estimated Free Cashflows to the Firm - Disney

Year	Expected Growth	EBIT	EBIT (1-t)	Reinvestment Rate	Reinvestment	FCFF
Current		$2,805				
1	6.38%	$2,984	$1,871	53.18%	$994.92	$876.06
2	6.38%	$3,174	$1,990	53.18%	$1,058.41	$931.96
3	6.38%	$3,377	$2,117	53.18%	$1,125.94	$991.43
4	6.38%	$3,592	$2,252	53.18%	$1,197.79	$1,054.70
5	6.38%	$3,822	$2,396	53.18%	$1,274.23	$1,122.00
6	5.90%	$4,047	$2,538	50.54%	$1,282.59	$1,255.13
7	5.43%	$4,267	$2,675	47.91%	$1,281.71	$1,393.77
8	4.95%	$4,478	$2,808	45.27%	$1,271.19	$1,536.80
9	4.48%	$4,679	$2,934	42.64%	$1,250.78	$1,682.90
10	4.00%	$4,866	$3,051	40.00%	$1,220.41	$1,830.62

In table 12.5, we estimate the present value of the free cashflows to the firm using the cost of capital Since the beta and debt ratio change each year from year 6 to 10, the cost of capital also changes each year.

Table 12.5: Present Value of Free Cashflows to Firm – Disney

Year	Cost of capital	FCFF	PV of FCFF
1	8.59%	$876.06	$806.74
2	8.59%	$931.96	$790.31
3	8.59%	$991.43	$774.21
4	8.59%	$1,054.70	$758.45
5	8.59%	$1,122.00	$743.00
6	8.31%	$1,255.13	$767.42
7	8.02%	$1,393.77	$788.91
8	7.73%	$1,536.80	$807.42
9	7.45%	$1,682.90	$822.90

10	7.16%	$1,830.62	$835.31
PV of cashflows during high growth =			$7,894.66

To compute the present value of the cashflows in years 6 through 10, we have to use the compounded cost of capital over the previous years. To illustrate, the present value of $1536.80 million in cashflows in year 8 is:

$$\text{Present value of cashflow in year 8} = \frac{1536.80}{(1.0859)^5(1.0831)(1.0802)(1.0773)}$$

The final piece of the valuation is the terminal value. To estimate the terminal value, at the end of year 10, we estimate the free cashflow to the firm in year 11:

$FCFF_{11} = EBIT_{11}\ (1\text{-}t)\ (1\text{- Reinvestment Rate}_{\text{Stable Growth}})/$

$= 4866\ (1.04)\ (1\text{-}.40) = \$1{,}903.84$ million

$\text{Terminal Value} = FCFF_{11}/\ (\text{Cost of capital}_{\text{Stable Growth}} - g)$

$= 1903.84/\ (.0716 - .04) = \$60{,}219.11$ million

The value of the firm is the sum of the present values of the cashflows during the high growth period, the present value of the terminal value and the value of the non-operating assets that we estimated in illustration 12.9.

PV of cashflows during the high growth phase	=$ 7,894.66
PV of terminal value= $\frac{60{,}219.11}{(1.0859)^5(1.0831)(1.0802)(1.0773)(1.0745)(1.0716)}$	=$ 27,477.81
+ Cash and Marketable Securities	=$ 1,583.00
+ Non-operating Assets (Holdings in other companies)	=$ 1,849.00
Value of the firm	=$ 38,804.48

Subtracting out the market value of debt (including operating leases) of $14,668.22 million and the value of the equity options (estimated to be worth $1,334.67 million in illustration 12.10) yields the value of the common stock:

Value of equity in common stock = Value of firm – Debt – Equity Options

= $38,804.48 - $14,668.22 - $1334.67 = $ 22,801.59

Dividing by the number of shares outstanding (2047.60 million), we arrive at a value per share o $11.14, well below the market price of $ 26.91 at the time of this valuation.

12.10. ☞: Net Capital Expenditures and Value

In the valuation above, we assumed that the reinvestment rate would be 40% in perpetuity to sustain the 4% stable growth rate. What would the terminal value have been if, instead, we had assumed that the reinvestment rate was zero, while continuing to use a stable growth rate of 4%?

In Practice: Adjusted Present Value (APV)

In chapter 8, we presented the adjusted present value approach to estimate the optimal debt ratio for a firm. In that approach, we begin with the value of the firm without debt. As we add debt to the firm, we consider the net effect on value by considering both the benefits and the costs of borrowing. To do this, we assume that the primary benefit of borrowing is a tax benefit and that the most significant cost of borrowing is the added risk of bankruptcy.

The first step in this approach is the estimation of the value of the unlevered firm. This can be accomplished by valuing the firm as if it had no debt, i.e., by discounting the expected free cash flow to the firm at the unlevered cost of equity. In the special case where cash flows grow at a constant rate in perpetuity, the value of the firm is easily computed.

$$\text{Value of Unlevered Firm} = \frac{FCFF_o(1+g)}{\rho_u - g}$$

where $FCFF_0$ is the current after-tax operating cash flow to the firm, ρ_u is the unlevered cost of equity and g is the expected growth rate. In the more general case, you can value the firm using any set of growth assumptions you believe are reasonable for the firm.

The second step in this approach is the calculation of the expected tax benefit from a given level of debt. This tax benefit is a function of the tax rate of the firm and is discounted at the cost of debt to reflect the riskiness of this cash flow. If the tax savings are viewed as a perpetuity,

$$\text{Value of Tax Benefits} = \frac{(\text{Tax Rate})(\text{Cost of Debt})(\text{Debt})}{\text{Cost of Debt}}$$
$$= (\text{Tax Rate})(\text{Debt})$$
$$= t_c D$$

The tax rate used here is the firm's marginal tax rate and it is assumed to stay constant over time. If we anticipate the tax rate changing over time, we can still compute the present value of tax benefits over time, but we cannot use the perpetual growth equation cited above.

The third step is to evaluate the effect of the given level of debt on the default risk of the firm and on expected bankruptcy costs. In theory, at least, this requires the estimation of the probability of default with the additional debt and the direct and indirect cost of bankruptcy. If π_a is the probability of default after the additional debt and BC is the present value of the bankruptcy cost, the present value of expected bankruptcy cost can be estimated.

$$\text{PV of Expected Bankruptcy cost} = (\text{Probability of Bankruptcy})(\text{PV of Bankruptcy Cost}) = \pi_a BC$$

This step of the adjusted present value approach poses the most significant estimation problem, since neither the probability of bankruptcy nor the bankruptcy cost can be estimated directly.

In theory, the APV approach and the cost of capital approach will yield the same values for a firm if consistent assumptions are made about financial leverage. The difficulties associated with estimating the expected bankruptcy cost, though, often lead many to use an abbreviated version of the APV model, where the tax benefits are added to the unlevered firm value and bankruptcy costs are ignored. This approach will over value firms.

Valuing Private Businesses

All of the principles that we have developed for valuation apply to private companies as well. In other words, the value of a private company is the present value of the expected cashflows that you would expect that company to generate over time, discounted back at a rate that reflects the riskiness of the cashflows. The differences that exist are primarily in the estimation of the cashflows and the discount rates:

- When estimating cashflows, we should keep in mind that while accounting standards may not be adhered to consistently in publicly traded firms, they can diverge dramatically in private firms. In small, private businesses, we should reconstruct financial statements rather than trust the earnings numbers that are reported. There are

also two common problems that arise in private firm accounting that we have to correct for. The first is the failure on the part of many owners to attach a cost to the time that they spend running their businesses. Thus, the owner of a store who spends most of every day stocking the store shelves, manning the cash register and completing the accounting will often not show a salary associated with these activities in her income statement, resulting in overstated earnings. The second is the intermingling of personal and business expenses that is endemic in many private businesses. When re-estimating earnings, we have to strip the personal expenses out of the analysis.

- When estimating discount rates for publicly traded firms, we hewed to two basic principles. With equity, we argued that the only risk that matters is the risk that cannot be diversified away by marginal investors, who we assumed were well diversified. With debt, the cost of debt was based upon a bond rating and the default spread associated with that rating. With private firms, both these assumptions will come under assault. First, the owner of a private business is almost never diversified and has his or her entire wealth often tied up in the firm's assets. That is why we developed the concept of a total beta for private firms in chapter 4, where we scaled the beta of the firm up to reflect all risk and not just non-diversifiable risk. Second, private businesses usually have to borrow from the local bank and do not have the luxury of accessing the bond market. Consequently, they may well find themselves facing a higher cost of debt than otherwise similar publicly traded firms.
- The final issue relates back to terminal value. With publicly traded firms, we assume that firms have infinite lives and use this assumption, in conjunction with stable growth, to estimate a terminal value. Private businesses, especially smaller ones, often have finite lives since they are much more dependent upon the owner/founder for their existence.

With more conservative estimates of cashflows, higher discount rates to reflect the exposure to total risk and finite life assumptions, it should come as no surprise that the values we attach to private firms are lower than those that we would attach to otherwise similar publicly traded firms. This also suggests that private firms that have the option of

becoming publicly traded will generally succumb to that temptation even though the owners might not like the oversight and loss of control that comes with this transition.

Illustration 12.12: Valuing a Private Business: Bookscape

To value Bookscape, we will use the operating income of $2 million that the firm had in its most recent year as a starting point. Adjusting for the operating lease commitments that the firm has, we arrive at an adjusted operating income of $2,368.88 million.[33] To estimate the cost of capital, we draw on the estimates of total beta and the assumption that the firm's debt to capital ratio would resemble the industry average of 16.90% that we made in chapter 4 (see illustration 4.16):

Cost of capital = Cost of equity (D/(D+E)) + After-tax cost of debt (D/(D+E))

= 13.93% (.831) + 5.50% (1-.4) (.169) = .1214 or 12.14%

The total beta for Bookscape is 2.06 and we will continue to use the 40% tax rate for the firm.

In chapter 5, we estimated a return on capital for Bookscape of 12.68% and we will assume that the firm will continue to generate this return on capital for the next 40 years, while growing its earnings at 4% a year. The resulting reinvestment rate is 31.54%:

Reinvestment rate = Growth rate/ Return on capital = 4%/12.68% = 31.54%

The present value of the cashflows, assuming perpetual growth, can be computed as follows:

Value of operating assets = 2,368.88 (1-.40)(1-.3154)/(.1214-.04)	= $12.483 million
Value of cash holdings	= $ 2.500 million
Value of the firm	= $ 14.939 million
- Value of debt (operating leases)	= $ 6.707 million
Value of equity	= $ 8.231 million

If we wanted to be conservative, and assume that the cashflows will continue for only 40 years, the value of the operating assets drops marginally to $12.3 million.

In Practice: Illiquidity Discounts in Private Firm Valuation

[33] In illustration 4.15, we estimated the present value of the operating lease commitments at Bookscape to be $6.7 million. To adjust the operating income, we add back the imputed interest expense on this debt,

If you buy stock in a publicly traded firm and change your mind and decide to sell, you face modest transactions costs. If you buy a private business and change your mind, it is far more difficult to reverse your decision. As a consequence, many analysts valuing private businesses apply an illiquidity discount that ranges from 20 to 40% of the value to arrive at a final value. While the size of the discount is large, there is surprisingly little thought that seems to go into the magnitude of the discount. In fact, it is almost entirely based on studies of restricted stock issued by publicly traded firms. These stock are placed with investors who are restricted from trading on the stock for 2 years after the issue and the price on the issue can be compared to the market price of the traded shares of the company to get a sense of the discount that investors demand for the enforced illiquidity. Since there are relatively few restricted stock issues, the sample sizes tend to be small and involve companies that may have other problems raising fresh funds.

While we concede the necessity of an illiquidity discounts in the valuation of private businesses, the discount should be adjusted to reflect the characteristics of the firm in question, Other things remaining equal, we would expect smaller firms with less liquid assets and in poorer financial health to have much larger illiquidity discounts attached to their values. One way to make this adjustment is to take a deeper look at the restricted stock issues for which we have data and look at reasons for the differences in discounts across stocks.[34] Another way is to view the bid-ask spread as the illiquidity discount on publicly traded companies and extend an analysis of the determinants of bid-ask spreads to come up with a reasonable measure of the bid-ask spread or illiquidity discount of a private business.[35]

Value Enhancement

In a discounted cashflow valuation, the value of a firm is the function of four key inputs – the cashflows from existing investments, the expected growth rate in these cashflows for the high growth period, the length of time before the company becomes a

obtained by multiplying the pre-tax cost of borrowing (5.5%) by the present value of the operating leases (6.7 m))

[34] Silber did this in a 1989 study, where he found that the discount tended to be larger for companies with smaller revenues and negative earnings.

[35] See Investment Valuation (John Wiley and Sons) for more details.

stable growth company and the cost of capital. Put simply, to enhance the value of a firm, we have to change one or more of these inputs:

a. *Increase cashflows from existing assets*: There are a number of ways in which we can increase cashflows from assets. First, we can use assets more efficiently, cutting costs and improving productivity. If we succeed, we should see higher operating margins and profits. Second, we can, within the bounds of the law, reduce the taxes we pay on operating income through good tax planning. Third, we can reduce maintenance capital expenditures and investments in working capital – inventory and accounts receivable – thus increasing the cash left over after these outflows.
b. *Increase the growth rate during the high growth period*: Within the structure that we used in the last section, there are only two ways of increasing growth. We can reinvest more in internal investments and acquisitions or we can try to earn higher returns on the capital that we invest in new investments. To the extent that we can do both, we can increase the expected growth rate. One point to keep in mind, though, is that increasing the reinvestment rate will almost always increase the growth rate but it will not increase value, if the return on capital on new investments lags the cost of capital.
c. *Increase the length of the high growth period*: It is not growth per se that creates value but excess returns. Since excess returns and the capacity to continue earning them comes from the competitive advantages possessed by a firm, a firm has to either create new competitive advantages – brand name, economies of scale and legal restrictions on competition all come to mind – or augment existing ones.
d. *Reduce the cost of capital*: In chapter 8, we considered how changing the mix of debt and equity may reduce the cost of capital, and in chapter 9, we considered how matching your debt to your assets can reduce your default risk and reduce your overall cost of financing. Holding all else constant, reducing the cost of capital will increase firm value.

Which one of these four approaches you choose will depend upon where the firm you are analyzing or advising is in its growth cycle. For large mature firms, with little or no growth potential, it is cashflows from existing assets and the cost of capital that offer the most promise for value enhancement. For smaller, risky, high growth firms, it is likely to

be changing the growth rate and the growth period that generate the biggest increases in value.

Illustration 12.13: Value Enhancement at Disney

In illustration 12.11, we valued Disney at $11.14 a share. In the process, though, we assumed that there would be no significant improvement in the return on capital that Disney earns on its existing assets, which at 4.42% is well below the cost of capital of 8.59%. To examine how much the value per share could be enhanced at Disney if it were run differently, we made the following changes:

- We assumed that the current after-tax operating income would increase to $3,417 million, which would be 8.59% of the book value of capital. This, in effect, would ensure that existing investments do not destroy value.
- We also assumed that the return on capital on new investments would increase to 15% from the 12% used in the status quo valuation. This is closer to the return that Disney used to make prior to its acquisition of Capital Cities. We kept the reinvestment rate unchanged at 53.18%. The resulting growth rate in operating income (for the first 5 years) is 7.98% a year.
- We assumed that the firm would increase its debt ratio immediately to 30%, which is its current optimal debt ratio (from chapter 8). As a result the cost of capital will drop to 8.40%.

Keeping the assumptions about stable growth unchanged, we estimate significantly higher cashflows for the firm for the high growth period in table 12.6.

Table 12.6: Expected Free Cashflows to the Firm- Disney

Year	Expected Growth	EBIT	EBIT (1-t)	Reinvestment Rate	Reinvestment	FCFF	Cost of capital	PV of FCFF
Current		$5,327						
1	7.98%	$5,752	$3,606	53.18%	$1,918	$1,688	8.40%	$1,558
2	7.98%	$6,211	$3,894	53.18%	$2,071	$1,823	8.40%	$1,551
3	7.98%	$6,706	$4,205	53.18%	$2,236	$1,969	8.40%	$1,545
4	7.98%	$7,241	$4,540	53.18%	$2,414	$2,126	8.40%	$1,539
5	7.98%	$7,819	$4,902	53.18%	$2,607	$2,295	8.40%	$1,533
6	7.18%	$8,380	$5,254	50.54%	$2,656	$2,599	8.16%	$1,605
7	6.39%	$8,915	$5,590	47.91%	$2,678	$2,912	7.91%	$1,667
8	5.59%	$9,414	$5,902	45.27%	$2,672	$3,230	7.66%	$1,717

9	4.80%	$9,865	$6,185	42.64%	$2,637	$3,548	7.41%	$1,756
10	4.00%	$10,260	$6,433	40.00%	$2,573	$3,860	7.16%	$1,783

The terminal value is also pushed up, as a result of the higher growth in the high growth period:

Terminal value = $FCFF_{11}$/(Cost of capital – g) = 6433 (1.04)/(.0716-.04) = $126,967 mil

The value of the firm and the value per share can now be estimated:

Present Value of FCFF in high growth phase =	$16,254.91
+ Present Value of Terminal Value of Firm =	$58,645.39
+ Value of Cash, Marketable Securities & Non-operating assets =	$3,432.00
Value of Firm =	$78,332.30
- Market Value of outstanding debt =	$14,648.80
- Value of Equity in Options =	$1,334.67
Value of Equity in Common Stock =	$62,348.84
Market Value of Equity/share =	$30.45

Disney's value per share increases from $11.14 per share in illustration 12.11 to $30.45 a share, when we make the changes to the way it is managed.[36]

In Practice: The Value of Control

The notion that control is worth 15% or 20% or some fixed percent of every firm's value is deeply embedded in valuation practice and it is not true. The value of control is the difference between two values – the value of the firm run by its existing management (status quo) and the value of the same firm run optimally.

Value of control = Optimal value for firm – Status quo value

Thus, a firm that takes poor investments and funds them with a sub-optimal mix of debt and equity will be worth more if it takes better investments and funds them with the right mix of debt and equity. In general, the worse managed a firm is the greater the value of control. This view of the world has wide ramifications in corporate finance and valuation:

- In a hostile acquisition, which is usually motivated by the desire to change the way that a firm is run, you should be willing to pay a premium that at best is equal to the

[36] You may wonder why the dollar debt does not change even though the firm is moving to a 30% debt raito. In reality, it will increase but the number of shares will decrease when Disney recapitalizes. The net effect is that the value per share will be close to our estimated value.

value of control. You would rather pay less, to preserve some of the benefits for yourself (rather than give them to target company stockholders).

- In companies with voting and non-voting shares, the difference in value should be a function of the value of control. If the value of control is high and there is a high likelihood of control changing, the value of the voting shares will increase relative to non-voting shares.

In the Disney valuation above, the value of control can be estimated by comparing the value of Disney, run optimally, with the status quo valuation done earlier in the chapter.

Value of control$_{Disney}$ = Optimal value – Status quo value = \$30.45 - \$11.14 = \$19.31

The fact that Disney trades at \$26.91 can be an indication that the market thinks that there will be significant changes in the way the firm is run in the near future, though it is unclear whether these changes will occur voluntarily or through hostile actions.

Relative Valuation

In discounted cash flow valuation, the objective is to find the value of assets, given their cash flow, growth and risk characteristics. In relative valuation, the objective is to value assets, based upon how similar assets are currently priced in the market. In this section, we consider why and how asset prices have to be standardized before being compared to similar assets, and how to control for differences across comparable firms.

Standardized Values and Multiples

To compare the values of "similar" assets in the market, we need to standardize the values in some way. They can be standardized relative to the earnings they generate, to the book value or replacement value of the assets themselves, or to the revenues that they generate. We discuss each method next.

1. Earnings Multiples

One of the more intuitive ways to think of the value of any asset is as a multiple of the earnings it generates. When buying a stock, it is common to look at the price paid as a multiple of the earnings per share generated by the company. This **price/earnings ratio** can be estimated using current earnings per share, which is called a **trailing PE**, or an expected earnings per share in the next year, called a **forward PE**. When buying a

business, as opposed to just the equity in the business, it is common to examine the value of the firm as a multiple of the operating income or the earnings before interest, taxes, depreciation and amortization (EBITDA). While, as a buyer of the equity or the firm, a lower multiple is better than a higher one, these multiples will be affected by the growth potential and risk of the business being acquired.

2. Book Value or Replacement Value Multiples

While markets provide one estimate of the value of a business, accountants often provide a very different estimate of the same business. The accounting estimate of book value is determined by accounting rules and is heavily influenced by the original price paid for the asset and any accounting adjustments (such as depreciation) made since. Investors often look at the relationship between the price they pay for a stock and the book value of equity (or net worth) as a measure of how over- or undervalued a stock is; the **price/book value** ratio that emerges can vary widely across industries, depending again upon the growth potential and the quality of the investments in each. When valuing businesses, we estimate this ratio using the value of the firm and the book value of all assets (rather than just the equity). For those who believe that book value is not a good measure of the true value of the assets, an alternative is to use the replacement cost of the assets; the ratio of the value of the firm to replacement cost is called[37].

3. Revenue Multiples

Both earnings and book value are accounting measures and are determined by accounting rules and principles. An alternative approach, which is far less affected by these factors, is to use the ratio of the value of an asset to the revenues it generates. For equity investors, this ratio is the **price/sales ratio (PS)**, where the market value per share is divided by the revenues generated per share. For firm value, this ratio can be modified as the **value/sales ratio (VS)**, where the numerator becomes the total value of the firm. This ratio, again, varies widely across sectors, largely as a function of the profit margins in each. The advantage of using revenue multiples, however, is that it becomes far easier

[37] See Chung and Pruitt for a simple approximation of Tobin's Q.

to compare firms in different markets, with different accounting systems at work, than it is to compare earnings or book value multiples.

Determinants of Multiples

One reason commonly given for the use of these multiples to value equity and firms is that they require far fewer assumptions than does discounted cash flow valuation. We believe this is a misconception. The difference between discounted cash flow valuation and relative valuation is that the assumptions we make are explicit in the former and remain implicit in the latter. It is important that we know what the variables are that cause multiples to change, since these are the variables we have to control for when comparing these multiples across firms.

To look under the hood, so to speak, of equity and firm value multiples, we will go back to fairly simple discounted cash flow models for equity and firm value and use them to derive our multiples. Thus, the simplest discounted cash flow model for equity, which is a stable growth dividend discount model, would suggest that the value of equity is:

$$\text{Value of Equity} = P_0 = \frac{DPS_1}{k_e - g_n}$$

where DPS_1 is the expected dividend in the next year, k_e is the cost of equity and g_n is the expected stable growth rate. Dividing both sides by the earnings, we obtain the discounted cash flow equation specifying the PE ratio for a stable growth firm:

$$\frac{P_0}{EPS_0} = PE = \frac{\text{Payout Ratio} * (1 + g_n)}{k_e - g_n}$$

Dividing both sides by the book value of equity, we can estimate the price/book value ratio for a stable growth firm:

$$\frac{P_0}{BV_0} = PBV = \frac{ROE * \text{Payout Ratio} * (1 + g_n)}{k_e - g_n}$$

where ROE is the return on equity. Dividing by the Sales per share, the price/sales ratio for a stable growth firm can be estimated as a function of its profit margin, payout ratio, profit margin, and expected growth.

$$\frac{P_0}{Sales_0} = PS = \frac{\text{Net Profit Margin} * \text{Payout Ratio} * (1 + g_n)}{k_e - g_n}$$

We can do a similar analysis from the perspective of firm valuation[38]. The value of a firm in stable growth can be written as:

$$\text{Value of Firm} = V_0 = \frac{FCFF_1}{k_c - g_n}$$

Dividing both sides by the expected free cash flow to the firm yields the Value/FCFF multiple for a stable growth firm:

$$\frac{V_0}{FCFF_1} = \frac{1}{k_c - g_n}$$

Since the free cash flow the firm is the after-tax operating income netted against the net capital expenditures and working capital needs of the firm, the multiples of EBIT, after-tax EBIT and EBITDA can also be estimated similarly. The value/EBITDA multiple, for instance, can be written as follows:

$$\frac{\text{Value}}{\text{EBITDA}} = \frac{(1 - t)}{k_c - g} + \frac{\text{Depr (t)/EBITDA}}{k_c - g} - \frac{\text{CEx/EBITDA}}{k_c - g} - \frac{\Delta \text{ Working Capital/EBITDA}}{k_c - g}$$

The point of this analysis is not to suggest that we go back to using discounted cash flow valuation but to understand the variables that may cause these multiples to vary across firms in the same sector. If we ignore these variables, we might conclude that a stock with a PE of 8 is cheaper than one with a PE of 12, when the true reason may be that the latter has higher expected growth or we might decide that a stock with a P/BV ratio of 0.7 is cheaper than one with a P/BV ratio of 1.5, when the true reason may be that the latter has a much higher return on equity. Table 12.7 lists the multiples that are widely used and the variables that determine each; the variable that, in our view, is the most significant determinant is highlighted for each multiple. This variable is what we would call the *companion variable* for this multiple, i.e., the one variable we need to know in order to use this multiple to find under or over valued assets.

[38] In practice, cash and marketable securities are subtracted from firm value to arrive at what is called enterprise value. All the multiples in the following section can be written in terms of enterprise value, and the determinants remain unchanged.

Table 12.7: Multiples and Companion Variables (in italics)

Multiple	*Determining Variables*
Price/Earnings Ratio	*Growth*, Payout, Risk
Price/Book Value Ratio	Growth, Payout, Risk, *ROE*
Price/Sales Ratio	Growth, Payout, Risk, *Net Margin*
Value/EBIT Value/EBIT (1-t) Value/EBITDA	Growth, *Net Capital Expenditure needs*, Leverage, Risk
Value/Sales	Growth, Net Capital Expenditure needs, Leverage, Risk, *Operating Margin*
Value/Book Capital	Growth, Leverage, Risk and *ROC*

 *eqmult.xls*: This spreadsheet allows you to estimate the equity multiples for a firm, given its fundamentals.

 *firmmult.xls*: This spreadsheet allows you to estimate the firm value multiples for a firm, given its fundamentals.

The Use of Comparable Firms

When we use multiples, we tend to use them in conjunction with "comparable" firms to determine the value of a firm or its equity. This analysis begins with two choices - the multiple that will be used in the analysis and the group of firms that will comprise the comparable firms. The multiple is computed for each of the comparable firms, and the average is computed. To evaluate an individual firm, we then compare its multiple to the average computed; if it is significantly different, we make a subjective judgment about whether the firm's individual characteristics (growth, risk or cash flows) may explain the difference. Thus, a firm may have a PE ratio of 22 in a sector where the average PE is only 15, but the analyst may conclude that this difference can be justified because the firm has higher growth potential than the average firm in the industry. If, in the analysts' judgment, the difference on the multiple cannot be explained by the variables listed in Table 12.7, the firm will be viewed as over valued (if its multiple is higher than the average) or undervalued (if its multiple is lower than the average). Choosing comparable firms, and adequately controlling for differences across these comparable firms, then

become critical steps in this process. In this section, we will consider both these decisions.

1. Choosing Comparables

The first step in relative valuation is usually the selection of comparable firms. A comparable firm is one with cash flows, growth potential, and risk similar to the firm being valued. It would be ideal if we could value a firm by looking at how an exactly identical firm - in terms of risk, growth and cash flows - is priced. In most analyses, however, analysts define comparable firms to be other firms in the firm's business or businesses. If there are enough firms in the industry to allow for it, this list is pruned further using other criteria; for instance, only firms of similar size may be considered. The implicit assumption being made here is that firms in the same sector have similar risk, growth, and cash flow profiles and therefore can be compared with much more legitimacy.

This approach becomes more difficult to apply when there are relatively few firms in a sector. In most markets outside the United States, the number of publicly traded firms in a particular sector, especially if it is defined narrowly, is small. It is also difficult to find comparable firms if differences in risk, growth and cash flow profiles across firms within a sector are large. Thus, there may be hundreds of computer software companies listed in the United States, but the differences across these firms are also large. The tradeoff is therefore a simple one. Defining a industry more broadly increases the number of comparable firms, but it also results in a more diverse group.

2. Controlling for Differences across Firms

In Table 12.7, we listed the variables that determined each multiple. Since it is impossible to find firms identical to the one being valued, we have to find ways of controlling for differences across firms on these variables. The process of controlling for the variables can range from very simple approaches, which modify the multiples to take into account differences on one key variable, to more complex approaches that allow for differences on more than one variable.

a. Simple Adjustments

Let us start with the simple approaches. In this case, we modify the multiple to take into account the most important variable determining it. Thus, the PE ratio is divided by the expected growth rate in EPS for a company to determine a growth-adjusted PE ratio or the **PEG ratio**. Similarly, the PBV ratio is divided by the ROE to find a **Value Ratio**, and the price sales ratio is divided by the net margin. These modified ratios are then compared across companies in a sector. The implicit assumption we make is that these firms are comparable on all the other measures of value, besides the one being controlled for.

Illustration 12.14: Comparing PE ratios and growth rates across firms: Entertainment companies

To value Disney, we look at the PE ratios and expected growth rates in EPS over the next 5 years, based on consensus estimates from analysts, for all entertainment companies where data is available on PE ratios and analyst estimates of expected growth in earnings over the next 5 years. Table 12.8 lists the firms and PE ratios.

Table 12.8: Entertainment firms – PE Ratios and Growth Rates – 2004

Company Name	*Ticker Symbol*	*PE*	*Expected Growth Rate*	*PEG*
Point 360	PTSX	10.62	5.00%	2.12
Fox Entmt Group Inc	FOX	22.03	14.46%	1.52
Belo Corp. 'A'	BLC	25.65	16.00%	1.60
Hearst-Argyle Television Inc	HTV	26.72	12.90%	2.07
Journal Communications Inc.	JRN	27.94	10.00%	2.79
Saga Communic. 'A'	SGA	28.42	19.00%	1.50
Viacom Inc. 'B'	VIA/B	29.38	13.50%	2.18
Pixar	PIXR	29.80	16.50%	1.81
Disney (Walt)	DIS	29.87	12.00%	2.49
Westwood One	WON	32.59	19.50%	1.67
World Wrestling Ent.	WWE	33.52	20.00%	1.68
Cox Radio 'A' Inc	CXR	33.76	18.70%	1.81
Beasley Broadcast Group Inc	BBGI	34.06	15.23%	2.24
Entercom Comm. Corp	ETM	36.11	15.43%	2.34
Liberty Corp.	LC	37.54	19.50%	1.92
Ballantyne of Omaha Inc	BTNE	55.17	17.10%	3.23
Regent Communications Inc	RGCI	57.84	22.67%	2.55
Emmis Communications	EMMS	74.89	16.50%	4.54
Cumulus Media Inc	CMLS	94.35	23.30%	4.05

Univision Communic.	UVN	122.76	24.50%	5.01
Salem Communications Corp	SALM	145.67	28.75%	5.07
Average for sector		47.08	17.17%	2.74

Source: Value Line

This simple view of multiples leads us to conclude that Disney should trade at 47.08 times earnings, since that is the average for similar publicly traded firms. The resulting value for the equity would be:

Value of Equity = Disney net income in 2003* Average PE ratio for sector

= $1,267 million * 47.08 = $59,650 million

In this valuation, we assume that Disney has a growth rate similar to the average for the sector. One way of bringing growth into the comparison is to compute the PEG ratio, which is reported in the last column. Based on the average PEG ratio of 2.79 for the sector and the analyst estimate of growth in earnings of 12% for the next 5 years, we obtain the following value for the equity in Disney:

Value of Equity = $ 1,267 million * 2.79 * 12 = $ 41,692 million

While this may seem like an easy adjustment to resolve the problem of differences across firms, the conclusion holds only if these firms are of equivalent risk. Implicitly, this approach assumes a linear relationship[39] between growth rates and PE.

12.11. ☞**: Underlying Assumptions in Comparable Valuation**

Assume that you are reading an equity research report where a buy recommendation for a company is being based upon the fact that its PE ratio is lower than the average for the industry. Implicitly, what is the underlying assumption or assumptions being made by this analyst?

a. The sector itself is, on average, fairly priced
b. The earnings of the firms in the group are being measured consistently
c. The firms in the group are all of equivalent risk
d. The firms in the group are all at the same stage in the growth cycle
e. The firms in the group are of equivalent risk and have similar cash flow patterns
f. All of the above

 *pe.xls*: There is a dataset on the web that summarizes PE ratios and PEG ratios by industry group in the United States for the most recent quarter.

b. Adjusting for more than one variable

When firms differ on more than one variable, it becomes difficult to modify the multiples to account for the differences across firms. We can run regressions of the multiples against the variables and then use these regressions to find predicted values for each firm. This approach works reasonably well when the number of comparable firms is large and the relationship between the multiple and the variables is stable. When these conditions do not hold, a few outliers can cause the coefficients to change dramatically and make the predictions much less reliable.

Illustration 12.15: Price to Book Value Ratios and Return on Equity: European Banks

Table 12.9 lists price/book value ratios of European banks and reports on their returns on equity and risk levels (measured using the standard deviation in stock prices over the previous 5 years):

Table 12.9: European Banks: Price to Book Value Ratio – 2003

Name	*PBV Ratio*	*Return on Equity*	*Standard Deviation*
Bayerische Hypo-Und Vereinsb	0.80	-1.66%	49.06%
Commerzbank Ag	1.09	-6.72%	36.21%
Deutsche Bank Ag -Reg	1.23	1.32%	35.79%
Banca Intesa Spa	1.66	1.56%	34.14%
Bnp Paribas	1.72	12.46%	31.03%
Banco Santander Central Hisp	1.86	11.06%	28.36%
Sanpaolo Imi Spa	1.96	8.55%	26.64%
Banco Bilbao Vizcaya Argenta	1.98	11.17%	18.62%
Societe Generale	2.04	9.71%	22.55%
Royal Bank Of Scotland Group	2.09	20.22%	18.35%
Hbos Plc	2.15	22.45%	21.95%
Barclays Plc	2.23	21.16%	20.73%
Unicredito Italiano Spa	2.30	14.86%	13.79%

[39] Only by assuming a linear relationship can we compare the PEG ratio of a 10% growth firm to a 20% growth firm.

Kredietbank Sa Luxembourgeoi	2.46	17.74%	12.38%
Erste Bank Der Oester Spark	2.53	10.28%	21.91%
Standard Chartered Plc	2.59	20.18%	19.93%
Hsbc Holdings Plc	2.94	18.50%	19.66%
Lloyds Tsb Group Plc	3.33	32.84%	18.66%
Sector Average	2.05	12.54%	24.99%

Source: Bloomberg

Since these firms differ on both risk and return on equity, we run a regression of PBV ratios on both variables:

PBV = 2.27 + 3.63 ROE -2.68 Standard Deviation R^2 = 79.48%

(5.56) (3.32) (2.32)

Firms with higher return on equity and lower standard deviations trade at much higher price to book ratios. The numbers in brackets are t-statistics and suggest that the relationships between PBV ratios and both variables in the regression are statistically significant. The R-squared indicates the percentage of the differences in PBV ratios that is explained by the independent variables. Finally, the regression[40] itself can be used to get predicted PBV ratios for the companies in the list. Thus, the predicted PBV ratio for Deutsche Bank, based upon its return on equity of 1.32% and its standard deviation of 35.79%, would be 1.36.

Predicted $PBV_{\text{Deutsche Bank}}$ = 2.27 + 3.63 (.0132) – 2..68 (.3579) = 1.36

Since the actual PBV ratio for Deutsche Bank at the time of the analysis was 1.23, this would suggest that the stock is undervalued by roughly 10%.

 pbv.xls: There is a dataset on the web that summarizes price to book ratios and returns on equity by industry group in the United States for the most recent quarter.

 *ps.xls*: There is a dataset on the web that summarizes price to sales ratios and margins by industry group in the United States for the most recent quarter.

[40] Both approaches described above assume that the relationship between a multiple and the variables driving value are linear. Since this is not always true, we might have to run non-linear versions of these regressions.

3. Expanding the Range of Comparable Firms

Searching for comparable firms within the sector in which a firm operates is fairly restrictive, especially when there are relatively few firms in the sector or when a firm operates in more than one sector. Since the definition of a comparable firm is not one that is in the same business but one that has the same growth, risk and cash flow characteristics as the firm being analyzed, we need not restrict our choice of comparable firms to those in the same industry. A software firm should be comparable to an automobile firm, if we can control for differences in the fundamentals.

The regression introduced in the previous section allows us to control for differences on those variables that we believe cause multiples to vary across firms. Based upon the variables listed in Table 12.7, we should be able to regress PE, PBV and PS ratios against the variables that should affect them:

Price Earnings = f (Growth, Payout ratios, Risk)

Price to Book Value = f (Growth, Payout ratios, Risk, ROE)

Price to Sales = f (Growth, Payout ratios, Risk, Margin)

It is, however, possible that the proxies that we use for risk (beta), growth (expected growth rate), and cash flow (payout) may be imperfect and that the relationship may not be linear. To deal with these limitations, we can add more variables to the regression - e.g., the size of the firm may operate as a good proxy for risk - and use transformations of the variables to allow for non-linear relationships.

We ran these regressions[41] for PE, PBV, and PS ratios across publicly listed firms in the United States in January 2004 against analyst estimates of expected growth in earnings per share and other financial indicators from the most recent year. The sample, which had more than 7000 firms in it, yielded the regressions reported below. These regressions can then be used to get predicted PE, PBV, and PS ratios for each firm, which, in turn, can be compared to the actual multiples to find under and over valued firms.

PE = 9.475 + 0.814 Expected growth + 0.06 Payout + 6.283 Beta (R2 = 22.1%)

PBV= 0.140 ROE + 0.599 Beta + 0.08 Expected Growth +.002 Payout (R2= 47.1%)

PS= 0.04 Expected Growth +0.011 Payout + 0.549 Beta + 0.234 Net Margin (R2= 71.0%)

The first advantage of this approach over the "subjective" comparison across firms in the same sector, described in the previous section, is that it does quantify, based upon actual market data, the degree to which higher growth or risk should affect the multiples. It is true that these estimates can be noisy, but noise is a reflection of the reality that many analysts choose not to face when they make subjective judgments. Second, by looking at all firms in the market, this approach allows us to make more meaningful comparisons of firms that operate in industries with relatively few firms. Third, it allows us to examine whether all firms in an industry are under- or overvalued, by estimating their values relative to other firms in the market.

Illustration 12.16: Applying Market Regression to Estimate Multiples - Disney

We will use the results of the market regression summarized above to estimate the appropriate value for Disney. Consider the regression for the PE ratio:

PE = 9.475 + 0.814 Expected growth + 0.06 Payout + 6.283 Beta

The corresponding values for Disney are as follows:

Expected Growth rate = 12.00% (Analyst consensus estimate for EPS growth)

Payout Ratio = 32.31%

Beta = 1.2456

The estimated price earnings ratio for Disney is:

PS = 9.475 + 0.814 (12) + 0.06 (32.31) + 6.283 (1.2456) = 29.01

Since Disney trades at an actual PE ratio of 29.87, it is slightly overvalued, relative to the market, by about 3%.

 *multregr.xls*: This dataset summarizes the latest regression of multiples against fundamentals for the United States for the most recent quarter.

[41] We ran the regression using absolute values for the independent variables and both with intercepts and without intercepts. .If the intercept is negative, we have reported the regression without the intercept.

Reconciling Different Valuations

The two approaches to valuation – discounted cash flow valuation and relative valuation – yield different values for Disney[42]. In fact, Disney is significantly overvalued using a discounted cashflow model but is closer to being fairly valued using relative valuation models. Even within relative valuation, we arrive at different estimates of value, depending upon which multiple we use and what firms we based the relative valuation on.

The differences in value between discounted cash flow valuation and relative valuation come from different views of market efficiency, or put more precisely, market inefficiency. In discounted cash flow valuation, we assume that markets make mistakes, that they correct these mistakes over time, and that these mistakes can often occur across entire sectors or even the entire market. In relative valuation, we assume that while markets make mistakes on individual stocks, they are correct on average. In other words, when we value Disney relative to other entertainment companies, we are assuming that the market has priced these companies correctly, on average, even though it might have made mistakes in the pricing of each of them individually. Thus, a stock may be over valued on a discounted cash flow basis but under valued on a relative basis, if the firms used in the relative valuation are all overpriced by the market. The reverse would occur, if an entire sector or market were underpriced.

To conclude, we suggest the following broad guidelines on gauging value using different approaches:

- The discounted cash flow models are built on the implicit assumption of long time horizons, giving markets time to correct their errors.
- When using relative valuation, it is dangerous to base valuations on multiples where the differences across firms cannot be explained well using financial fundamentals – growth, risk, and cash flow patterns. One of the advantages of using the regression approach described in the later part of this chapter is that the R-squared and t-statistics from the regressions yield a tangible estimate of the strength (or weakness) of this relationship.

12.12. ☞**: Valuing an Initial Public Offering**

If you were an investment banker, pricing an initial public offering, would you primarily use discounted cash flow valuation, relative valuation or a combination of the two?

a. Relative valuation, because the buyers of the IPO will look at comparables
b. Discounted cash flow valuation, because it reflects intrinsic value
c. The higher of the two values, since it is my job to get the highest price I can for my client
d. None of the above

Explain.

Conclusion

There are two basic approaches to valuation. The first is discounted cash flow valuation, where the value of any asset is estimated by computing the present value of the expected cash flows on it. The actual process of estimation, in either case, generally requires four inputs –

- the length of the period for which a firm or asset can be expected to generate growth greater than the stable growth rate (which is constrained to be close to the growth rate of the economy in which the firm operates),
- the cash flows during the high growth period,
- the terminal value at the end of the high growth period and
- a discount rate.

The expected growth potential will vary across firms, with some firms already growing at a stable growth rate and others for which the expectation, at least, is that growth will last for some period into the future. We can value the operating assets of a firm by discounting cashflows before debt payments, but after reinvestments, at the cost of capital. Adding the value of cash and non-operating assets give us firm value, and subtracting out debt yields the value of equity. We can also value equity directly by discounting cash flows after debt payments and reinvestment needs at the cost of equity.

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[42] Kaplan and Ruback (1995) examine valuations in acquisitions and find that discounted cash flow models better explain prices paid than relative valuation models.

The other approach to valuation is relative valuation, where the value of any asset is estimated by looking at how "similar" assets are priced in the market. The key steps in this approach are defining "comparable" firms or assets and choosing a standardized measure of value (usually value as a multiple of earnings, cash flows or book value) to compare the firms. To compare multiples across companies, we have to control for differences in growth, risk and cash flows, just as we would have in discounted cash flow valuation.

Live Case Study

Valuation

Objective: To value your firm, based upon its existing management, and your expectations for the future.

Key Questions:

- What type of cash flow (dividends, FCFE or FCFF) would you choose to discount for this firm?
- What growth pattern would you pick for this firm? How long will high growth last?
- What is your estimate of value of equity in this firm? How does this compare to the market value?

Framework for Analysis:

1. *Cash Flow Estimation*
    - What is this firm's accounting operating income? Would you adjust it for your valuation?
    - What is your firm's effective tax rate? What is its marginal tax rate? Which would you use in your valuation?
    - How much did your firm reinvest last year in internal investments, acquisitions, R&D and working capital?

2. *Growth Pattern Choice*
    - How fast have this company's earnings grown historically?
    - How fast do analysts expect this company's earnings to grow in the future?
    - What do the fundamentals suggest about earnings growth at this company? (How much is being reinvested and at what rate of return?)
    - If there is anticipated high growth, what are the barriers to entry that will allow this high growth to continue? For how long?
3. *Valuation*
    - What is the value of the operating assets of the firm, based upon a discounted cash flow model?
    - Does the firm have cash and non-operating assets and what is their value?

- Are there equity options outstanding (management options, convertible bonds) and how much are they worth?
- What is the value of equity per share?

4. *Relative Valuation*

- What multiple would you use to value the firm or its equity?
- What industry does the firm belong to, and what are the comparable firms?
- How does your firm's valuation (in multiple terms) compare to those of the other firms in the industry?
- What value would you assign your firm (or its equity), given how comparable firms are valued?

Getting Information for valuation

Most of the information that you need for valuation come from your current or past financial statements. You will also need a beta (see risk and return section) and a debt ratio (see risk and return section) to estimate the free cash flows to equity. You can get analyst estimates of growth in several sources including Zacks and I/B/E/S.

Online sources of information:

http://www.stern.nyu.edu/~adamodar/cfin2E/project/data.htm

Problems

1. Vernon Enterprises has current after-tax operating income of $ 100 million and a cost of capital of 10%. The firm earns a return on capital equal to its cost of capital.

a. If we assume that the firm is in stable growth, growing 5% a year forever, estimate the firm's reinvestment rate.

b. Given this reinvestment rate, estimate the value of the firm.

c. What is the value of the firm, if you assume a zero reinvestment rate and no growth?

2. Assume, in the previous example with Vernon Enterprises, that the firm will earn a return on capital of 15% in perpetuity.

a. If we assume that the firm is in stable growth, growing 5% a year forever, estimate the firm's reinvestment rate.

b. Given this reinvestment rate, estimate the value of the firm.

3. Cello Inc. is a manufacturer of pianos. It earned an after-tax return on capital of 10% last year and expects to maintain this next year. If the current years after-tax operating income is $ 100 million and the firm reinvests 50% of this income back, estimate the free cash flow to the firm next year.(After-tax Operating Income = EBIT (1-t)]

4. Cell Phone Inc. is a cellular firm that reported net income of $50 million in the most recent financial year. The firm had $ 1 billion in debt, on which it reported interest expenses of $ 100 million in the most recent financial year. The firm had depreciation of $ 100 million for the year, and capital expenditures were 200% of depreciation. The firm has a cost of capital of 11%. Assuming that there is no working capital requirement, and a constant growth rate of 4% in perpetuity, estimate the value of the firm.

5. Netsoft is a company that manufactures networking software. In the current year, the firm reported operating earnings before interest and taxes of $ 200 million (Operating earnings does not include interest income), and these earnings are expected to grow 4% a year in perpetuity. In addition, the firm has a cash balance of $ 250 million on which it earned interest income of $ 20 million. The unlevered beta for other networking software

firm is 1.20, and these firms, have, on average, cash balances of 10% of firm value. If Netsoft has a debt ratio of 15%, a tax rate of 40%, a return on capital of 10% on operating assets, and a cost of debt of 10%, estimate the value of the firm. [The riskfree rate is 6% and you can assume a market risk premium of 5.5%.]

6. Gemco Jewellers earned $ 5 million in after-tax operating income in the most recent year. The firm also had capital expenditures of $ 4 million and depreciation of $ 2 million during the year, and the non-cash working capital at the end of the year was $ 10 million.

a. Assuming that the firm's operating income will grow 20% next year, and that all other items (capital expenditures, depreciation and non-cash working capital) will grow at the same rate, estimate the free cash flow to the firm next year.

b. If the firm can grow at 20% for the next 5 years, estimate the present value of the free cash flows to the firm over that period. You can assume a cost of capital of 12%.

c. After year 5, the firm's capital expenditures will decline to 125% of revenues, and the growth rate will drop to 5% (in both operating income and non-cash working capital). In addition, the cost of capital will decline to 10%. Estimate the terminal value of the firm at the end of year 5.

d. Estimate the total value of the operating assets of the firm.

7. Now assume that Gemco Jewellers has $ 10 million in cash and non-operating assets and that the firm has $ 15 million in outstanding debt.

a. Estimate the value of equity in the firm.

b. If the firm has 5 million shares outstanding, estimate the value of equity per share.

c. How would your answer to (b) change if you learn that the firm has 1 million options oustanding, with an exercise price of $ 5 and 5 years to maturity. (The estimated value per option is $ 7)

8. Union Pacific Railroad reported net income of $770 million in 1993, after interest expenses of $320 million. (The corporate tax rate was 36%). It reported depreciation of $960 million in that year, and capital spending was $1.2 billion. The firm also had $4 billion in debt outstanding on the books, rated AA (carrying a yield to maturity of 8%), trading at par (up from $3.8 billion at the end of 1992). The beta of the stock is 1.05, and

there were 200 million shares outstanding (trading at $60 per share), with a book value of $ 5 billion. Union Pacific paid 40% of its earnings as dividends and working capital requirements are negligible. (The treasury bond rate is 7%.)

a. Estimate the free cash flow to the firm in 1993.

b. Estimate the value of the firm at the end of 1993.

c. Estimate the value of equity at the end of 1993, and the value per share

9. Lockheed Corporation, one of the largest defense contractors in the US, reported EBITDA of $1290 million in 1993, prior to interest expenses of $215 million and depreciation charges of $400 million. Capital Expenditures in 1993 amounted to $450 million, and working capital was 7% of revenues (which were $13,500 million). The firm had debt outstanding of $3.068 billion (in book value terms), trading at a market value of $3.2 billion, and yielding a pre-tax interest rate of 8%. There were 62 million shares outstanding, trading at $64 per share, and the most recent beta is 1.10. The tax rate for the firm is 40%. (The treasury bond rate is 7%)

The firm expects revenues, earnings, capital expenditures and depreciation to grow at 9.5% a year from 1994 to 1998, after which the growth rate is expected to drop to 4%. (Capital spending will offset depreciation in the steady state period.) The company also plans to lower its debt/equity ratio to 50% for the steady state (which will result in the pre-tax interest rate dropping to 7.5%).

a. Estimate the value of the firm.

b. Estimate the value of the equity in the firm, and the value per share.

10. In the face of disappointing earnings results and increasingly assertive institutional stockholders, Eastman Kodak was considering the sale of its health division, which earned $560 million in earnings before interest and taxes in 1993, on revenues of $5.285 billion. The expected growth in earnings was expected to moderate to 6% between 1994 and 1998, and to 4% after that. Capital expenditures in the health division amounted to $420 million in 1993, while depreciation was $350 million. Both are expected to grow 4% a year in the long term. Working capital requirements are negligible.

The average beta of firms competing with Eastman Kodak's health division is 1.15. While Eastman Kodak has a debt ratio (D/(D+E)) of 50%, the health division can

sustain a debt ratio (D/(D+E)) of only 20%, which is similar to the average debt ratio of firms competing in the health sector. At this level of debt, the health division can expect to pay 7.5% on its debt, before taxes. (The tax rate is 40%, and the treasury bond rate is 7%.)

a. Estimate the cost of capital for the division.

b. Estimate the value of the division.

11. You have been asked to value Alcoa and have come up with the following inputs.

- The stock has a beta of 0.90, estimated over the last 5 years. During this period, the firm had an average debt/equity ratio of 20% and an average cash balance of 15%.
- The firm's current market value of equity is 1.6 billion and its current market value of debt is $800 million. The current cash balance is $ 500 million.
- The firm earned earnings before interest and taxes of $ 450 million, which includes the interest income on the current cash balance of $ 50 million. The firm's tax rate is 40%.
- The firm is in stable growth, and its earnings from operations are expected to grow 5% a year. The net capital expenditures next year are expected to be $ 90 million.

Estimate the value of the non-cash assets of the firm, its total value, and the value of its equity.

12. You are analyzing a valuation done on a stable firm by a well-known analyst. Based upon the expected free cash flow to firm, next year, of $30 million, and an expected growth rate of 5%, the analyst has estimated a value of $750 million. However, he has made the mistake of using the book values of debt and equity in his calculation. While you do not know the book value weights he used, you know that the firm has a cost of equity of 12%, and an after-tax cost of debt of 6%. You also know that the market value of equity is three times the book value of equity, while the market value of debt is equal to the book value of debt. Estimate the correct value for the firm.

13. You have been asked to value Office Help Inc., a private firm providing office support services in the New York area.

- The firm reported pre-tax operating income of $ 10 million in its most recent financial year on revenues of $ 100 million. In the most recent financial year, you note that the

owners of the business did not pay themselves a salary. You believe that a fair salary for their services would be $ 1.5 million a year.

- The cost of capital for comparable firms that are publicly traded is 9%. (You can assume that this firm will have similar leverage and cost of capital).
- The firm is in stable growth and expects to grow 5% a year in perpetuity. The tax rate is 40%.

While the average illiquidity discount applied to private firms is 30%, you have run a regression and arrived at the following estimate for the discount:

Illiquidity Discount = 0.30 – 0.04 (ln (Revenues in millions))

Estimate the value of Office Help for sale in a private transaction (to an individual).

14. National City Corporation, a bank holding company, reported earnings per share of $2.40 in 1993, and paid dividends per share of $1.06. The earnings had grown 7.5% a year over the prior five years, and were expected to grow 6% a year in the long term (starting in 1994). The stock had a beta of 1.05 and traded for ten times earnings. The treasury bond rate was 7%.

a. Estimate the P/E Ratio for National City Corporation.

b. What long term growth rate is implied in the firm's current PE ratio?

15. The following were the P/E ratios of firms in the aerospace/defense industry at the end of December, 1998, with additional data on expected growth and risk:

Company	*P/E Ratio*	*Expected Growth*	*Beta*	*Payout*
Boeing	17.3	3.5%	1.10	28%
General Dynamics	15.5	11.5%	1.25	40%
General Motors - Hughes	16.5	13.0%	0.85	41%
Grumman	11.4	10.5%	0.80	37%
Lockheed Corporation	10.2	9.5%	0.85	37%
Logicon	12.4	14.0%	0.85	11%
Loral Corporation	13.3	16.5%	0.75	23%
Martin Marietta	11.0	8.0%	0.85	22%
McDonnell Douglas	22.6	13.0%	1.15	37%

Northrop	9.5	9.0%	1.05	47%
Raytheon	12.1	9.5%	0.75	28%
Rockwell	13.9	11.5%	1.00	38%
Thiokol	8.7	5.5%	0.95	15%
United Industrial	10.4	4.5%	0.70	50%

a. Estimate the average and median P/E ratios. What, if anything, would these averages tell you?

b. An analyst concludes that Thiokol is undervalued, because its P/E ratio is lower than the industry average. Under what conditions is this statement true? Would you agree with it here?

c. Using the PEG ratio, assess whether Thiokol is under valued. What are you assuming about the relationship between value and growth, when you use PEG ratios?

c. Using a regression, control for differences across firms on risk, growth, and payout. Specify how you would use this regression to spot under and overvalued stocks. What are the limitations of this approach?

16. NCH Corporation, which markets cleaning chemicals, insecticides and other products, paid dividends of $2.00 per share in 1993 on earnings of $4.00 per share. The book value of equity per share was $40.00, and earnings are expected to grow 6% a year in the long term. The stock has a beta of 0.85, and sells for $60 per share. The treasury bond rate is 7%.

a. Based upon these inputs, estimate the price/book value ratio for NCH.

b. How much would the return on equity have to increase to justify the price/book value ratio at which NCH sells for currently?

17. You are trying to estimate a price per share on an initial public offering of a company involved in environmental waste disposal. The company has a book value per share of $20 and earned $3.50 per share in the most recent time period. While it does not pay dividends, the capital expenditures per share were $2.50 higher than depreciation per share in the most recent period, and the firm uses no debt financing. Analysts project that

earnings for the company will grow 25% a year for the next five years. You have data on other companies in the environment waste disposal business:

Company	*Price*	*BV/Share*	*EPS*	*DPS*	*Beta*	*Exp.Growth*
Air & Water	$9.60	$8.48	$0.40	$0.00	1.65	10.5%
Allwaste	$5.40	$3.10	$0.25	$0.00	1.10	18.5%
Browning Ferris	$29.00	$11.50	$1.45	$0.68	1.25	11.0%
Chemical Waste	$9.40	$3.75	$0.45	$0.15	1.15	2.5%
Groundwater	$15.00	$14.45	$0.65	$0.00	1.00	3.0%
Intn'l Tech.	$3.30	$3.35	$0.16	$0.00	1.10	11.0%
Ionics Inc.	$48.00	$31.00	$2.20	$0.00	1.00	14.5%
Laidlaw Inc.	$6.30	$5.85	$0.40	$0.12	1.15	8.5%
OHM Corp.	$16.00	$5.65	$0.60	$0.00	1.15	9.50%
Rollins	$5.10	$3.65	$0.05	$0.00	1.30	1.0%
Safety-Kleen	$14.00	$9.25	$0.80	$0.36	1.15	6.50%

The average debt/equity ratio of these firms is 20%, and the tax rate is 40%.

a. Estimate the average price/book value ratio for these comparable firms. Would you use this average P/BV ratio to price the initial public offering.

b. What subjective adjustments would you make to the price/book value ratio for this firm and why?

18. Longs Drug, a large U.S. drugstore chain operating primarily in Northern California, had sales per share of $122 in 1993, on which it reported earnings per share of $2.45 and paid a dividend per share of $1.12. The company is expected to grow 6% in the long term, and has a beta of 0.90. The current T.Bond rate is 7%.

a. Estimate the appropriate price/sales multiple for Longs Drug.

b. The stock is currently trading for $34 per share. Assuming the growth rate is estimated correctly, what would the profit margin need to be to justify this price per share.

19. You have been asked to assess whether Walgreen Company, a drugstore chain, is correctly priced relative to its competitors in the drugstore industry at the end of 1993.

The following are the price/sales ratios, profit margins, and other relative details of the firms in the drugstore industry.

Company	*P/S Ratio*	*Profit Margin*	*Payout*	*Expected Growth*	*Beta*
Arbor Drugs	0.42	3.40%	18%	14.0%	1.05
Big B Inc.	0.30	1.90%	14%	23.5%	0.70
Drug Empor.	0.10	0.60%	0%	27.5%	0.90
Fay's Inc.	0.15	1.30%	37%	11.5%	0.90
Genovese	0.18	1.70%	26%	10.5%	0.80
Longs Drug	0.30	2.00%	46%	6.0%	0.90
Perry Drugs	0.12	1.30%	0%	12.5%	1.10
Rite Aid	0.33	3.20%	37%	10.5%	0.90
Walgreen	*0.60*	*2.70%*	*31%*	*13.5%*	*1.15*

Based entirely on a subjective analysis, do you think that Walgreen is overpriced because its price/sales ratio is the highest in the industry? If it is not, how would you rationalize its value?

20. Time Warner is considering a sale of its publishing division. The division had earnings before interest, taxes and depreciation of $ 550 million in the most recent year (depreciation was $ 150 million), growing at an estimated 5% a year (You can assume that depreciation grows at the same rate). The return on capital in the division is 15%, and the corporate tax rate is 40%. If the cost of capital for the division is 9%, estimate the following:

a. Value/FCFF multiple

b. Value/EBIT multiple

c. Value/EBITDA multiple