The Internal Rate of Return

When you are evaluating an investment, a useful number to know is the **internal rate of return**.

For some investments, like bank accounts, the internal rate of return is easy to figure because the bank tells you what it is. For example, a 5% simple interest bank account has an internal rate of return of 5%.

For other investments, you have to do some work to calculate the internal rate of return. This is especially true of investments like building a factory or getting an education. These kinds of investments generally don't pay money in nice even amounts like a bank account does. Nevertheless, you can calculate an internal rate of return for these investments, and use it to decide which investments pay best.

To evaluate investments and calculate an internal rate of return, we need the concept of **income stream**.

Income Stream

In the grubby world of economic theory, where money is everything, any investment can be expressed as an income stream. An income stream lists years (or months or whatever) and the amounts of money that flow in or out during those years (or months or whatever).

Here is a 5% simple interest bank account, expressed as an income stream.

Income Stream for 5% Bank Account

Year	0	1	2	3	4	5	6
Income	-Rs.1000	Rs.50	Rs.50	Rs.50	Rs.50	Rs.50	Rs.1050

In Year 0, which represents now, we put Rs.1000 in the bank. We put a negative number, -Rs.1000 in for Income in Year 0, because the Rs.1000 flows out from us.

In years 1 through 5, we will get paid Rs.50, 5% of Rs.1000. The 50's above are positive numbers, showing that the money flows to us.

Note: To keep things simple, we imagine that interest is paid annually. Most real life bank accounts pay interest monthly. Also, we imagine that we withdraw each year's interest payment from the bank. We don't leave it in the bank to compound (earn interest on the accumulated interest) during the following years.

Imagine that at the end of year 6 we take our Rs.1000 back. Our total income in year 6 is Rs.1050, the Rs.1000 principal plus the Rs.50 interest we get in year 6.

Alternative Investments

Now let's consider an investment that our financial vice president has proposed as an alternative to putting our Rs.1000 in the bank for six years. This investment involves buying a machine that will cost Rs.1000. It will give us Rs.200 in operating profit per year for six years, starting the year after we buy it. At the end of six years, the machine will have no value, due to wear and obsolescence. There's no lump of money waiting for us at the end, as there is with the bank account.

This table shows the bank income stream and the machine income stream.

Year	0	1	2	3	4	5	6
Bank	-Rs.1000	Rs.50	Rs.50	Rs.50	Rs.50	Rs.50	Rs.1050

Machine	-Rs.1000	Rs.200	Rs.200	Rs.200	Rs.200	Rs.200	Rs.200

It's not obvious which investment is better, is it? We could try adding up the income streams ...

Bank Account: -1000 + 50 + 50 + 50 + 50 + 50 +1050 = 300
Machine: -1000 +200 +200 +200 +200 +200 + 200 = 200

The income stream from the bank account adds up to Rs.300. The income stream from the machine adds up to Rs.200. Does this make the bank account better?

We use the "present value" concept. The point is: The bank account income stream pays more money in total, but most of that income is in the big lump of Rs.1050 in year 6. The machine pays less in total, but it pays more money per year in the years that come sooner. Getting the money sooner is what may make the machine's income stream have a higher present value than the banks.

The Present Value of an Income Stream

Present Value = (Future Value) / (1 + Discount Rate)^a,

Where the exponent ^{a} is the number of years in the future that the future value will be received. The discount rate is the same as the interest rate.

An income stream is a series of future values. The present value of an income stream is calculated by adding up the present values of all the items in the income stream.

To calculate a present value, we need to pick a discount rate. Since one of our alternative investments is a 5% per year bank account, let's pick 5% per year as the discount rate.

Year ^{a}	0	1	2	3	4	5	6	Total
Machine income stream	-Rs.1000	Rs.200	Rs.200	Rs.200	Rs.200	Rs.200	Rs.200	
1.05^a	1.0000	1.0500	1.1025	1.1576	1.2155	1.2763	1.3401	
Present value, at 5% rate of discount	-Rs.1000	Rs.190.48	Rs.181.41	Rs.172.77	Rs.164.54	Rs.156.71	Rs.149.24	Rs.15.14

Each of the numbers in the Present values row is the number above it in the Machine income stream row, divided by 1.05^a, where the exponent ^{a} is the year number.

The total of these present values is Rs.15.14. This is the present value of the machine income stream at a 5% discount rate. (If you check the addition, you'll get Rs.15.15, due to round-off error.)

The fact that this Rs.15.14 total is bigger than Rs.0 is enough to tell us that the machine is a better-paying investment than a 5% interest bank account.

Not convinced yet? Let's apply the same calculation to the 5% interest bank account.

Year	0	1	2	3	4	5	6	Total
5% account income stream	-Rs.1000	Rs.50	Rs.50	Rs.50	Rs.50	Rs.50	Rs.1050	
Present value, 5% discount rate	-Rs.1000	Rs.47.62	Rs.45.35	Rs.43.19	Rs.41.14	Rs.39.18	Rs.783.53	Rs.0.00

These present values add up to Rs.0. (Actually, they add to Rs.0.01, but that's due to round-off error.) The present value of an X% bank account, evaluated at an X% discount rate, has to be Rs.0, no matter what X is.

At a 5% discount rate, the machine has a higher present value (Rs.15.14) than the 5% bank account (with its present value of Rs.0), so the machine is the better-paying investment.

So far, so good, but what if we have other alternative investments? How do we compare them? How do we decide what discount rate to use?

Calculating the Internal Rate of Return

One way to compare investments is to calculate an internal rate of return for each one. The calculation we just made for the bank account gives us an idea of what we want to do. The bank account's interest rate (5%) was the discount rate that made the present value of its income stream total zero.

We can define the machine's internal rate of return the same way. **The internal rate of return for the machine is the discount rate that makes the present value of the machine's income stream total to zero.**

Consider this table:

Year	0	1	2	3	4	5	6	Total
Income stream	-Rs.1000	Rs.200	Rs.200	Rs.200	Rs.200	Rs.200	Rs.200	
Present value, at a 5% discount rate	-Rs.1000	Rs.190.48	Rs.181.41	Rs.172.77	Rs.164.54	Rs.156.71	Rs.149.24	Rs.15.14
Present value, at a 6% discount rate	-Rs.1000	Rs.188.68	Rs.178.00	Rs.167.92	Rs.158.42	Rs.149.45	Rs.140.99	-Rs.16.54

At a 6% per year discount rate, the machine investment's present value is less than Rs.0. At a 5% discount rate, the present value is greater than Rs.0. The Intermediate Value Theorem implies that there is a discount rate between 5% and 6% at which the present value is Rs.0. Let's find that discount rate.

Change the discount rate to anything between 5% and 6% with an increment of 0.10 %. NPV for 5.4 % would be positive while that for 5.5 % would be negative. Now change the discount rate between 5.4 % & 5.5 % by increments of 0.01%. Play with this a little. Then adjust the discount rate to get the total present value as close as you can to Rs.0. You won't be able to hit Rs.0 exactly. When you have gotten the present value as close to Rs.0 as you can, stop.

Year	0	1	2	3	4	5	6	Total
Income stream	-Rs.1000	Rs.200	Rs.200	Rs.200	Rs.200	Rs.200	Rs.200	
Present value, at a 5.47% discount rate	-Rs.1000	Rs.189.63	Rs.179.79	Rs.170.74	Rs.161.63	Rs.153.24	Rs.145.30	Rs.0.06

A discount rate of 5.47% makes the total present value Rs.0.06, which is as close as you can get to zero without going to the next decimal place. Thus, 5.47% is our best approximation the internal rate of return for the machine investment.

The machine investment's 5.47% internal rate of return is higher than the bank account's 5% rate of return. This is sufficient to tell us that the machine is a better-paying investment.

Two cautionary notes:

The idea that better investments have higher internal rates of return is appropriate for comparing investments that have their costs first and their positive incomes later, and which have about the same initial costs. Our imaginary bank account and machine fit this criterion, so we are OK to use the internal rate of return for comparison.

Risk can complicate the comparison of investments. For this tutorial let us assume that the risks of our alternative investments are the same. In particular, we will assume that the machine is just as safe as the bank account.

In real life, investments that offer better payback generally carry greater risks that the future income won't be paid. If the machine is riskier than the back account, you may prefer the bank account, even if its internal rate of return is lower. Even so, the internal rate of return is useful to know. It tells you how much caution would cost you, or how much reward there is if you choose to assume some risk.

Let's look at the two investments again. Here are the income streams and each investment's internal rate of return (IRR):

Year	0	1	2	3	4	5	6	IRR
Machine	-Rs.1000	Rs.200	Rs.200	Rs.200	Rs.200	Rs.200	Rs.200	5.47%
5% bank account	-Rs.1000	Rs.50	Rs.50	Rs.50	Rs.50	Rs.50	Rs.1050	5.00%

Suppose you don't need any money until year 6? Doesn't that make the bank account better? The total of the income stream (not discounted) is higher for the bank, and it gives you the money when you need it.

The answer is: Even if the times when you'll need money don't match when the investment pays, you should still go by the internal rate of return. That's especially true if the investment pays you money before you need it. That's because you can use the bank, even if you buy the machine. You can deposit the extra income from the machine into a 5% account. At the end of Year 6, you'll have a bigger lump of money than you would have had if you had put your Rs.1000 in the bank.

You buy the machine for Rs.1000. At the end of Year 1, you get Rs.200. You keep Rs.50 for spending, just like you would do for the bank account (according to what we assumed), and put the extra Rs.150 into the bank	**End of year 1:** You get Rs.200.00. You take Rs.50.00. You add to bank account Rs.150.00. Bank balance is Rs.150.00.
At the end of Year 2, the bank pays you 5% interest on your Rs.150. That makes your bank balance Rs.157.50. At the same time, you get another Rs.200 from the machine. You keep Rs.50 of that for spending, and put Rs.150 in the bank. Your bank balance is Rs.157.50 + Rs.150 = Rs.307.50.	**End of year 2:** Bank adds 5% of Rs.150.00, Rs.7.50. You get Rs.200.00. You take Rs.50.00. You add to bank account Rs.150.00. Bank balance is Rs.307.50.
At the end of Year 3, the bank pays you 5% interest on your Rs.307.50. That makes your bank balance Rs.322.88. The machine pays you another Rs.200. You keep Rs.50 and put Rs.150 in the bank. Your bank balance is Rs.322.88 + Rs.150 = Rs.472.88.	**End of year 3:** Bank adds 5% of Rs.307.50, Rs.15.38. You get Rs.200.00. You take Rs.50.00. You add to bank account Rs.150.00. Bank balance is Rs.472.88.
At the end of Year 4, the bank pays you 5% interest on your Rs.472.88. That makes your bank balance Rs.496.52. The machine pays you another Rs.200. You keep Rs.50 and put Rs.150 in the bank. Your bank balance is Rs.496.52 + Rs.150 = Rs.646.52.	**End of year 4:** Bank adds 5% of Rs.472.88, Rs.23.64. You get Rs.200.00. You take Rs.50.00. You add to bank account Rs.150.00. Bank balance is Rs.646.52.
At the end of Year 5, the bank pays you 5% interest on your Rs.646.52. That makes your bank balance Rs.678.84. The machine pays you another Rs.200. You keep Rs.50 and put Rs.150 in the bank. Your bank balance is Rs.678.84 + Rs.150 = Rs.828.84.	**End of year 5:** Bank adds 5% of Rs.646.52, Rs.32.32. You get Rs.200.00. You take Rs.50.00. You add to bank account Rs.150.00. Bank balance is Rs.828.84.
Finally, at the end of Year 6, the bank pays you 5% interest on your Rs.828.84 That makes your bank balance Rs.870.28. The machine pays you its last Rs.200. You withdraw the Rs.870.28 from the bank, and you have Rs.870.28 + Rs.200 = Rs.1070.28. By comparison, at the end of six years with the bank alone you get Rs.1050. With the machine, you're ahead by Rs.20.28. OK, it's not that much, but it does show that even if you don't need most of your money until Year 6, you wind up with more if you buy the machine.	**End of year 6:** Bank adds 5% of Rs.828.84, Rs.41.44. Bank balance is Rs.870.28. You get Rs.200.00. The total you have is Rs.1070.28.

If the machine investment pays you money after you need it (say, you need Rs.300 in Year 1) then you should compare the interest rate you'd pay to borrow money with the machine's internal rate of return.

A digression: We keep talking about the "internal" rate of return. Were you wondering if there is such a thing as an "external" rate of return? There is, and the above analysis is an example, because it takes into account the interest that you can earn from machine's payments in the bank, which is an investment separate from, and thus "external" to, the machine investment itself.

Internal Rate of Return Summary (so far)

- The internal rate of return is the interest rate that makes the present value of the investment's income stream -- its costs and payoffs -- add up to 0.
- The internal rate of return is a measure of the worth of an investment. If the risks are equal investments with higher internal rates of return pay better.

Some Applications

- Let's look at three applications of the internal rate of return.
- Evaluating a bond sold at a discount.
- Detecting an economic shortage.
- The effect of regulation on innovation.

Application 1. Evaluating a bond sold at a discount.

Here's a common type of investment, a bond sold at a discount.
The Sam's Software Corporation is selling 5-year Rs.1000 bonds that pay 10% interest per year. The bonds are selling at Rs.900. How do you evaluate the bonds as an investment?

The first step is to figure out what the income stream is. Which of these is the income stream for a 5-year Rs.1000 10% bond that's selling for Rs.900?

Income stream for Rs.1000 5-year bond, paying 10% interest, bought at Rs.900.

Year	0	1	2	3	4	5
Income stream	-Rs.900	Rs.100	Rs.100	Rs.100	Rs.100	Rs.1100

This is the income stream for the bond. The income stream goes 5 years into the future because it's a 5-year bond. The bond costs you Rs.900 initially. It pays Rs.100 (10% of Rs.1000) in years 1-5. In year 5 it also pays its face value, Rs.1000.

Since the nominal interest rate is 10%, we evaluate the present value at 10%, and see what we get.

Year	0	1	2	3	4	5	Total
Income	-Rs.900	Rs.100	Rs.100	Rs.100	Rs.100	Rs.1100	
Income discounted at 10%	-Rs.900	Rs.90.91	Rs.82.64	Rs.75.13	Rs.68.30	Rs.683.01	Rs.100.00

The present value of this bond, evaluated at a 10% discount rate, is Rs.100. Notice that this equals the amount of the initial price discount. That's because we discounted future income at the bond's stated interest rate. If the price of the bond had been Rs.1000, instead of Rs.900, then the present value evaluated at a 10% discount rate would have been Rs.0, instead of Rs.100.

Just to be sure we're all together here, please answer this:

The Rs.683.01 number in the above table comes from the formula $Rs.683.01 = F/(1 + i)^a$. Fill in the entry boxes below with the correct numbers for F, i, and a. (For each entry box, click in it to type in it, then press Enter to see if you're right. You can also use the Tab key to move from one box to the next.)

Let's find the internal rate of return for this bond. The applet below allows you to change the discount rate to anything between 0% and 20% by clicking and dragging the slide. To change the discount rate by 10%, click in the space between the slider and the end. To change the discount rate by 0.1%, click on the scrollbar's ends. As you change the discount rate, the applet will recalculate the present value of the income stream.

See how close you can get the total present value to Rs.0. The discount rate at that the present value closest to Rs.0 will be our approximation of the internal rate of return. As before, make a mental note of this discount rate, and then go on.

Year	0	1	2	3	4	5	Total
Income stream	-	Rs.100	Rs.100	Rs.100	Rs.100	Rs.1100	

	Rs.900						
Present value, 12.8% discount rate	-Rs.900	Rs.88.65	Rs.78.59	Rs.69.67	Rs.61.77	Rs.602.35	Rs.1.04

The discount rate that makes the total present value come closest to zero is 0.128 (12.8%), so the internal rate of return is 12.8%. This internal rate of return is the number to use to compare this investment with others. For bonds, the internal rate of return is also called the yield to maturity. It may be confusing, but the yield to maturity differs from the bond's nominal interest rate of 10%. That's because the bond is not selling for its face value.

Application 2: Detecting an Economic Shortage

Economists use the term economic shortage to mean a situation in which the quantity supplied of some goods or service is less than what it would be if the market were functioning freely and competitively.

Shortages can manifest themselves two ways. One is with long lines of buyers or empty store shelves. The other is with prices higher than cost plus normal profit. (More on "normal profit" later.)

Detecting the first type of shortage is easy -- you just observe the lines or empty shelves. Detecting the second type is harder.

How do you judge when prices are high? Use the internal rate of return! We'll illustrate this with our machine investment example.

Suppose that there are two investments available to large numbers of investors. One is a machine like this.

Year	0	1	2	3	4	5	6	IRR
Income	-Rs.1000	Rs.200	Rs.200	Rs.200	Rs.200	Rs.200	Rs.200	5.47%

The other is the 5% bank account. The machine has a higher internal rate of return, at 5.47%, than the bank's 5%. What happens? Firms like yours take money out of the bank to buy machines.

As more and more firms buy machines, put them to work, and try to sell the machine's products, what would you expect to happen to the typical machine's income stream?

As the supply of the machine's products expands, and assuming that the demand curve stays the same, the prices of the machine's products should fall. This will cause the amount of income earned in years 2-6 to shrink. That will make the internal rate of return fall.

For example, suppose that annual income falls by just Rs.1 to Rs.199.

Here's the machine investment after competition starts to drive prices down:

Year	0	1	2	3	4	5	6	IRR
Income	-Rs.1000	Rs.199	Rs.199	Rs.199	Rs.199	Rs.199	Rs.199	5.31%

The internal rate of return falls to 5.31%, from the 5.47% it was before.
How far would you expect the income amounts to fall?

Market equilibrium income for the machine under competition

Year	0	1	2	3	4	5	6	IRR
Income	-Rs.1000	Rs.197.02	Rs.197.02	Rs.197.02	Rs.197.02	Rs.197.02	Rs.197.02	5.00%

Once income falls this far, people will no longer want to take money out of the bank to buy machines. The supply of machine products will stop expanding, and the income you can get from a machine will stop falling. (Given the time lag in decision-making, income may fall further than this and then later come back up. Equilibrium will be reached when there's no incentive to shift money either from bank to machine or from machine to bank.)

How economic shortage fits with this:

If an investment has a high rate of return that persists over a long period of time, economists infer that competition must not be working, as it should to lower income and equalize rates of return.

Normal profit is profit equivalent to what you could earn in the bank. No business would persist that didn't earn normal profit. Why put money into a business if you can do better in a no-effort bank account? But if the return to an investment stays higher than normal for a long time, that's an indication of an economic shortage, and some kind of monopoly restriction.

Some economists used to argue that the high rate of return to a physician's education showed that there was an economic shortage of physicians. In the late 1990s, managed care may reduce physicians' incomes to the point that there is an economic surplus, with below-normal returns, particularly to specialty training.

Application 3: Regulation Delays and the Internal Rate of Return

Changing the timing of future payoffs changes the internal rate of return.

Let's go back to our original machine investment again. Imagine that we have a patent to protect us against competitors and their price cuts.

Our machine investment, as before.

Year	0	1	2	3	4	5	6	IRR
Income	-Rs.1000	Rs.200	Rs.200	Rs.200	Rs.200	Rs.200	Rs.200	5.47%

Suppose, however, that meeting government safety regulations forces a delay in the start of sales of machine products. The income stream might change to this:

Machine with delayed income.

Year	0	1	2	3	4	5	6	7	IRR
Income	-Rs.1000	Rs.0	Rs.200	Rs.200	Rs.200	Rs.200	Rs.200	Rs.200	?????

The amounts in the income stream are the same, but the income is shifted forward by one year.
What happens to the internal rate of return?

Find the internal rate of return for this machine now. The discount rate that makes the total present value as close as you can get to Rs.0 with this number of decimal places is 4.19%. The regulatory delay drops the internal rate of return to below what you can get from the bank. No one will invest in one of these machines now.

One way this machine's products could come to market would be if the prices for the machine's products go up. The prices would have to rise enough to make the internal rate of return as high as the bank's 5%. In this way, consumers wind up paying for the regulatory delay in higher prices. Notice that I didn't add any paperwork or testing cost to the calculations. The higher prices are purely because of the delay and the need to compete with other investments. Adding in the paperwork cost of complying with regulations would make the internal rate of return even lower.

Another way this machine's products could come to market would be if the price of the machine goes down. Like the discounted bond, a discount in the price of the machine would raise the internal rate of return. In this circumstance, the machine's manufacturer would pay the delay cost of the regulation.

Application 4. Does Regulation Discourage Research and Innovation?

The pharmaceutical industry argues that something much like this happened to them after 1962, when Congress tightened regulation of new drugs. New drug applications began taking longer, putting off the start of the payoffs from research. The regulations also made research more expensive, by adding requirements, and shortened the whole income stream by taking up time, cutting into the period of patent-protected high profits. The rate of return to research fell. By the above reasoning, that should cause less to be spent on research, so fewer new drugs would be developed.

There are two problems with this argument. One is that drug industry research spending has increased since 1962, even allowing for inflation. The rate of return is evidently still good enough to attract money into research.

The second problem is that regulation is there for a purpose: to keep unsafe and ineffective drugs off of the market. The industry's record is not spotless. "Research" is not necessarily good, unless it's research that might lead to safe and effective products.

Any testing system, whether adopted voluntarily by industry or imposed by government, discourages investment. The price you pay for safety and effectiveness assurance is a reduction in innovation. How much assurance do we get? Is the price too high?

These are issues that are better handled in class discussion than on a computer. I hope that you now understand what the internal rate of return is and how the concept figures in the controversy.

Internal Rate of Return -- Review and Summary

An income stream for an investment is all of the investment's costs and payoffs, along with when each cost and payoff will happen.

The present value of an investment is the amount of money you'd need now to be able to duplicate the investment's income stream. The present value is calculated using a discount rate, which you set to equal your bank's interest rate or the rate of return of some other alternative investment.

The internal rate of return is the discount rate that makes the present value of the investment's costs and payoffs add up to 0.

From the applications:

The yield to maturity of a discounted bond equals its internal rate of return.
Investments with higher internal rates of return attract money away from investments with lower internal rates of return.
If a kind of investment has a persistently high internal rate of return, something is preventing the market from reaching a competitive equilibrium.
Regulation can reduce the rate of return to innovation, just by delaying the payoffs. How to protect the public without stifling innovation is a major problem of regulation, in pharmaceuticals, for example.

Perils of Using the Internal Rate of Return

The internal rate of return is not a good way evaluate an investment that has costs later rather than just earlier. An example of that would be an investment that generates an environmental problem that will require a cleanup at the end of the income stream. For some such investments, the worse investments have higher internal rates of return.
